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DETERMINING DISPERSION CHARACTERISTICS OF RECTANGULAR WAVEGUIDE WITH NARROW IMPEDANCE WALLS

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This study explores a rectangular metal waveguide with narrow impedance walls, described by equivalent impedance-type boundary conditions. The task addressed is to build an effective mathematical model for analyzing waveguides with non-ideally conducting and irregular boundary surfaces, by determining their dispersion characteristics and wave propagation constants.

An approach based on the Fourier method and Leontovych impedance boundary conditions has been proposed. This has made it possible to avoid the complications associated with the vector statement of the problem and obtain transcendental equations for determining the propagation constants of bulk and surface waves. The dispersion equation was analytically solved and the eigenwave parameters were calculated in a wide range of surface impedance values.

The analytical results made it possible to verify correctness of the approach from a physical point of view; they could facilitate the optimization of parameters for the basic structure to the requirements of a specific microwave device. This is due to the use of an impedance boundary condition model, which adequately takes into account the influence of losses and reactive properties of the surface on electromagnetic fields and wave propagation processes in the waveguide.

In practice, the proposed approach could be used for the analysis and design of complex periodic microwave structures, in particular, filters, directional couplers, as well as power distribution elements between phased array antenna elements. Through the generalization of research results in the form of normalization of the impedance and spectral characteristics of the basic waveguide structure, the obtained characteristics could be used to design microwave devices in the range from decimeter to millimeter wavelengths

Keywords: propagation constant, boundary conditions, surface waves, dispersion characteristics, surface impedance, filters of harmonics.

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1. Introduction

Periodic and non-periodic directional structures consisting of waveguides of rectangular and circular cross-sections with various inhomogeneities are widely used in microwave technology. Newer frequencies are being mastered in radio engineering and telecommunications applications. These include, for example, 5G/6G communication systems, global positioning, as well as complexes for their suppression.

The development of new ranges and systems requires taking into account the level of increasing losses and the enhancement of surface effects associated with the roughness of the walls or the application of metamaterials on them, and through-holes in directional systems. Known numerical methods are capable of performing appropriate calculation; however, only by burdening the full model against the background of a gap in correspondence between the mathematical and physical interpretations of the processes.

Instead, impedance boundary conditions make it possible to adequately take into account these effects without overcomplicating the model: the impedance approach enables modeling these factors through computationally efficient boundary conditions. Analytical modeling using the Fourier method together with impedance boundary conditions provides quick estimates of key parameters and makes it possible to easily identify physical patterns. This makes it extremely useful both at the initial design stages and when optimizing filters, directional couplers, phase shifters, etc.

Therefore, it is a relevant task to conduct studies aimed at analyzing the dispersion characteristics of a rectangular waveguide with narrow impedance walls under a multimode mode.

2. Literature review and problem statement

The analysis of electromagnetic wave propagation in guide structures usually leads to the formulation of boundary value problems for Maxwell's equations with complex boundary conditions. In most practical cases, these problems do not have rigorous analytical solutions, which requires the use of either approximate or numerical methods.

The main disadvantages of approximate methods [1] include the limited range of variation of model parameters. Numerical methods also have their specific advantages and disadvantages or limitations. Universal numerical methods are widely used for guide systems with complex geometry, but they are disconnected from the physical model. Therefore, under certain conditions, their application causes the emergence of solutions that do not correspond to the modes present in the waveguide. This requires either a posteriori filtering or the imposition of a priori constraints. The assignment of additional conditions that would suppress fictitious modes requires the researcher to complicate the algorithm for finding solutions that have physical meaning and correspond to the waves actually existing in the system.

The authors of [2] provide a cumbersome complex solution for complex geometry. In [3], a metamaterial is used as an impedance structure. The application of the geometric theory of diffraction (GTD) method for impedance surfaces is proposed in [4] but, in its pure form without modifications, the GTD method leads to significant errors. The finite element method is discussed in [5, 6], but it is resource-intensive and therefore impractical. The impedance method in combination with the finite element method for calculating losses in the structure of an H-plane waveguide [7] requires a careful choice of impedance and mesh values for reliable results. Studies [8, 9] consider the numerical analysis of bandpass filters and low-pass filters based on waveguides with complex geometry and require cumbersome calculations. The same applies to the numerical analysis of multiband filters and multiplexers with higher harmonics [10, 11].

Depending on the type and magnitude of the waveguide unevenness, one chooses among narrowly specialized methods. For small unevenness, asymptotic approaches are usually used. As a result of the analysis of the non-strict Galerkin equation in the mixed statement for Maxwell's equations [12], an incorrect reproduction of boundary conditions on complex boundaries occurs. In [13], the boundary value problem is solved by the method of parabolic equations. This method gives significant errors due to the basic assumption of unidirectional paraxial wave propagation.

In [14], an infinite waveguide with inhomogeneous losses is considered, where the problem is the inhomogeneity of the boundaries. The authors of [15] propose a semi-analytical solution to the diffraction problem but, in this case, there is both slow convergence of the series and the presence of global errors of the solution. Numerical methods for solving the scattering problem in periodic waveguides are considered in [16]. At the same time, it was not possible to avoid the general drawback of numerical methods, namely, sensitivity to space discretization, as well as the danger associated with the loss of physical clarity of the process. In work [17], the results of the study on the spectral characteristics of guided modes in smoothly irregular waveguides using the symbolic method are reported. The authors were unable to get rid of the cumbersomeness of analytical expressions. In addition, the instability of the convergence of the result is observed in the presence of large-scale calculations.

From a practical point of view, work [18] is interesting; it combines modern methods of machine learning and domain decomposition. All this was intended to solve partial differential equations. But such a combination of methods failed to overcome the violation of strict agreement at the boundaries of subdomains and the accumulation of errors when "gluing" subdomains. In addition, a large amount of training data is required for training. The complex numerical statement of the boundary value problem for a waveguide using the impedance method in surface acoustic modeling is discussed in [19]. Here, poor reproduction of the mode structure of the waveguide and the dependence of accuracy on frequency are observed. The full-wave Galerkin method in the time domain in combination with the use of the finite element method for analyzing electromagnetic field modes was used by the authors of [20]. However, the temporal nature of the method, the presence of time dispersion, the appearance of parasitic modes during the solution, together with high computational costs, make it less effective and less accurate than the direct spectral statement of the eigenproblem in boundary value methods.

