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IMPROVING A METHOD OF RATIONAL PLACEMENT OF BASE STATIONS IN A GLOBAL NAVIGATION SATELLITE SYSTEM DIFFERENTIAL CORRECTION SYSTEM

This study investigates the process of deploying base stations of the global navigation satellite system's differential correction system. The task addressed is to reduce complexity of the differential correction system by optimizing the number and placement of base stations in the global navigation satellite system.

The method for the rational placement of base stations of the global navigation satellite system's differential correction system has been improved. In contrast to known ones, it involves the following:

– initially filling the predefined area with Voronoi diagram pole points;

– constructing the Voronoi diagram;

– finding the centroids of the resulting cells as the positions of the centers of the Voronoi cells;

– verifying that the condition for changing the positions of the centroids is satisfied;

– verifying that the condition for finding the centroids within prohibited zones is met;

– adjusting the positions of the centroids.

An experimental study was conducted on the optimal placement of base stations for the differential correction system in the global navigation satellite system. A visual analysis was performed of the results of base station placement using the improved method and known coverage methods. A comparative visual analysis reveals better coverage of the target area when using the same number of base stations. A quantitative analysis showed that using the improved method reduces the number of base stations by 21% compared to known methods. At the same time, the condition of full coverage of the specified area was met. This, in turn, has made it possible to reduce the complexity and cost of the global navigation satellite system's differential correction system.

The scope of application of the improved method is the placement of base stations for the global navigation satellite system's differential correction system

Keywords: base station, global navigation satellite system, navigation corrections, optimal placement

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1. Introduction

It is known [1, 2] that modern global navigation satellite systems (GNSSs) provide an accuracy in determining the location of a consumer within 10 m or more. However, for many applications such as:

1) automated vehicle control;
2) geodesy;
3) marine and river navigation;
4) precision agriculture, such accuracy is insufficient.

One of the ways to improve the characteristics of the navigation system, such as accuracy, reliability, and availability, is the use of differential correction systems [3]. The essence of most differential correction methods is that the navigation equipment takes into account various corrections obtained from alternative sources. The sources of corrective information are base stations (BSs) [4] whose coordinates are known with high accuracy. When comparing the known coordinates of BSs obtained as a result of geodetic surveying with the coordinates of the same BSs measured by navigation equipment, navigation corrections are formed [5]. These navigation corrections are transmitted to consumers via communication channels.

Nowadays, differential correction methods based on obtaining navigation corrections in real time (Real Time Kinematic (RTK) [6] are widely used. These methods make it possible to achieve centimeter accuracy of navigation measurements when the BS is located at a distance of no more than 70...80 km from the navigation receiver [6]. To provide

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navigation corrections to subscribers located within a significant area of the earth's surface, several units or dozens of BSs are combined into local and regional networks [7].

Therefore, there is a relevant task to place BSs within the predefined area so that from any point in this area the distance to the nearest BS does not exceed a given value. This can be achieved by increasing the number of BSs, which does not seem rational since it leads to an increase in the complexity and cost of the system. It is more acceptable to search for a rational location of BSs that would provide coverage of the predefined area with the characteristics required by consumers.

2. Literature review and problem statement

In [8], a comparative analysis of several common methods for placing nodes in wireless sensor networks to improve their coverage and connectivity was conducted. The analyzed methods are as follows:

– the Voronoi diagram method;
– the Delaunay triangulation method;
– the Voronoi tessellation method;
– the convex hull method.

It is shown that the Voronoi diagram method is recommended specifically for solving the problem of minimizing the number of sensors in the network while ensuring complete coverage of the predefined area and the maximum network connectivity. However, questions regarding the operation of the method at the border of the coverage area and in areas where there are obstacles remain unresolved. The most likely reason is problems of detecting coverage and connectivity of such areas. This is the approach used in [9].

In [9], the task of effective placement of nodes in wireless sensor networks was solved using the Voronoi diagram method. The study of this approach showed that, unlike conventional methods, the Voronoi diagram method makes it possible to significantly increase the accuracy of sensor localization in networks, including those with a high density of reference nodes. It is noted that an important advantage of the Voronoi diagram method is that with an increase in the number of reference nodes in the network, the accuracy of determining the coordinates of the sensors increases. At the same time, there is no significant increase in computational costs. In [9], the results of evaluating the efficiency of wireless networks of different topologies depending on the distance between its reference nodes are reported. However, questions regarding the operation of the method [9] for differential correction of GNSS remain unresolved.

In [10], a comparative analysis of a number of common algorithms and tools for optimal placement of charging stations for electric vehicles is given. Among the analyzed ones:

– Genetic algorithm (GA);
– Particle swarm optimization (PSO);
– Ant colony optimization (ACO);
– Greedy algorithm (GrA);
– Integer (linear) programming (ILP);
– optimization capabilities of the CPLEX software (United States of America).

The advantages and disadvantages of the considered algorithms and optimization tools are as follows [10]:

1. GA:
– advantages – easy to implement; better suited for placement problems;
– disadvantages – takes a long time to solve the placement problem.

2. PSO:
– advantages – simple calculations and the ability to find a close to optimal solution;
– disadvantages – premature convergence; higher probability of hitting a local maximum.

3. ACO:
– advantages – positive feedback explains the rapid discovery of feasible solutions;
– disadvantages – convergence time is uncertain.

4. GrA:
– advantages – fast and guaranteed to give a valid solution;
– disadvantages – the obtained solution is usually suboptimal.

5. ILP:
– advantages – simplicity; solves many different combinations of problems;
– disadvantages – works only with linear variables; potentially cannot solve stochastic problems.

6. CPLEX:
– advantages – effectively solves linear, convex, or non-convex problems with constraints;
– disadvantages – difficulty in modifying optimization procedures.

But the methods in [10] were used for the optimal placement of charging stations for electric vehicles. Therefore, questions regarding the operation of these methods for differential correction of GNSS remained unresolved. This approach is considered in [11].

