

This study investigates artillery fire control processes under conditions of uncertain disturbances when using the "shoot-and-scoot" tactic. The task addressed is to increase the efficiency of artillery fire and reduce the time for performing fire tasks by optimally choosing the number of shots in a series.

To assess the efficiency of firing, the time for performing tasks, and the risk of hitting an artillery installation for different values of the length of the firing series, probabilistic-analytical modeling methods have been used. To select the best solution according to a set of contradictory criteria, a fuzzy logic apparatus was applied, which makes it possible to formalize the decision-making process under conditions of uncertainty.

A method for determining the rational length of firing series has been devised, which involves the formation of a generalized criterion, the search for a set of local maxima, and their further evaluation using a fuzzy model. The proposed approach makes it possible to take into account the influence of random disturbances, in particular, a possible sudden increase in barrel wear during firing series.

The results of computational experiments have confirmed the effectiveness of the proposed method. It was found that the firing efficiency increased by 11.6–20.8% compared to the fixed-length firing series approach, as well as the reduction in the time to complete firing tasks by up to 31.6%. In addition, in most cases, a decrease in the total time spent by installations at firing positions was observed, which indicates a decrease in the risk of damage by counter-battery means.

The devised method could be applied in control systems for modern artillery installations to increase the effectiveness of their combat use under conditions of uncertainty

Keywords: *artillery firing, firing series, probabilistic-analytical modeling, fuzzy logic, multi-criteria evaluation*

DEVISING A METHOD FOR DETERMINING THE LENGTH OF A SERIES OF ARTILLERY FIRE BASED ON PROBABILITY-ANALYTICAL MODELING AND FUZZY MULTI-CRITERION EVALUATION

Maksym Maksymov

Doctor of Technical Sciences, Professor*

ORCID: <https://orcid.org/0000-0002-7536-2570>

Oleksiy Kozlov

Corresponding author

Doctor of Technical Sciences, Professor

Department of Intelligent Information Systems

Petro Mohyla Black Sea National University

68 Desantnykiv str., 10, Mykolaiv, Ukraine, 54003

E-mail: kozlov_ov@ukr.net

ORCID: <https://orcid.org/0000-0003-2069-5578>

Oleksiy Maksymov

Doctor of Philosophy (PhD), Associate Professor

Department of Radio Engineering Armament,

Communications and Robotics

Institute of Naval Forces of the National

University "Odesa Maritime Academy"

Didrikhsona str., 8, Odesa, Ukraine, 65029

ORCID: <https://orcid.org/0000-0003-2504-0853>

Ruslan Riaboshapka*

ORCID: <https://orcid.org/0009-0004-2068-0290>

*Department of Computer Technologies of Automation

Odesa Polytechnic National University

Shevchenko ave., 1, Odesa, Ukraine, 65044

Received 02.03.2026

Received in revised form 08.05.2026

Accepted 18.05.2026

Published 30.06.2026

How to Cite: Maksymov, M., Kozlov, O., Maksymov, O., Riaboshapka, R. (2026). Devising a method for determining the length of a series of artillery fire based on probability-analytical modeling and fuzzy multi-criterion evaluation.

Eastern-European Journal of Enterprise Technologies, 3 (3 (141)), 97–110.

<https://doi.org/10.15587/1729-4061.2026.361486>

1. Introduction

Armed conflicts of the last decade are characterized by a rapid growth in the role of advanced technologies in the conduct of military operations. First, this is manifested in the widespread use of unmanned aerial vehicles of various classes, high-precision weapons, as well as intelligent military control systems operating in real time [1]. The use of such technologies significantly changes the nature of modern combat, increasing its dynamics, accuracy, and ability to adapt to rapidly changing conditions. At the same time, despite the active

evolution of unmanned and missile weapons, artillery, both field and long-range, retains its important role as an effective means of fire [2].

Modern approaches to the use of artillery systems have undergone significant changes, moving away from the concept of area fire to the implementation of precise fire tasks on individual targets [3]. Since such targets can be located both in the tactical and operational depth of the enemy's defense, the requirements for accuracy of fire are significantly increased. Also, the importance of making operational decisions and increasing the efficiency of fire correction is growing [4].

In addition, the use of artillery systems and complexes under conditions of full-scale high-intensity combat operations requires additional consideration of a number of specific factors and limitations. These include constantly acting disturbing influences of a random or uncertain nature, including gradual and abrupt deterioration of the technical condition of guns, as well as the shortage and unstable quality of ammunition. Another very important factor is the significant risk of detection and subsequent defeat of artillery installations by enemy counter-battery fire, which imposes additional restrictions on the time spent at firing positions. In this regard, there is a particularly acute need for new approaches to artillery control, which should include procedures for optimizing parameters and firing modes taking into account random and uncertain disturbances. These approaches should be aimed at increasing fire efficiency, reducing the time spent in positions and the total time of completing tasks, as well as minimizing the risks of damage and destruction of artillery installations.

Therefore, studies aiming at devising methods for controlling artillery systems based on optimizing parameters and firing modes under the influence of random disturbances and the limitations of modern high-intensity combat operations are relevant.

2. Literature review and problem statement

Increasing the efficiency of artillery systems under current conditions is achieved through comprehensive improvement of both technical and algorithmic solutions [5]. One of the important directions is the use of intelligent optimization and control methods that make it possible to improve the characteristics of various artillery systems and their components. In particular, hybrid approaches have been proposed that combine neural networks with radial basis functions and genetic algorithms to determine rational design parameters of artillery installations [6]. Also, multi-agent heuristic methods have been devised, focused on optimizing the parameters of guidance mechanism control systems, which ensures increased accuracy and speed of the corresponding processes [7]. However, studies [6, 7] do not take into account the effects of random and uncertain disturbances, which significantly affect artillery systems when used under real firing conditions.

The existence of these limitations necessitates the improvement of the mathematical description of the firing processes by taking into account various types of disturbing influences and external factors that affect the firing result. In particular, simulation and analytical models have been built that take into account variable weather conditions [8], as well as errors associated with the discreteness of aiming angles and the influence of the Coriolis force on the projectile flight trajectory [9]. Also, the use of more detailed motion models (in particular, with five degrees of freedom) provides an additional increase in the accuracy of trajectory prediction [10]. At the same time, a significant drawback of those models is the lack of consideration of the possibility of deviation in the parameters of the technical condition of the guns from nominal ones. In addition, they are focused on simplified model scenarios that do not fully reflect the features of the use of artillery under combat conditions, associated with counter-battery combat and limitations on the time of execution of fire tasks.

To take into account the features of real combat, significant attention is paid to the development of tactical approaches to the organization and conduct of fire, which make it possible

to increase the capabilities of artillery systems under current conditions [11]. Within the framework of this direction, methods of combining artillery with unmanned systems and radar networks are considered. In particular, new solutions for conducting counter-battery combat based on the use of unmanned aerial vehicles [12], as well as approaches involving the use of distributed radar surveillance systems [13], have been proposed.

However, the approaches reported in [11–13], as well as the vast majority of other advancements discussed above, have significant limitations when applied under conditions of large-scale and protracted high-intensity war. These conditions are characterized by a significant load on the existing artillery systems, the lack of serviceable guns, and a large number of fire missions that require operational execution [14]. Additional complicating factors are limited ammunition supplies and their unstable quality, as well as active enemy resistance in the form of counter-battery fire and advanced aerial reconnaissance.

The combined effect of the above factors leads to a significant decrease in the effectiveness of artillery units, and in some cases to the impossibility of completing the assigned tasks [15]. First, the limited number of guns suitable for use, together with the high intensity of combat operations, determines the need to fire a large number of shells in a short time. This, in turn, leads to rapid wear of the barrels, which is a significant disturbance in the firing process. An increase in its degree leads to a decrease in the initial velocity of the shell and a deterioration in ballistic characteristics, which negatively affects the probability of damage even under nominal all other firing conditions [16].

In parallel, the shortage of ammunition leads to increased requirements for its rational use, especially the number of shots required to hit one target. At the same time, the unstable quality of ammunition introduces new random disturbances that affect the result of individual shots. For example, the use of ammunition with characteristic deviations can lead to a significant decrease in the effectiveness of a particular shot. In addition, the presence of advanced reconnaissance and counter-battery effects of the enemy limits the permissible time for an artillery installation to remain in a firing position. This increases the risk of its destruction even after the first shots and also increases the requirement for the speed of execution of the fire task.

Thus, under the conditions of the above-mentioned factors and disturbances, the most appropriate is to use the "shoot-and-scoot" tactic [17]. It involves firing in small series with subsequent operational change of firing positions. The artillery installation fires a limited number of shots, after which it immediately leaves the position and moves to another one [18]. An important advantage of this tactic is that all the necessary calculations, settings, and corrections of the firing parameters for the next series are carried out during the movement. This makes it possible to minimize the time the installation is in the firing position, which is extremely important from the point of view of its survival under conditions of active counter-battery resistance. In addition, to take into account the effect of barrel wear, during the last shot of each series it is advisable to assess its current state. The data obtained can be used to calculate the appropriate corrections during the movement to the next position. This would ensure proper accuracy of fire in the next series without additional delays. In [19], a method was proposed that provides a fairly accurate determination of the degree of barrel wear, which makes it possible to effectively introduce adjustments to the firing parameters. However, in that study, the wear of the gun barrel was considered in isolation without taking into account

different firing modes and conditions of combat use of the artillery system.