The estimation of the threshold value of the penalty coefficient and the limiting value of the time step for the Galerkin method with a gap in the time domain using the Helmholtz equation is discussed in [21]. This method provides stability and accuracy only with a very careful selection of parameters. Limiting the time step increases computational costs. The difficulties of using this approach are associated with increased sensitivity to geometry, grid cell size, and electrophysical properties of the medium.

The superconvergent Galerkin method with a gap in the simulation of high-frequency waves [22] requires careful mesh preparation and preservation of high-order elements during calculations. It is sensitive to stabilization parameters and is limitedly effective for complex, multi-scale, or very high-frequency problems. The approach in [23] based on Green's functions for Neumann boundary conditions turns out to be numerically complex due to the singularity of the derivatives and is sensitive to geometry, especially in the high-frequency region. Accordingly, it requires dense discretization and special numerical schemes, in particular, for the implementation of the process of regularization of singular integrals.

In [24], the definition of constant propagation is considered, but in a conical segment of a slotted rectangular waveguide. The analysis of integral equations of three-dimensional rectangular waveguide microwave structures using Green's functions, the calculation of which is accelerated by the Ewald method, is given in work [25]. The method has geometric and frequency limitations and requires significant computational resources. At the same time, it is quite complex to implement and sensitive to the settings of the accelerating parameters of the computational process.

For many practical applications, it is extremely important to determine the dispersion characteristics as accurately as possible. In such cases, the impedance method [1] offers an effective and physically meaningful approach to the analysis of the propagation of electromagnetic waves in complex waveguide systems. However, an analytical solution to the electrodynamic problem of two coupled irregular lossy waveguides operating under a multimode mode has not yet been obtained. The calculation of the electrical characteristics of such structures is complicated by the presence of a large number of auxiliary waveguides and coupling holes of different diameters. Such complexity of the design requires the development of a computational model that makes it possible to analyze factors affecting the surface impedance parameters of a narrow wall of an irregular waveguide.

One of the possible options for overcoming the specified difficulties may be the use of analytical modeling. This is the approach used in [1]. However, in the case of its direct use, this does not deprive the final result of analytical cumbersomeness and, as a result, leads to the lack of obviousness of its physical interpretation.

Summarizing the tasks not yet solved in the reviewed literature [1–25], which concern the determination of the influence of different types of impedance on the dispersion characteristics of the key structure in a wide range of frequency changes and impedance parameters, two main groups can be identified. A first issue is the extreme complexity and cumbersomeness of analytical solutions, especially for problems in vector statement. A second issue concerns the high requirements for computational resources to obtain numerical results of the required level of accuracy. One of the possible ways to overcome these problems is to use a combination of Leontovych impedance boundary conditions with the Fourier method.

3. The aim and objectives of the study

The purpose of our study is to determine the influence of different types of impedance on the dispersion characteristics of a rectangular waveguide with narrow impedance walls in a wide range of frequency and impedance parameters using the Fourier method and Leontovych impedance boundary conditions. This will make it possible not only to reduce the complexity of analytical expressions but also derive a solution analytically. At the same time, the requirements for the calculation process are reduced.

To achieve this goal, it is necessary to solve the following tasks:

– by setting and further solving the boundary value problem, derive a dispersion equation for a key structure with one and two impedance walls;

– to determine the frequency dependence of the phase velocity of surface and bulk waves for a wide range of generalized parameters of the impedance structure;

– to categorize waves and define their critical values (perturbation and transformation conditions in the key structure);

– to calculate dispersion characteristics of the waveguide's eigenwaves for different types of surface impedance in a wide range of its possible values, corresponding to both dielectrics of natural origin and artificial media.

4. The study materials and methods

The object of our study is a rectangular waveguide with narrow impedance walls, described by equivalent impedance-type boundary conditions. The hypothesis of the study assumes the validity of applying approximate Leontovych boundary conditions to the selected key structure (Fig. 1).

The assumptions adopted before formulating the boundary value problem in the form of the Helmholtz equation concern the restrictions on the variation of the structure parameters, under the existence of which the impedance boundary conditions are valid. To ensure the validity of the application of approximate boundary conditions, the periodic structure in the side narrow walls was replaced by an equivalent smooth impedance surface characterized by a uniform, isotropic surface impedance Z_s. In general, the surface impedance of a narrow wall is a complex quantity, which makes it possible to take into account, among other things, losses in the waveguide.

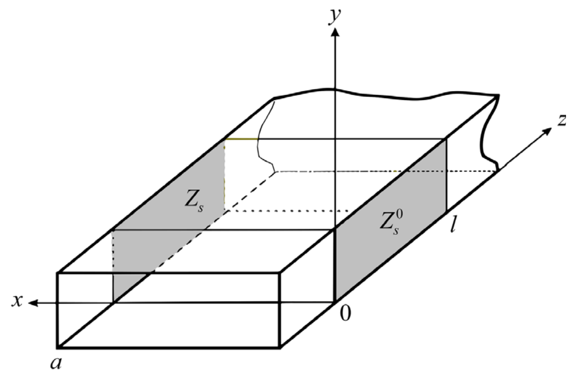

Fig. 1. Rectangular waveguide with narrow impedance walls

The simplifications of the model concern the homogenization of the boundaries along the narrow walls, which, accordingly, requires small variations in the geometry of their surface relative to the wavelength.

A rectangular coordinate system was used, the origin of which coincides with one of the vertices of the rectangle. The following notations were used: a is size of the wide wall of the main waveguide, b is the size of the narrow wall of the main waveguide. Let the fundamental wave of H_{10} type be incident on the waveguide entrance.

It is expedient to consider the propagation of transverse electric waves in a rectangular waveguide, the impedance of the narrow wall of which $(x=0, x=a)$ is not zero. The impedance of the narrow walls is due to the presence of, for example, round holes in them. Let the wide walls of the waveguide $(y=0,\ y=b)$ be characterized by ideal conductivity, and the medium inside the waveguide by the relative dielectric permittivity $\varepsilon_r=1$ and magnetic permeability $\mu_r=1$. The process of vibrational perturbation is considered stationary, and fields of any type have the form of waves propagating along the z axis. All components of the fields periodically change according to the $\exp[j\cdot(\omega\cdot t-\beta\cdot z)]$ law. In further calculations, this factor is omitted.

