UDC 621.9
DOI: 10.15587/1729-4061.2026.362733

DETERMINING AN ANALYTICAL CRITERION FOR THE EFFECTIVE SPEED OF A SPINDLE UNIT FOR THE AUTOMATIC MACHINING OF LONG WORKPIECES

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This study considers a long bar workpiece fixed in the spindle assembly of a bar lathe according to the "clamping chuck-intermediate radial support" scheme. The task addressed relates to the limitation in the effective spindle speed because of the operating speed approaching the first critical frequency of transverse vibrations of the bar.

A distinctive feature of the results is the representation of the critical speed as a function of three controlled parameters: the diameter of the bar, the length of the calculated span, and the stiffness of the intermediate radial support of the bar. That makes it possible to obtain an engineering criterion for the effective speed of the spindle assembly when machining a long bar workpiece.

Practically significant results are attributed to the features of the adopted model in which the clamping chuck is represented as an equivalent clamping, the intermediate support is considered to be a rigid or elastic-flexible radial support and the dimensionless parameter of relative stiffness μ is used. That has made it possible to reflect the transition from virtually no support to perfectly rigid support and to establish a nonlinear saturable effect of the support stiffness on the critical frequency.

For a solid round steel bar, a compact dependence $n_{kp} = K_{cm} \cdot d/L^2$ ***was obtained, where*** $K_{cm} \approx 1.86 \cdot 10^5$ ***rev m/min. It was established that the span length has a quadratic effect on the critical frequency, the diameter has a linear effect, and an increase in the support stiffness is effective only up to the saturation zone. Based on the resulting relations, the limiting dependences of the effective operation of the system on the frequency load coefficient were constructed.***

The results are suitable for the preliminary selection of the inter-support distance, the bar diameter, as well as the conditions of its support when designing, modernizing, and adjusting bar automatic lathes

Keywords: critical frequency, long workpiece, bar turning, elastic-flexible support, spindle speed

Received 11.03.2026
Received in revised form 18.05.2026
Accepted 27.05.2026
Published 30.06.2026

How to Cite: *Prydalnyi, B., Seleznov, D., Seleznov, E., Muravynets, Y. (2026). Determining an analytical criterion for the effective speed of a spindle unit for the automatic machining of long workpieces. Eastern-European Journal of Enterprise Technologies, 3 (1 (141)), 64–72. https://doi.org/10.15587/1729-4061.2026.362733*

1. Introduction

One way to improve the productivity of automatic processing is to use long metal rods as blanks. This makes it possible to reduce auxiliary time and increase the continuity factor of the part manufacturing cycle. At the same time, an increase in the spindle speed, which is a natural reserve for increasing productivity, for a long rod is accompanied by a complication of the dynamic behavior of the "spindle-rod-support" system. Unlike common short and relatively rigid blanks, a long rod has increased bending flexibility. Therefore, its rotation may be accompanied by an increase in deflections, eccentricity, and transverse vibrations, which are due to its own weight, initial imbalance, clamping flexibility, rigidity of the spindle supports, and conditions of intermediate support. The scheme in which the rod is clamped in the chuck and has one intermediate support is special since it combines local support with the presence of a section prone to the largest bending movements.

The operating speed approaching the natural frequency of such a system creates prerequisites for resonantly dangerous and destructive operating modes. Under these modes, the radial runout of the bar increases, it becomes more difficult to ensure a stable position of the workpiece in the zone of further processing, the conditions for forming the accuracy and quality of the surface deteriorate, and dynamic loads on the elements of the spindle assembly and its supports increase. As a result, the reserve for improving productivity by increasing the speed is limited not only by the drive power or the strength of the machine elements but also by the vibrational stability of the system with the bar. In practice, this means the need to determine in advance, even before the start of cutting, such speed modes under which the rotation of a long bar workpiece does not lead to a dangerous increase in vibrations. Of particular importance in this regard is the establishment of the influence of the diameter of the bar, the distance between the points of its fastening and support, and the rigidity of the supports on the critical speed. It is precisely such dependences that are needed for a well-founded choice of design and operating parameters for the "spindle-bar" system in bar automatic lathes.

Therefore, determining the critical rotation frequency of a long bar in the "chuck-intermediate support" system is an urgent task. This makes it possible, at the stage of development, modernization, and adjustment of equipment, to reasonably determine safe and productive operating modes for the spindle assembly.