Work [11] describes the results of field tests of GNSS equipment under RTK mode under conditions of large base length. During the measurements, the base station is located on the roof of a 4-story building at a distance of 80 km from the stationary rover. The rover is installed on a flat terrain without significant obstacles that obscure the signal. Coordinate measurements were made for one hour every second for three Global Navigation Satellite System (GNSS) configurations:

– Global Positioning System (GPS) (United States);
– GPS + GloNavSatSystem (GLONASS) (Russian Federation);
– GPS + GLONASS + GALILEO (European Union) + BEIDOU (China).

The elevation angle mask was chosen to be 10 degrees. Field tests showed high stability of the equipment under RTK mode in these settings, even when using only GPS navigation satellites. The coordinate measurement error did not exceed 4 cm in plan and 12 cm in height. Work [11] considers practical tests. Therefore, questions regarding the methodology and algorithm for optimizing the placement of RTK BSs in the predefined area remained unresolved. This approach is considered in [12].

The methodology and algorithm for optimizing the placement of RTK BSs in the predefined area, taking into account the probable number and distribution of GNSS users, are proposed in [12]. During optimization, a placement option is determined in which all users are served without exception by the minimum number of BSs. The condition is taken into account that the base length does not exceed 70 km. In [12], it is proposed to search for the extremum of the objective function using a memetic algorithm. The memetic algorithm provides a faster and more accurate search for the extremum than the conventional genetic algorithm. However, questions regarding the performance of the memetic algorithm for optimal RTK base station location in the predefined area remain unresolved. This approach is considered in [13].

In [13], a method for finding the optimal location of 5G microbase stations of the intelligent street lighting system is proposed. The interaction conditions between 5G stations and installed devices are taken into account. A universal indicator of the quality of station location selection is developed to implement the rational use of urban space resources and intelligent communication between subsystems. Since 5G has a small range, in [13], a model for station placement based on stationary lampposts of "smart" street lighting is proposed. The model takes into account peak data transfer rates and user location density.

But the method proposed in [13] was used for the optimal location of street lighting system stations. Therefore, questions regarding the operation of the method [13] for differential GNSS correction remain unresolved.

In [14], a review of methods for deploying base stations in wireless sensor networks is given from the point of view of maximizing their coverage and energy efficiency. The authors note that conventional single-criteria methods optimize either coverage or energy consumption without balancing them. Therefore, a multi-criteria optimization method was proposed (coverage maximization and energy consumption minimization). The multi-criteria optimization method can also be applied to BS GNSS networks (accuracy maximization and cost minimization). The evolutionary algorithm and the clustering algorithm are investigated as algorithms for finding the optimal solution. In [14], specific geometric or GNSS factors are not considered, only general optimization principles are provided and experience in related areas is summarized.

In [15], a comparative analysis of the effectiveness of several heuristic algorithms is carried out:

– GA;
– PSO;
– simulated annealing;
– gray wolf.

for the deployment of 5G BSs in urban and rural environments. It is shown that PSO and GA provide an optimal balance between coverage and throughput, eliminating redundant BSs. The work concerns wireless telecommunication networks, where the distribution of subscribers is characterized only by population density. Under the conditions of taking into account the influence of the geometric factor of GNSS, relief, atmospheric model and specific characteristics of radio channels, the considered algorithms can be adapted to the analysis of GNSS efficiency. However, the methods in [15] were used for the optimal placement of cellular BSs. Therefore, questions regarding the operation of the methods of [15] for differential correction of GNSS remained unresolved.

In [16], a multi-criteria optimization algorithm for the deployment of BSs in ultra-dense 5G networks is proposed. The optimization of BS topology is carried out according to such criteria as coverage maximization, minimization of the level of mutual interference, and energy consumption. To find optimal solutions, an evolutionary method is used, which makes it possible to obtain compromise Pareto-optimal configurations. As constraints, the algorithm takes into account the specified density of network users, radio signal characteristics, and quality of service level. Unfortunately, the method above:

– does not optimize the accuracy of GNSS positioning;

– does not take into account the features of the geometry of BS networks of navigation systems;

– does not ensure their uniform placement on the terrain.

In [17], the use of a heuristic algorithm for multi-criteria optimization of a moth is proposed to solve the problem of rational placement of BSs. The method is based on a stochastic search for the global maximum of the objective function by simulating the behavior of moths moving in the direction of the flame. The objective function takes into account indicators of completeness of territory coverage and communication quality. The algorithm adaptively changes the search parameters to avoid local minima and adapts well to complex conditions. The disadvantage of [17] is the stochastic nature of the algorithm, which leads to instability of the obtained results. The method does not provide geometrically uniform coverage of the territory, does not take into account the features of positioning in GNSS.

Work [18] considers the problem of optimizing the placement of BSs of a positioning system. A genetic algorithm is used to minimize the maximum error in determining coordinates for a given number of stations. Various node location configurations and their impact on positioning accuracy indicators are considered. The algorithm takes into account the network geometry specific to positioning systems and the conditions for ensuring interaction between BSs. An objective function is proposed that evaluates the quality of the BS network configuration. The algorithm makes it possible to find the best solutions to complex spatial problems. The results are confirmed by numerical experiments. However, due to the rejection of fast geometric optimization methods, the algorithm has a high computational complexity. This makes the algorithm unsuitable for optimizing networks over large areas. In addition, the optimization results depend on the initial optimization parameters, the algorithm is unable to adapt to the complex shape of the region.

In work [19], the task of optimal placement of sensors in wireless networks is investigated. The main goal is to maximize the completeness of coverage and achieve the best network connectivity. A genetic optimization algorithm is used to solve the problem. The limitations on the communication range and energy consumption are taken into account, an algorithm for evaluating the efficiency of the network configuration is proposed, and experimental results for various scenarios are presented. However, the proposed optimization criteria are not related to the GNSS error, and the influence of the spatial position of the navigation receiver and satellites on the positioning accuracy is also not taken into account. The method is focused on optimizing the completeness of coverage, not the positioning accuracy.

In [20], the task of choosing the optimal location of GNSS BSs is considered. It is proposed to compare alternative BS positions by the value of the complex BS quality indicator. The method [20] takes into account environmental factors important for positioning systems. These are the degree of probable influence of the multipath effect on navigation determinations and the average duration of direct visibility sessions of navigation satellites at a particular position. The disadvantage of the method is that it optimizes only the positions of each individual BS without optimizing the network as a whole. There is no general criterion for minimizing the error, the interaction between BSs is not taken into account, and their uniform coverage of the predefined area is not ensured.