5. Results of investigating the dispersion characteristics of a rectangular waveguide with narrow impedance walls

5. 1. Stating the boundary value problem and obtaining the dispersion equation

In the case of TE waves, the longitudinal component H_z is determined from the Helmholtz equation

$$\frac{\partial^2 H_z}{\partial x^2}+\frac{\partial^2 H_z}{\partial y^2}+\gamma_\perp^2\cdot H_z=0, \qquad (1)$$

where $\gamma_\perp^2=k_0^2-\beta^2$ is the square of the transverse wave number; $k_0=\omega\cdot\sqrt{\varepsilon_0\cdot\mu_0}$ – wave number of unlimited free space with dielectric ε_0 and magnetic μ_0 constants of vacuum; $\omega=2\cdot\pi\cdot c/\lambda$ – cyclic frequency; c – speed of light; λ – wavelength; β – longitudinal wave number (propagation constant) in the directional structure.

By solving the Helmholtz equation (1) for a given directional system by the Fourier method, an expression for the H_z component was obtained

$$H_z = \left(A\cos\gamma_x x + B\sin\gamma_x x\right)\cdot\left(C\cos\gamma_y y + D\sin\gamma_y y\right), \qquad (2)$$

where A, B, C, D are unknown constants to be determined; γ_x, γ_y are transverse wave numbers along coordinates x, y, respectively: $\gamma_\perp^2 = \gamma_x^2 - \gamma_y^2$.

Given that the wide walls of the waveguide under consideration are perfectly conducting, the boundary condition for the tangential component of the electric field E_x takes the following form

$$E_x \approx \left.\frac{\partial H_z}{\partial y}\right|_{\substack{y=b\\y=0}} = 0. \qquad (3)$$

Solution to (2) can be written as

$$H_z = \left(A\cos\gamma_x x + B\sin\gamma_x x\right)\cdot\cos\frac{n\cdot\pi}{b}y\cdot e^{-j\beta z}, \qquad (4)$$

where from the defined boundary conditions the values are $\gamma_y = n\cdot\pi / b$, $n = 0, 1, 2, \ldots$

For narrow impedance walls the Leontovych boundary conditions can be written as

$$\left[n,E\right] = Z_s\left[n,\left[n,H\right]\right], \qquad (5)$$

where n is the normal directed into the impedance wall.

For the problem under consideration, condition (5) will take the following form:

– at $x = 0$

$$E_y = -Z_s^0\cdot H_z; \qquad (6)$$

– at $x = a$

$$E_y = Z_s\cdot H_z, \qquad (7)$$

where Z_s^0, Z_s are in the general case the complex impedances of the side walls, depending on the frequency.

Boundary conditions (6) and (7) include the E_y component, which is to be determined. We obtain an expression for the E_y component in terms of the longitudinal component H_z.

For this purpose, the following system of equations was used to couple the longitudinal and transverse components:

$$\begin{cases} -\gamma_\perp^2\cdot E_y = j\cdot\beta\cdot\dfrac{\partial E_z}{\partial y} - j\cdot\omega\cdot\mu_a\cdot\dfrac{\partial H_z}{\partial x}, \\ -\gamma_\perp^2\cdot H_x = j\cdot\beta\cdot\dfrac{\partial H_z}{\partial x} - j\cdot\omega\cdot\varepsilon_a\cdot\dfrac{\partial E_z}{\partial y}, \end{cases} \qquad (8)$$

where ε_a and μ_a are the absolute dielectric and magnetic constants.

For transverse electric waves, the following expressions are valid:

$$\begin{cases} E_y = j\cdot\omega\cdot\mu_0\cdot\dfrac{1}{\gamma_\perp^2}\cdot\dfrac{\partial H_z}{\partial x}, \\ H_x = -j\cdot\beta\cdot\dfrac{1}{\gamma_\perp^2}\cdot\dfrac{\partial H_z}{\partial x}, \end{cases} \qquad (9)$$

where $\gamma_\perp^2 = \gamma_x^2 - \left(\dfrac{n\cdot\pi}{b}\right)^2$ is the transverse wave number.

For the convenience of performing further numerical analysis, a normalized impedance was introduced into the equation, referred to the characteristic impedance of free space

$$Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}},$$

and impedances Z_s^0, Z_s, which are normalized in the same way:

$$\begin{cases} \dfrac{Z_s^0}{Z_0} = \tilde{Z}_s^0, \\ \dfrac{Z_s}{Z_0} = \tilde{Z}_s. \end{cases} \qquad (10)$$

Now, taking into account (10), we can write the dispersion equation in the form

$$\begin{aligned} &\left(\tilde{Z}_s^0 + \tilde{Z}_s\right)\cdot\cos\gamma_x a + \\ &+ j\cdot\left(\frac{\gamma_x\cdot\omega\cdot\mu_0}{Z_0\cdot\gamma_{\perp_n}^2} + \frac{Z_s\cdot Z_s^0}{Z_0}\cdot\frac{\gamma_{\perp_n}^2}{\gamma_x\cdot\omega\cdot\mu_0}\right)\cdot\sin\gamma_x a = 0, \end{aligned} \qquad (11)$$

where:

$$\begin{cases} \tilde{\gamma}_{\perp_n}^2 = \dfrac{\tilde{\gamma}_\perp^2}{k_0^2}, \\ \tilde{\gamma}_x = \dfrac{\gamma_x}{k_0}, \end{cases}$$

are quantities normalized to the wave number of free space $k_0 = \omega\cdot\sqrt{\varepsilon_0\cdot\mu_0}$.

Now, after standard formal transformations, equation (11) can be written as follows

$$\begin{aligned} &\tilde{\gamma}_{\perp_n}^2\cdot\tilde{\gamma}_x\cdot\left(\tilde{Z}_s^0 + \tilde{Z}_s\right)\cdot\cos\gamma_x a + \\ &+ j\cdot\left(\tilde{\gamma}_x^2 + \tilde{Z}_s\cdot\tilde{Z}_s^0\cdot\tilde{\gamma}_{\perp_n}^4\right)\cdot\sin\gamma_x a = 0. \end{aligned} \qquad (12)$$

The resulting expression (12) is a dispersion equation with respect to the transverse wavenumber γ_x.

5. 2. Solving the dispersion equation for bulk and surface waves

When the impedance has only an imaginary part, the dispersion equation (12) takes the form of a transcendental equation of the following form

$$\begin{aligned} &\tilde{\gamma}_{\perp n}^2\cdot\tilde{\gamma}_x\cdot\left(\tilde{X}_s^0 + \tilde{X}_s\right)\cdot\cos\gamma_x a + \\ &+ \left(\tilde{\gamma}_x^2 - \tilde{X}_s\cdot\tilde{X}_s^0\cdot\tilde{\gamma}_{\perp n}^4\right)\cdot\sin\gamma_x a = 0. \end{aligned} \qquad (13)$$

By solving this equation, the propagation constants of fast (bulk) waves in a waveguide with impedance walls were obtained.

