This study investigates natural harmonic vibrations of a working implement equipped with a lever-type hitching system to the traction mechanisms and two support points on the surface. The task addressed relates to the absence of a generalized theoretical model for quantitative prediction of amplitude-frequency characteristics of such systems. This complicates the substantiation of rational structural and geometric parameters to enable technologically acceptable vertical deviations of working bodies under field irregularities and variable soil properties.

A calculation scheme of a two-support lever mechanism has been devised, separating the implement's intrinsic dynamics from the power unit motion and considering roller-soil contact as an excitation source. An energy-based mathematical model using Lagrange equations of the second kind has been constructed; parametric modeling revealed the decisive influence of attachment geometry and made it possible to determine rational parameter values. The length of the hitch lever to the power unit is 0.5 m, and the distance from the oscillatory system's center of mass to the hitch point is 0.25 m.

The advantage of the results involves the analytical relationship between the mechanism geometry, contact parameters, and vibration characteristics, which enables structural minimization of vertical deviations without mandatory use of active control. The identified regularities are explained by changes in the reduced inertial-elastic characteristics of the system when varying the lever arms and the position of the hitch point relative to the center of mass.

The results could be practically applied at the stages of designing and adjusting implements provided that equivalent elastic-damping parameters of the "roller-soil" contact are identified for specific field and soil conditions and operating speeds

Keywords: tillage (seeding) implement, aggregation, support rollers, parallelogram oscillatory system, soil surface following, tillage (seeding) depth accuracy

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CONSTRUCTION OF A THEORETICAL MODEL OF OSCILLATIONS OF A TILLAGE (SEEDING) IMPLEMENT WITH INDEPENDENT MOUNTING AND TWO SUPPORT ROLLERS

Viacheslav Padalka
PhD, Associate Professor*
ORCID: https://orcid.org/0000-0002-4135-3318

Oleksandr Gorbenko
Corresponding author
PhD, Associate Professor*
E-mail: oleksandr.gorbenko@pdau.edu.ua
ORCID: https://orcid.org/0000-0003-2473-0801

Dmytro Khvostenko
PhD Student*
ORCID: https://orcid.org/0009-0007-3664-021X

Andrii Lazorenko
Assistant*
ORCID: https://orcid.org/0009-0002-3586-5656

Vladyslav Alpidovskyi
PhD Student*
ORCID: https://orcid.org/0009-0009-5797-7983

*Department of Agricultural Engineering and Road Transport
Poltava State Agrarian University
Skovorody str., 1/3, Poltava, Ukraine, 36003

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1. Introduction

Modern crop production is characterized by increased requirements for the accuracy of technological operations of tillage and sowing, an increase in the speed of operation of assemblies, and the need to ensure stable quality under conditions of heterogeneous microrelief of the field and variable physical and mechanical properties of the soil. Under such conditions, the stability of the course of tillage and sowing tools in the vertical plane becomes a determining factor since vertical deviations lead to unstable depth of cultivation/seed wrapping, unevenness of the technological process, increased energy consumption, and accelerated wear of components.

The need for research in this area is due to the fact that oscillatory processes in the lever mechanisms of tools with two points of support and independent fastening determine the real trajectories of the working bodies and, accordingly, the quality of the implementation of the technological operation. The transition to more productive units and the desire to minimize tolerances in depth and profile of cultivation or sowing makes empirical approaches to the selection of design parameters insufficient [1]. It is necessary to theoretically substantiate the spectrum of natural oscillations and the sensitivity of the system to changes in kinematic parameters (lengths of attachment links, position of the attachment point relative to the center of mass), as well as to the characteristics of contact of the support rollers with the soil. Without such a justification, there is a risk of undesirable oscillation modes that may fall into the operating range and impair the accuracy of the process.

Under the conditions of modern evolution of crop production technologies, the justification of the natural harmonic oscillations of a tool with an independent mount and two support rollers is necessary. Therefore, studies on the theoretical analysis of the natural harmonic oscillations of such lever vibration systems are relevant.

2. Literature review and problem statement

For tillage and seeding machines, the stability of the trajectory of the working bodies in the vertical plane is critical [2] since it determines the maintenance of the depth of cultivation/seeding and the uniformity of the operation. Irregularities of the support surface and the variability of the physical and mechanical properties of the soil form disturbances that manifest themselves in the form of vibrations of the "tractor-implement" system and worsen the quality of work. The issue of functional separation of the vibrations of the trailed unit system (sections of the working bodies) from the elastic vibrations of the power tool system to maintain a constant depth of travel of the tool has remained unconsidered. In this regard, modeling the dynamics of the units taking into account the profile of the support surface and interaction with the soil environment remains a relevant area of research.

Study [3] considered the force processes in the three-point hitch of the tractor and showed that the mounted implement significantly affects the redistribution of vertical reactions between the axes of the power tool. The authors focused on the traction and coupling properties of the tractor and the change in its loaded state. At the same time, the natural vibrational dynamics of the implement itself in the vertical plane is not distinguished as a separate object of analysis.

Paper [4] also investigated the influence of the parameters of the mounted unit on the power state of the tractor and the implementation of the traction force. The results are important for assessing the efficiency of the aggregation, but the implement in this formulation is actually represented as an external load on the tractor. With this approach, it is impossible to determine the natural forms and frequencies of oscillations of the implement itself, as well as to assess the influence of the geometry of its attachment mechanism on the amplitude of vertical movements.

Thus, studies [3, 4] reveal the force interaction in the "tractor-attachment-implement" system but do not form an analytical basis for studying the natural harmonic oscillations of an implement with an independent mount. This is explained by the fact that those studies are primarily focused on the energy source, and not on the implement as an independent oscillatory system.

In paper [5], a system for measuring the depth of cultivation in real time based on a combination of signals from several sensors is proposed. This approach makes it possible to quickly assess the actual depth and its relationship with the traction resistance. At the same time, the work is aimed primarily at monitoring the parameter of the technological process, rather than theoretically determining the causes of oscillations and their structural reduction.