In [21], a hybrid algorithm for searching for rational placement of system nodes is proposed, which combines the geometric algorithm of Voronoi diagrams with the genetic algorithm. The Voronoi diagram is used for the initial division of the area into service zones, which makes it possible to obtaining the initial network configuration. Further optimization is performed using a genetic algorithm, which improves the location of nodes according to the specified criteria.

The main purpose of the hybrid algorithm is to increase the uniformity of coverage and efficiency of the network. In [21] it is shown that the combined approach makes it possible to achieve better results than using each of the algorithms separately. The disadvantage of [21] is that the search for the optimal configuration is actually performed by the genetic algorithm, which has its own certain limitations. Therefore, the hybrid algorithm [21] does not provide fast convergence, which is characteristic of the Voronoi diagram algorithm.

In [22], a method for optimal deployment of a heterogeneous sensor network on a limited area of the terrain to create a single arched barrier detection zone of a non-convex shape was proposed. A method for forming an arched barrier detection zone was devised, aimed at minimizing the total costs of deploying and operating a sensor network. To find the minimum of the objective function, an algorithm combining ILP and the cuckoo search algorithm (CSA) was proposed. The effectiveness of the developed algorithm was studied by simulation modeling under various conditions. CSA demonstrated a number of advantages, namely fast and easy convergence to the global maximum, simple calculation procedure, small number of parameters and optimal route for finding the extremum. However, the method [22] does not take into account the non-convex shape of the BS deployment area. Therefore, questions regarding the consideration of such a structure of the BS deployment area remained unresolved. This approach was considered in [23].

In [23], to solve the task of the most efficient use of a limited non-convex territory when deploying a network of multi-position radars, it is proposed to apply the harmonic search algorithm. In this case, the parameters are adjusted based on learning by contrast. The method reported in [23] was devised to overcome the limitations of the conventional harmonic search algorithm (slow convergence and looping in local extrema). The simulation results showed that the method is effective and practical in solving the problem. Despite the large volume of calculations, the method could be used for other similar tasks. For example, optimizing the placement of BS navigation systems, rational selection of a location for a new observation and communication point, and deploying mobile radars. But the method [23] is cumbersome and does not work with a small number of BSs.

In [24], a method and algorithm for stochastic optimization of BS placement in wireless cellular networks based on the gradient of the objective function are proposed. The algorithm effectively calculates stochastic gradients based on the estimates of the nearest neighbors' positions using the Voronoi diagram method. This contributes to balanced coverage and reduces the iteration execution time by more than two orders of magnitude compared to conventional integral optimization. This makes the method [24] useful only when designing real communication networks. However, the method was used for optimal placement of wireless cellular BSs. Therefore, questions regarding the operation of the method for differential GNSS correction remain unresolved.

In [25], a three-stage algorithm for solving the task of covering a flat convex polygon with circles of the same radius with complete overlap is presented. At the first stage, an initial approximation of the centers of the circles is given. The second (construction of Voronoi diagrams) and the third (finding the centroids of the cells obtained in the second stage) stages of the algorithm are performed repeatedly in an iterative process. In [25], the radii of the circles enclosing the cells, with centers at the centroids, decrease and approach some minimum value.

The main disadvantages of [25] are:

– failure to take into account the sphericity of the earth's surface, which limits the application of the algorithm within small areas;

– it is not determined how the algorithm works in the case when the cells of the Voronoi diagram within the predefined area are not convex polygons.

Thus, our review of related literature data showed that existing methods of rational placement of BSs in the GNSS differential correction system (hereinafter, BS systems) are often built on the basis of the Voronoi diagram method.

This is due to the following obvious advantages of the method [8, 9, 21]:

– effective solution of the problem of dividing the surface into adjacent zones around a given set of points;

– completeness of coverage (the surface is divided without gaps and overlapping of adjacent zones);

– clarity;

– low computational complexity;

– high convergence.

In known works, certain limitations of the Voronoi diagram method are noted, which are critical for the optimization problems of the BS position in limited areas, namely:

– failure to take into account the sphericity of the Earth's surface [25];

– failure to take into account natural and artificial obstacles, within which the BS placement is impossible [8, 24];

– problematic work in limited areas, especially in "non-convex" zones. As a result, to create a continuous coverage zone, the method places the BS outside the given area. This may be impossible (for example, when the given area is limited by state borders) [25].

The above limitations are usually tried to be compensated for by developing hybrid or multi-stage iterative algorithms. In such algorithms, the Voronoi diagram method is used for the initial division of the surface into zones of approximately the same plane. Then, using another, usually heuristic algorithm, the location of the BS in the problem areas is improved. This complicates the algorithm and the optimization process, reduces the speed of finding the extremum, increases the computational complexity, reduces the stability of the algorithm, etc. Such algorithms do not guarantee obtaining an acceptable result in all necessary cases. They find solutions that are certainly better than the initial ones, but are not optimal [21, 24].

Therefore, our analysis of known methods for placing BSs in the GNSS differential correction system [8–25] reveals a number of disadvantages, namely:

– complexity of the optimization process;

– low speed of finding the extremum of the objective function;

– increased computational complexity;

– low stability of operation;

– not guaranteeing an acceptable result, etc.

The above gives grounds to argue that it is advisable to conduct research on improving the method of rational placement of BSs for a GNSS differential correction system.

3. The aim and objectives of the study

The purpose of our study is to improve the method of rational placement of BSs of a GNSS differential correction system. This will make it possible to reduce the number of BSs through their rational placement.

To achieve the goal, the following tasks were set:

– to list the main stages of the method of rational placement of BSs;

– to conduct an experimental study using the improved method of rational placement of the system.

4. The study materials and methods

The object of our study is the process of placing base stations for the differential correction system in the global navigation satellite system.

The main hypothesis of the study assumed that improving the method of rational placement of a BS system would make it possible to enable full coverage of the predefined area, reduce the number of BS systems to some minimum value. And, as a result, reduce the cost and complexity of the system, take into account the sphericity of the earth's surface within the predefined area and the presence of obstacles within which the placement of the BS is impossible.