In the case of slow (surface) waves, the transverse wave number γ_x is imaginary and, therefore, in (13) we can introduce the substitution $\gamma_x \to j\cdot\gamma_x$. Now

$$\gamma_{\perp_n}^2 = -\gamma_x^2 - \left(\frac{n\cdot\pi}{b}\right)^2,$$

and equation (13) takes the form of a dispersion equation that depends on the transverse wavenumber

$$\begin{aligned} &\tilde{\gamma}_{\perp_n}^2\cdot\tilde{\gamma}_x\cdot\left(\tilde{X}_s^0 + \tilde{X}_s\right)\cdot \mathrm{ch}\gamma_x a - \\ &- \left(\tilde{\gamma}_x^2 + \tilde{X}_s\cdot\tilde{X}_s^0\cdot\tilde{\gamma}_{\perp_n}^4\right)\cdot \mathrm{sh}\gamma_x a = 0. \end{aligned} \qquad (14)$$

For a waveguide with one narrow impedance wall, equations (13), (14) are simplified. Under the condition $\tilde{X}_s^0 = 0$, equation (14) takes the form

$$\mathrm{tg}\gamma_x a = -\tilde{X}_s \frac{\tilde{\gamma}_{\perp n}^2}{\tilde{\gamma}_x}. \qquad (15)$$

The conditions for the propagation of electromagnetic waves in waveguides of different cross-sections are the fulfillment of the inequality $\lambda < \lambda_{cr}$, where λ_{cr} is understood as the wavelength measured in free space, at which the propagation of a wave of this type in the waveguide stops.

5. 3. Classification of wave types and critical values of wavelengths

From equation (15) an analytical relation was obtained from which the critical wavelengths in a rectangular waveguide with two impedance walls can be determined. The critical wavelengths correspond to condition $\beta^2 = k_0^2 - \gamma_{\perp n}^2$ and, therefore, after substitution $k_0 \to k_{cr}$,

$$\mathrm{tg}k_{cr}a = -\frac{\tilde{X}_s + \tilde{X}_s^0}{1 - \tilde{X}_s \cdot \tilde{X}_s^0}, \qquad (16)$$

the solution to which is

$$\frac{a}{\lambda_{cr}} = \frac{1}{2\pi}\mathrm{arctg}\left(-\frac{\tilde{X}_s + \tilde{X}_s^0}{1 - \tilde{X}_s \cdot \tilde{X}_s^0}\right). \qquad (17)$$

For a single impedance wall, equation (16) simplifies to:

$$\mathrm{tg}\sqrt{k_{cr}^2 - \left(\frac{n\pi}{b}\right)^2} \cdot a = -\tilde{X}_s \frac{k_{cr}}{\sqrt{k_{cr}^2 - \left(\frac{n\pi}{b}\right)^2}},$$

and takes the following form for waves of type $TE_{m,0}$, when $\tilde{X}_s^L = +\tilde{X}_s$:

$$\mathrm{tg}k_{cr}a = -\tilde{X}_s^L. \qquad (18)$$

For inductive reactive impedance, we obtain a solution in the form

$$\frac{a}{\lambda_{cr}} = \frac{1}{2\pi}\mathrm{arctg}\left(\tilde{X}_s^L\right). \qquad (19)$$

For capacitive reactance $\left(\tilde{X}_s^C = -\tilde{X}_s\right)$, equation (16) takes the form

$$\mathrm{tg}k_{cr}a = +\tilde{X}_s^c.$$

Its solution is

$$\frac{a}{\lambda_{cr}} = \frac{1}{2\pi}\mathrm{arctg}\left(\tilde{X}_s^C\right). \qquad (20)$$

Fig. 2 illustrates the results of a graphical analysis of the critical wavelengths of a waveguide with one impedance wall. Here, the functions $F = \mathrm{tg}k_{cr}a$, $F = -\tilde{X}_s^L$, $F = +\tilde{X}_s^c$. are plotted:

As follows from Fig. 2, for ideal metal walls, when the surface impedance is zero, the plots intersect at point $a/\lambda_{cr} = 0.5$, which corresponds to the $\lambda_{cr} = 2a$ value – the critical wavelength of the H_{10} type. With an increase in the absolute value of the capacitive impedance (line 1), the value a/λ_{cr} also increases, which corresponds to a decrease in the critical wavelength λ_{cr}. In the case of the inductive impedance of the waveguide wall (line 2), with an increase in impedance, the a/λ_{cr} value decreases, which corresponds to an increase in the critical wavelength λ_{cr}.

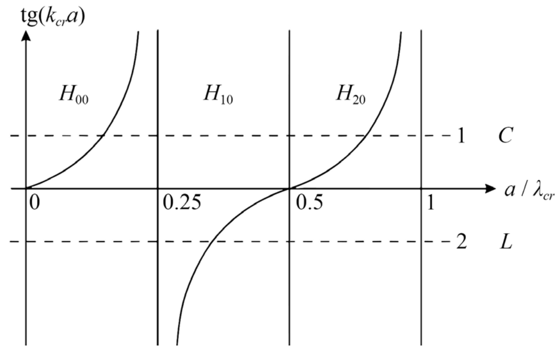

Fig. 2. Graphical analysis of critical wavelengths of a waveguide with one impedance wall

The conclusions regarding the critical wavelength are confirmed by numerical calculations of the normalized values of the critical wavelengths of the quasi-H_{10} and quasi-H_{20} types (for a wall with inductive and capacitive impedance) from the impedance of one wall of the waveguide (Fig. 3) according to formulae (17) and (20).

Fig. 2 shows that in the structure under consideration, with the capacitive impedance of the wall, the propagation of a wave of the H_{00} type is possible, which does not propagate in waveguides with ideal walls. Physically, this is due to the fact that in such a structure, the existence of a transverse component of the electric field E_y on the impedance wall is possible, which is not possible in a waveguide with ideal walls.

In the case of an inductive impedance of the wall $\tilde{X}_s^L$, the existence of a wave of the H_{00} type becomes impossible (Fig. 2, line 2). With increasing $\tilde{X}_s^L$ a/λ_{cr} decreases, which occurs with increasing critical wavelength λ_{cr}.

In the structure under consideration, with capacitive impedance, the propagation of both bulk and surface waves is possible.