Typically, when using the "shoot-and-scoot" tactic, a fixed length of a series of shots is used, which is previously set as a parameter of the fire task or a resource limit, as given in [20, 21]. However, this approach has significant limitations, since with different initial data of the fire task (the required number of hits, rate of fire, etc.), a fixed length of the series leads to instability of the quality indicators of the obtained results. In work [18], several heuristic optimization algorithms were developed that make it possible to reduce the time of moving an artillery installation between positions by compiling optimal routes. However, due to the use of a fixed length of the series at each position, which is a given unchanging parameter of the optimization task, the overall efficiency of the fire task will be reduced, regardless of the chosen route. On the one hand, the use of too short series, despite the optimization of the route, requires significant time losses due to frequent changes of positions, which increases the total time of the fire task. On the other hand, increasing the length of the series leads to an increase in the duration of the installation's stay in one position, which significantly increases the risk of its detection and destruction by enemy counter-battery fire. Therefore, to overcome the considered limitations when applying the "shoot-and-scoot" tactic, it is necessary to solve an important and rather complex problem, which is to determine the most rational length of the series of shots.

In [17], an approach to determining the optimal duration of the installation's stay in position is proposed, which partially makes it possible to solve the problem of choosing the length of a series of shots. However, this approach does not consider the presence of internal disturbing influences associated with changes in the technical condition of the artillery system. In particular, during the execution of a series of shots, a sudden increase in barrel wear is possible (for example, due to a rupture of the shell belt and damage to the rifling). In such a situation, all subsequent shots of the current series are executed with reduced characteristics until its completion, after which the barrel condition will be assessed and adjustments made. Thus, the use of too long series leads to a decrease in the overall efficiency of fire due to an increase in the number of ineffective shots after the appearance of the specified disturbances, which is not taken into account in the approach reported in [17].

Therefore, there is a need to devise an improved method for determining the length of a series of shots, which could provide a rational compromise between three criteria: fire efficiency, duration of the fire task, and the risk of damage from enemy counter-battery fire. In this case, it is advisable to evaluate the specified criteria for different firing scenarios and initial conditions based on the probabilistic-analytical modeling approach, which was used in the development of artillery control methods in [22]. Since in this case it is assumed to determine the rational value of the length of a series of shots based on several conflicting criteria, the task acquires an obvious multi-criteria nature. In the optimization process, several alternatives can be obtained that provide close values of target indicators, but differ in the ratio of fire efficiency, task execution time, and risk level. In such a case, the use of artillery system control methods proposed in [22] will not make it possible to find an adequate compromise solution. Therefore, there is a need to apply mathematical tools that allow for flexible and informed decision-making under conditions of uncertainty and incomplete information.

Thus, in this case, it is advisable to use fuzzy logic in the process of choosing optimal solutions [23]. The mathematical apparatus of fuzzy logic allows for fairly effective processing of heterogeneous data that may be specified inaccurately or contain noise [24]. The use of linguistic variables and membership functions makes it possible to formalize qualitative assessments and approximate complex nonlinear dependences between the parameters of the object [25]. Representing existing knowledge in the form of a set of logically transparent rules contributes to better interpretability of the decision-making process [26]. In addition, the construction of a fuzzy-logical system based on expert knowledge makes it possible to take into account practical experience in the use of artillery installations and complexes, which ensures the validity of the selected solutions [27].

Our review of the literature showed that existing models and methods for controlling artillery systems have a number of significant limitations when used under conditions of large-scale and protracted high-intensity war. Namely, the methods in [5–7] do not take into account random and uncertain disturbing influences that are present in real firing conditions. The models constructed in [8–10] make it possible to take into account only individual external influences and do not reflect internal disturbances associated with random changes in the technical condition of guns and the quality of ammunition. The approaches reported in [11–13] consider the features of real combat use of artillery, but internal disturbing influences characteristic of prolonged high-intensity combat operations are also not taken into account. In study [19], internal disturbances in the form of gun barrel wear are evaluated, but this is considered in isolation without taking into account different firing modes and conditions of combat use of the artillery installation. Also, the methods reported in [22] take into account different types of external and internal disturbing influences, but do not provide a simultaneous increase in the contradictory indicators of the effectiveness of the artillery installation. In addition, existing methods of artillery control when using the "shoot-and-scoot" tactic are based on a fixed series length [18, 20, 21] or on its change without taking into account internal disturbing influences [17]. Thus, in total, the above approaches do not make it possible to adequately take into account the influence of random disturbances and do not provide a coordinated increase in the contradictory indicators of the efficiency of the artillery installation when performing a fire task.

Therefore, a scientific problem arises consisting in the lack of a method for determining the rational length of the firing series under the conditions of the action of random disturbances when using the "shoot-and-scoot" tactic. Such a method should provide a simultaneous increase in the efficiency of artillery firing and a reduction in the time of performing fire tasks by combining probabilistic-analytical modeling of uncertain disturbances and fuzzy multi-criteria evaluation of alternative solutions.

3. The aim and objectives of the study

The aim of our research is to devise a method for determining the rational length of a series of shots under conditions of random disturbances when using the "shoot-and-scoot" fire tactics based on a combination of probabilistic-analytical modeling and fuzzy multi-criteria evaluation. This will allow for a well-founded choice of the length of the series of shots, taking into account the conflicting requirements for firing

efficiency, the time of execution of the fire task, and the risk of hitting the artillery installation by enemy counter-battery fire.

To achieve the set goal, it is necessary to solve the following tasks:

- to propose a formalization procedure and the basic steps of the method for determining the rational length of a series of artillery shots under conditions of random disturbances;
- to build a fuzzy model for evaluating and selecting the best alternative for the length of the series based on a set of conflicting criteria;
- to conduct a series of computational experiments to study the effectiveness of the proposed method for determining the rational length of a series of shots.

4. The study materials and methods

The object of our study is the processes of fire control of artillery installations under conditions of uncertain disturbances when using the "shoot-and-scoot" tactic.

The principal hypothesis assumes that in the presence of limiting factors there is a certain set of permissible values of the length of the firing series, within which the most rational value is located. When the length of the series increases, the limiting factors are a decrease in the effectiveness of firing as a result of the action of disturbing influences and an increase in the risk of hitting or destroying the artillery installation by enemy counter-battery fire. When the length of the firing series is reduced, the limitation is an increase in the total time for completing the fire task due to the addition of unnecessary movements between positions and procedures for deploying, aiming, and folding at positions. It is also assumed that the indicators of firing effectiveness, task completion time, and the risk of defeat can be combined into one universal criterion, which will have local extrema (maxima), depending on the value of the series length. In turn, one of the given local maxima will correspond to the best compromise solution, which will provide a significant increase in the efficiency of artillery fire and a reduction in the time of execution of fire tasks at an acceptable risk. In addition, probabilistic-analytical modeling will ensure proper consideration of the influence of such random disturbances as abrupt wear of the gun barrel, the use of poor-quality charges and the possibility of damage to the installation. In turn, the fuzzy-logical model will provide an adequate multi-criteria assessment of existing alternative options for choosing the most rational series length.

The study adopts the following system of assumptions. The artillery firing is considered with a single installation using the "shoot-and-scoot" tactic, which is a characteristic feature of the modern use of artillery systems in high-intensity combat. This approach involves performing fire missions in short series of shots with subsequent operational change of firing positions. At the same time, all firing positions are predetermined and fully prepared for conducting aimed fire. Such an organization of fire makes it possible to significantly reduce the probability of detection of an artillery installation by the enemy and increase its survivability in the counter-battery counteraction zone [17]. When using this tactic, a key advantage is the transfer of computational procedures related to the preparation of the next series of shots outside the firing position. In particular, the calculation of corrective settings for aiming before the next series is performed when moving the installation to a new position. This makes it possible to minimize the time spent at the firing position and, accordingly,

reduce the risk of being hit by enemy counter-battery fire. To ensure high accuracy of firing, the above calculations are carried out on the basis of adequate ballistic models, taking into account the topographic reference of the new position, current meteorological data, and other available parameters.

An additional element of ensuring accuracy of firing is taking into account the technical condition of the artillery system, in particular the degree of barrel wear. For this purpose, the current wear value is assessed on the last shot of each series. The information obtained is used to refine the calculations of aiming parameters during movement before the next series. In particular, corrections are introduced that compensate for the decrease in the initial velocity of the projectile under other nominal firing conditions. This ensures increased accuracy of target engagement without increasing the time the installation spends at the firing position.