2. Literature review and problem statement

Approaches to vibration resistance are generalized in [1], but no engineering criterion for preliminary determination of the critical frequency of the bar is proposed. In studies [2, 3], the stability of the position of the system elements during turning is considered, taking into account the change in the stability limits during the cutting process and the compliance of the system. However, these models do not provide an explicit engineering expression for assessing the influence of the inter-support distance and the characteristics of the supports on the critical spindle speed. In [4], a model of turning stability for workpieces supported on the tailstock is constructed, taking into account the compliance of the tool and workpiece. However, the considered structural scheme corresponds to the "chuck-rear center" system and does not cover the "clamping chuck-intermediate radial support" scheme. In [5], a model of a flexible system of the "tool-workpiece" type is built and the influence of the clamping conditions on the parameters displayed in the stability diagram is shown. However, the result is formulated through the stability limits of the cutting process, and not through the analytical determination of the critical rotation frequency of the bar as a separate dynamic system. In [6] it is proven that the real boundary conditions of the "spindle-workpiece-tailstock" system significantly affect the stiffness and natural frequency of the flexible workpiece. This confirms the fundamental role of the support system, but it is focused on the scheme with support in the tailstock and does not contain a compact criterion for local intermediate radial support of the bar. Study [7] offers an approach to avoiding excessive vibrations for low-stiffness workpieces, taking into account the variable dynamic properties of the system during material removal. Despite the high informativeness of such an approach, it is numerically complex and is intended for building multiparameter stability maps, and not for a quick engineering assessment of the effective speed of the spindle with a bar. In [8], the nonlinear dynamics of the "flexible workpiece-tool" system with delay and internal resonance were investigated. This formulation provides a detailed description of unstable regimes, but its mathematical complexity limits its use for the operational selection of the rotation frequency and the inter-support distance under real equipment setup conditions.

In [9], a continuous model of the turning process was constructed with a refined consideration of cutting forces, system compliance, and position-dependent modal parameters. However, the model is focused on the dynamic characteristics of the turning process and is not aimed at obtaining a simple analytical criterion for a system with a local intermediate radial support. In [10], it was shown that periodic axial perturbation is capable of changing the limits of vibration stability in non-rigid workpieces. This confirms that the dynamic properties of the system can be purposefully modified, but the work does not solve the problem of preliminary determination of the critical rotation speed in the bar support scheme. In [11], the characteristics of a new type of bar workpiece clamping mechanism were determined. However, the nature of their influence on the critical rotation speed of a long workpiece was not established. In [12], an approach to designing automatic clamping mechanisms for machine tool spindle assemblies was proposed. At the same time, there is no consideration of the parameters that determine the critical rotation speed of the bar workpiece. In study [13], a methodology and means of computerized calculation of the characteristics of the electromechanical drive for clamping bar workpieces were devised. However, the possibility of using these characteristics to pre-determine the critical rotation speed of a long workpiece was not shown. The results reported in [14] indicate that the characteristics of new metal materials can affect the dynamic behavior of long workpieces during turning. At the same time, the issue of their influence on the critical rotation speed of the workpiece in the "chuck-intermediate support" scheme requires a separate analytical consideration.

Therefore, known studies mainly focus on vibration stabilization during turning, which is formed on the basis of taking into account the compliance of the "tool-workpiece" system and on a numerically detailed description of the cutting process. At the same time, no simple analytical approach has been found for the "clamping chuck-intermediate radial support" scheme. Such an approach would be appropriate for a preliminary assessment of the critical speed and the influence of the bar diameter, the span length, and the stiffness of the intermediate support.

The above allows us to argue that it is advisable to conduct a study aimed at constructing such an engineering criterion.

3. The aim and objectives of the study

The purpose of our study is to devise an analytical approach to assessing the critical speed of a long bar workpiece in a spindle assembly when it is fixed according to the "clamping chuck-intermediate radial support" scheme. This will make it possible to reasonably determine the effective speed modes of the spindle assembly of an automatic machine tool, taking into account the diameter of the processed bar, the length of the calculated span and the stiffness of the intermediate support.

To achieve the goal, the following tasks were set:

– to form a sequence of analytical calculation of the critical speed of a long bar workpiece in the "clamping chuck-intermediate radial support" system;

– to derive characteristic equations for the natural transverse vibrations of the bar for cases of ideally rigid and elastic-flexible intermediate radial support;

– to establish the patterns of influence of the bar diameter, the length of the calculated span, and the relative stiffness of the intermediate support on the critical speed;

– to perform a numerical estimate of the critical frequency and frequency load factor for typical parameters of steel bars and determine the conditions for preliminary selection of effective speed regimes.

4. The study materials and methods

4. 1. The object and hypothesis of the study

The object of our study is a long bar workpiece rotating in the spindle assembly of an automatic lathe (Fig. 1) according to the "chuck-intermediate radial support" scheme.

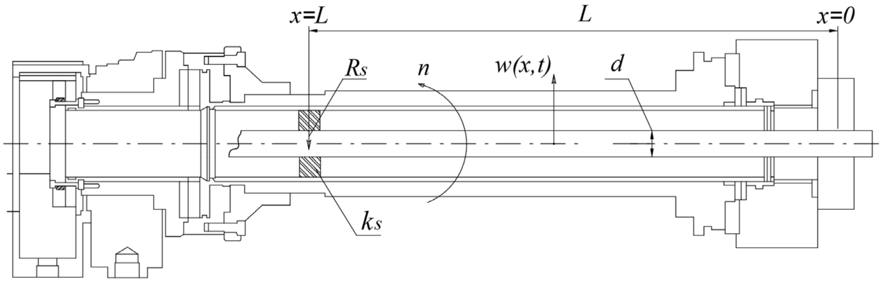

Fig. 1. Simplified diagram of a spindle assembly with a long rod: the chuck is represented as an equivalent clamping, the intermediate support is represented as a radial support with stiffness k_s, L is the length of the calculated span between the clamping point and the point of contact with the support, d is the rod diameter, $w(x, t)$ is the transverse displacement of the rod, $R_s = k_s w(L,t)$ is the reaction of the elastic-flexible support

The hypothesis of the study assumes that the critical speed of rotation of a long bar workpiece in the "clamping chuck-intermediate radial support" system can be estimated using the Euler-Bernoulli beam model if the intermediate support is represented as a rigid or elastic-flexible radial support. Under such conditions, the critical speed can be pre-determined through the first natural frequency of transverse vibrations of the bar. This allows us to link the speed of the spindle assembly with the diameter of the bar, the length of the calculated span, and the rigidity of the bar support, and obtain an engineering criterion for the effective spindle speed.