When conducting the study, the following assumptions were adopted:

– BS systems provide navigation corrections when at least one BS is located at a distance not exceeding a given value (*R*) from the subscriber;

– BSs are located within the predefined area of the earth's surface;

– part of the area of the given area is occupied by obstacles within which the placement of BS is unacceptable.

When conducting an experimental study, the following were used:

– hardware: DELL Intel(R) Core (TM) i7-8650U CPU @ 1.90GHz (2.11 GHz) laptop (USA);

– software: object-oriented programming language Python 3.12 (The Netherlands);

– geographic information system QGIS Desktop 3.36.1 (European countries).

The following general methods were used in the study:

– probability theory and mathematical statistics;

– empirical research;

– mathematical modeling;

– comparative research.

– research methods were selected taking into account the formulated research tasks.

The following theoretical methods were used in determining the main stages of the method for rational placement of a BS system:

– computational geometry and spatial modeling;

– probability theory and mathematical statistics.

The choice of the above theoretical research methods is due to:

– construction of the Voronoi diagram;

– finding the centroid of the cells;

– checking the fulfillment of the condition for changing the position of the centroid;

– geometric construction of forbidden zones.

When conducting an experimental study on the rational placement of BS systems, the following theoretical and practical research methods were applied:

– empirical research;

– mathematical modeling;

– probability theory and mathematical statistics.

– comparative research.

The choice of the above theoretical and practical research methods is due to:

– the use of an iterative process;

– experimental determination of the positions of the centers of the smallest enclosing circles of Voronoi cells;

– conducting an experiment on the rational placement of BSs;

– conducting a comparative analysis of improved and known methods of placement of BSs.

5. Results of investigating the method of rational placement of base stations

5. 1. Main stages in the method of rational placement of base stations

The main stages of the method of rational placement of BS systems are shown in Fig. 1.

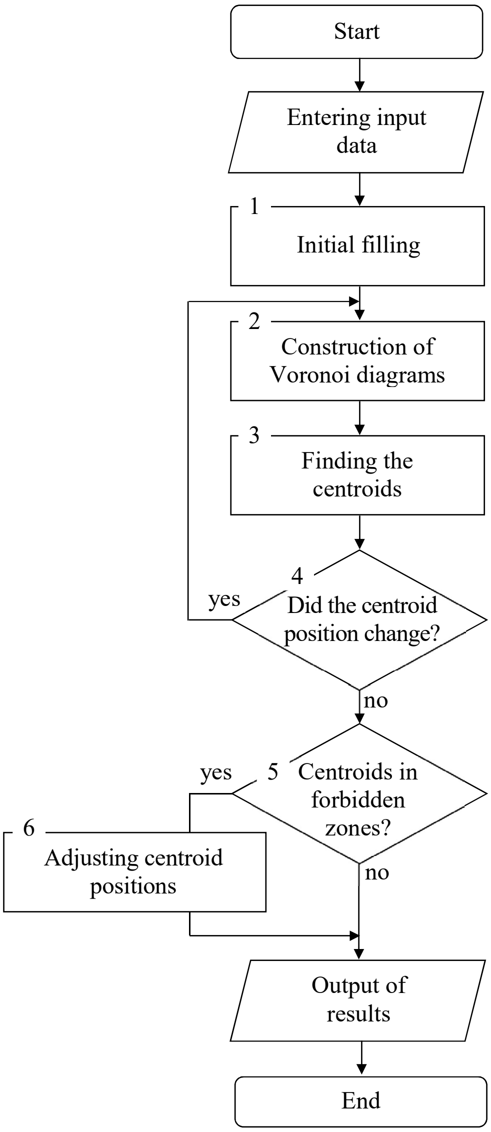

Fig. 1. Basic stages of the method for rational placement of base stations in the differential correction system for a global navigation satellite system

The main stages of the method of rational placement of the BS system are:

1) initial filling of the predefined area with points-poles of the Voronoi diagram;

2) construction of the Voronoi diagram. As a result, so-called Voronoi cells are obtained around each pole point – polygons. The sides of the polygons represent the locus of points equidistant from two poles;

3) finding the centroids of the obtained cells as the positions of the centers of the smallest enclosing circles (LEC) of the Voronoi cells. Voronoi cells are circles of minimum radius, within which all points of the corresponding cells are located;

4) checking the fulfillment of the condition for changing the centroid positions. If the centroid positions have changed, stages 2 and 3 are repeated. At each iteration, new centroid positions are obtained. For this purpose, the value of the radius of the smallest enclosing circle (LEC) was calculated for each cell. NOC is a circle of minimum radius, within which all points of the Voronoi cell are located. The center of NOC coincides with the position of the corresponding centroid;

5) checking the fulfillment of the condition of the centroid being within the forbidden zones. If the condition is met, the transition to stage 6 is made;

6) adjusting the position of the centroid.

Steps 1–4 are performed similarly to the corresponding steps of the algorithm [25]. The features of the improved method of rational location of BS within a certain area of the earth's surface are that the boundaries of the predefined area of the earth's surface are generally not a convex polygon. In addition:

– the boundaries of the predefined area of the earth's surface are determined by points on the earth's surface, given in the geographical coordinate system;

– the coordinates of NOC centers are determined in the same geographical coordinate system;

– the Voronoi cells obtained as a result of constructing a Voronoi diagram on an ellipsoid or geoid are not flat figures.

These problems can be solved using a projection coordinate system (SC). This makes it possible to consider Voronoi cells and the predefined area as flat figures and use simple relationships in calculations. In this case, the selected SC must satisfy the requirement of no distortion of distances when switching to a geographic SC. That is, if the distance between points A and A_1 is equal to the distance between points B and B_1 when measuring in a projection SC, this equality must be preserved when determining the distance between these points in a geographic SC. This is done regardless of the position of points A, A_1, B, B_1 within the predefined area. Not every projection SC is suitable for solving the problem. Thus, using the Mercator projection SC is not advisable since in this SC the distances are significantly distorted when moving away from the equator.

In further research, the User Coordinate System (UCS)-2000 (European Petroleum Survey Group (EPSG):6385) (hereinafter referred to as UCS) was used. In the case where the boundaries of the earth's surface area are specified in the World Geodetic System (WGS)-84, a transition from WGS-84 to UCS was made using known relations or using the rp.transform procedure from the pyreproj library in the Python programming environment.