During firing, the installation may be subject to disturbing influences in the form of wear of the gun barrel, hit of a low-quality charge and damage to the installation by enemy counter-battery fire. Under nominal operating conditions, barrel wear has the character of a controlled disturbance, which accumulates gradually and relatively slowly [28]. Within one series of fire, such a process, as a rule, does not cause significant changes in the initial velocity of the projectiles and, accordingly, does not have a significant impact on the effectiveness of the fire. At the same time, during firing, random jump-like disturbances associated with a sharp increase in barrel wear may appear [29]. Such a situation may arise, in particular, as a result of a rupture of the projectile belt and local damage to the rifling. In the event of such a disturbance occurring on a certain shot within the series, this shot and all subsequent shots will most likely have deteriorated ballistic parameters and low effectiveness. At the same time, detection and consideration of such a sharp increase in wear is possible only after the end of the current series, which limits the possibilities of operational correction of fire parameters. If a poor-quality charge appears, only the current shot will have low efficiency, and this disturbance will not affect the subsequent shots of the series. In turn, the probability of a disturbance in the form of damage to the installation by enemy counter-battery fire exponentially depends on the time the installation is in the firing position after the first shot of the series.

A shot is considered effective if the projectile hits within the permissible dispersion circle around the target, which is taken as 1% of the firing range.

In this study, methods of probabilistic-analytical modeling were used to determine the main criteria for the effectiveness of artillery fire under conditions of random disturbances. In particular, the appropriate approach was used to formalize the influence of stochastic factors on the effectiveness of fire, the duration of the fire task, and the risk of damage to the artillery installation while it was in position. The obtained criteria estimates were used to form a set of rational alternatives regarding the length of the series of shots. Further selection of the optimal solution is carried out using fuzzy logic methods, which allow for multi-criteria assessment under conditions of uncertainty and inconsistency of input data. The use of the fuzzy logic apparatus provides the possibility of integrating quantitative and qualitative indicators, as well as forming a generalized solution based on a system of linguistic rules.

When conducting computational experiments, a personal computer with an Intel Core i7 3.6 GHz processor, 16 GB of RAM, and a 512 GB SSD drive was used as hardware. MATLAB R2026a (trial version) with standard computational

tools and Fuzzy Logic Toolbox was employed as software. The simulation was carried out for a generalized self-propelled artillery installation, the parameters of the rate of fire, movement speed, deployment and retraction of which correspond to the characteristics of modern 155 mm class systems (in particular CAESAR, M109, Archer), which are given in open sources. The initial data for the simulation were given as the values of the probabilities of the emergence of disturbances in the form of a projectile belt failure and the use of a low-quality charge in one arbitrary shot of the series. Also, the desired number of hits to destroy the target, the rate of fire, the probability of hitting in the absence and presence of appropriate disturbances, the value of the time of movement between positions, deployment, guidance, and closing were set. To calculate the probability of defeat of the installation, the values of the intensity of the threat increase and the critical time of the installation's stay in position after the first shot were given.

In addition, when performing calculations, the same values of the time of movement of the installation between all available firing positions and the same rate of fire for all firing series were accepted as simplifications.

5. Results of investigating the method for determining the rational length of a series of shots under conditions of random disturbances

5.1. The formalization procedure and the main steps of the method for determining the rational length of a series of artillery shots

Determining the effectiveness of firing a series of shots when using the "shoot-and-scoot" tactic is reduced to the following. Given the probability of a projectile belt failure in one random shot p_{sa0} , the probability of such a stepwise disturbance occurring at least once in one series of shots p_{sa1} is defined as

$$p_{sa1} = 1 - (1 - p_{sa0})^n, \quad (1)$$

where n is the number of shots (length) of the current series.

Moreover, if such an event occurred, then it could have happened on any k -th shot of the series ($k = 1, \dots, n$) with the same probability $1/n$. In this case, the mathematical expectation of the number of successful shots (hits) E_{Ca1} when a given stepwise disturbance appears at least once in the firing series is determined from the following dependence (2)

$$E_{Ca1} = \frac{1}{n} \sum_{k=1}^n [(k-1)p_{wa1} + (n-k+1)p_{a1}], \quad (2)$$

where p_{wa1} is the probability of hitting for shots before the projectile belt rupture occurs; p_{a1} is the probability of hitting for the shot in which the projectile belt rupture occurred and for all subsequent shots of the current series.

In this case, the probability of hitting p_{a1} is quite low and is set, for example, in the range of 0.05...0.2. In turn, the calculation of the probability of hitting p_{wa1} (before the projectile belt rupture occurs) is carried out as follows. In the absence of a projectile belt rupture, one of two mutually exclusive states may occur: firing a shot without disturbances and firing a shot with another type of random disturbance – the use of a poor-quality charge. In the first case, the probability of hitting p_{wa} is quite high, for example, in the range of 0.8...0.95. In the second case, the initial velocity of the projectile is significantly reduced, which with a high probability corresponds

to the low efficiency of the current shot. Therefore, the probability of hitting p_{a2} of such a shot has approximately the same low value as p_{a1} . The probabilities of the occurrence of the two states are given as p_{swa} and p_{sa2} ($p_{swa} + p_{sa2} = 1$), respectively. Therefore, the probability of hitting p_{wa1} is given on the basis of the total probability formula as the sum of the products of the probabilities of the occurrence of the considered states by the corresponding probabilities of hitting

$$p_{wa1} = p_{swa}p_{wa} + p_{sa2}p_{a2}. \quad (3)$$

For the convenience of calculating the mathematical expectation E_{Ca1} , formula (2) is simplified using the following simple transformations. For this purpose, the sum in expression (2) is represented as two separate sums

$$E_{Ca1} = \frac{p_{wa1}}{n} \sum_{k=1}^n (k-1) + \frac{p_{a1}}{n} \sum_{k=1}^n (n-k+1). \quad (4)$$

Next, using the formula for the sum of an arithmetic progression and simple transformations, we shall represent the individual sums from expression (4) as follows:

$$\sum_{k=1}^n (k-1) = \frac{n(n-1)}{2}; \quad (5)$$

$$\sum_{k=1}^n (n-k+1) = \frac{n(n+1)}{2}. \quad (6)$$

After that, expressions (5) and (6) are substituted into equation (4) and a simplified expression for calculating the mathematical expectation E_{Ca1} is obtained

$$E_{Ca1} = \frac{n-1}{2} p_{wa1} + \frac{n+1}{2} p_{a1}. \quad (7)$$

If the shell belt rupture did not occur in any shot of the firing series, then the mathematical expectation E_{Cwa1} of such a series is determined by the product np_{wa1} . The probability of such an event, respectively, is $1 - p_{sa1}$. Therefore, based on the above equations and statements, an expression has been constructed to calculate the general mathematical expectation $E_C(n)$ of the firing series under the action of the two considered random disturbances. This expression is represented as a function of $E_C(n)$ on the number of shots n in the series

$$E_C(n) = (1 - p_{sa1})np_{wa1} + p_{sa1} \left(\frac{n-1}{2} p_{wa1} + \frac{n+1}{2} p_{a1} \right). \quad (8)$$

After dividing expression (8) by the number of shots n in the series, a formula was obtained for calculating the firing efficiency $\eta_s(n)$ in the current series as a function of the number of shots n

$$\eta_s(n) = \frac{E_C(n)}{n} = (1 - p_{sa1})p_{wa1} + p_{sa1} \left(\frac{n-1}{2n} p_{wa1} + \frac{n+1}{2n} p_{a1} \right). \quad (9)$$

After certain simple transformations, dependence (9) is represented in a simplified form

$$\eta_s(n) = (1 - p_{sa1})p_{wa1} + p_{sa1} \left(\frac{p_{wa1} + p_{a1}}{2} + \frac{p_{a1} - p_{wa1}}{2n} \right). \quad (10)$$

Fig. 1 shows dependences of the firing efficiency of a series η_s on its length (number of shots n), which are constructed using expression (10) at different values of the probability p_{sa0} . In this case, the values of the hit probabilities for all curves were $p_{wa1} = 0.82$, $p_{a1} = 0.1$.

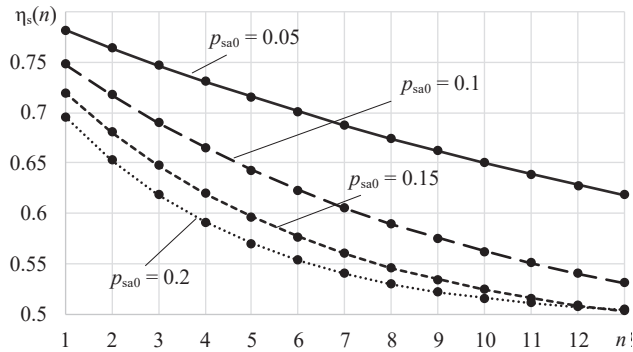


Fig. 1. Dependences of the firing efficiency of a series η_s on its length n

As can be seen from Fig. 1, under the action of the two above-mentioned types of random disturbances in the process of artillery firing, an increase in the length of the firing series n leads to a significant decrease in its efficiency η_s . On the one hand, this justifies the expediency of using shorter series when firing and more frequent movement between positions. However, on the other hand, splitting the fire task into a large number of short series causes additional time costs for performing the processes of movement, deployment, aiming, and closing. This, of course, leads to a significant increase in the total time of task execution.

Next, the determination of the time of execution of the fire task when using the "shoot-and-scoot" tactic is considered.