The subject of our study is the dependence of critical rotation frequency on the diameter of the bar, the length of the calculated span, and the stiffness of the intermediate support.

The following assumptions were adopted in the study. The bar was considered as a straight, uniform rod of circular cross-section with constant mechanical characteristics. In the section between the chuck and the intermediate support, it was modeled by a Euler-Bernoulli beam with small transverse displacements.

The following simplifications were accepted in the work. The influence of the cutting force, damping, contact stiffness of the clamp, as well as gyroscopic and centrifugal effects in the first approximation were not taken into account. The critical rotation frequency was identified with the first natural frequency of transverse vibrations.

In our paper, the terms "pinching", "pivot support", and "elastically compliant support" are used as idealized boundary conditions of the beam model, corresponding, respectively, to a spindle chuck, a perfectly rigid intermediate radial support, and an intermediate support with finite radial stiffness.

4. 2. Differential equation of transverse vibrations of a long bar billet

Transverse vibrations of a bar in the area between the chuck and the intermediate support are described by the Euler-Bernoulli beam model.

We considered a bar of diameter d and span length L with modulus of elasticity E and density ρ, where L is the distance between the equivalent clamping in the chuck and the point of contact with the intermediate support. The cross-sectional area A and the axial moment of inertia of cross section I relative to the neutral axis are defined as

$$A = \frac{d^2\pi}{4},\ I = \frac{d^4\pi}{64}. \quad (1)$$

The transverse vibrations of the rod are described by the following differential equation

$$\rho A \frac{\partial^2 w(x,t)}{\partial t^2} + EI \frac{\partial^4 w(x,t)}{\partial x^4} = 0,\ 0 < x < L, \quad (2)$$

where $w(x, t)$ is the transverse displacement of the point of the rod with coordinate x at time t.

We searched for a solution using the method of separation of variables. Solution in the form of separation of variables is

$$w(x,t) = W(x)\cdot\sin(\omega t), \quad (3)$$

where $W(x)$ is the shape function of oscillations, ω is the circular natural frequency, rad/s.

Substituting (3) into (2) gives the ordinary differential equation for function $W(x)$

$$EI\frac{d^4W(x)}{dx^4} - \rho A\omega^2 W(x) = 0. \quad (4)$$

Dividing by EI produces

$$\frac{d^4W(x)}{dx^4} - \beta^4 W(x) = 0, \quad (5)$$

where

$$\beta^4 = \frac{\rho A\omega^2}{EI}. \quad (6)$$

A general solution to (5) takes the form

$$W(x) = C_1\cosh(\beta x) + C_2\sinh(\beta x) + C_3\cos(\beta x) + C_4\sin(\beta x), \quad (7)$$

where C_1, C_2, C_3, C_4 are the integration constants, which are determined from the boundary conditions of the fixing and support of the bar.

Since the critical rotation frequency in the adopted formulation is identified with the first natural frequency of transverse vibrations, the further solution is associated with the construction of the characteristic equation for the two considered support schemes: rigid and elastic-flexible.

4. 3. Determining the first natural and critical rotation frequency

After determining the smallest positive root Λ_1 of the corresponding characteristic equation, the first natural angular frequency of transverse vibrations was determined from

$$\omega_1 = \frac{\lambda_1^2}{L^2}\sqrt{\frac{EI}{\rho A}}. \qquad (8)$$

Within the accepted formulation, the critical rotation frequency was identified with the first natural frequency, therefore n_{kp} was calculated from

$$n_{kp} = 60 f_1 = \frac{30}{\pi}\omega_1. \qquad (9)$$

and for a round rod – from

$$n_{kp} = \frac{15}{2\pi}\lambda_1^2\sqrt{\frac{E}{\rho}}\cdot\frac{d}{L^2}. \qquad (10)$$

For a rigid support, Λ_1 was substituted into (10), and for an elastic-flexible support, $\Lambda_1(\mu)$. Within the accepted formulation, the critical rotation frequency was identified with the first natural frequency of transverse vibrations of the rod.

Taking into account the relations for the cross-sectional area and axial moment of inertia of a round rod given in (1), formula (9) takes the form.

Expression (10) is the basic analytical dependence for determining the first critical rotation frequency of a long rod in the adopted calculation scheme. It shows that, for fixed mechanical characteristics of the material, the critical rotation frequency is determined by three main factors: the dimensionless root Λ_1, which reflects the conditions of fastening and support, the diameter of the bar d, and the span length L. For a scheme with a perfectly rigid intermediate radial support, the constant root Λ_1 is substituted into formula (10), and for a scheme with an elastically flexible intermediate radial support, the root $\Lambda_1(\mu)$, which is determined from the equation taking into account the relative stiffness parameter μ.