The authors of work [6] devised an approach to determining the depth based on assessing the spatial orientation of the working body. This increases the accuracy of information for further control but does not answer the question of how exactly the choice of geometric parameters of the attachment mechanism affects the dynamics of the implement.

In [7], online calibration of model parameters is proposed to adapt the control system to changes in operating conditions. The advantage of this approach is the possibility of improving the accuracy of depth maintenance in real time. However, the design parameters of the attachment mechanism are considered only as part of the control object, and not as the subject of a separate parametric synthesis.

The results reported in [8] give an experimental comparison of automatic depth control strategies for plow systems. It is shown that the use of feedback and improved control laws helps reduce depth deviations. However, in this case, the problem is solved mainly by active control actions, and not by choosing a rational geometry and parameters of the mechanical system itself.

Thus, studies [5–8] confirm the effectiveness of information-control approaches to depth stabilization but do not solve the problem at the design level. The question remains unresolved as to which structural and geometric parameters of the attachment mechanism enable the minimization of the implement's own oscillations even before the application of active control. The reason is that most of the research in this group is focused on the control and compensation of deviations, and not on the analytical synthesis of the parameters of the oscillation system.

In papers [9, 10], no-till and high-speed seeding systems were investigated and the dependence of depth stability on the kinematics of the mechanisms and the parameters of the elastic elements was shown. It was shown that the configuration of the links and damping reduce the depth variation. However, the results are mostly tied to specific designs and experimental settings. A generalized analytical model for a mechanism with two support rollers remains unresolved. A likely reason is the difficulty of simultaneously taking into account the geometry of the links, mass-inertial parameters, and the contact interaction "roller-soil". The nonlinearity of the contact and its variability in the field additionally complicates the task.

In [11, 12], coupled DEM-MBD (Discrete Element Method + Multi-Body Dynamics) models were built to describe the contact of the soil with the working bodies and the processes of material movement in the working zone. It is shown that such models increase the physical adequacy of the description of disturbances. However, DEM-MBD approaches are computationally expensive and usually focused on a specific geometry. The problem of obtaining compact generalized regularities for parametric synthesis remains unsolved.

Therefore, our review of the literature allows us to make the following generalizations. Attachment models describe force flows well but do not give a parametric picture of the natural oscillations of a tool with two support points and an independent attachment. Depth control systems increase the accuracy of the process but do not replace the rational choice of the attachment geometry, which reduces oscillations at the design level. DEM-MBD and controlled damping are effective but the generalized analytical formulation for a lever mechanism with two support rollers is covered fragmentarily.

Thus, the unsolved problem is the lack of a generalized theoretical model of the natural harmonic oscillations of a tool with independent mounting in a vertical plane and two support rollers. Such a model should take into account the geometry of the links, mass-inertial characteristics, and equivalent elastic-damping properties of the roller-soil contact.

3. The aim and objectives of the study

The purpose of our study is to construct a generalized theoretical model of natural harmonic oscillations of a tool with independent mounting in a vertical plane and two support rollers based on the energy approach. This will provide an opportunity to substantiate rational structural and geometric parameters of the lever mechanism and connection schemes that reduce oscillations in the working range and provide technologically permissible vertical deviations.

To achieve this aim, the following objectives were accomplished:

– to substantiate the calculation scheme of the lever mechanism with two support rollers and independent attachment in a vertical plane, as well as the assumptions adopted for the description of small oscillations;

– to build an energy mathematical model of natural harmonic oscillations taking into account the elastic-damping properties of the contact "roller-soil" and derive the equation of motion/analytical solution;

– to perform a parametric analysis of the influence of parameters on the amplitude-frequency characteristics and justify rational parameters that enable technologically permissible deviations.

4. The study materials and methods

The object of our study is the natural harmonic oscillations of the working tool, which has a lever system of connection to the traction mechanisms and two points of support on the surface. The oscillatory motion in the vertical plane is considered, which determines the deviation of the tool position from the given depth of the technological operation.

The principal hypothesis assumes that the dynamics of vertical deviations of the tool with two support rollers and independent attachment can be adequately described by an equivalent mechanical model of lumped parameters. The influence of the design parameters of the lever mechanism and the contact interaction "roller-soil" determines the spectrum of natural frequencies and amplitude-frequency properties of oscillations in the vertical plane.

The following assumptions are accepted in the work:

1) small oscillations: vertical displacements and angular deviations are small, which allows the use of a linearized description of the oscillatory process;

2) stationary contact with the surface: the track rollers/wheels are in contact with the support surface within the studied regimes; separation is not considered;

3) equivalence of the soil environment: the interaction of the "track rollers-soil" is given in the form of equivalent elastic-damping elements that reflect the elastic-plastic properties of the soil in the vicinity of the working position;

4) rigidity of the tool elements: the structural elements of the lever mechanism are considered absolutely rigid, and the deformations are concentrated in the elements of contact interaction and/or elastic connections of the model;

5) independence of vertical movement from the power source: the influence of the power source on the vertical movements of the implement is taken into account through kinematic connections and/or connection conditions, without detailed modeling of tractor vibrations as a separate subsystem.

To enable reproducibility and reduce the complexity of the model, the following simplifications were adopted:

– movement is considered only in the vertical plane without taking into account transverse and spatial vibrations;

– the contact "roller-soil" is described by linear equivalent parameters of stiffness and damping without a detailed nonlinear rheological description of the soil;

– the influence of backlash, friction in the joints and local elastic deformations of the structure is taken into account in a generalized manner or is considered secondary within the accepted formulation;

– the geometric parameters of the mechanism within one calculation option are considered constant.

The study used a set of theoretical methods: mathematical modeling, analytical mechanics, small oscillation theory, and parametric analysis.