When establishing the desired mathematical expectation of the number of hits $E_{\Sigma D}$ required to destroy the target, the minimum required number of series $m_{\min}(n)$ is determined as the ratio of $E_{\Sigma D}$ to $E_C(n)$, rounded up to the nearest integer. In turn

$$m_{\min}(n) = \left\lceil \frac{E_{\Sigma D}}{E_C(n)} \right\rceil. \tag{11}$$

Next, using expression (12), the time $t_s(n)$ required to execute one firing series is determined

$$t_s(n) = t_{dp} + t_{ep} + \frac{n}{v_{fr}} + t_{dpp}, \tag{12}$$

where t_{dp} is the average time of moving the installation between positions; t_{ep} is the time of deployment and aiming of the installation; v_{fr} is the rate of fire; t_{dpp} is the time of closing.

After that, the total time of execution of the fire task $t_{\Sigma}(n)$ is defined as the product of the minimum number of series $m_{\min}(n)$ and the time of execution of one series $t_s(n)$

$$t_{\Sigma}(n) = m_{\min}(n) t_s(n). \tag{13}$$

When analyzing expressions (11) to (13), it is obvious that the total time of movement, deployment, aiming, and closing ($t_{dp} + t_{ep} + t_{dpp}$) is always much greater than the time of direct fire n / v_{fr} . This means that an increase in the length of the firing series n leads to a decrease in the total time of execution of the fire task t_{Σ} .

When determining the optimal length of the series n , in addition to the firing efficiency η_s and the total time of execution of the fire task t_{Σ} , another very important aspect must also be taken into account. When conducting fire and further staying at firing positions, the risk of being hit or destroyed by enemy counter-battery fire for an artillery installation is constantly increasing [30]. The probability of hitting or destroying the installation p_{ds} is determined by the exponential dependence on the time of stay at the firing position after the start of firing t_{ps}

$$p_{ds} = 1 - e^{-\lambda_{ds} t_{ps}^{k_{cr}}}, \tag{14}$$

where λ_{ds} is the intensity of threat growth; k_{cr} is the critical threat growth coefficient.

In turn:

$$t_{ps} = \frac{n}{v_{fr}} + t_{dpp}; \tag{15}$$

$$k_{cr} = \begin{cases} 1, & \text{at } t_{ps} < t_{cr}; \\ > 1, & \text{at } t_{ps} \geq t_{cr}, \end{cases} \tag{16}$$

where t_{cr} is the critical time of being in the firing position after the start of firing.

The values of parameters λ_{ds} , t_{cr} and k_{cr} after time t_{cr} are determined and set empirically.

Since the above indicators (firing efficiency, task completion time and hit risk) are contradictory, partially uncertain, and do not have clear limits of acceptability, their formalization is carried out on the basis of fuzzy logic. This mathematical apparatus allows for quite effective use of linguistic assessments and degrees of membership without rigid thresholds [31]. Unlike traditional convolution methods, fuzzy-logical models make it possible to operate with expert rules in the form of "If-Then" and adequately take into account nonlinear dependences among compromise criteria [32]. This makes the use of a fuzzy model the most appropriate for solving the task of evaluating and choosing the optimal value of the length of a series of shots.

When evaluating various alternative options corresponding to certain numbers of shots n in a series, the input of the fuzzy model is given the values of their efficiency $\eta_s(n)$, time $t_{\Sigma}(n)$, and the probability of destruction of the installation $p_{ds}(n)$. In turn, the variable t_{Σ} is previously normalized by its maximum value $t_{\Sigma \max}^*$ ($t_{\Sigma}^* = t_{\Sigma} / t_{\Sigma \max}$). A certain number of linguistic terms (LTs) with certain membership functions (MFs) are used to fuzzify each of these input variables. Moreover, in this case, the most suitable are Gaussian MFs of the first and second kind, which are described by equations (17) and (18), respectively:

$$\mu(x_{in}) = e^{-\frac{(x_{in}-b)^2}{2a^2}}; \tag{17}$$

$$\mu(x_{in}) = \begin{cases} e^{-\frac{(x_{in}-b)^2}{2a^2}}, & \text{at } x_{in} < b; \\ 1, & \text{at } b \leq x_{in} \leq d; \\ e^{-\frac{(x_{in}-d)^2}{2c^2}}, & \text{at } x_{in} > d; \end{cases} \tag{18}$$

$$b < d,$$

where x_{in} is a certain input variable of the model (η_s , t_{Σ}^* or p_{ds}); a , b , c and d are adjustable coefficients.

The suitability of the alternative S_a , which has a working range from 0 to 1, is chosen as the output variable of this fuzzy model. Linguistic terms with triangular membership functions are used for this variable:

$$\mu(S_a) = \begin{cases} 0, & \text{at } S_a \leq a; \\ \frac{S_a - a}{b - a}, & \text{at } a < S_a \leq b; \\ \frac{c - S_a}{c - b}, & \text{at } b < S_a < c; \\ 0, & \text{at } S_a \geq c, \end{cases} \quad (19)$$

where a , b , and c are the vertices of the term that are subject to adjustment.

For fuzzy inference procedures, it is most appropriate to use Mamdani [33] or Larsen [34] mechanisms, which make it possible to build intuitive rule bases (RBs) and easily interpret the drawn conclusions. In turn, the RB of this model is constructed as a set of certain rules composed of antecedents (conditions) and consequents (consequences), which make it possible to reproduce the logic of rational choice of the best alternative according to the values of its criteria. Each rule within this RB has a construct represented by the following generalized expression (20)

$$\begin{aligned} &\text{IF } " \eta_s = LT_{1ini} " \text{ AND } " t_{\Sigma}^* = LT_{2inj} " \\ &\text{AND } " p_{ds} = LT_{3ini} " \text{ THEN } " S_a = LT_{outk} ", \end{aligned} \quad (20)$$

where LT_{1ini} , LT_{2inj} , LT_{3ini} – i -th, j -th, and l -th LT of the corresponding input variables of the model; LT_{outk} – k -th term of the output variable.

The total number of rules in this RB depends on the number of LTs for each input variable and is determined by their product. This RB reproduces the following logic in the process of selecting the best alternatives. Variants with a high risk of defeat or destruction are unacceptable, since in addition to the potential loss of the installation, their selection jeopardizes the overall possibility of completing the task. If the alternative has high efficiency and short duration, but also, at the same time, a high risk of defeat, then most likely the task will not be completed at all as the installation can be destroyed during the execution of this fire task. Also, among the variants with low risk or below average, those that have a certain balance between efficiency and speed should be selected.

To narrow down the set of alternative options for the series length, the proposed method introduces a universal criterion $J_f(n)$ and finds the values of n corresponding to its maxima. This criterion combines the firing efficiency, the normalized total time of the firing task, and the probability of hitting the installation as follows

$$J_f(n) = \frac{\eta_t(n)(1 - p_{ds}(n))}{t_{\Sigma}^*(n)}. \quad (21)$$

Moreover, for better generalization, this criterion does not use the efficiency of one series $\eta_s(n)$, but the actual efficiency when performing the entire task $\eta_t(n)$ for a certain value of the series length n . To calculate this firing efficiency, expression (22) is used

$$\eta_t(n) = \frac{E_{\Sigma D}}{m_{\min}(n)n}. \quad (22)$$

So, the best value of the number of shots in the series n_{best} can be found as

$$n_{best} = \arg \max J_f(n). \quad (23)$$

However, given the multi-extreme (multimodal) nature of the $J_f(n)$ dependence, direct selection of the n_{best} value corresponding to the global maximum will not always be appropriate. Therefore, this method provides for the analysis of all found local maxima with the evaluation of their individual indicators (η_t , t_{Σ}^* , p_{ds}) based on a fuzzy-logical model. The value n that corresponds to the highest fitness value S_a will be selected as the best series length. This will significantly reduce computational costs since the evaluation based on the fuzzy model will not be carried out for all possible options n , but only for those that correspond to the maxima of the $J_f(n)$ function.

Thus, based on all of the above, a method has been devised for determining the rational length of a firing series taking into account the effectiveness of firing, the time of execution of the fire task, and the risk of damage to the installation. The basic steps of this method are given below:

Step 1. Initialization. In this step, the values of all parameters necessary for determining the criterion J_f and individual indicators η_t , t_{Σ}^* and p_{ds} are set. In particular, the maximum possible value of shots in a firing series n_{max} , the values of the probabilities of occurrence of disturbances p_{sa0} , p_{sa2} and the probabilities of hits p_{wa} , p_{a1} , p_{a2} are set. Also, the desired mathematical expectation of the number of hits $E_{\Sigma D}$, the values of time t_{dp} , t_{ep} , t_{dpp} and the rate of fire v_{fr} are set. In addition, the values of the threat growth intensity λ_{ds} , the critical time t_{cr} and the value of the coefficient k_{cr} when the critical time t_{cr} is reached are set.

Step 2. Calculation of efficiency values $\eta_t(n)$ for all possible values of the series length n . In this step, for each possible value of the number of shots in the series ($n = 1, \dots, n_{max}$), the value of the actual efficiency when performing the entire task $\eta_t(n)$ is calculated. For this purpose, expressions (1), (3), (8), (11), and (22) are used.

