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DEVISING A TECHNIQUE FOR CONSTRUCTING APPROXIMATE SWEEPS OF HELICOIDS BASED ON THE THEORY OF SURFACE BENDING

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This study investigates the process of constructing approximate helicoid sweeps based on the classical theory of their continuous bending in the surface of revolution. Straight helicoids are non-sweep surfaces; therefore, a flat workpiece for fabricating them can only be an approximate sweep. Such an approximate sweep is a flat ring bounded by the inner and outer arcs of circles whose radii are tabular data. The dimensions of the ring must be such as to ensure a minimum of plastic deformations when forming them into a helicoid coil.

To find the dimensions of the ring, the classical theory of bending of non-sweep surfaces has been applied. According to the theorem of differential geometry, any helical surface can be bent into a surface of revolution. Such bending is carried out by reducing the pitch of the surface to zero: that makes it possible to visually observe the deformation of the surface. The resulting surface of revolution can be approximated by a truncated cone. The exact sweep of the truncated cone will be an approximate sweep of the helicoid turn. This approach is based not on experimental data but on theoretical approaches to the bending process. Depending on the type of helicoid, the surface of revolution can be a catenoid or a single-cavity hyperboloid of revolution. This makes it possible to choose such sections of the surface of revolution for approximation by a truncated cone where it most closely fits it. This will correspond to the minimum of plastic deformations when forming the sweep of the cone into a helicoid turn.

In this work, approximate sweeps have been constructed for straight closed and open helicoids with the same design data: surface pitch, $H = 100$ linear units; radii of the cylinders bounding the surface, $r = 20$ linear units; and $R = 60$ linear units. The results are attributed to a new approach to finding approximate sweeps using the theory of bending of non-sweeping surfaces

Keywords: surface pitch, flat ring, flat workpiece, screw turn, straight helicoid

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1. Introduction

The most common helical surface in technology is a straight closed helicoid, known as a screw. Screws are widely used in various industries for transporting materials [1, 2], for preparing mixtures [3], etc. The straight helicoid has two varieties: open and closed, which are similar to each other but have certain differences. Usually, in engineering practice, these differences are not paid attention to, although they can significantly affect the construction of an approximate spiral sweep. In mechanical engineering practice, approximate methods for constructing a flat workpiece are used, the accuracy of which directly affects the amount of plastic deformations during the formation of the turn [4]. When screws operate under difficult conditions at a large load, then the deviation of the spiral geometry significantly worsens the operating characteristics [5].

The lack of an accurate analytical method for determining the geometric parameters of the workpiece leads to a number of technological problems in real production. The use of approximate empirical or graph-analytical models leads to an uneven distribution of internal stresses when stretching steel rings into a screw turn. In turn, this causes a decrease in the thickness of the screw in critical areas, deviations from the design pitch of the screw, and the emergence of microcracks on the inner edge, which increases the percentage of defects. In addition, minimizing deformation forces is one of the main factors in reducing the energy intensity of production processes. With the evolution of computer technology and numerical methods, the task of finding a flat workpiece for manufacturing a screw turn remains relevant both in theoretical and practical aspects.

2. Literature review and problem statement

Screw turns can be formed from a straight strip, which does not require finding flat blanks. There are machines for winding a strip into a helical surface of a given length without welding individual turns. Such a technology is considered in [6]. It is characterized by technological versatility and high productivity. However, from a geometric point of view and the stress-strain state of the turn, such a technology does not provide high accuracy of the turn without additional corrective operations. In addition, it is energy-consuming due to overcoming significant resistance that occurs during plastic deformation of the strip.

In [7, 8], methods for generating helicoids of various types (including inclined) using geometric algorithms were proposed. However, the authors' attention focused on related computer design using CAD (Computer-Aided Design) and BIM (Building Information Modeling) technologies. The issues of manufacturing such surfaces and studying their operational characteristics were left out of consideration.

In [9], a technology for winding circular sector blanks to form helicoidal surfaces was proposed. In this case, a significant reduction in the deformation force was recorded. However, when using sector blanks, the reliability of the product during its further operation is reduced.

Unlike classical methods that consider the blank as a simple ring, current approaches [10] take into account the internal geometry of the surfaces, which makes it possible to reduce residual stresses in the metal. The authors of [10] emphasize that for accurate design of flat blanks it is advisable to use the differential geometry apparatus to take into account the nonlinear dependence between deformation and helicoid pitch. It should be noted that the methodology proposed by the authors is limited to the case of a constant pitch of the helix. This makes it unsuitable for designing highly efficient screw systems with variable pitch and amplitude parameters.

In [11], the methodology and results of experiments on forming screws from screw blanks are described. For this purpose, forming rollers and screw blanks with different outer and inner diameters were used. In this case, experimental studies play a leading role in finding blanks and manufacturing turns from them. Theoretical studies based on bending a blank for a closed helicoid were conducted in [12]. This study is a continuation of it and concerns two surfaces – closed and open helicoids and their comparison.

The above allows us to state that it is advisable to conduct a study aimed at improving the accuracy of constructing approximate turns of closed and open straight helicoids.

3. The aim and objectives of the study

The aim of our research is to devise a technique for constructing approximate sweeps of turns of closed and open straight helicoids based on continuous bending of these helical surfaces in the surface of revolution. This will make it possible to improve the accuracy of constructing the approximate sweep, which, in turn, will reduce plastic deformations during the manufacture of the turn.