Mathematical modeling was used to construct an equivalent calculation scheme of the tool as a mechanical system with lumped parameters. Analytical mechanics was employed to derive the equations of motion of the system. The linearization method was applied assuming small displacements and angular deviations. Methods of the theory of oscillations and parametric analysis were used to determine the natural frequencies, amplitude-frequency properties, and the influence of design parameters on the dynamics of the tool.

When solving the mathematical model, the static equilibrium position was taken as the initial state of the system. The oscillatory process was given by small initial deviations of generalized coordinates and zero or given small initial velocities. To determine the natural frequencies, the initial conditions were used only as a means of perturbing the system since the values of the natural frequencies are determined by the model parameters and do not depend on the specific choice of small initial deviations.

5. Results of theoretical modeling of oscillations of a tool with an independent mount and two support rollers

5. 1. Calculation scheme of the lever mechanism and statement of the problem of small oscillations

The construction and study of the mathematical model of oscillations of the lever system of attachment of the working tool [13] were performed on the basis of the energy method using Lagrange equations of the second kind. To take into account the kinematic conditions imposed on the system, the approach of Lagrange equations of the second kind with undefined multipliers was applied. This allows us to correctly describe the holonomic relations associated with the geometry of the lever mechanism and the conditions of contact of the support rollers with the surface (Fig. 1) [14–16].

The generalized coordinates were chosen in such a way that they uniquely determined the position of the oscillatory system in the vertical plane and allowed us to record the kinetic and potential energies of the mechanism, as well as the dissipative function (to take into account damping). On this basis, a system of differential equations of motion was built, which was subsequently used to analyze the natural harmonic oscillations and amplitude-frequency properties.

To solve the system of differential equations of motion and perform numerical modeling, methods of computer mathematics and numerical integration were used.

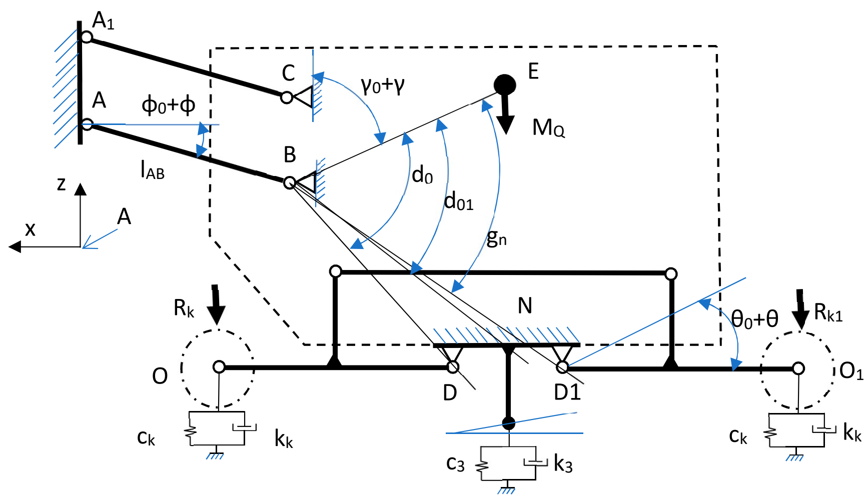

Fig. 1. Scheme of the lever mechanism of a working tool with two support points: A, A_1 – attachment points to the unit (movable connection); C, B – attachment points of the hitch to the tool (movable connection); O, O_1 – attachment points of the support wheels of the tool (movable connection); D, D_1 – attachment points of the levers of the support wheels of the tool (movable connection); E – point of the center of mass of the tool; I – moment of inertia of the system, kg m^2; l_{AB}, l_{BE}, l_{BD}, l_{BD1}, l_{BC}, l_{DO} – lengths of the corresponding links, m; k_3 – viscosity coefficient of the working body of the tool, kg/m s; k_k – viscosity coefficient of the support wheel, kg/m s; c_3 – elasticity coefficient of the working body of the tool, H/m^2; φ_0 – angle of deviation of the AB link, degrees; γ_0 – angle of deflection of link BE, degrees; d_0 – angle of deflection of link BD, degrees; θ_0 – angle of deflection of link D_1O_1, degrees; d_{01} – angle of deflection of link BD_1, degrees; ρ_0 – angle of deflection of link, degrees; M_Q – weight of tool, N; P_z – resistance force of working body of tool in vertical plane, kN; P_x – resistance force of working body of tool in horizontal plane, kN; R_k, R_{k1} – resistance force of working body of support wheels, kN

5. 2. Energy mathematical model of natural harmonic oscillations and equations of motion

A lever mechanism of a working tool with two points of support on the surface is considered (Fig. 1). The dynamics of oscillations are analyzed in a vertical plane in the vicinity of the equilibrium position under the condition of small angular deviations. For this purpose, generalized coordinates are introduced that determine the position of the lever system and the working tool, and the imposed holonomic connections are taken into account through the system connection equation.

Let the unit move uniformly with a speed V_0 = const. At time t, due to the angular deviations of the mechanism elements, the coordinates of the center of mass of the working tool are determined from the following relation:

$$x_1 = V_0 t + l_{AB}\cos(\varphi_0+\varphi) + l_{BE}\sin(\gamma_0+\gamma), \qquad (1)$$

$$z_1 = l_{AB}\sin(\varphi_0+\varphi) + l_{BE}\cos(\gamma_0+\gamma).$$

The velocities of the center of mass in the projections on the x and z axes are obtained by differentiating the kinematic relationships, which leads to the following expressions:

$$\dot{x}_1 = V_0 - \dot{\varphi} l_{AB}\sin(\varphi_0+\varphi) + \dot{\gamma} l_{BE}\cos(\gamma_0+\gamma), \qquad (2)$$

$$\dot{z}_1 = \dot{\varphi} l_{AB}\cos(\varphi_0+\varphi) - \dot{\gamma} l_{BE}\sin(\gamma_0+\gamma).$$