Step 3. Calculation of normalized values of the total time of execution of the firing task $t_{\Sigma}^*(n)$ for all possible values of the series length n . In this step, first for each possible value of n ($n = 1, \dots, n_{max}$) the absolute value of the total time $t_{\Sigma}(n)$ is calculated according to formulae (12) and (13). After that, their normalization is carried out by the maximum $t_{\Sigma max}$ value. Accordingly, $t_{\Sigma}^*(n) = t_{\Sigma}(n) / t_{\Sigma max}$.

Step 4. Calculation of the values of the probability of defeating the installation $p_{ds}(n)$ for all possible values of the series length n . In this step, for each possible value of n ($n = 1, \dots, n_{max}$), the value of the probability of defeat or destruction of the installation $p_{ds}(n)$ is calculated using formulae (14) to (16).

Step 5. Calculation of values for the universal criterion $J_f(n)$ for all possible values of the series length n . In this step, for each possible value of n ($n = 1, \dots, n_{max}$), the value of criterion $J_f(n)$ is calculated using expression (21).

Step 6. Finding local maxima of function $J_f(n)$. In this step, all existing local maxima of function $J_f(n)$ are found. For this purpose, the value of the criterion $J_f(n)$ for each value of n , starting from $n = 2$, is checked for compliance with condition (24)

$$J_f(n-1) < J_f(n) > J_f(n+1). \quad (24)$$

If for a certain value of n , condition (24) is fulfilled, then this value is selected as the local maximum of function $J_f(n)$. After that, the found values of n are selected for further evaluation using a fuzzy model in the next step.

Step 7. Evaluation of local maxima of function $J_f(n)$ based on a fuzzy logic model. In this step, based on the indicators η_t , t_Σ^* and p_{ds} obtained for all n corresponding to the local maxima of function $J_f(n)$, the fitness value S_a is calculated using a fuzzy model.

Step 8. Selection of the best value of the series length n_{best} . In this step, the most rational number of shots in the series n_{best} is selected according to the highest fitness value S_{abest} among the calculated values in the previous step.

Below is the construction of a fuzzy logic model, which is used in the proposed method for evaluating the values of length of the firing series corresponding to the local maxima of function $J_f(n)$.

5. 2. Construction of a fuzzy logic model for estimating values of the firing series length

The Mamdani mechanism was chosen as the fuzzy logic inference mechanism for the constructed model. In turn, this fuzzy logic model is built according to the traditional structure of Mamdani-type models (Fig. 2).

The input variables η_t , t_Σ^* , p_{ds} and the output variable S_a of the fuzzy model have working ranges [0, 1]. For the first two input variables (η_t, t_Σ^*), 4 linguistic terms are selected, and for the input variable p_{ds} , 3 LTs are set. For the output variable S_a , it is advisable to set 7 terms. In turn, Table 1 gives the types of membership functions and the values of their parameters for all selected LTs.

After setting LT parameters, a rule base of a fuzzy logic model was synthesized, which reproduces the

basic logic of determining the suitability S_a of the found local maximum, depending on the corresponding indicators η_t , t_Σ^* and p_{ds} . Each rule in this RB is formed in the form of expression (20). In turn, the synthesized rule base is represented in matrix form in Table 2.

In the cells of the leftmost column, the top row and the rightmost column, the LTs for the corresponding input variables are given, the combinations of which form the antecedents of the rules. In the inner cells of Table 2, the LTs of the output variable are given, which act as the corresponding consequents.

Three-dimensional diagrams of dependences of fitness S_a on the efficiency η_t and the normalized time t_Σ^* are given in Fig. 3, a-c.

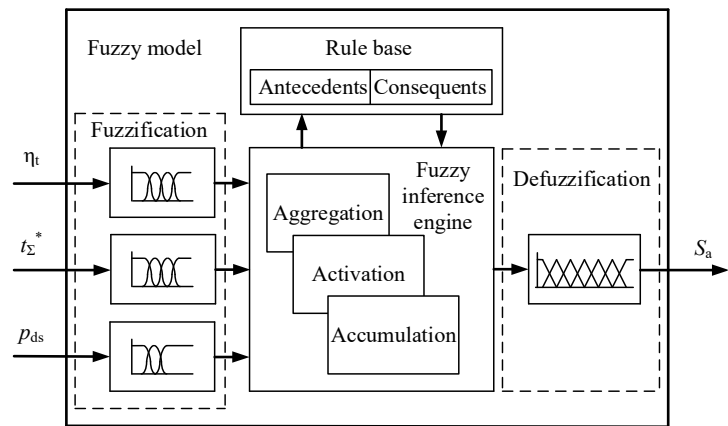


Fig. 2. Structure of a fuzzy logic model for estimating firing series length values

Table 1

Parameters of linguistic terms for the fuzzy-logical model

#	LT	MF	Parameter value
Input variable η_t (shooting efficiency)			
1	L - low	Gaussian of the 2nd kind	$a = 0.04; b = 0; c = 0.04; d = 0.4$
2	A - average	Gaussian of the 1st kind	$a = 0.04; b = 0.5$
3	AA - above average	Gaussian of the 1st kind	$a = 0.04; b = 0.6$
4	H - high	Gaussian of the 2nd kind	$a = 0.04; b = 0.7; c = 0.04; d = 1$
Input variable t_Σ^* (normalized time of task completion)			
1	S - short	Gaussian of the 2nd kind	$a = 0.04; b = 0; c = 0.04; d = 0.2$
2	M - medium	Gaussian of the 1st kind	$a = 0.04; b = 0.3$
3	L - long	Gaussian of the 1st kind	$a = 0.04; b = 0.4$
4	VL - very long	Gaussian of the 2nd kind	$a = 0.04; b = 0.5; c = 0.04; d = 1$
Input variable p_{ds} (probability of damage to the installation)			
1	L - low	Gaussian of the 2nd kind	$a = 0.06; b = 0; c = 0.06; d = 0.2$
2	A - average	Gaussian of the 1st kind	$a = 0.08; b = 0.35$
3	H - high	Gaussian of the 2nd kind	$a = 0.06; b = 0.5; c = 0.06; d = 1$
Output variable S_a (suitability of the alternative)			
1	VL - very low	triangular	$a = 0; b = 0; c = 0.167$
2	L - low	triangular	$a = 0; b = 0.167; c = 0.333$
3	BA - below average	triangular	$a = 0.167; b = 0.333; c = 0.5$
4	A - average	triangular	$a = 0.333; b = 0.5; c = 0.667$
5	AA - above average	triangular	$a = 0.5; b = 0.667; c = 0.833$
6	H - high	triangular	$a = 0.667; b = 0.833; c = 1$
7	VH - very high	triangular	$a = 0.833; b = 1; c = 1$

Table 2

Fuzzy logic model rule base matrix

LT for t_{Σ}^*	LT for η_t				LT for p_{ds}
	L	A	AA	H	
S	A	H	VH	VH	L
M	BA	AA	H	VH	
L	VL	L	L	BA	
VL	VL	L	L	L	
S	BA	AA	H	H	A
M	L	A	AA	H	
L	VL	VL	VL	L	
VL	VL	VL	VL	VL	
S	VL	L	A	A	H
M	VL	VL	L	BA	
L	VL	VL	VL	VL	
VL	VL	VL	VL	VL	

In turn, these diagrams were constructed for different values of the probability of damage ($p_{ds} = 0.1$; $p_{ds} = 0.5$; $p_{ds} = 0.8$) based on the constructed model using Fuzzy Logic Toolbox, MATLAB R2026a (trial version).

To investigate the effectiveness of our method and the constructed fuzzy model, computational experiments are de-

scribed in the following subsection with a detailed discussion of the results.

5. 3. Conducting computational experiments to confirm the effectiveness of the method for determining the rational length of a firing series

To confirm the effectiveness of the proposed method, computational experiments were conducted in this work that simulated the performance of three different firing tasks. To simulate each of the considered tasks, different values of the desired number of hits $E_{\Sigma D}$ and the probability of a projectile belt failure in one arbitrary shot p_{sa0} were set. Namely, for the first task $E_{\Sigma D} = 7$, $p_{sa0} = 0.1$; for the second – $E_{\Sigma D} = 10$, $p_{sa0} = 0.12$; for the third – $E_{\Sigma D} = 14$, $p_{sa0} = 0.07$. Also, for all three experiments, the same values of other adjustable parameters were set, which are given in Table 3.

When applying the proposed method during computational experiments, the following results were obtained.

For the first experiment, the dependence of change in the universal criterion $J_f(n)$ on the values of the series length n is shown in Fig. 4.

From Fig. 4 it is seen that the local maxima of function $J_f(n)$ in this case are the points at which n is 2, 4, 6, and 14. In turn, Table 4 gives indices $J_f(n)$, η_t , t_{Σ}^* and p_{ds} for given n values, as well as the corresponding fitness values S_a , calculated using the constructed fuzzy model.