5. Results of devising an analytical approach to assessing the critical speed of a long bar billet

5. 1. Sequence of analytical calculation of the critical speed of a long bar billet

To identify the influence of the geometric and stiffness parameters of the system on the critical speed, parametric analysis was used based on the obtained analytical dependences for schemes with a rigid and elastic-flexible intermediate support. The initial data for the calculation are the modulus of elasticity of material E, density ρ, rod diameter d, span length L, and radial stiffness of the intermediate support k_s.

For each given combination of parameters, it is first necessary to determine the type of intermediate support model. For the case of an ideally rigid intermediate radial support, it is advisable to use the characteristic equation and the constant root Λ_1. For the case of an elastic-compliant intermediate radial support, it is first necessary to calculate the relative stiffness parameter μ, and then numerically find the smallest positive root $\Lambda_1(\mu)$ of the characteristic equation. After determining the root Λ_1, the first critical rotation frequency must be calculated from formula (10). This sequence allows for each set of parameters d, L, and k_s to obtain a single value n_{kp}, which characterizes the boundary of the system's transition to a resonantly dangerous regime.

Parametric analysis was performed by alternately varying one of the system parameters with fixed values of the others. This allows us to separately evaluate:

– the influence of the bar diameter on the critical speed;

– the influence of the span length between the chuck and the intermediate radial support;

– the influence of the stiffness of the intermediate radial support on the change in the critical speed.

For engineering interpretation of the results, it is advisable to compare the calculated critical speed with the specified operating frequency of the spindle n_{rob}. For this purpose, the frequency load coefficient is introduced

$$\eta = \frac{n_{rob}}{n_{kp}}. \qquad (11)$$

The value of coefficient η is used to assess the proximity of the operating mode to the critical one. For $\eta < 1$, the operating frequency is lower than the critical one, for $\eta \approx 1$, the system operates near the resonantly dangerous zone, and for $\eta > 1$, the specified mode should be considered unacceptable from the point of view of vibrational stability.

The arrays of n_{kp} and η values obtained as a result of parametric analysis can be used to construct calculated dependences that reflect the change in the critical rotation frequency depending on the diameter of the bar, the length of the span, and the stiffness of the intermediate support. These dependences form the basis for further representation of the results and the formation of engineering recommendations for the selection of safe speed modes of operation.

For the scheme "clamping chuck-ideally rigid intermediate radial support", the smallest positive root of the characteristic equation is

$$\Lambda_1 = 3.9266. \qquad (12)$$

Substituting the Λ_1 value into formula (10) gives a compact analytical expression for the first critical speed

$$n_{kp} = K\frac{d}{L^2}, \qquad (13)$$

where

$$K = \frac{15}{2\pi}\lambda_1^2\sqrt{\frac{E}{\rho}}. \qquad (14)$$

For long cylindrical steel billets in the form of bars with $E \approx 2\times10^{11}$ Pa, $\rho \approx 7.8\times10^3$ kg/m^3, the numerical value of coefficient K is

$$K_{cm} \approx 1.86\times10^5 \text{ rpm} \qquad (15)$$

Then, for steel bars, formula (13) takes the form

$$n_{kp} \approx 1.86\cdot10^5\frac{d}{L^2}, \qquad (16)$$

where d and L are expressed in meters, and n_{kp} is in rpm.

For a visual representation of the result, formula (16) can be written in terms of the flexibility ratio $\Lambda = L/d$

$$n_{kp} \approx 1.86\times10^5\frac{1}{\Lambda^2 d}. \qquad (17)$$

5. 2. Characteristic equations of the natural transverse vibrations of the bar

In the adopted calculation scheme, the real elements of the system are replaced by the corresponding idealized boundary conditions of the beam model. The spindle chuck in the cross section $x = 0$ is represented as a rigid clamping. The intermediate support of the bar in the cross section $x = L$ is considered in two versions: as an ideally rigid radial support, which in the beam model corresponds to a hinged support, and as an elastic-flexible radial support with a finite stiffness k_s.

In the cross section $x=0$, the conditions of rigid clamping are adopted. Therefore, the boundary conditions take the form

$$W(0)=0,\ \frac{dW(0)}{dx}=0. \tag{18}$$

Taking into account conditions (18), the general solution to (7) takes the form

$$W(x)=C_1(\cosh\beta x-\cos\beta x)+ \\ +C_2(\sinh\beta x-\sin\beta x), \tag{19}$$

Therefore, after taking into account the pinching conditions, the solution to (19) is determined by two integration constants C_1 and C_2.

Scheme with a perfectly rigid intermediate radial support.

For a perfectly rigid intermediate support at the point $x=L$, zero deflection and zero bending moment were assumed. Therefore, in the section $x=L$, the boundary conditions are written as

$$W(L)=0,\ \frac{d^2W(L)}{dx^2}=0. \tag{20}$$

Substituting expression (19) into conditions (20) gives a homogeneous system of algebraic equations with respect to C_1 and C_2. The condition of existence of its nontrivial solution leads to the characteristic equation

$$\sin\lambda\cdot\cosh\lambda-\cos\lambda\cdot\sinh\lambda=0,\ \lambda=\beta L. \tag{21}$$

Equation (21) can be represented in an equivalent form

$$\tan\lambda=\tanh\lambda. \tag{22}$$

The smallest positive root of this equation, $\Lambda_1=3.9266$, corresponds to the first eigenmode of transverse vibrations of the bar for the scheme with a perfectly rigid intermediate radial support.