The kinetic energy of the system is given by the translational motion of the center of mass and the rotational motion of the tool relative to the center of mass. After substituting velocities (2) into the general expression and performing the transformations, the kinetic energy in compact form is obtained

$$T = \frac{1}{2}M\begin{bmatrix} V_0^2 - 2V_0\dot{\varphi} l_{AB}^2 \sin(\varphi_0+\varphi) + \\ +2V_0\dot{\gamma} l_{BE}\cos(\gamma_0+\gamma) + \\ +\dot{\varphi}^2 l_{AB}^2 \sin^2(\varphi_0+\varphi) - \\ -2\dot{\varphi}\dot{\gamma} l_{AB} l_{BE}\sin(\varphi_0+\varphi)\cos(\gamma_0+\gamma) + \\ +\dot{\gamma} l_{BE}^2\cos^2(\gamma_0+\varphi) + \dot{\varphi} l_{AB}^2\cos^2(\varphi_0+\varphi) - \\ -2\dot{\varphi}\dot{\gamma} l_{AB} l_{BE}\cos(\varphi_0+\varphi)\times \\ \times\sin(\gamma_0+\gamma) + \dot{\gamma}^2 l_{BE}^2\sin^2(\gamma_0+\gamma) \end{bmatrix} +$$

$$+\frac{1}{2}I_Q\dot{\gamma}^2 = \frac{1}{2}M\begin{bmatrix} V_0^2 - 2V_0\dot{\varphi} l_{AB}\sin(\varphi_0+\varphi) + \\ +2V_0\dot{\gamma} l_{BE}\cos(\gamma_0+\gamma) + \\ +\dot{\varphi}^2 l_{AB}^2 - 2\dot{\varphi}\dot{\gamma} l_{AB} l_{BE}\times \\ \times\sin(\varphi_0+\varphi+\gamma_0+\gamma) + \gamma^2 l_{BE}^2 \end{bmatrix} + \frac{1}{2}I_Q\dot{\gamma}^2. \qquad (3)$$

The potential energy is taken as the sum of the deformation energies of the suspension elements and the equivalent elastic deformation of the soil under the track rollers. The deformations are determined from the geometric relationships of the positions of the characteristic points of the system and linearization at small angles of deviation. As a result, the potential energy takes the form

$$U = c_n l_{AB} l_{BC}\cos(\varphi) + \frac{c_3}{2}\times$$

$$\times\begin{bmatrix} 3\varphi^2 l_{AB} - \gamma^2\left(l_{BD}^2 - l_{DN}^2 - l_{O1D1}^2\right) + \Theta^2\left(l_{OD}^2 - l_{O1D1}^2\right) + \\ +\varphi\gamma\begin{Bmatrix} 2l_{AB}l_{BD}\cos(\varphi_0 - 180 + \gamma_0 + d_0) - \\ -2l_{AB}l_{BN}\cos(180 + \varphi_0 - \gamma_0 - \rho_0) - \\ -l_{AB}l_{BD1}\cos(\varphi_0 - 180 + \gamma_0 + d_{01}) \end{Bmatrix} - \\ -\gamma\Theta\begin{Bmatrix} 2l_{BD1}l_{DO1}\cos(180 + \theta_0 - \gamma_0 - d_{01}) - \\ -2l_{OB}l_{OD}\cos(\theta_0 - 180 + \gamma_0 + d_0) \end{Bmatrix} - \\ -\varphi\theta\begin{Bmatrix} 2l_{AB}l_{OD}\cos(\varphi_0 - \theta_0) + \\ +2l_{AB}l_{O1D1}\cos(\varphi_0 + \theta_0) \end{Bmatrix} \end{bmatrix}. \qquad (4)$$

To take into account energy losses (damping in the system and/or in the roller-soil contact), a dissipative function was introduced. After transformations, its final expression in the adopted generalized coordinates was obtained

$$\psi = k_1 l_{AB} l_{BC} \cos\dot{\varphi} + \frac{1}{2} k_3 \times \times \left[\begin{array}{l} 3\dot{\varphi}^2 l_{AB} + \dot{\gamma}^2 \left(l_{BD}^2 - l_{DN}^2 + l_{O1D1}^2 \right) + \dot{\Theta}^2 \left(l_{OD}^2 + l_{O1D1}^2 \right) + \\ + \dot{\varphi}\dot{\gamma} \left\{ \begin{array}{l} 2 l_{AB} l_{BD} \cos\left(\varphi_0 - 180 - d_0 - \gamma_0 \right) - \\ - l_{AB} l_{BN} \cos\left(180 + \varphi_0 - \rho_0 - \gamma_0 \right) - \\ - 2 l_{AB} l_{BD1} \cos\left(\varphi_0 - 180 + d_{01} + \gamma_0 \right) \end{array} \right\} - \\ - \dot{\varphi}\dot{\Theta} \left\{ \begin{array}{l} 2 l_{BD1} l_{DO1} \cos\left(180 - \theta_0 - d_0 - \gamma_0 \right) - \\ 2 l_{OB} l_{BD} \cos\left(\theta_0 - 180 - d_0 - \gamma_0 \right) \end{array} \right\} - \\ - \dot{\varphi}\dot{\Theta} \left\{ \begin{array}{l} 2 l_{AB} l_{OD} \cos\left(\varphi_0 - \theta_0 \right) + \\ + 2 l_{AB} l_{O1D1} \cos\left(\varphi_0 + \theta_0 \right) \end{array} \right\} \end{array} \right]. \quad (5)$$