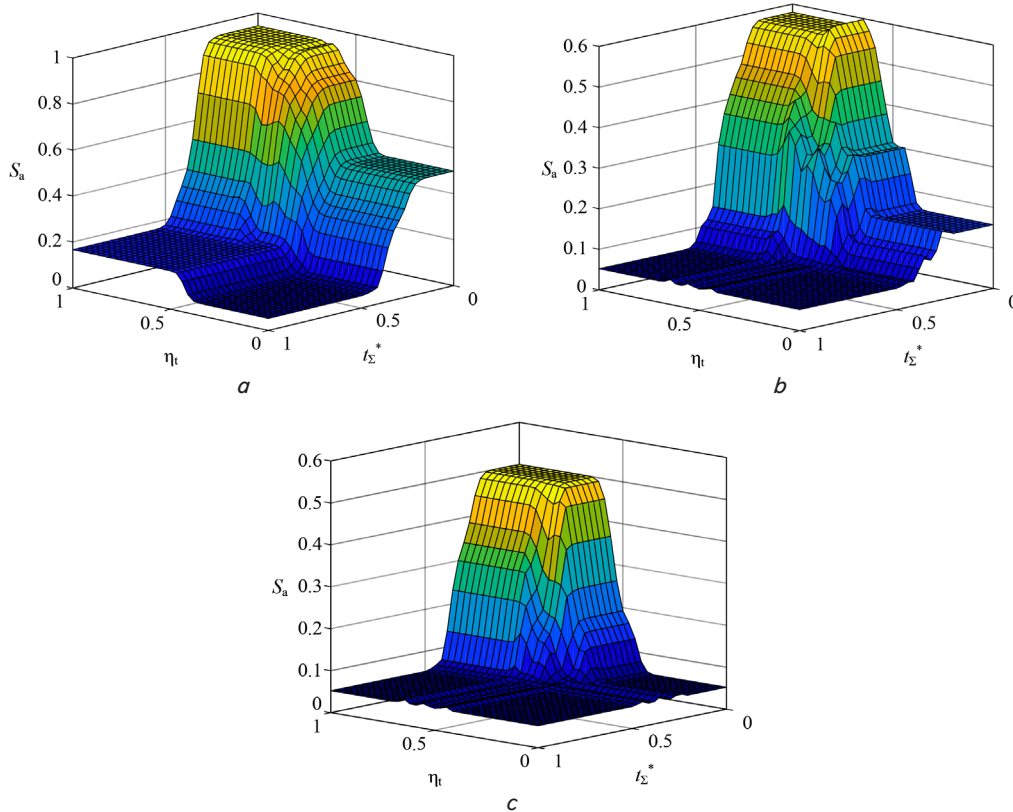


Fig. 3. Three-dimensional diagrams of dependence of suitability S_a on the efficiency η_t and normalized time t_{Σ}^* at different values of the probability of damage: a – $p_{ds} = 0.1$; b – $p_{ds} = 0.5$; c – $p_{ds} = 0.8$

Table 3

Values of adjustable parameters for conducting experiments

Parameter	$n_{max, shot}$	p_{sa2}	p_{wa}	p_{a1}	p_{a2}	$t_{dp, min}$	$t_{ep, min}$	$t_{dpp, min}$	$v_{fr, shot/min}$	λ_{ds}	$t_{cr, min}$	$k_{cr}(t_{ps} \geq t_{cr})$
Value	25	0.1	0.9	0.1	0.1	7	1	1	4	0.25	5	1.3

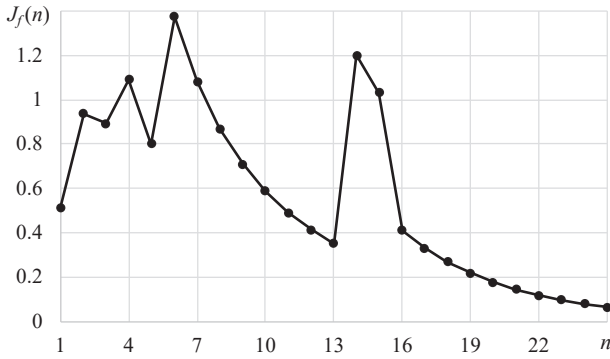


Fig. 4. Dependence of change in the universal criterion $J_f(n)$ on the values of series length n (1st experiment)

Table 4

The $J_f(n)$, η_t , t_{Σ}^* , p_{ds} and S_a indices for the values of n corresponding to the local maxima of function $J_f(n)$ (1st experiment)

n , shot	$J_f(n)$	η_t	t_{Σ}^*	p_{ds}	S_a
2	0.937	0.7	0.513	0.313	0.111
4	1.091	0.583	0.324	0.393	0.527
6	1.375	0.583	0.227	0.465	0.565
14	1.201	0.5	0.135	0.675	0.193

As can be seen from Table 4, the most rational value of the series length determined using the proposed method is $n_{best} = 6$. In this case, this value also corresponds to the global maximum of function $J_f(n)$.

For the second experiment, the dependence of change in $J_f(n)$ on the values of n is shown in Fig. 5.

In this case, the local maxima of function $J_f(n)$ are the points at which n is 4, 6, 10, and 21. Table 5 gives the $J_f(n)$, η_t , t_{Σ}^* and p_{ds} indices, as well as the corresponding fitness values S_a for these n .

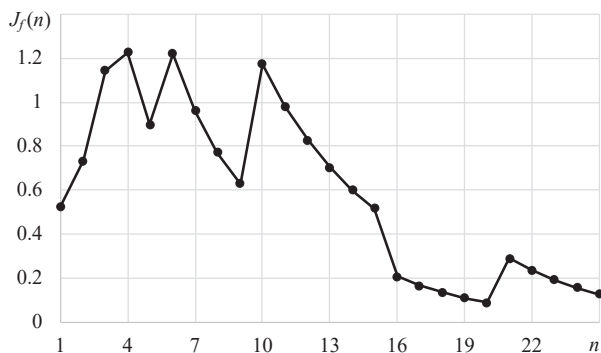


Fig. 5. Dependence of change in the universal criterion $J_f(n)$ on the values of series length n (2nd experiment)

Table 5

The $J_f(n)$, η_t , t_{Σ}^* , p_{ds} and S_a indices for values of n corresponding to local maxima of function $J_f(n)$ (2nd experiment)

n , shot	$J_f(n)$	η_t	t_{Σ}^*	p_{ds}	S_a
4	1.227	0.625	0.309	0.393	0.56
6	1.225	0.555	0.243	0.465	0.495
10	1.174	0.5	0.178	0.583	0.211
21	0.289	0.476	0.11	0.933	0.165

As can be seen from Table 5, the most rational value of series length is $n_{best} = 4$. In this case, this value also corresponds to the global maximum of function $J_f(n)$.

Similarly, for the third experiment, the dependence of $J_f(n)$ on n and the $J_f(n)$, η_t , t_{Σ}^* , p_{ds} , S_a indices for local maxima are shown in Fig. 6 and Table 6.

In this case, the local maxima of function $J_f(n)$ are the points at which n is 4, 6, 8, and 12.

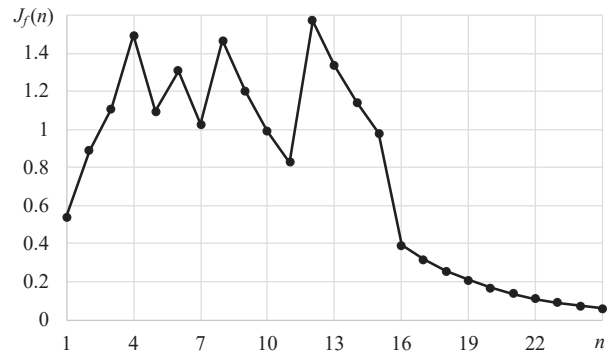


Fig. 6. Dependence of change in the universal criterion $J_f(n)$ on the values of series length n (3rd experiment)

Table 6

The $J_f(n)$, η_t , t_{Σ}^* , p_{ds} and S_a indices for values of n corresponding to local maxima of function $J_f(n)$ (3rd experiment)

n , shot	$J_f(n)$	η_t	t_{Σ}^*	p_{ds}	S_a
4	1.492	0.7	0.284	0.393	0.677
6	1.307	0.583	0.239	0.465	0.521
8	1.467	0.583	0.188	0.528	0.484
12	1.571	0.583	0.137	0.632	0.441

As can be seen from Table 6, the most rational value of series length is also $n_{best} = 4$. In this case, this value does not correspond to the global maximum of function $J_f(n)$.

In addition, when conducting computational experiments, the devised method was also compared with the traditional approach that uses a fixed length of the firing series. In particular, the quality indicators for the n_{best} values obtained using the proposed method were compared with the corresponding indicators for 2 other fixed values of series length: $n = 5$; $n = 8$.

Tables 7–9 give the η_t , t_{Σ}^* , p_{ds} indicators obtained in three experiments for the corresponding values of n .

Also, in Tables 7–9, the absolute values of the total time $t_{ps\Sigma}$ of the installation's stay at all firing positions after the start of firing at each were additionally compared.