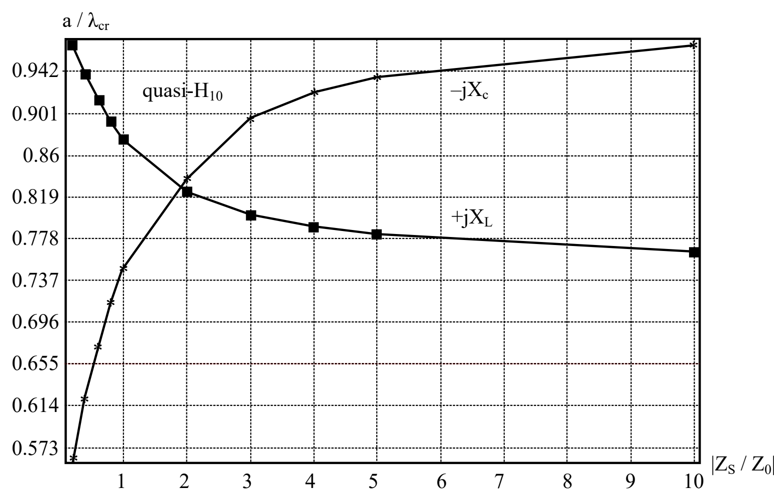

Fig. 3. Dependence of the ratio of value of the wide wall of a waveguide to the critical wavelength a/λ_{cr} on the value of the free space normalized to the wave resistance of the surface impedance modulus $|Z_s / Z_o|$ for the quasi-H_{10} wave for the case of capacitive impedance: (increasing curve) and inductive impedance (decreasing curve) for one impedance wall

Similar calculations were performed for a waveguide with two impedance walls. The results of these calculations using formula (17) are shown in Fig. 4.

Fig. 5 shows the results of the calculation for two impedance walls according to formula (17) and according to formula (19) for one impedance wall with an inductive impedance in both cases. The dependences of the ratio of the size of the wide waveguide wall to the critical wavelength a / λ_{cr} for the quasi-H_{20} wave on the value of the impedance modulus normalized to the free space wave resistance for different numbers of impedance walls are shown.

The results of investigating the dependence of the ratio of the size of the wide wall of the waveguide to the critical wavelength on the modulus of the normalized surface impedance lead to the following generalization. For the quasi-H_{10} wave, the curves have a decreasing character for the inductive impedance, and an increasing one for the capacitive one, and this tendency is preserved for both the model with one impedance wall and with two. In contrast, for the quasi-H_{20} wave, the curves decrease for the inductive surface impedance in the case of both one and two impedance walls.

5. 4. Dispersion characteristics of the waveguide's eigenwaves

In the considered structure, the propagation of both fast (bulk) and slow (surface) waves is possible. The existence of fast waves is determined by the condition

$$k_{cr} < k < k_{tr},$$

where k_{tr} is the wave number of transformation (conversion of fast waves into surface waves).

The k_{cr} value was determined from (18) provided that $\tilde{\gamma}_x = 0$.

Since $\tilde{\gamma}_{\perp n}^2 = \tilde{\gamma}_x^2 + (n\pi / k_0 b)^2$, at $n = 0$ and at capacitive impedance $\tilde{X}_s^C$

$$\frac{\mathrm{tg}\tilde{\gamma}_x a}{\tilde{\gamma}_x a} = \tilde{X}_s^c \frac{1}{k_{tr} a}. \qquad (19)$$

The limit of the relation in (19) is equal to

$$\lim_{\gamma_x \to 0} \frac{\mathrm{tg}\tilde{\gamma}_x a}{\tilde{\gamma}_x a} = 1.$$

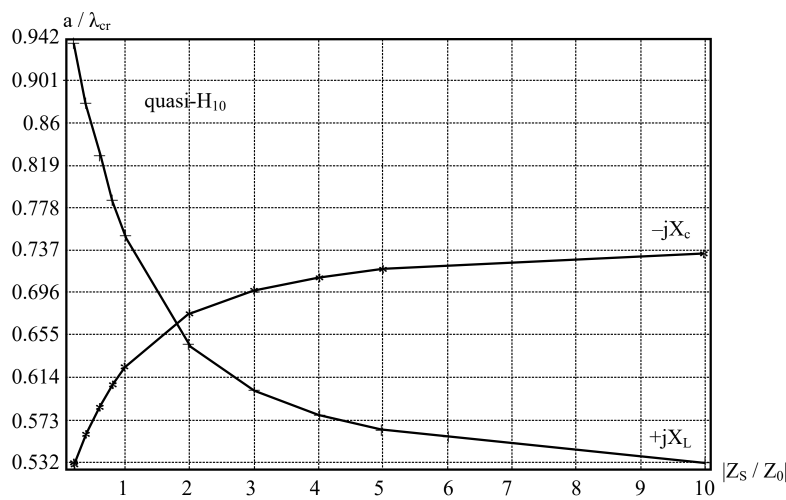

Fig. 4. Dependence of the ratio of value of the wide wall of the waveguide to the critical wavelength a / λ_{cr} on the value of the inductive impedance modulus normalized to the free space wave resistance (decreasing curve) and the capacitive impedance modulus normalized to the free space wave resistance (increasing curve) for the quasi-H_{10} wave in the case of two impedance walls

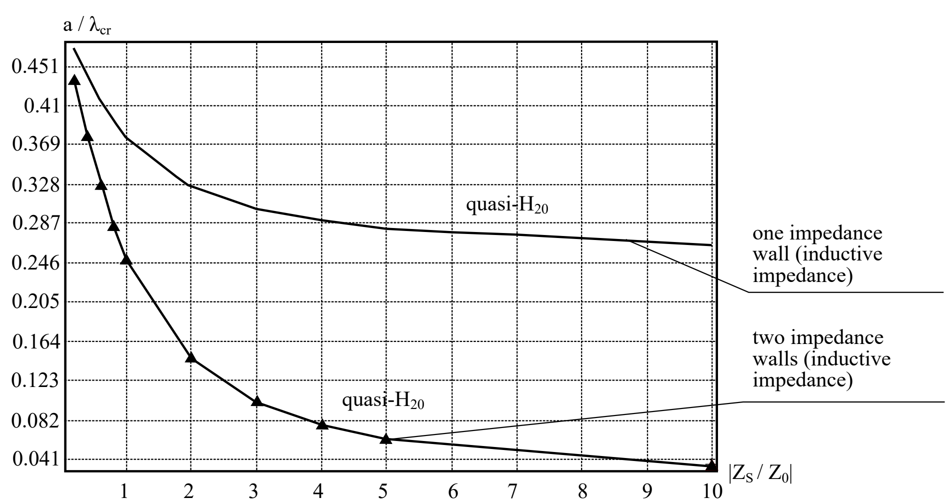

Fig. 5. Dependence of the ratio of value of the wide wall of the waveguide to the critical wavelength a / λ_{cr} for the quasi-H_{20} wave on the value of the impedance modulus normalized to the wave resistance of free space for one and two impedance walls

Therefore, an analytical expression was obtained for k_{tr}

$$k_{tr} = \frac{\tilde{X}_s^c}{a}. \qquad (20)$$

One of the most important characteristics of any transmission line is the dispersion characteristic, which shows the dependence of the phase velocity of the wave v_f or the normalized propagation constant $\tilde{\beta}$ on the frequency or wavelength. The dispersion characteristics have features related to the structure of the transmission line (its cross-section, the nature of the change in properties, both in the cross-section and longitudinal sections, as well as the type of filling of the transmission line with a certain material).