The stages of the method are discussed in more detail below.

The set J of points $P = \{p_1, p_2, ..., p_J\}$ is located within a given region S. A uniform distribution of points within the given region is used. It is known [26] that one of the optimal fillings of an infinite plane is filling with equilateral triangles. The first filling point O with coordinates (x_1, y_1) is located in the southwest corner of a spherical rectangle circumscribed around the given region (Fig. 2). We determine the coordinates of the filling points x_{ik}, y_{ik} (i is the column number, starting from the west, k is the row number, starting from the south) as (expressions (1), (2)):

$$x_{ik} = x_1 + (i-1)R\sqrt{3} + ((k-1)\mathrm{mod}2)0.5R\sqrt{3}, \qquad (1)$$

$$y_{ik} = y_1 + (k-1)1{,}5R, \qquad (2)$$

where mod means the operation of calculating the remainder during integer division. It is not difficult to show that with such a filling, the distances between the vertices of equilateral triangles, measured in UCS, are equal to $a = R\sqrt{3}$. Moreover, if circles of radius R are placed at the vertices of the triangles, these circles will completely cover the given area.

The results of filling the predefined area (the territory of Ukraine) with equilateral triangles with a side of 50√3 km are shown in Fig. 2. The vertices of the triangles that are located outside the given area are removed before starting the next stage of the method.

It was determined how much the distances between the vertices of the filling are distorted when converting the coordinates of the vertices from UCS to the WGS-84 coordinate system. Using the rp.transform procedure from the pyreproj library in the Python programming environment, the values of the vertex coordinates obtained in UCS were converted to the WGS-84 coordinate system. As a result, the geographical (latitude and longitude) coordinates of the vertices of spherical triangles in the WGS-84 coordinate system were obtained, which correspond to the equilateral triangles obtained earlier in UCS. The geodetic distances between points A and A_1, as well as between B and B_1, selected on the southern and northern borders of the given area (Fig. 2) were calculated. The results of our calculations are given in Table 2. Analysis of Table 2 revealed that when converting the coordinate systems from UCS to WGS-84, the linear dimensions are slightly distorted (the error does not exceed 0.12%).

Similar studies were conducted for 44 control points randomly located across Europe (Fig. 3).

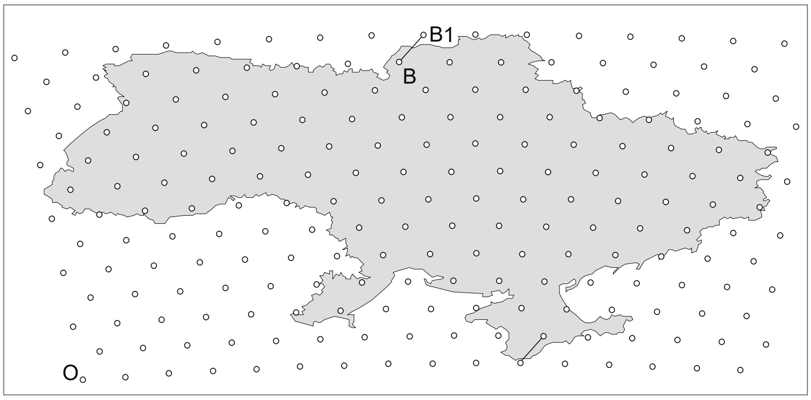

Fig. 2. Filling the predefined area (territory of Ukraine) with equilateral triangles

The distance distortion was checked in the EPSG:3034, EPSG:3035, ESRI:102014, ESRI:102031 control points. To this end, the coordinates of points located at a distance $a = R\sqrt{3}$ from the given ones were calculated using expression (1). The coordinates of the obtained pairs of points were converted to WGS-84, after which the geodetic distances between them were calculated. The smallest distance distortions were obtained when using the EPSG:3035 control point (Table 3). Table 3 gives examples of calculations for points C, D, E (Fig. 3). The value of the average relative deviation calculated for 44 control points in the EPSG:3035 control points is 0.09%.

In some cases, the initial filling of the area can ensure its complete coverage with circles of radius R. However, in the general case, this is not the case. In Fig. 4, the shaded areas that are not covered when using filling (expression (1)) are shown.

From the analysis of Fig. 4 it is seen that at the initial filling for some points on the borders of the predefined area the distances to the BS are almost twice the value of R.

The Voronoi diagram $VOR(P)$ divides the area S into polygons $vor(p_j)$ in such a way that each point inside the polygon $vor(p_j)$ is closer to the point p_j than to other points of the set P (Fig. 3). The points p_j represent the poles of the Voronoi diagram, and the polygons themselves are Voronoi cells. The edges of the polygons $vor(p_j)$ represent the locus of points in the area S equidistant from the two poles.

If the accuracy of measuring the subscriber's coordinates is proportional to the distance to a BS, to ensure the highest accuracy of measurements it is necessary to place the BS within the given area. In this case, the maximum distance ρ from any point of the j-th cell to the pole p_j must be minimal for all points of the set P.

The algorithm for constructing a Voronoi diagram is described in detail in [27]. To construct a Voronoi diagram, the Voronoi diagram construction procedure built into QGIS was used (menu items Vector – Geometry tools – Voronoi polygons). The peculiarity of using this procedure in QGIS is that it is necessary to use a projection coordinate system, for example UCS. When using a geographic coordinate system, for example WGS-84, the results will be incorrect. An example of constructing a Voronoi diagram for the territory of Ukraine and the initial distribution of points calculated from expression (1) is shown in Fig. 5. The points in Fig. 5 show the positions of the poles.

Voronoi cells located near the boundaries of the predefined area are open polygons (Fig. 5). Therefore, it is necessary to limit them to the boundaries of the predefined area. This can be done in QGIS by using the intersection operation of the corresponding vector layers.