Table 7

Comparison of quality indicators for different values of n (1st experiment)

Method for determining n	n , shot	η_t	t_{Σ}^*	p_{ds}	$t_{ps\Sigma}$, min
Proposed method	6	0.583	0.227	0.465	5
Fixed series length approach	5	0.467	0.332	0.43	6.75
	8	0.438	0.238	0.528	6

Table 8

Comparison of quality indicators for different values of n (2nd experiment)

Method for determining n	n , shot	η_t	t_{Σ}^*	p_{ds}	$t_{ps\Sigma}$, min
Proposed method	4	0.625	0.309	0.393	8
Fixed series length approach	5	0.5	0.317	0.43	9
	8	0.417	0.255	0.528	9

Table 9

Comparison of quality indicators for different values of n (3rd experiment)

Method for determining n	n , shot	η_t	t_{Σ}^*	p_{ds}	$t_{ps\Sigma}$, min
Proposed method	4	0.7	0.284	0.393	10
Fixed series length approach	5	0.56	0.292	0.43	11,25
	8	0.583	0.188	0.528	9

6. Discussion of results based on investigating the method for determining the rational length of a series of artillery shots

Analysis of the results given in Tables 7–9 reveals a significant advantage of the proposed method in terms of the firing efficiency criterion compared to the traditional approach of a fixed series length, which was used in [20, 21]. In all three experiments, the η_t values obtained for the best value found n_{best} exceed the corresponding values for fixed series lengths. This confirms that the adaptive selection of the number of shots in the series used in the devised method allows for better consideration of changes in the initial data and existing limitations when performing a fire task. Also, an increase in firing efficiency is achieved by taking into account the influence of internal random disturbances acting on the artillery installation, which is not used in the approach reported in [17].

Comparison by normalized time shows that the proposed method also provides a reduction in the time to complete the firing task compared to the option of a fixed value of the series length $n = 5$. At the same time, for a fixed length $n = 8$, in some cases a smaller time value is observed, but this is achieved at the expense of a significant increase in the probability of hitting the installation. Thus, reducing the time at large values of the number of shots n leads to an unacceptable increase in the risk of hitting, which limits the practical feasibility of using too long series.

Also, special attention should be paid to the values of the probability of defeat or destruction of the installation p_{ds} in the conducted experiments. Our experimental results show that for the series length $n = 8$ this indicator is the highest in all considered cases, which is due to the longer stay of the artillery installation at one firing position, while the proposed method makes it possible to provide lower values of p_{ds} , maintaining a balance between the effectiveness of firing, the time of execution of fire tasks, and the safety of the use of the artillery system. Additional confirmation of the risk reduction are the calculated absolute values of the total time of stay of the installation at all firing positions $t_{ps\Sigma}$. In most cases, when using the proposed method, this indicator is lower than for

options with fixed values of n , which also indicates a reduction in the risk of potential defeat after the first shot. This certainly increases the survivability of the artillery installation in the presence of counter-battery combat.

In general, the results from all computational experiments confirm that the proposed method makes it possible to eliminate existing limitations of current approaches, which are associated with the lack of adaptive determination of the length of the firing series taking into account the influence of random disturbances. This is achieved by combining probabilistic-analytical modeling of random disturbing influences and fuzzy multi-criteria evaluation of the found alternative solutions. The use of the devised method ensures the improvement of the most important indicators of the quality of functioning of the artillery system by ensuring a compromise between firing efficiency, task completion time, and the risk of damage, which confirms the achievement of the set goal. From the point of view of practical application, the devised method could be effectively used in decision support systems and artillery installation control during preliminary planning and fire control under conditions of uncertain disturbances.

The limitation of our method is the need to specify specific values of the probabilities of disturbances (projectile belt rupture and the use of a poor-quality charge), as well as the probabilities of hitting in the absence and presence of these disturbances.

The disadvantages of the proposed method for determining the length of the series include the dependence of accuracy of the results on the specified parameters of the probabilistic-analytical models used and the reliability of the initial data. In particular, with significant uncertainty in the parameters of the external environment and in the presence of significant errors in the initial estimates of the characteristics of the disturbances, it is possible to reduce the accuracy of determining local maxima and, accordingly, the effectiveness of the decisions made. This disadvantage can be further eliminated by adaptive evaluation and identification of the probabilistic parameters of the models based on experimentally measured and accumulated statistical data in the process of functioning of the artillery system.

In further studies, increasing the reliability of the method can be achieved by using modern approaches to solving the problems of identifying model parameters based on experimental data under noise conditions [35]. Along with this, a promising direction for the development of the method is the construction of a fuzzy-logical model of multi-criteria evaluation using parametric optimization procedures to adjust membership functions [36], as well as structural optimization to form an effective rule base [37]. This could increase the accuracy of management decisions and the adaptability of the method to the changed conditions of artillery use.

7. Conclusions

1. A formalization procedure and basic steps of the method for determining the rational length of a series under the action of random disturbances when using the "shoot-and-scoot" tactic have been proposed. This method is based on the assessment of three indicators for the quality of functioning: firing efficiency, time to complete the firing task, and the risk of damage to the artillery system. In the process of implementing the main steps of the method, probabilistic-analytical modeling was used to form a generalized criterion and

determine the set of local maxima that correspond to potential rational values of the series length. That has made it possible to take into account the influence of random disturbances, in particular, a sharp increase in barrel wear, on the efficiency of firing within one series.

2. A fuzzy multi-criteria evaluation model has been built, which makes it possible, when applying the considered method, to choose the best version of the series length among the obtained set of local maxima. The model provides a combination of contradictory quality indicators based on a system of linguistic rules and membership functions, which makes it possible to formalize the process of making management decisions under uncertainty. This allows one to choose a series length value that corresponds to a reasonable compromise between firing efficiency, task duration, and the level of risk of damaging the installation.

3. A number of simulation experiments were conducted, the results of which confirm the effectiveness of the proposed method compared to the approach using a fixed series length. It was found that in all our experiments, an increase in firing efficiency was observed, which is 11.6–14.0% compared to the fixed value of $n = 5$ and 11.7–20.8% compared to $n = 8$. Also, the total time for completing the firing task is significantly less than in the case of the $n = 5$ variant, which indicates an increase in the speed of using the artillery installation. In particular, in the first experiment, a decrease in the time spent by 31.6% is observed. At the same time, although for a fixed value of $n = 8$ in some cases the task completion time is somewhat shorter, a noticeable increase in the probability of hitting the installation is observed. Namely, the probability of hitting reaches the level of 0.528, which is significantly higher than the corresponding values (0.393–0.465) obtained when using the proposed method. This fact demonstrates the obvious limitations and inexpediency of using too long series. In addition, it is shown that when using the proposed method, the total time the installation is at all firing positions is in most cases

less, namely up to 25.9%, which confirms the reduction in the risk of counter-battery damage.

Conflicts of interest

The authors declare that they have no conflicts of interest in relation to the current study, including financial, personal, authorship, or any other, that could affect the study and the results reported in this paper.

Funding

The study was conducted without financial support.

Data availability

All data are available in the main text of the manuscript.

Use of artificial intelligence

The authors confirm that they did not use artificial intelligence technologies when creating the current work.

Authors' contributions

Maksym Maksymov: Conceptualization, Validation, Writing – review and editing, Supervision; **Oleksiy Kozlov:** Conceptualization, Methodology, Formal analysis, Writing – review and editing; **Oleksiy Maksymov:** Methodology, Software, Formal analysis, Writing – original draft; **Ruslan Riaboshapka:** Methodology, Software, Writing – original draft.

References

- Chávez, K., Swed, O. (2023). Emulating underdogs: Tactical drones in the Russia-Ukraine war. *Contemporary Security Policy*, 44 (4), 592–605. <https://doi.org/10.1080/13523260.2023.2257964>
- Ganjiyev, S. J., Usmonov, S. R., Karimov, A. Kh. (2023). Use of artillery in modern war: a brief analysis of the Ukrainian conflict. *Galaxy International Interdisciplinary Research Journal*, 11 (3), 118–121. Available at: <https://internationaljournals.co.in/index.php/giirj/article/view/3646>
- Oprean, L.-G. (2020). Artillery from the Perspective of Firing Effects and Ensured Capabilities. *Scientific Bulletin*, 25 (2), 107–113. <https://doi.org/10.2478/bsaft-2020-0015>
- Zha, Q., Rui, X., Liu, F., Yu, H. (2017). Study On the Dynamic Modeling and the Correction Method of the Self-Propelled Artillery. *Proceedings of the 2017 7th International Conference on Mechatronics, Computer and Education Informationization (MCEI 2017)*. Atlantis Press, 385–393. <https://doi.org/10.2991/mcei-17.2017.84>
- Kislitsyn, A., Dorofeev, N. (2021). Directions for the development of domestic self-propelled artillery systems based on the analysis of samples of artillery weapons from the leading countries of the world. *Social Development and Security*, 11 (6), 98–107. <https://doi.org/10.33445/sds.2021.11.6.7>
- Xiao, H., Yang, G., Ge, J. (2017). Surrogate-based multi-objective optimization of firing accuracy and firing stability for a towed artillery. *Journal of Vibroengineering*, 19 (1), 290–301. <https://doi.org/10.21595/jve.2016.17108>
- Wang, X., Li, X., Sun, Q., Xia, C., Chen, Y.-H. (2025). Improved Manta Ray Foraging Optimization for PID Control Parameter Tuning in Artillery Stabilization Systems. *Biomimetics*, 10 (5), 266. <https://doi.org/10.3390/biomimetics10050266>
- Zhuravlev, A., Orlov, S., Shuliakov, S. (2020). Mathematical model of the flight path of a projectile of a long-range artillery system. *Systems of Arms and Military Equipment*, 3 (63), 62–68. <https://doi.org/10.30748/soivt.2020.63.09>
- Khalil, M. (2022). Study on modeling and production inaccuracies for artillery firing. *Archive of Mechanical Engineering*, 69 (1), 165–183. <https://doi.org/10.24425/ame.2021.139802>