The calculation of the propagation constants in this work was implemented in the form of a program to numerically solve the dispersion equations (13), (14). The values of the propagation constants of bulk and surface waves were obtained depending on different values of the surface impedance of the walls and the nature of its reactance, as well as for different values of the frequency (wavelength) of the source. The wavelength range was chosen from the condition of single-mode propagation of the wave H_{10} in a waveguide with smooth metal walls.

Fig. 6 shows the dependence of the constant of propagation of the quasi-H_{10} wave on a / λ in a wide range of values of the inductive impedance $Z_s / Z_0 = +0.5j$; $+1.0j$; $+5.0j$; $+10.0j$.

The dependences of the propagation constant on the normalized to the wavelength size of the wide wall for different types of waves existing in the structure (Fig. 6, 7) show the presence of normal dispersion, the value of which depends on the wavelength and the impedance value, decreasing with increasing impedance value and decreasing wavelength.

In Fig. 7, similar dependences are plotted, but in the case of two walls with capacitive impedance for the same values of the normalized surface impedance. Unlike inductive impedance, with capacitive impedance of the walls, all dispersion curves are shifted to the region of shorter wavelengths, since in such structures the critical wavelengths are smaller. The shift to the region of shorter wavelengths depends on the impedance value, increasing with its increase, and can be quite large.

The dispersion equation was also solved for surface waves. Its analysis revealed that surface waves in the considered structure can propagate only if the narrow walls of the waveguide have a capacitive impedance (Fig. 8).

Fig. 8 shows the dependence of the normalized propagation constant $\tilde{\beta}$ on different $|Z_s / Z_o|$ values for $a / \lambda = 0.8$. It can be seen that for changes in $|Z_s / Z_o|$ from 0.1 to 10 in a rectangular waveguide with one impedance wall, two waves are disturbed: bulk (quasi-H_{10}) and combined (surface, which turns into bulk). Analyzing these plots, we can say that for the quasi-H_{10} wave, for changes in $|Z_s / Z_o|$ from 0 to 2, $\tilde{\beta}$ changes by 33%, and for changes in $|Z_s / Z_o|$ from 2 to 10 – by 32%. In this interval of $|Z_s / Z_o|$ values, $\tilde{\beta}$ changes weakly. The nature of change in the combined wave is as follows: in the $|Z_s / Z_o|$ interval from 0 to 2, a sharp $\tilde{\beta}$, change is visible, and then $\tilde{\beta}$ changes weakly, approaching the value of the propagation constant of a regular waveguide.

In the case of two impedance walls (Fig. 9), in this structure at $a / \lambda = 0.8$, the propagation of three types of waves is possible: bulk (quasi-H_{10}) and two surface waves.

When $|Z_s / Z_o| \to 0$, the quasi-H_{10} wave transforms into an H_{10} wave with ideal walls while the surface waves degenerate. Thus, for certain $|Z_s / Z_o|$ values, the bulk wave undergoes clipping. With increasing $|Z_s / Z_o|$, $\tilde{\beta}$ decreases, and for the first wave it tends to unity, and the second wave transforms from surface to bulk. The splitting of surface waves occurs under the condition $|Z_s / Z_o| = 1$.

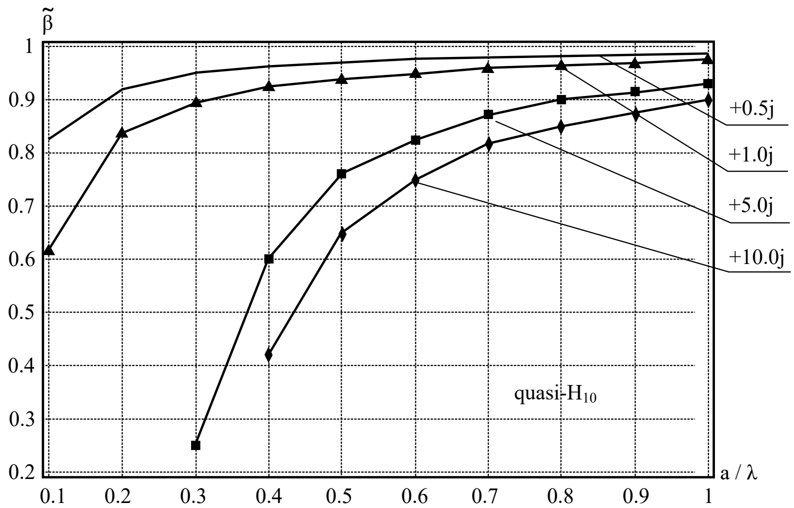

Fig. 6. Dependence of the propagation constant $\tilde{\beta}$ of the quasi-H_{10} wave on the ratio of the size of the wide waveguide wall to wavelength a / λ for different values of the inductive impedance normalized to the free space wave resistance in the case of a single impedance wall