The next step of the iterative process is to calculate the centroids of Voronoi cells as the positions of the centers of NOC and the radii of NOC for each Voronoi cell. When searching for NOC, it is taken into account that each cell $vor(p_j)$ is given by a set of vertices T. The task of constructing the NOC is reduced to finding a circle of the smallest radius such that all points of the set T are within the circle whose boundary is the given circle. As shown in [28], NOC is unique. This is either a circle passing through three points of the set T, or some pair of points of this set is a diameter of this circle. It is obvious that the simplest way to find the smallest enclosing circle is to search for all pairs and triples of points in the set T. This method, despite its simplicity, is not the most efficient since with the increase in the number of points in the set T, the number of combinations of their pairs and triples increases. There are more efficient algorithms than the direct search algorithm [29–31]. However, given that the number of points in the cells $vor(p_j)$ is usually within five to seven, in this work the direct search method was used to search for NOC.

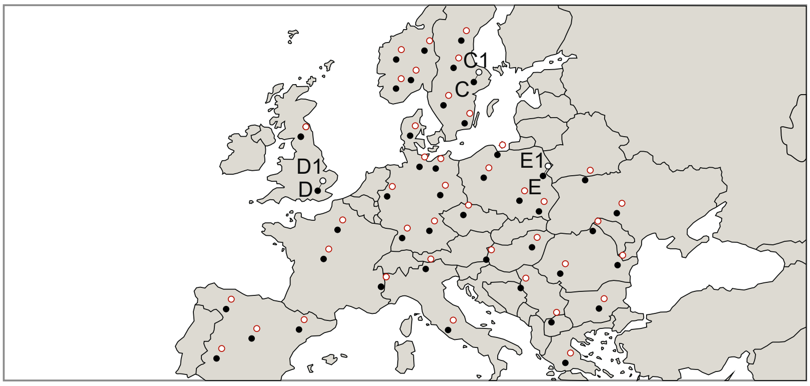

Fig. 3. Location of checkpoints in Europe

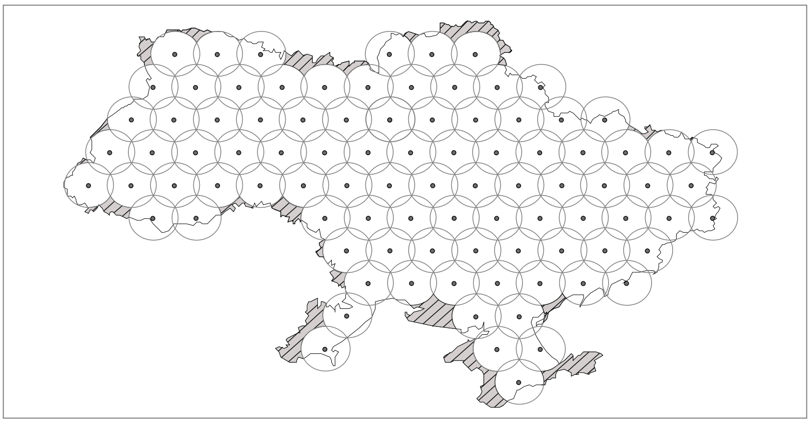

Fig. 4. Uncovered areas of the predefined area (shaded) after initial filling

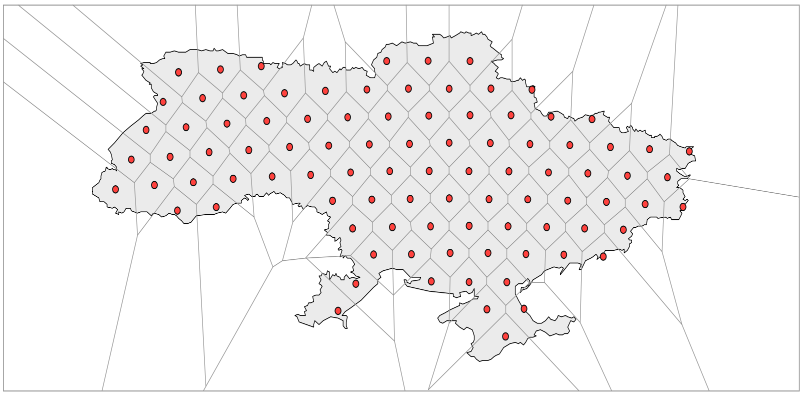

Fig. 5. Example of constructing a Voronoi diagram (done in QGIS 3.6)

Table 2

Distances between control points measured in different coordinate systems

Point	Coordinates in UCS, degrees	Distance in UCS, km	Coordinates in WGS-84, degrees	Distance in WGS-84, km
A	4906001.00, 372249.40	86.603	33.9037, 44.2851	86.595
A_1	4981001.00, 415550.67		34.4628, 44,9542	
B	5731001.00, 155743.05	86.603	30.9122, 51.6898	86.503
B_1	5806001.00, 199044.32		31.5159, 52.3731	

Table 3

Distances between control points located within Europe

Point	Coordinates in UCS, degrees	Distance in UCS, km	Coordinates in WGS-84, degrees	Distance in WGS-84, km
C	3204323.59, 3622265.98	86.603	51.5134, –0.1025	86.462
C_1	3279323.59, 3665567.25		52.2329, 0.3742	
D	3324584.43, 5191652.08	86.603	52.3364, 22.8503	86.444
D_1	3399584.43, 5234953.35		52.9267, 23.6832	
E	4069221.59, 4711226.39	86.603	59.5530, 16.8972	86.676
E_1	4144221.59, 4754527.66		60.1833, 17.8100	

Determining the radius and center of the enclosing circles when enumerating pairs of points in the set T is a trivial problem. Determining the radius and center of the enclosing circles when enumerating triples of points in the set T is carried out using expressions (3), (4):

$$x = -\begin{pmatrix} y_1\left(x_2^2+y_2^2-x_3^2-y_3^2\right)+ \\ +y_2\left(x_3^2+y_3^2-x_1^2-y_1^2\right)+ \\ +y_3\left(x_1^2+y_1^2-x_2^2-y_2^2\right) \end{pmatrix} / Q, \tag{3}$$

$$y = \begin{pmatrix} x_1\left(x_2^2+y_2^2-x_3^2-y_3^2\right)+ \\ +x_2\left(x_3^2+y_3^2-x_1^2-y_1^2\right)+ \\ +x_3\left(x_1^2+y_1^2-x_2^2-y_2^2\right) \end{pmatrix} / Q, \tag{4}$$

where $Q = 2\left(x_1\left(y_2-y_3\right)+x_2\left(y_3-y_1\right)+x_3\left(y_1-y_2\right)\right)$, x_1, y_1, x_2, y_2, x_3, y_3 are the coordinates of the points of the selected triple of points of the set T.