The equation of motion is derived from the Lagrange equations of the second kind taking into account holonomic constraints. After substituting the expressions for the kinetic (3), potential (4) energies and the dissipative function (5), as well as performing the necessary differentiations, the equation is reduced to a standard linear second-order differential equation

$$a\ddot{\varphi} + b\dot{\varphi} + c\varphi = d, \quad (6)$$

where the coefficients in the equation are functions of the design parameters of the mechanism, mass-inertial characteristics, and equivalent elastic-damping parameters of the contact. The analytical expressions of the coefficients take the following form:

$$a = \frac{4 l_{AB}^2 l_{BE} M_3 \sin\left(\varphi_0 + \gamma_0 \right)}{\left(l_{BD} + L_{ON} \right) \cos\left(180 - \gamma_0 - d_0 - \theta_0 \right)} + l_{AB}^2 M_3, \quad (7)$$

$$b = \left[3 k_3 l_{AB} + 4 l_{AB}^2 k_1 \cos\varphi_0 \cos\theta_0 \right], \quad (8)$$

$$\begin{aligned} c = {} & l_{AB} l_{BN} \left(c_1 + k_1 \right) + 4 l_{AB} c_3 \cos\varphi_0 \cos\theta - \\ & - \frac{4 c_3 l_{AB}^2 l_{BN} \cos\left(\varphi_0 - 180 - d_0 - \gamma_0 \right)}{\left(l_{BN} + L_{ON} \right) \cos\left(180 - \gamma_0 - d_0 - \theta_0 \right)} - \\ & - \frac{4 c_3 l_{AB}^2 l_{BD} \cos\left(\varphi_0 + 180 - \rho_0 - \gamma_0 \right)}{\left(l_{BD} + L_{OD} \right) \cos\left(180 - \gamma_0 - d_0 - \theta_0 \right)} - \\ & - \frac{4 c_3 l_{AB}^2 l_{BD1} \cos\left(\varphi_0 - 180 - d_{01} - \gamma_0 \right)}{\left(l_{BD1} + L_{O1D1} \right) \cos\left(180 - \gamma_0 - d_0 - \theta_0 \right)} - 3 c_3 l_{AB}, \end{aligned} \quad (9)$$

$$\begin{aligned} d = {} & \left(Q - P_z - R_k - R_{k1} \right) l_{AB} \cos\varphi_0 - R_x \sin\varphi_0 - \frac{4 l_{AB} l_{BD1} \cos\left(180 - \rho_0 - \gamma_0 \right) R_x}{\left(l_{BD} + l_{OD} \right) \cos\left(180 - \gamma_0 - d_0 - \theta_0 \right)} + \\ & + \frac{4 R_z l_{AB} l_{ON} \cos\left(180 - \rho_0 - \gamma_0 \right) - 4 l_{AB} l_{BE} Q \cos\varphi_0 - 4 R_x l_{AB} l_{OD} \sin\gamma_0 - 4 l_{AB} l_{O1D1} \sin\gamma_0}{\left(l_{BD} + L_{BD1} \right) \cos\left(180 - \gamma_0 - d_0 - \theta_0 \right)}. \end{aligned} \quad (10)$$

For an analytical description of the transient process, equation (6) is solved by the operational method (Laplace transform), which allows us to take into account the initial conditions and reduce the problem to algebraic transformations. The operational form of the equation is written as

$$X(p) = h \frac{1}{p\left(p^2 + 2np + k^2 \right)}. \quad (11)$$

After standard transformations (expansion into simple fractions and inverse Laplace transformation), the time dependence of the generalized coordinate is obtained in the following form

$$\varphi(t) = \frac{h}{k^2} \left[1 - e^{-nt} \left(\begin{array}{l} \cos\sqrt{k^2 - n^2}\, t + \\ + \frac{n}{\sqrt{k^2 - n^2}} \sin\sqrt{k^2 - n^2}\, t \end{array} \right) \right]. \quad (12)$$

The first term in (12) corresponds to the set (static) displacement of the system, and the other components describe damped oscillations around the new equilibrium position. The parameters included in (12) are determined through coefficients (7)–(10) and, accordingly, allow us to analyze the influence of the design parameters of the lever mechanism and the roller-soil contact on the nature of the oscillatory process.

5. 3. Parametric analysis of amplitude-frequency characteristics and justification of rational parameters

To study the functional regularities of vertical vibrations of the working tool suspension, the design parameters and physical and mechanical indicators given in Table 1 (according to the calculation scheme in Fig. 1) were used.

The disturbing forces P_z and P_x are formed during the interaction of the working body (cultivator paw) with the soil environment. The resistance force P_x acts in the horizontal plane [17] and, at the adopted working depth of 0.05 m, is considered constant and has little influence on oscillations in the vertical plane, therefore further analysis is focused on the vertical component of the oscillation process.

The evaluation was carried out by the amplitude H and the period/frequency characteristic T in the characteristic currents of the extremum of the first oscillation wave (A, B), using phase-frequency dependences and normalized spectral densities.

The phase-frequency characteristics of vertical oscillations in the l_{AB} function are given in Table 2 (for three values of l_{AB} = 1.0; 0.5; 0.1) and in Fig. 2.

The obtained values show that the smallest amplitudes of oscillations are achieved at l_{AB} = 0.5 m: for point A_1 the amplitude decreases to H = 0.03 m, and for point B_1 – to H = 0.01 m. For other l_{AB} lengths, the amplitude increases (for example, at l_{AB} = 1.0 m for A_1 H = 0.14 m; at l_{AB} = 0.1 m for A_1 H = 0.10 m). The normalized spectral densities (Fig. 3) confirm the presence of a pronounced dependence of the amplitude and frequency range of oscillations on l_{AB}: the optimal value of l_{AB} = 0.5 m corresponds to the mode with the smallest deviations at characteristic points.

Thus, the length of the link l_{AB} is one of the determining parameters; its deviation from 0.5 m leads to an increase in the amplitude of vertical oscillations, which potentially worsens the stability of the working body movement in depth.

The study on the influence of the l_{BE} distance (position of the center of mass relative to the attachment node) was performed for l_{BE} = 0; 0.1; 0.25; 0.5 m. The phase-frequency characteristics are given in Table 3 and are shown in Fig. 4.