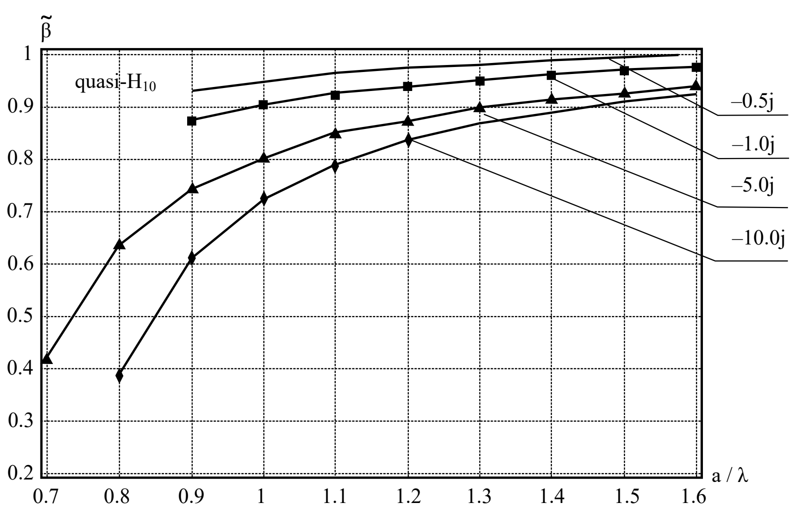

Fig. 7. Dependence of the propagation constant $\tilde{\beta}$ of the quasi-H_{10} wave on the ratio of the size of the wide wall of a waveguide to wavelength a / λ for different values of the capacitive impedance in the case of a single impedance wall

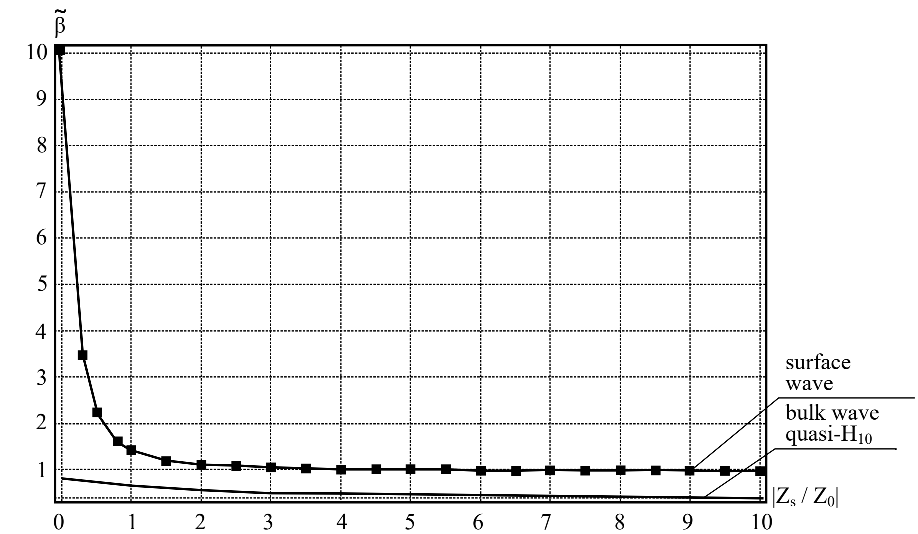

Fig. 8. Dependence of the propagation constant $\tilde{\beta}$ normalized to the free space wavenumber on the surface impedance modulus $|Z_s / Z_o|$ at the ratio of the size of the wide waveguide wall to wavelength $a / \lambda = 0.8$ for bulk and surface waves in the case of a single impedance wall

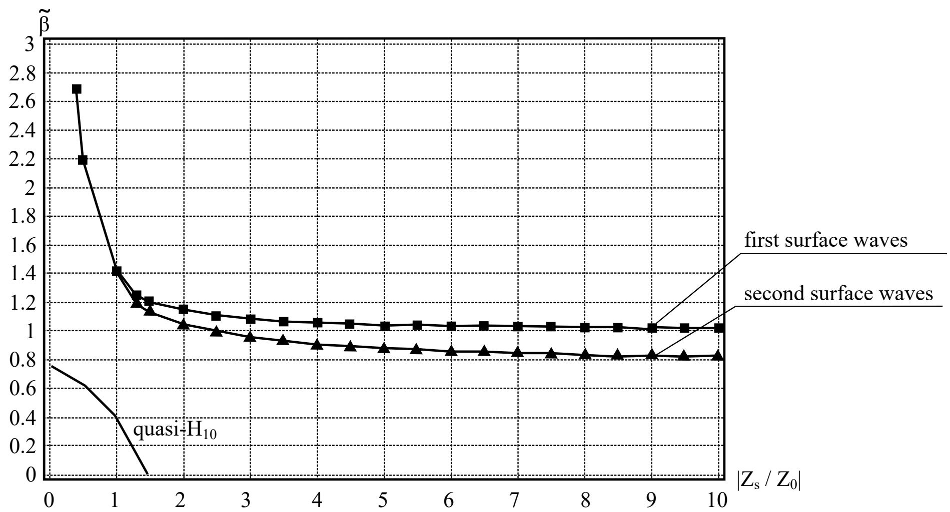

Fig. 9. Dependence of the propagation constant $\tilde{\beta}$ normalized to the free space wavenumber on the modulus of surface impedance normalized to the free space impedance $|Z_s / Z_0|$ at the ratio of the size of the wide waveguide wall to wavelength $a / \lambda = 0.8$ for surface waves with two impedance walls

6. Discussion of results based on investigating the dispersion characteristics of a rectangular metal waveguide with narrow impedance walls

The peculiarities of the approach to solving the problem of eigenwaves of the key structure consisted in combining the Fourier method with the use of Leontovych impedance boundary conditions. This allowed us to construct two dispersion equations for two types of waves: bulk (13) and surface (14).

To obtain the main result – an analytical solution to the dispersion equations – certain simplifications were introduced into the key structure, namely, first, one impedance wall was left, and then two. It was this simplification that made it possible to analytically find an expression for determining the critical wavelengths in such a structure (17), (19), (20) and to carry out a numerical analysis of the structure.

Analysis of the results of our computational experiment of ratios (17), (19), (20) was used to determine the types of waves that can propagate in the key model structure in the presence of one or two impedance walls. From Fig. 3, 4, it is possible to determine the transformation point – the transition from a bulk wave to a surface wave, and vice versa. In the process of research, it was found that in such a structure, unlike a metal waveguide, a H_{00} wave can propagate, which physically means the presence of a transverse component E_y in the structure of the electric field.

The presence of inductive walls in the structure leads to a shift of the critical frequency to the lower frequency region (Fig. 2, 6). This corresponds to the effect of the waveguide expansion without its physical expansion. The presence of walls with capacitive impedance leads to a shift of the frequency to the higher frequency region (Fig. 2, 7), which corresponds to the narrowing of the waveguide, but without an actual corresponding change in geometry. The magnitude of the shift depends on the impedance of the waveguide walls and practically does not change for large (more than 10) values of the inductive impedance modulus normalized to the wave resistance.

The dependences of propagation constants of bulk and surface waves on different values of the surface impedance of both inductive (Fig. 7) and capacitive (Fig. 8) nature were obtained. In a wide range of changes in the normalized wavelength a / λ (from 0.1 to 1.6), the presence of normal dispersion for the quasi-H_{10} wave was revealed.