Scheme with an elastically compliant intermediate radial support.

In the more general case, the intermediate support has a finite radial stiffness k_s. Then at the point $x=L$ the bending moment is still considered to be absent, but the transverse force is balanced by the reaction of the elastic support. For this case, the boundary conditions take the form

$$\frac{d^2W(L)}{dx^2}=0,\ EI\frac{d^3W(L)}{dx^3}=k_sW(L). \tag{23}$$

Substituting expression (19) into condition (23) leads to the characteristic equation

$$1+\cosh\lambda\cdot\cos\lambda+ \\ +\frac{\mu}{\lambda^3}(\cosh\lambda\cdot\sin\lambda-\sinh\lambda\cdot\cos\lambda)=0, \tag{24}$$

where

$$\mu=\frac{k_sL^3}{EI}, \tag{25}$$

is the dimensionless parameter of relative stiffness of the intermediate support.

The parameter μ characterizes the degree of influence of elastic-yielding support on the dynamic behavior of the bar. As $\mu\to\infty$, the elastic support approaches an ideally rigid radial support, and equation (24) transforms into equation (21). As $\mu\to 0$, the supporting action of the intermediate support disappears, and the system approaches the "clamping chuck-free end of the clamped beam" scheme.

5. 3. Influence of bar diameter, span length, and support stiffness on critical speed

For the scheme "clamp chuck-ideally rigid intermediate radial support", parametric analysis was performed on the basis of a compact analytical expression for the first critical speed. Calculations showed that, with fixed mechanical characteristics of the material, the first critical speed monotonically increases with increasing bar diameter. At the same time, it monotonically decreases with increasing length of the calculated span between the clamp chuck and the point of contact of the bar with the intermediate radial support.

For a relative representation of the results, it is advisable to consider the basic calculation case with parameters d_0 and L_0. Then, with a fixed span $L=L_0$, we obtained

$$\frac{n_{kp}(d,L_0)}{n_{kp}(d_0,L_0)}=\frac{d}{d_0}, \tag{26}$$

and at a fixed diameter $d=d_0$

$$\frac{n_{kp}(d_0,L)}{n_{kp}(d_0,L_0)}=\left(\frac{L_0}{L}\right)^2. \tag{27}$$

Relation (26) shows that an increase in the diameter of the rod by m times leads to the same, i.e., m-fold, increase in the first critical frequency of rotation. At the same time, it follows from relation (27) that an increase in the length of the calculated span by m times reduces the critical frequency by m^2 times. Therefore, among the two geometric parameters considered, the span length has a stronger effect on the change in n_{kp} than the diameter of the rod.

For typical design cases, it was found that at $d=40$ mm, the first critical rotation frequency decreased from 4.66×10^4 rpm at $L=0.4$ m to 2.9×10^3 rpm at $L=1.6$ m. At a fixed span of $L=1.0$ m, the decrease in the rod diameter from 40 mm to 20 mm is accompanied by a decrease in the critical frequency from 7.5×10^3 to 3.7×10^3 rpm. These values are consistent with the linear dependence on d and the quadratic dependence on L^{-1}, which follow from formula (16).

The resulting dependences form the base surface $n_{kp}(d, L)$, which is then used to compare with the case of an elastic-flexible support. The results obtained for the scheme with a perfectly rigid intermediate support are further used as reference when analyzing the influence of the finite stiffness of the intermediate radial support on the first critical speed.

For the scheme with an elastic-flexible intermediate radial support, the first critical speed is determined by the value of the root $\Lambda_1(\mu)$ in the characteristic equation (24), where μ is the dimensionless parameter of the relative stiffness of the support. For the relative representation of results, the ratio of the critical frequency for the elastic-flexible support to the corresponding value for the limiting case of an ideally rigid support was used

$$\frac{n_{kp}(\mu)}{n_{kp}^{\infty}} = \varphi(\mu) = \frac{\lambda_1^2(\mu)}{3.9266^2}. \quad (28)$$

Accordingly, the relative reduction in the critical speed compared to the limiting case of a perfectly rigid support was determined from the following expression

$$\Delta_n(\mu) = 1 - \frac{n_{kp}(\mu)}{n_{kp}^{\infty}} = 1 - \varphi(\mu). \quad (29)$$

Calculations have shown that with increasing parameter μ, the $n_{kp}(\mu)$ value monotonically increases, and the $\Delta_n(\mu)$ value monotonically decreases. For large values of μ, the function $\varphi(\mu)$ approaches unity, and the critical rotation frequency approaches the limiting value n_{kp}^{∞}. For small values of μ, the largest deviation from the limiting case of a perfectly rigid intermediate radial support is observed. For fixed values of rod diameter d and the length of calculated span L, the dependence of $n_{kp}(\mu)$ on the stiffness of the intermediate radial support retains the same qualitative character: with increasing k_s, the critical rotation frequency increases, and with decreasing k_s, it decreases. The relative form of this dependence is completely determined by function $\varphi(\mu)$, while the absolute level of the critical frequency is given by parameters d and L according to formula (10).

The resulting dependences $n_{kp}(\mu)$, $\varphi(\mu)$ and $\Delta_n(\mu)$ were used to construct calculation curves characterizing the influence of the stiffness of an intermediate radial support on the critical rotation frequency of a long bar.