10. STANAG 4355 The Lieske modified point mass and five degrees of freedom trajectory models -AOP-4355 EDITION A. (No enabled versions). (2022) Washington: United States Department of Defense. Available at: <https://publishers.standardstech.com/content/military-dod-stanag-4355> Last accessed: 14.11.2023
11. Bartulović, V., Trzun, Z., Hoić, M. (2023). Use of Unmanned Aerial Vehicles in Support of Artillery Operations, *Strategos*, 7 (1), 71–92. Available at: <https://hrcak.srce.hr/305562>
12. Oprean, L.-G. (2023). Artillery and Drone Action Issues in the War in Ukraine. *Scientific Bulletin*, 28 (1), 73–78. <https://doi.org/10.2478/bsaft-2023-0008>
13. Khudov, H., Yuzova, I., Lisohorskyi, B., Solomonenko, Y., Mykus, S., Irkha, A. et al. (2021). Development of methods for determining the coordinates of firing positions of roving mortars by a network of counter-battery radars, *EUREKA: Physics and Engineering*, 3, 140–150. <https://doi.org/10.21303/2461-4262.2021.001821>
14. Świętochowski, N. (2023). Field Artillery in the defensive war of Ukraine 2022–2023. Part I. Combat potential, tasks and tactics. *Scientific Journal of the Military University of Land Forces*, 210 (4), 341–358. <https://doi.org/10.5604/01.3001.0054.1631>
15. Świętochowski, N. (2024). Field Artillery in the defensive war of Ukraine 2022–2023. Part II. Methods of task implementation. *Scientific Journal of the Military University of Land Forces*, 211 (1), 57–76. <https://doi.org/10.5604/01.3001.0054.4136>
16. Shen, C., Zhou, K.-D., Lu, Y., Li, J.-S. (2019). Modeling and simulation of bullet-barrel interaction process for the damaged gun barrel. *Defence Technology*, 15(6), 972–986. <https://doi.org/10.1016/j.dt.2019.07.009>
17. Shim, Y., Atkinson, M. P. (2018). Analysis of artillery shoot-and-scoot tactics. *Naval Research Logistics (NRL)*, 65 (3), 242–274. <https://doi.org/10.1002/nav.21803>
18. Damgaard, T. J., Rittri, M. (2025). Optimizing Firing Position Usage for Survivability and Effectiveness in Artillery Shoot-and-Scoot Tactics. *SAE Technical Paper Series*, 1. <https://doi.org/10.4271/2025-01-0431>
19. Dobrynin, Y., Maksymov, M., Boltentkov, V. (2020). Development of a method for determining the wear of artillery barrels by acoustic fields of shots. *Eastern-European Journal of Enterprise Technologies*, 3 (5 (105)), 6–18. <https://doi.org/10.15587/1729-4061.2020.206114>
20. Choi, Y. B., Yun, H. Y., Kim, J. y., Jin, S. H., Kim, K. S. (2019). Robust Optimization Approach Using Scenario Concepts for Artillery Firing Scheduling Under Uncertainty. *Applied Sciences*, 9 (14), 2811. <https://doi.org/10.3390/app9142811>
21. Sun, Y., Zhang, S., Lu, G., Zhao, J., Tian, J., Xue, J. (2022). Research on a Simulation Algorithm for Artillery Firepower Assignment According to Region. 2022 3rd International Conference on Computer Science and Management Technology (ICCSMT). Shanghai, 353–356. <https://doi.org/10.1109/iccsmt58129.2022.00082>
22. Grishyn, M., Maksymova, O., Kirkopulo, K., Klymchuk, O. (2025). Development of methods of artillery control for suppression of an enemy amphibious operation in video game simulations. *Technology Audit and Production Reserves*, 1 (2 (81)), 26–33. <https://doi.org/10.15587/2706-5448.2025.321797>
23. Raskin, L., Sira, O. (2020). Development of methods for extension of the conceptual and analytical framework of the fuzzy set theory. *Eastern-European Journal of Enterprise Technologies*, 6 (4 (108)), 14–21. <https://doi.org/10.15587/1729-4061.2020.217630>
24. Congxiang, L., Kozlov, O., Kondratenko, G., Aleksieieva, A.; Kondratenko, Y. P., Shevchenko, A. I. (Eds.) (2024). Decision Support System for Maintenance Planning of Vortex Electrostatic Precipitators Based on IoT and AI Techniques. *Research Tendencies and Prospect Domains for AI Development and Implementation*. New York: River Publishers, 87–105. <https://doi.org/10.1201/9788770046947-5>
25. Skakodub, O., Kozlov, O., Kondratenko, Y. (2021). Optimization of Linguistic Terms' Shapes and Parameters: Fuzzy Control System of a Quadrotor Drone. 2021 11th IEEE International Conference on Intelligent Data Acquisition and Advanced Computing Systems: Technology and Applications (IDAACS), 566–571. <https://doi.org/10.1109/idaacs53288.2021.9660926>
26. Kozlov, O. (2021). Information Technology for Designing Rule bases of Fuzzy Systems using Ant Colony Optimization. *International Journal of Computing*, 20 (4), 471–486. <https://doi.org/10.47839/ijc.20.4.2434>
27. Kozlov, O., Kondratenko, G., Aleksieieva, A., Maksymov, M., Tarakhtij, O. (2024). Swarm optimization of the drone's intelligent control system: comparative analysis of hybrid techniques, *CEUR Workshop Proceedings*, 3790. Available at: <https://ceur-ws.org/Vol-3790/paper01.pdf>
28. Kumar, D., Kalra, S., Jha, M. S. (2022). A concise review on degradation of gun barrels and its health monitoring techniques. *Engineering Failure Analysis*, 142, 106791. <https://doi.org/10.1016/j.engfailanal.2022.106791>
29. Litsman, A., Nesterov, D. (2020). Definitions degree of influence of individual factors on mechanical equipment failure rate during artillery operation. *Collection of Scientific Works of the National Academy of the State Border Guard Service of Ukraine. Series: Military and Technical Sciences*, 80 (2), 283–299. <https://doi.org/10.32453/3.v80i2.204>
30. Boltentkov, V., Brunetkin, O., Dobrynin, Y., Maksymova, O., Kuzmenko, V., Gultsov, P. et al. (2021). Devising a method for improving the efficiency of artillery shooting based on the Markov model. *Eastern-European Journal of Enterprise Technologies*, 6 (3 (114)), 6–17. <https://doi.org/10.15587/1729-4061.2021.245854>
31. Zheng, Y., Wang, J., Kozlov, O., Kondratenko, G., Aleksieieva, A.; Shevchenko, A. I., Kondratenko, Y. P. (Eds.) (2026). Optimization-oriented Synthesis of Rule Bases of Intelligent Systems: Application Features for Complex Plants' Control. *Artificial Intelligence: Achievements and Recent Developments*. River Publishers, 83–111. <https://doi.org/10.1201/9788743800989-4>

32. Rakityanska, G. B. (2015). Fuzzy classification knowledge base construction based on trend rules and inverse inference. *Eastern-European Journal of Enterprise Technologies*, 1 (3 (73)), 25–32. <https://doi.org/10.15587/1729-4061.2015.36934>
33. Pujaru, K., Adak, S., Kar, T. K., Patra, S., Jana, S. (2024). A Mamdani fuzzy inference system with trapezoidal membership functions for investigating fishery production. *Decision Analytics Journal*, 11, 100481. <https://doi.org/10.1016/j.dajour.2024.100481>
34. Chaudhari, T. U., Patel, V. B., Thakkar, R. G., Singh, C. (2023). Comparative analysis of Mamdani, Larsen and Tsukamoto methods of fuzzy inference system for students' academic performance evaluation. *International Journal of Science and Research Archive*, 9 (1), 517–523. <https://doi.org/10.30574/ijrsra.2023.9.1.0443>
35. Maksymov, M., Kozlov, O., Shynder, A., Maksymova, O., Aleksieieva, A. (2025). Development of mathematical models for temperature control objects in thermal destruction systems based on transient process identification. *EUREKA: Physics and Engineering*, 3, 207–220. <https://doi.org/10.21303/2461-4262.2025.003802>
36. Kondratenko, Y., Kozlov, O., Zheng, Y., Wang, J., Kuzmenko, V., Aleksieieva, A. (2024). Bio-inspired optimization of fuzzy control system for inspection robotic platform: comparative analysis of hybrid swarm methods. *CEUR Workshop Proceedings*, 3711. Available at: <https://ceur-ws.org/Vol-3711/paper7.pdf>
37. Kozlov, O., Kondratenko, G., Aleksieieva, A., Maksymov, M. (2025). Complex Structural-Parametric Optimization of Fuzzy Control Systems Based on Bioinspired Algorithms, *CEUR Workshop Proceedings*, 4048. Available at: <https://ceur-ws.org/Vol-4048/paper01.pdf>