This frequency behavior of the eigenwaves of the key structure can be explained by the presence of dispersion in the characteristics of impedance materials. At the same time, it becomes possible to control the frequency characteristics of the structure without changing its geometry.

For the model with two impedance walls, more complex physical processes of transformation of wave types into one another are inherent. Therefore, in this case, both normal and anomalous dispersions are observed. As can be seen from Fig. 8, when changing the $|Z_s / Z_o|$ values from 0.1 to 10, two waves are excited in a rectangular waveguide with one impedance wall: bulk (quasi-H_{10}) and combined (surface, which passes into bulk). In the case of two impedance walls, a whole spectrum of both bulk and surface waves is excited. For example, quasi-H_{10} is cut off at the value of the normalized impedance $|Z_s / Z_0| = 1.5$, but with an increase in $|Z_s / Z_o|$, the phase velocity of surface waves normalized to the wave number of free space decreases, and for the first wave it tends to unity, and for the second wave it passes from surface to bulk. In this case, the splitting of surface waves occurs under the condition $|Z_s / Z_o| = 1$ (Fig. 9).

As a result of the numerical solution of the transcendental equations, the values of the propagation constants of bulk and surface waves were obtained depending on different values of the surface impedances in a wide range of changes in the normalized width of the waveguide to wavelength a / λ. A strong anomalous dispersion of the bulk wave is observed in the second harmonic region. Here, the value of the propagation constant decreases with increasing frequency.

Our principal results do not contradict the physical principles and laws of the interaction of electromagnetic waves and a rectangular waveguide with narrow impedance walls. In particular, the key structure in the limiting case of metal side walls gives a result that is identical to a hollow metal waveguide [2].

The proposed model could be applied to the analysis of complex aperiodic structures such as directional couplers, filters, as well as phased array antenna feed networks. The analytical approach allows for accurate calculation of coupling coefficients for power transfer between waveguides by mode conversion (e.g., TE_{10} to TM_{mn}). It also facilitates the analysis of aperture-related losses in high-frequency radar and communication systems. The statement is valid over the entire operating wavelength range (decimeter, millimeter, and submillimeter) of waveguide structures. The key structure and its mathematical model could be used to analyze the field structure, diffraction coefficients, and to estimate the magnitude of dissipative losses in the multimode regime. These tasks will be the subject of future research.

A limitation of our study is the consideration of the detection of regularities in the dependence of the $\tilde{\beta}$ delay coefficients only on ratio $a / \lambda = 0.8$, which corresponds to the first harmonic, and in the range of surface impedance values $|Z_s|$ from 0 to 10. At higher values of surface impedance in such structures, the disappearance of dispersion for the main wave type is observed.

The disadvantages of the study include the fact that not all inhomogeneities can be modeled by a homogeneous impedance structure, but only those that are small in size relative to the wavelength.

Further development of our study involves obtaining dispersion characteristics for waves that are excited in the key model structure (Fig. 1) in a wider wave range, namely $a / \lambda = 1.6$; 2.4; 3.2, which corresponds to the second, third, and fourth harmonics of the fundamental wave H_{10}. Of certain practical interest are also studies on the field structure of the excited wave types and determination of losses taking into account the complex nature of the surface impedance in the presence of one and two impedance walls. The latter research will be useful, in particular, to developers of absorption filters, namely, harmonic filters of microwave transmitters.

7. Conclusions

1. We have solved a boundary value problem analytically by reducing it to dispersion equations. The results of investigating the influence of different types of impedance on the dispersion characteristics of a rectangular waveguide with narrow impedance walls in the range of changes in the relative frequency and modulus of normalized impedance are presented. This was achieved by combining the Fourier method with Leontovych boundary conditions. The issue of extreme complexity and cumbersomeness inherent in analytical methods was overcome by reducing the boundary value problem to transcendental dispersion equations.

2. Two types of perturbed waves in a rectangular waveguide with narrow impedance walls have been determined: bulk and surface. Dispersion equations for these waves were built. Factors that affect the parameters of such structures were determined, in particular, the magnitude and nature of the surface impedance and the operating wavelength. Dispersion equations were derived for two types of key structures – with one and two impedance walls.

3. The classification of wave types and analytical determination of critical wavelength values for a model with one and two narrow walls has been carried out. It is shown that bulk (fast) and surface (slow) waves can exist in such a structure. The presence of wave types propagating in it is indicated. The problem related to high requirements for computational resources to obtain numerical results of the required level of accuracy does not even arise in this case because the critical wavelengths are determined precisely analytically. It is established that for the fundamental wave of the quasi-H_{10} structure, with an increase in the normalized impedance modulus at small impedance values, the deceleration coefficient initially decreases quite quickly. However, with a further increase in impedance, the rate of decrease of the deceleration coefficient decreases sharply due to the approximation of the quasi-H_{10} wave propagation conditions to the corresponding conditions in a regular waveguide.

4. When determining the dependence of the propagation constants of fast and slow waves on the surface impedance, the fact of the presence of a strong anomalous dispersion of the fast wave for the second harmonic has been established. In this case, the value of the propagation constant decreases with increasing frequency by 33–35%. When the surface impedance tends to zero, the quasi-H_{10} wave turns into the H_{10} wave with ideally conducting side walls, and the surface waves degenerate. Thus, when the normalized modulus of the surface impedance is equal to 1.5, the bulk wave undergoes clipping. With a further increase in the surface impedance, the first wave turns into a wave of an empty waveguide, and the second wave is transformed from surface to bulk. The splitting of surface waves occurs at the point where the normalized value of the modulus of the surface impedance is equal to unity.

Conflicts of interest

The authors declare that they have no conflicts of interest in relation to the current study, including financial, personal, authorship, or any other, that could affect the study and the results reported in this paper.

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Data availability

All data are available in the main text of the manuscript.

Use of artificial intelligence

The authors confirm that they did not use artificial intelligence technologies when creating the current work.

Authors' contributions

Ludmila Logachova: Conceptualization, Methodology, Validation, Formal analysis, Visualization, Writing – original draft; **Tetiana Bugrova**: Conceptualization, Methodology, Validation, Formal analysis, Visualization, Writing – original draft; **Mikhail Chornoborodov**: Validation, Supervision, Writing – original draft; **Sergii Morshchavka**: Validation, Formal analysis, Visualization; **Natalia Chornoborodova**: Formal analysis, Visualization, Writing – review & editing.

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