The results show that at l_{BE} = 0.25 m the amplitudes at the characteristic points decrease sharply: for A_2 H = 0.033 m, and for B_2 – H = 0.001 m. For comparison, at l_{BE} = 0.1 m the amplitudes are significantly larger (A_2: $H = 0.21$ m; B_2: $H = 0.067$ m), and at $l_{BE} = 0$ m an increased level of oscillations is also observed (A_2: $H = 0.15$ m; B_2: $H = 0.055$ m).

Table 1

Design parameters and physical indicators in theoretical modeling

Indicator	l_{AB}, m	l_{BE}, m	l_{BD}, m	l_{BD1}, m	l_{ON}, m	l_{BN}, m	l_{OD}, m	l_{O1D1}, m	k_3, kg/m·s	k_k, kg/m·s	c_3, N/m^2
Value	0.5	0.25	0.24	0.3	0.24	0.25	0.22	0.37	10	18	1,000
Indicator	φ_0, degree	γ_0, degree	θ_0, degree	ρ_0, degree	d_0, degree	d_{01}, degree	M_3, N	P_z, kN	R_k, R_{k1}, kN	P_x, kN	
Value	15	40	0	75	70	80	100	100	100	20	

Table 2

Phase-frequency characteristic of vertical oscillations depending on link length l_{AB}

l_{AB}, m	Link length, m					
	1.0		0.5		0.1	
	H, m	T, s	H, m	T, s	H, m	T, s
A_1	0.14	1.1	0.03	0.7	0.1	0.4
B_1	0.04	2.4	0.01	1.6	0.05	0.6

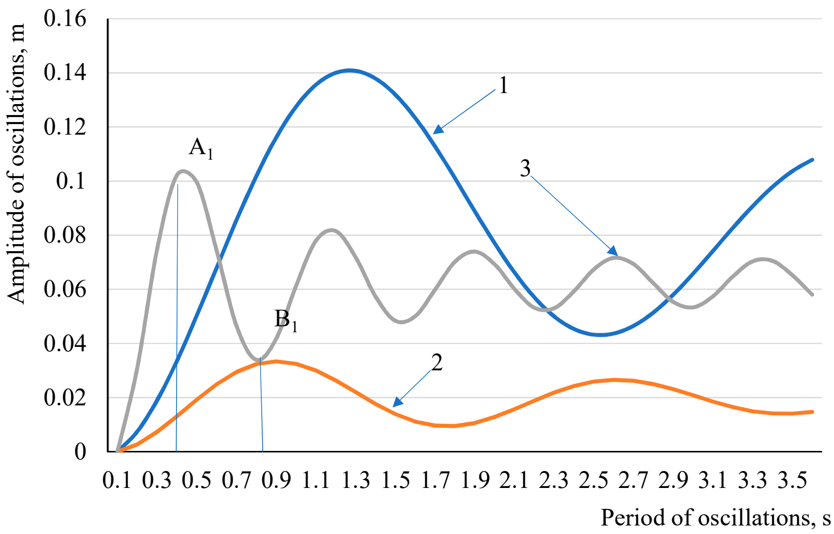

Fig. 2. Phase-frequency characteristic of vertical vibrations of a working tool with independent connection to the energy source and two support points: 1 – $l_{AB} = 1.0$ m; 2 – $l_{AB} = 0.5$ m; 3 – $l_{AB} = 0.1$ m; A_1, B_1 – characteristic points

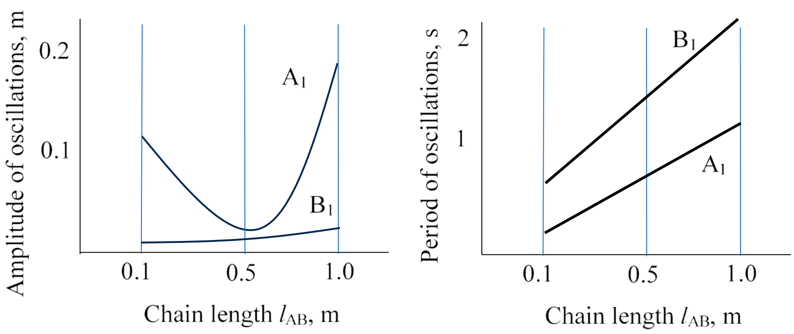

Fig. 3. Normalized spectral densities of change in the amplitude and period of oscillations of the system depending on the length of the chain: *a* – change in the amplitude of oscillations at the chain length $l_{AB} = 0.1$; 0.5; 1.0 m; *b* – change in the period of oscillations at the chain length $l_{AB} = 0.1$; 0.5; 1.0 m; A_1, B_1 – characteristic points of influence of the length of the link l_{AB}

The normalized spectral densities (Fig. 4) confirm that the change in l_{AE} primarily affects the amplitude level of oscillations, while the characteristic values of T change in a limited range.

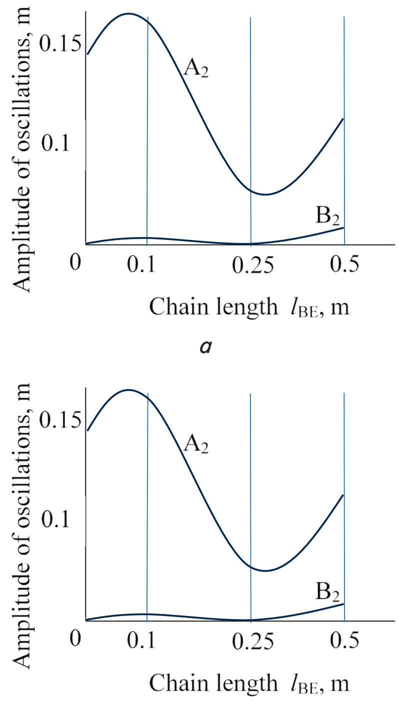

Fig. 4. Normalized spectral densities of change in the amplitude and period of oscillations of the system depending on the chain length: *a* – change in the amplitude of oscillations at chain length $l_{BE} = 0.1$; 0.25; 0.5; *b* – change in the period of oscillations at chain length $l_{BE} = 0.1$; 0.25; 0.5 m; A_2, B_2 – characteristic points of influence of the link length l_{BE}