5. 4. Numerical evaluation of effective speed regimes for long workpieces in the form of steel bars

For practical representation of the results of parametric analysis, the frequency load coefficient was used

$$\eta = \frac{n_{rob}}{n_{kp}}, \quad (30)$$

where n_{rob} is the operating spindle speed, n_{kp} is the first critical speed, determined from formula (10) for the corresponding combination of parameters d, L, and k_s.

Taking into account formula (10), the frequency load factor for the system under consideration takes the form

$$\eta = \frac{2\pi}{15}\frac{n_{rob}L^2}{\lambda_1^2}\sqrt{\frac{\rho}{E}}\frac{1}{d}. \quad (31)$$

Relation (31) shows that at a fixed operating spindle speed, the η value increases with increasing length of the calculated span L, decreases with increasing diameter of the rod d and depends on conditions of the intermediate support through parameter Λ_1. For a scheme with a perfectly rigid intermediate radial support, $\Lambda_1 = 3.9266$ is substituted into formula (31), and for a scheme with an elastic-flexible support, the $\Lambda_1(\mu)$ value is determined from the characteristic equation (24).

The boundary condition of the safe speed regime is determined from the following relation

$$\eta = 1. \quad (32)$$

Under this condition, for a given operating spindle speed, the maximum allowable length of the calculated span is equal to

$$L_{all} = \sqrt{\frac{15}{2\pi}\lambda_1^2\sqrt{\frac{E}{\rho}}\frac{d}{n_{rob}}}. \quad (33)$$

Accordingly, for given L and n_{rob}, the minimum allowable rod diameter was determined as

$$d_{\min} = \frac{2\pi}{15}\frac{n_{rob}L^2}{\lambda_1^2}\sqrt{\frac{\rho}{E}}. \quad (34)$$

Based on relations (30) to (34), the limits of permissible and impermissible operating modes of the system "clamping chuck-intermediate radial support-rod" in the coordinates n_{rob}-L, n_{rob}-d and d-L were constructed. For region $\eta < 1$, permissible speed modes were obtained, for region $\eta > 1$, modes corresponding to exceeding the first critical speed were obtained. The limit curves $\eta = 1$ were used as a calculation basis for constructing maps of safe operation of the system.

The form of record (17) is equivalent to formula (16) after replacing $L = \Lambda d$. In this case, the $n_{kp} \sim 1/(\Lambda^2 d)$ dependence is valid for the case when the flexibility parameter Λ is considered as an independent variable. According to formula (16), for a rod with a diameter of $d = 40$ mm, it was obtained at: $L = 0.4$ m, $n_{kp} \approx 4.66 \times 10^4$ rpm; $L = 1.0$ m, $n_{kp} \approx 7.5 \times 10^3$ rpm; $L = 1.6$ m, $n_{kp} \approx 2.9 \times 10^3$ rpm. For a rod with a diameter $d = 20$ mm at $L = 1.0$ m, $n_{kp} \approx 3.7 \times 10^3$ rpm was obtained.

Based on the relationships among obtained dependences, generalized parameter maps (Fig. 2) of the safe operation of the "clamping chuck-intermediate radial support-rod" system were constructed. The resulting dependences have made it possible to form such maps for various combinations of rod diameter d, design span length L, spindle operating speed n_{rob} and stiffness of the intermediate radial support k_s.

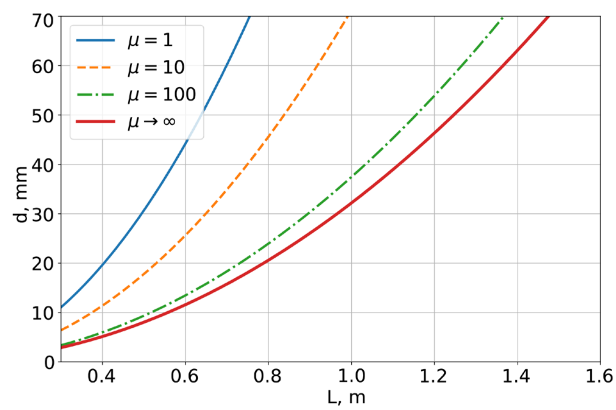

Fig. 2. Map of the limiting curves of mode separation in coordinates d-L for different values of parameter for relative stiffness of the intermediate radial support μ

The dependences (30) to (34) allowed the construction of a number of maps in the main coordinate planes, for example: n_{rob}-L, n_{rob}-d, and d-L. The n_{rob}-L plane allows the display of the limits of permissible values of the operating speed for given rod diameters and stiffness of the intermediate support. The n_{rob}-d plane allows the display of permissible combinations of the speed and rod diameter for given lengths of the calculated span and stiffness of the support. The d-L plane displays (Fig. 3) the limiting combinations of the rod diameter and the calculated span length for a given operating spindle speed.

For the scheme with an ideally rigid intermediate radial support (Fig. 3, a), limit maps corresponding to the maximum level of the first critical rotation frequency within the adopted model were obtained. For a scheme with an elastic-pliable intermediate radial support, similar maps (Fig. 3, b) are constructed for different values of the relative stiffness parameter, which makes it possible to trace the change in the limits of permissible modes with a decrease or increase in support stiffness.