From the obtained set of enclosing circles, the circle with the smallest radius is selected as NOC.

The method quickly finds a rational location of the BS network from the point of view of full coverage of the given area and the minimum distance from the subscriber to the nearest BS. In practice, additional requirements may be imposed on the location of the BS on the ground, such as the presence of cellular coverage and electrification, the absence of natural obstacles, the possibility of providing security, etc. Within the given area, there may be zones within which the location of the BS is impossible, or this will lead to additional financial costs (hereinafter referred to as prohibited zones). During stages 2 and 3 of the method, the points – poles of the Voronoi diagram – may fall into these zones. Therefore, the position of the points is corrected in two ways. In the first method, the position of the points is corrected once during the last iteration by transferring the points to the nearest place outside the prohibited zone. In the case when the size of the prohibited zones is comparable to the value R, such a one-time correction may result in part of the territory of the prohibited zones being uncovered by the BS coverage zones. With the second technique, a forced correction of the location of points relative to the forbidden zones is carried out at each iteration. Such correction provides a more correct result of covering the forbidden zones without the need to place points in them. At the same time, the use of this method significantly increases the calculation time.

Thus, the method of rational placement of BSs of the differential correction system of the global navigation satellite system has been improved, which, unlike known ones, provides for the following:

– initial filling of the predefined area with points-poles of the Voronoi diagram;

– construction of the Voronoi diagram;

– finding the centroids of the obtained cells as the positions of the centers of the NOC of the Voronoi cells;

– checking the fulfillment of the condition of changing the centroid positions;

– checking the fulfillment of the condition of finding the centroid within the prohibited zones;

– correction of the centroid position.

5. 2. Experimental study on rational placement

To conduct an experimental study on rational placement of a BS system, the territory of Ukraine was selected as the coverage area (Fig. 6). Water obstacles (reservoirs, lakes, large rivers) were selected as prohibited zones – they are shown in Fig. 6 in dark color.

The initial filling of the area shown in Fig. 2 was used; the distance between the points $a = R\sqrt{3}$ at $R = 50$ km. Stages 2, 3 of the method were performed in an iterative process, a total of 100 iterations were performed. Fig. 7 shows the northern part of the given area enlarged. The dark color in Fig. 7 shows the prohibited zones (reservoirs).

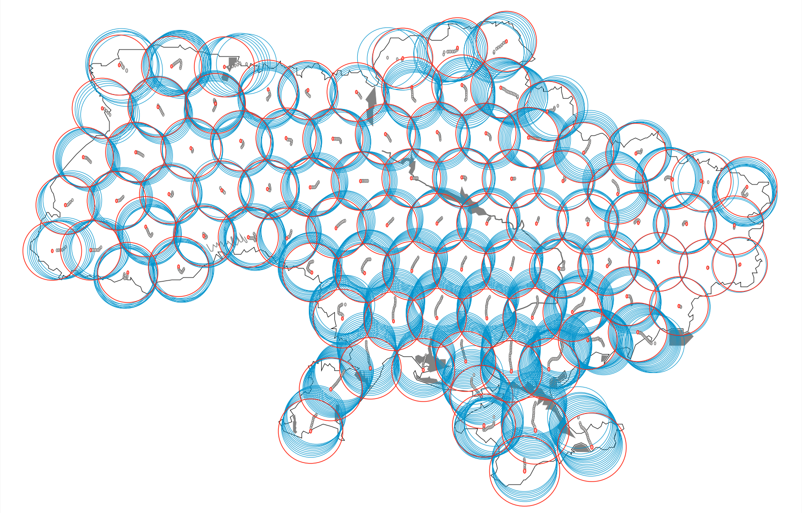

Fig. 6. Example of how the method works (displayed in the WGS-84 coordinate system)

Light dots show intermediate positions of NOC centers, dark dots – the positions of the NOC centers at the last iteration. The NOCs themselves are displayed as ellipses in the figures, which is explained by the use of the WGS-84 datum when displaying the results. At the first iteration, the NOC radii slightly increased to 54…56 km, at subsequent iterations there was a decrease in the NOC radii. After 60 iterations, the NOC radii were in the range of 50.6…50.7 km, and during the continuation of the iterations they almost did not change. At the same time, naturally, the position of the NOC centers almost did not change. This can be used as a sign of the possibility of the end of the iteration process. The NOC centers at the last iteration are outside the prohibited zones and within the predefined area. This is an advantage of the method since the placement of the BS outside the predefined area is not always possible.

As a result of the method, the average value of the NOC radii increased compared to the specified value R, which can be neglected if this excess is within acceptable limits. Otherwise, it is necessary to repeat the method by reducing the value R in expression (1), while the number of BSs will increase.

For comparative analysis, Fig. 4 was analyzed. Fig. 4 shows the uniform filling of the predefined area, which is used in modern methods of BS placement. Fig. 4 and the results of the improved method (Fig. 6) were compared. The comparative analysis reveals better filling of the predefined area (Fig. 6) when using the same number of BSs.

For quantitative analysis, an additional twenty BSs were added to the BS system shown in Fig. 4 for complete coverage of the predefined area (Fig. 8).