Table 3

Phase-frequency characteristic of vertical oscillations depending on the chain length, A_2, B_2 are characteristic points

l_{BE}, m	Link length, m							
	0		0.1		0.25		0.5	
	H, m	T, s	H, m	T, s	H, m	T, s	H, m	T, s
A_2	0.15	0.5	0.21	0.7	0.033	0.8	0.1	0.8
B_2	0.055	1.2	0.067	1.4	0.001	1.7	0.028	1.9

Thus, the value is $l_{BE} = 0.25$ m, which corresponds to the minimum amplitudes of oscillations and, accordingly, the best conditions for stabilization of the working body in the vertical plane.

An analysis was also performed for $M_3 = 10$; 30; 50; 80; 100 N (Table 4) using phase-frequency characteristics and spectral dependences (Fig. 5).

Based on our data, the amplitude at point A_3 remains practically constant for the entire weight range (at the level of $H \approx 0.03$ m), while the period/frequency characteristic changes moderately (from $T = 0.2$ to 0.7). At point B_3, the amplitude also changes slightly (approximately from $H = 0.015$ to 0.01 m), but the frequency

Table 4

Phase-frequency characteristic of vertical vibrations depending on the total weight of the tool M_3, A_3, B_3 – characteristic points

M_3, N	Tool weight, N									
	10		30		50		80		100	
	H, m	*T*, s	*H*, m	*T*, s	*H*, m	*T*, s	*H*, m	*T*, s	*H*, m	*T*, s
A_3	0.03	0.2	0.03	0.3	0.03	0.4	0.03	0.6	0.03	0.7
B_3	0.015	0.4	0.012	0.8	0.013	1.0	0.01	1.4	0.01	1.6

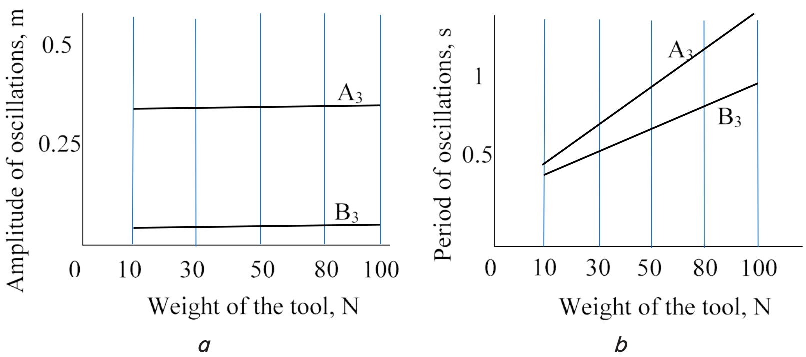

Fig. 5. Normalized spectral densities of change in the amplitude and period of oscillations of the system depending on the total weight of the tool M_3: *a* – change in the amplitude of oscillations; *b* – change in the period of oscillations; A_3, B_3 – characteristic points of influence of the weight of the tool M_3

Thus, in the adopted range M_3, the mass-inertia factor is not decisive for the amplitude of vertical oscillations, but it affects the frequency parameters of the process.

The parametric analysis performed reveals that the stabilization of the vertical movement of the working tool is most sensitive to the geometric parameters of the lever system, in particular l_{AB} and l_{BE}. According to the criterion of minimizing amplitudes at characteristic points, the values $l_{AB} = 0.5$ m and $l_{BE} = 0.25$ m are rational, while the variation of M_3 within the studied range does not lead to a significant increase in the amplitude of oscillations.

6. Discussion of results based on theoretical modeling of oscillations of a tool with an independent mount and two support rollers

Our second-order linear differential equation for the generalized coordinate of the oscillatory system is explained by the adopted formulation of small oscillations in the vertical plane and the use of an energy approach.

The kinematic relations for the position and velocities of the center of mass determine the kinetic energy of the system, and the deformations of the elastic elements of the suspension and the equivalent contact "roller-soil" form the potential energy (4). Taking into account the dissipation of energy through damping leads to the appearance of a term proportional to the velocity in the equation of motion (6). Thus, the structure of equation (6) is a direct consequence of the balance of energies T, Π and dissipative losses in the linearized formulation. The decisive factor for the interpretation is that coefficients (7) to (10) directly relate the dynamic properties of the system (frequency and damping) to the geometry of the lever links, mass-inertia parameters, and elastic-damping characteristics of the contact. The solution to the transient process (12) explains the presence of a fixed component and damped harmonic oscillations as a typical response of a second-order linear system with dissipation.

Our results are generally consistent with the conclusions from studies [9, 10], which show that the stability of the position of the working bodies depends on the kinematics of the mechanism and the parameters of the elastic elements. Similar to those studies, this work also found that changing the geometric parameters of the mechanism significantly affects the amplitude of vertical deviations. At the same time, unlike [9, 10], where the results are mainly tied to specific structures and experimental settings, in this study generalized analytical dependences were obtained, suitable for parametric synthesis of the mechanism with two support rollers.

The revealed strong sensitivity of the amplitude of oscillations to l_{AB} is explained by the fact that this link determines the arm of inertial moments application and the effective stiffness of the transmission of disturbances from the attachment to the tool body. The minimization of the amplitude for $l_{AB} = 0.5$ m is a consequence of the system approaching the regime where the ratio of the inertial and elastic components provides the minimum deviation at the characteristic points. This result builds on studies [9, 10], which also show the importance of the kinematic scheme, but do not provide an analytical criterion for choosing rational parameters.