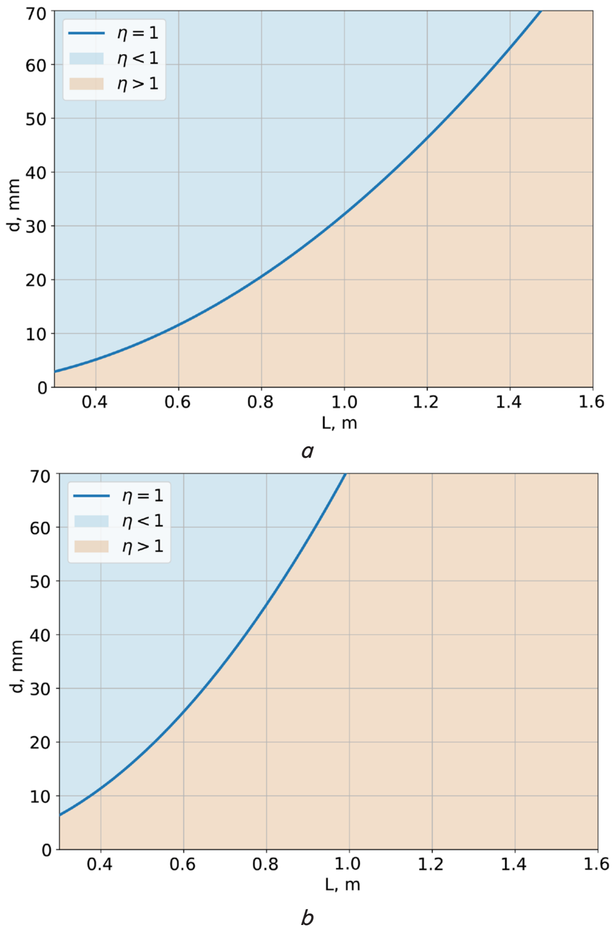

Fig. 3. Acceptability maps of combinations of bar diameter d and design span length L: *a* — for a perfectly rigid intermediate radial support; *b* — for an elastically compliant intermediate radial support at $\mu = 10$

6. Discussion of results based on studying the critical rotation frequency of a long bar billet

The obtained compact expression (16) for the first critical rotation frequency of a long bar metal billet in the spindle assembly of the machine reflects the structure of the dependence $n_{kp} \sim d / L^2$. This is consistent with the general relation (10), in which the natural frequency is determined by the combination of the bending stiffness EI and the linear mass ρA, and for a round section of a bar billet $I \sim d^4$, $A \sim d^2$. Therefore, an increase in the diameter of the bar increases the critical frequency, while an increase in the length of the calculated span reduces it much more. Therefore, even a moderate increase in L causes a significant decrease in n_{kp}, while the influence of diameter d is linear.

For a scheme with an elastically compliant intermediate radial support, the critical frequency is determined not only by the geometry of the bar but also by the relative stiffness parameter μ, introduced by formula (25). Through this parameter, the stiffness of the support affects the root of the characteristic equation (24), and therefore the critical frequency according to the general expression (10). Therefore, the $n_{kp}(\mu)$ dependence has a nonlinear character: for small μ, even a moderate increase in stiffness significantly changes the result, while for large μ, the system asymptotically approaches the limiting case of ideally rigid support. This effect is reflected through functions $\varphi(\mu)$ and $\Delta_n(\mu)$ in formulae (28), (29).

The practical significance of our results is the functional separation of system parameters. The length of span L sets the basic level of the critical frequency, diameter d scales it, and the stiffness of the intermediate support determines the degree of approximation of the real system to the limit scheme with an ideally rigid support. Therefore, the frequency load coefficient η according to formula (30) can be used as a simple criterion for preliminary separation of permissible speed modes from resonantly dangerous ones.

In contrast to approaches in [2, 3, 5, 7–10], which are mainly focused on the limits of vibration resistance of the turning process, regenerative chatter, compliance of the "tool-workpiece" system, or position-dependent modal parameters in the cutting process, this work considers a different statement of the problem: a preliminary analytical determination of the basic critical rotation frequency of a long bar even before the onset of cutting forces. This makes it possible to use the obtained ratios as an engineering criterion of the first approximation, and not as a numerical model of the cutting process itself.

Compared with the models for schemes "chuck-back center" or "spindle-workpiece-back headstock" [4, 6], the proposed model is focused specifically on the scheme "clamping chuck-intermediate radial support", which is important for automatic bar lathes. Its feature is the explicit introduction of local intermediate support and evaluation of its influence through the dimensionless parameter μ. In known works, the emphasis is mainly on the variable stiffness of the system along the cutting path or on the construction of stability diagrams, while a compact analytical criterion for a long-dimensional bar with local intermediate support is missing.

The advantage of the proposed approach is the combination of simplicity of calculation and reproducibility. Using the characteristic equations and analytical dependences for the critical frequency, it is possible to determine the n_{kp} value and estimate the admissibility of the mode through coefficient η. Unlike multiparameter numerical models, this approach is practical for engineering use at the stage of development, modernization, and adjustment of equipment. A certain confirmation of the adequacy of the model within the accepted assumptions is the fact that it is built on the classical Euler-Bernoulli beam formulation, has dimensionally consistent dependences, and reproduces physically expected trends: an increase in d increases n_{kp}, an increase in L decreases n_{kp}, and an increase in μ brings the system closer to the case of an ideally rigid support.