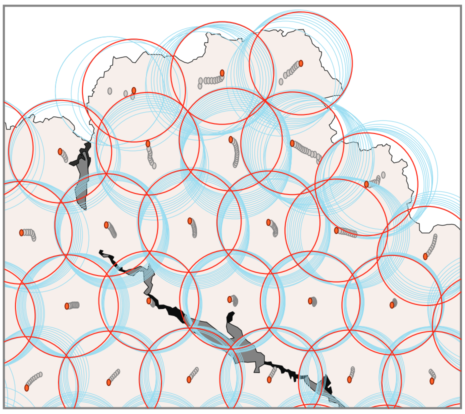

Fig. 7. Example of the method performance (magnified northern part of the predefined area)

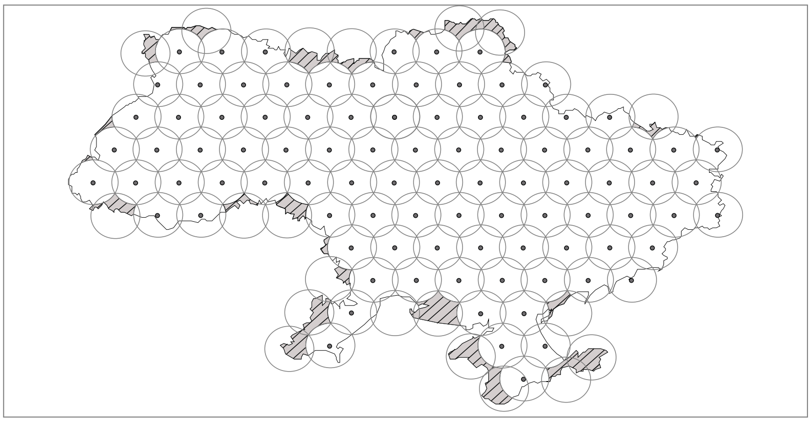

Fig. 8. Initial coverage with additional base stations added to fully cover the predefined area

At the same time, the total number of BSs is 112, which is 21% higher than the number of BSs (92), whose coverage areas completely covered the specified area as a result of the improved method (Fig. 6).

6. Discussion of results based on improving the method of rational placement of base stations

The method of rational placement of BS of the GNSS differential correction system (Fig. 1) has been improved, which, unlike known ones (for example, [8–10, 16–18, 20, 21, 23]), provides for the following:

– initial filling of the predefined area with points-poles of the Voronoi diagram (expressions (1), (2), Figs. 2–4);

– construction of the Voronoi diagram (Fig. 5);

– finding the centroids of the obtained cells as the positions of the centers of the NOC of the Voronoi cells (expressions (3), (4));

– checking the fulfillment of the condition of changing the centroid positions;

– checking the fulfillment of the condition of finding the centroid within the prohibited zones;

– adjusting the centroid position.

The features of the improved method are:

– the possibility of placing BS in a territory that, in the general case, is described by a non-convex polygon;

– when the dimensions of the predefined area are quite large and it is necessary to take into account the sphericity of the Earth;

– prohibited zones are taken into account (terrain relief, forest areas, hydrography, etc.) where the BS cannot be placed.

An experimental study was conducted on the rational placement of BSs for the GNSS differential correction system. The territory of Ukraine was selected as the coverage area (Fig. 6). Water obstacles (reservoirs, lakes, large rivers) were selected as prohibited zones – shown in Fig. 6 in dark color. The initial filling of the area was used, shown in Fig. 2, the distance between points $a = R\sqrt{3}$ at $R = 50$ km. Stages 2, 3 of the method were performed in an iterative process, a total of 100 iterations were performed. Fig. 7 shows the northern part of the predefined area in an enlarged manner. As a result of the method, the average value of the NOC radii increased compared to the given value R, which can be neglected if this excess is within acceptable limits. Otherwise, it is necessary to repeat the method by reducing the value R in expression (1), while the number of BSs will increase.

For comparative analysis, Fig. 4 was examined (uniform coverage of the predefined area, which is used in modern methods of BS placement). A comparison of Fig. 4 and the results of the improved method (Fig. 6) was performed. The comparative analysis reveals better coverage of the predefined area (Fig. 6) when using the same number of BSs. For quantitative analysis, an additional twenty BSs were added to the BS system shown in Fig. 4 to fully cover the predefined area (Fig. 8). Comparative visual analysis shows better coverage of the predefined area when using the same number of base stations. Quantitative analysis revealed that the use of the improved method makes it possible to reduce the number of base stations by 21% compared to known methods. At the same time, the condition of complete coverage of the predefined area is met. This, in turn, allowed us to reduce the complexity and cost of the differential correction system of the global navigation satellite system.

Our results are explained by the use of an improved method for rational placement of BSs for a GNSS differential correction system.

The improved method has the following limitations:

– BSs must be placed at a certain distance from the subscriber;

– the method does not provide for operation in conditions of a large number of prohibited zones;

– takes into account operation only with known (existing) GNSS.

The improved method has the following disadvantages:

– inoperability of the method in desert and water surfaces;

– does not take into account the possible mobility of BSs.

Future studies involve experimental testing of the improved method within another given area.

7. Conclusions

The main stages of the improved method for rational placement of base stations of the system are:

– initial filling of the predefined area with points-poles of the Voronoi diagram;

– construction of the Voronoi diagram;

– finding the centroids of the obtained cells as the positions of the centers of the NOC of the Voronoi cells;

– checking the fulfillment of the condition for changing the centroid positions;

– checking the fulfillment of the condition for finding the centroid within the prohibited zones;

– adjusting the centroid position.

The features of the stages of the improved method are:

– placement of BS in the territory described by a non-convex polygon;

– taking into account the sphericity of the Earth;

– taking into account the zones prohibited for placement of BS.

2. An experimental study was conducted on the rational placement of BSs in a GNSS differential correction system. A visual analysis of the results of BS placement using the improved method and known filling methods was conducted. Comparative visual analysis reveals better coverage of the predefined area when using the same number of BSs. Quantitative analysis showed that the use of the improved method makes it possible to reduce the number of base stations by 21% compared to known methods. At the same time, the condition of complete coverage of the predefined area was met. This, in turn, allowed us to reduce the complexity and cost of the differential correction system for a global navigation satellite system.

Conflicts of interest

The authors declare that they have no conflicts of interest in relation to the current study, including financial, personal, authorship, or any other, that could affect the study and the results reported in this paper.

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Data availability

The data will be provided upon reasonable request.

Use of artificial intelligence

The authors confirm that they did not use artificial intelligence technologies when creating the current work.

Authors' contributions

Hennadii Khudov: Conceptualization, Methodology, Software, Investigation, Writing-review & editing, Supervision, Project administration; **Igor Taran:** Validation, Data curation, Investigation, Software; **Serhii Lohachov:** Validation, Data curation, Writing-original draft, Visualization; **Kostiantyn Kulahin:** Data curation, Writing-original draft, Visualization; **Oleh Rybachuk:** Resources, Writing-review & editing, Funding Acquisition

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