Similarly, the influence of l_{BE} is explained by a change in the position of the center of mass relative to the connection node, which affects the reduced moments of inertia and static moments of the system. As a result, the coefficients of the equation of motion change, and therefore the frequency and amplitude characteristics. Compared with papers [5–8], in which most attention is on measuring and adjusting the depth, our results show another side of the problem. That is, even before the application of active control, the behavior of the system is largely determined by the structural geometry of the mechanism. So, whereas in [5–8] accuracy is achieved mainly by compensating for deviations, then in this work the possibility of their primary reduction at the design level has been shown.

Our results also provide grounds for a clearer comparison with studies [3, 4], which investigated the force processes in the three-point hitch and the redistribution of the load between the tractor axles. In those papers, the tool is actually considered as an external load that changes the force state of the energy vehicle. In contrast to this approach, in our work, the implement is isolated as an independent oscillatory system. This is what allowed us to obtain the equation of motion (6) and explicit analytical coefficients (7) to (10), which directly relate the design parameters to its dynamic characteristics. Thus, the results do not contradict the conclusions from [3, 4] but complement them, shifting the emphasis from the tractor's power state to the implement's natural dynamics.

Separately, it is necessary to compare our results with the DEM-MBD approaches reported in [11, 12]. Similar to those studies, this work takes into account the influence of the roller-soil contact on the vibration process. However, whereas in [11, 12] the contact is described with a high level of physical detail, then in our work it is represented through equivalent elastic-damper parameters. This reduces the physical detail of the model but, at the same time, makes it possible to obtain compact analytical dependences suitable for fast parametric analysis. Therefore, the proposed model is inferior to the DEM-MBD approaches in the completeness of the description of local contact phenomena but has an advantage at the stage of sketch design and selection of rational structural parameters.

When analyzing the literature, it was determined that the key unsolved problem is the lack of a generalized theoretical model of natural harmonic oscillations of a tool with independent mounting in a vertical plane and two support rollers, taking into account the contact properties of the roller-soil. Therefore, the derived equation of motion (10) with explicit coefficients (7) to (10) directly implements such a model, and the parametric dependences for l_{AB}, l_{BE} and M_3 (Fig. 4, 5) demonstrate that the model not only describes oscillations but also provides a tool for substantiating rational parameters of the mechanism. Thus, the problem part is closed to the extent that a formalized description, a connection with design parameters and quantitative recommendations for minimizing vertical deviations are provided.

The limitations and conditions of practical application concern the elastic-damper contact parameters. In the model they are given but in practice they may depend on humidity, density, soil structure, and speed of movement. Therefore, to transfer the recommendations, correct identification of these parameters is required under specific conditions.

The disadvantage of this study is the lack of experimental verification specifically for a tool with two support rollers in different types of soil. It should also be noted that the model uses equivalent contact parameters without a detailed nonlinear description of soil rheology. This may limit accuracy under extreme modes.

Further research should be directed to experimental identification of the parameters of the "roller-soil" contact and validation of the model by field measurements of vertical deviations and spectral characteristics. It is also promising to expand the model to a spatial setting, taking into account possible detachment, soil nonlinearity, and spatial fluctuations, since it is these factors that most often determine the limits of applicability of linearized models.

7. Conclusions

1. A calculation scheme of the lever mechanism of a working tool with two support points and independent attachment in the vertical plane has been built. This allows us to separate the tool's natural vibrational dynamics from the motion of the energy source and take into account the contact interaction "support roller-soil" as a source of disturbances. A distinctive feature of the result is that the scheme is directly focused on minimizing vertical deviations, i.e., it closes part of the problem of insufficient formalization of the natural vibrations of tools with two supports. This is explained by the fact that two support points provide "averaging" of the microrelief and reduce geometrically determined deviations compared to one support, which is confirmed by further quantitative estimates.

2. A generalized energy mathematical model of harmonic oscillations of the implement has been constructed in the form of a second-order linear differential equation with constant coefficients. The result is an advantage over known approaches focused mainly on force processes in the attachment of the unit since the model directly connects the dynamics of oscillations with the design parameters of the lever system and the equivalent elastic-damping properties of the "roller-soil" contact. This is explained by the use of Lagrange equations of the second kind and the energy approach, which provides a compact form of the model, as well as the possibility of further parametric synthesis without excessive computational complexity.

3. A parametric analysis of the amplitude-frequency characteristics of vertical displacements at characteristic points of the tool was performed and rational values of the geometric parameters of the mechanism were established. It was shown that the length of the link l_{AB} is decisive, at $l_{AB} = 0.5$ m the minimum amplitudes $H = 0.01$ m are achieved, while the deviation to $l_{AB} = 1.0$ m increases the amplitude to $H = 0.14$ m. It was also established that the optimal position of the attachment point is $\mathbf{l_{BE}} = 0.25$ m, and the variation of the tool weight $M_3 = 10...100$ N practically does not change the amplitude, although it affects the frequency parameters (the period increases to 1.6). Together, this provides a structural reduction of inherent deviations to a level close to technologically permissible (±0.02 m for rational parameters). This eliminates the problem of the lack of generalized recommendations on parameters that guarantee permissible vertical deviations of a tool with two support rollers.

Conflicts of interest

The authors declare that they have no conflicts of interest in relation to the current study, including financial, personal, authorship, or any other, that could affect the study and the results reported in this paper.

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Data availability

All data are available, either in numerical or graphical form, in the main text of the manuscript.

Use of artificial intelligence

When creating this manuscript, GPT-5.2 was used to edit grammar and search for sources using keywords and relevant criteria.

Authors' contributions

Viacheslav Padalka: Conceptualization, Methodology, Investigation; **Oleksandr Gorbenko:** Conceptualization, Methodology, Validation, Investigation, Supervision, Project administration**; Dmytro Khvostenko:** Resources, Validation, Investigation; **Andrii Lazorenko:** Formal analysis, Visualization, Writing – original draft; **Vladyslav Alpidovskyi:** Visualization, Writing – original draft.

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