The limits of application of the proposed model are determined by the accepted assumptions for the beam formulation. The model is suitable for preliminary engineering assessment of the first critical speed of a long bar workpiece under conditions of small transverse displacements, linear-elastic behavior of the material, constant bar geometry, and idealized chuck and intermediate support as boundary conditions. Therefore, our dependences should be used primarily for the selection of safe speed regimes before the start of cutting or for preliminary verification of the design parameters of the "chuck-intermediate radial support" system.

With significant transverse deformations, noticeable radial runout, changes in the actual contact of the bar with the support, or significant nonlinearity of the clamp, the given model loses its adequacy since the real stiffness and boundary conditions of the system can change during rotation. In such cases, it is necessary to take into account geometric nonlinearity, contact compliance, damping, possible gyroscopic and centrifugal effects, as well as experimentally refine the parameters of the supports.

The limitation of our research is predetermined by the presence of idealization of boundary conditions. In a real system, the clamping chuck, intermediate support, and contact of the bar with the support have finite and often nonlinear compliance. In the current work, these elements are reduced to simplified equivalent boundary conditions, which facilitates the analytical solution but reduces the completeness of the reflection of the real structure.

The disadvantage of this study is the lack of experimental verification of the analytical dependences derived. Although the model is internally consistent and gives physically expected trends of the influence of d, L, and k_s, its practical applicability should be confirmed by comparing the calculated and measured values of the first critical frequency for real bars.

A practical area for advancing the proposed approach could be the construction of a system for adaptive adjustment of the position of an intermediate support. Such a system could use the calculated determination of the critical frequency for automated separation of operating modes from the resonantly dangerous zone for different actual parameters of the bar and its fastening conditions.

7. Conclusions

1. A sequence of analytical calculation of the critical speed of a long bar blank in the "clamping chuck-intermediate radial support" system has been defined. The sequence involves the transition from the structural scheme to the beam model with equivalent clamping in the cross section $x = 0$ and intermediate radial support in the cross section $x = L$, writing the equation of transverse vibrations, setting boundary conditions, obtaining the characteristic equation and transition from the first natural frequency to the critical speed. This provided a reproducible calculation route for further assessment of the safe speed through the frequency load coefficient $\eta = n_{rob} / n_{kp}$.

2. Characteristic equations have been derived for schemes with an ideally rigid and elastically flexible intermediate support. Based on them, a compact analytical expression for the first critical speed was built. Unlike approaches focused on the stability of the cutting process, the proposed model allows us to determine the basic critical frequency even before the action of cutting forces. This is achieved by isolating the determining root of the characteristic equation and the parameter of the relative stiffness of the support.

3. It was found that the system parameters affect the critical speed unequally. With an increase in the diameter of the bar, it increases linearly, and with an increase in the length of the calculated span, it decreases inversely squared. The effect of the stiffness of the intermediate support is nonlinear and saturable. This is explained by the fact that the parameter L sets the base frequency level, d scales it, and k_s determines the degree of approximation of the system to the case of ideally rigid support.

4. A numerical evaluation was performed for typical steel bars and dependences were constructed that are suitable for selecting safe speed modes for the critical speed and frequency load coefficient. The resulting values confirm the practical importance of the proposed approach, since for long workpieces, dynamic limitations can determine not only the efficiency but also the admissibility of the machining process itself. For example, for a rod with a diameter of $d = 40$ mm, the critical frequency decreases from $n_{kp} \approx 4.66 \times 10^4$ rpm at $L = 0.4$ m to $n_{kp} \approx 7.5 \times 10^3$ rpm at $L = 1.0$ m and $n_{kp} \approx 2.9 \times 10^3$ rpm at $L = 1.6$ m. For a rod with a diameter of $d = 20$ mm at $L = 1.0$ m, $n_{kp} \approx 3.7 \times 10^3$ rpm was obtained. The $\eta = n_{rob} / n_{kp}$ coefficient allows us to quantitatively separate the effective modes ($\eta < 1$) from the modes exceeding the first critical frequency ($\eta > 1$), and the boundary condition $\eta = 1$ is the basis for choosing permissible combinations of d, L, n_{rob} and k_s. In other words, the practical value of our results is the possibility of a reasonable choice of inter-support distance, bar diameter, and intermediate support stiffness when designing, modernizing, and setting bar automatic lathes.

Conflicts of interest

The authors declare that they have no conflicts of interest in relation to the current study, including financial, personal, authorship, or any other, that could affect the study and the results reported in this paper.

Funding

The study was conducted without financial support.

Data availability

All data are available in the main text of the manuscript.

Use of artificial intelligence

"Language of the nation's DNA" https://ukr-mova.in.ua/perevirka-tekstu.

Checking grammar, spelling, punctuation without changing the text.

The entire manuscript was checked for spelling, grammatical, and lexical errors.

The result provided by the AI tool did not affect the conclusions in the study.

Authors' contributions

Borys Prydalnyi: Conceptualization; **Dmytro Seleznov**: Methodology; **Eduard Seleznov**: Formal analysis; **Yulia Muravynets**: Writing – original draft.

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