To achieve this goal, the following tasks were set:

– based on the approximation of the surface of revolution into which the straight closed helicoid is bent, to construct an approximate sweep of its turn;

– based on the approximation of the surface of revolution into which the straight open helicoid is bent, to construct an approximate sweep of its turn;

– to compare the sweeps for both helicoids with the same design parameters.

4. The study materials and methods

The object of our study is the process of constructing approximate helicoid sweeps based on the classical theory of their continuous bending in the surface of revolution. It was hypothesized that the application of the theory of continuous bending would make it possible to more accurately determine the parameters of the approximate sweep and compile recommendations for reducing plastic deformations when forming it into a coil. The assumption is that the workpiece material is homogeneous, isotropic, and behaves during deformation in the same way as the theoretical surface during bending. The simplification is that the thickness of the coil and workpiece material is taken equal to zero.

To solve the tasks set in the work, a comprehensive methodology was used, based on a combination of theoretical and applied methods: methods of differential geometry, methods of geometric modeling and approximation, methods of integral and differential calculus, as well as a comparative-analytical method.

To construct intermediate positions when bending a closed helicoid into a known surface of revolution – a catenoid, the following parametric equations were used [13]:

$$X=\sqrt{u^2+b^2-p^2}\cos\left[\varphi-\operatorname{arctg}\frac{pu}{\sqrt{\left(b^2-p^2\right)\left(u^2+b^2\right)}}\right],$$

$$Y=\sqrt{u^2+b^2-p^2}\sin\left[\varphi-\operatorname{arctg}\frac{pu}{\sqrt{\left(b^2-p^2\right)\left(u^2+b^2\right)}}\right],$$

$$Z=\frac{\sqrt{b^2-p^2}}{2}\ln\frac{\sqrt{u^2+b^2}+u}{\sqrt{u^2+b^2}-u}+p\varphi, \qquad (1)$$

where u, φ are independent surface variables, and u is the length of the straight-line generatrix, the reference of which starts from the helicoid axis; φ is the angle of rotation, which for one helicoid varies within $\varphi = 0...2\pi$; b and p are constants. The pitch H of the helicoid is determined by the helical parameter b: $H = 2\pi b$. The shape of the surface depends on the value of the bending parameter p: at $p = b$ – a closed helicoid, at $p = 0$ – a catenoid, at $b > p > 0$ – intermediate positions of the surface during continuous bending.

Analogous equations for bending an open helicoid take the following form [14], and a component of these equations is a directing helix on a cylinder of radius a with a helical parameter b:

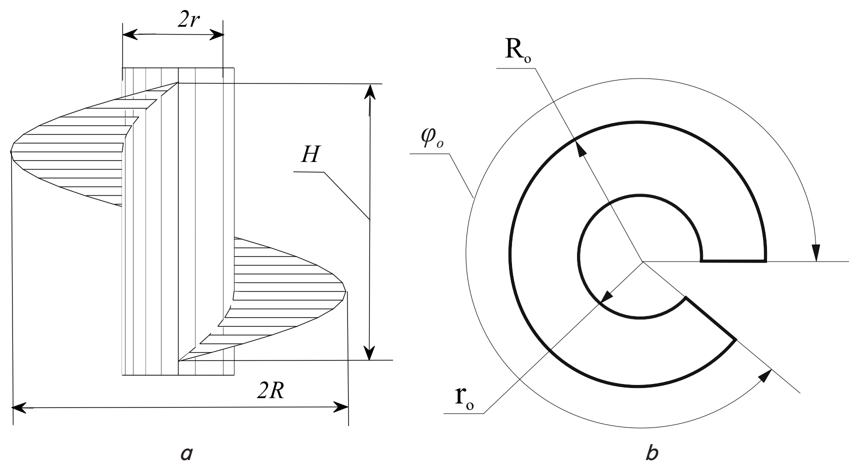


Fig. 1. Graphic illustrations for constructing an approximate sweep of a straight closed helicoid: *a* — frontal projection of the turn with an indication of its dimensions; *b* — approximate sweep of the turn

$$a = r_a \frac{(1-p)\sin^2\beta + \cos^2\beta}{1 - 2p\sin^2\beta + p^2\sin^2\beta},$$

$$b = r_a \frac{p\sin\beta\cos\beta}{1 - 2p\sin^2\beta + p^2\sin^2\beta}. \quad (2)$$

The shape of the helix depends on constants r_a, β, p, where p is the bending parameter, β is the helix elevation angle, r_a is the initial radius value at $p = 1$. A helical linear surface passes through the guide helix (2), which is deformed during bending, and is described by the following parametric equations:

$$X = a\cos\frac{s}{\sqrt{a^2+b^2}} - \frac{u(a\cos\beta + b\sin\beta)}{\sqrt{a^2+b^2}}\sin\frac{s}{\sqrt{a^2+b^2}},$$

$$Y = a\sin\frac{s}{\sqrt{a^2+b^2}} + \frac{u(a\cos\beta + b\sin\beta)}{\sqrt{a^2+b^2}}\cos\frac{s}{\sqrt{a^2+b^2}},$$

$$Z = \frac{bs}{\sqrt{a^2+b^2}} + \frac{u(b\cos\beta - a\sin\beta)}{\sqrt{a^2+b^2}}, \quad (3)$$

where s and u are independent surface variables, and s is the length of the helical guide line, and u is the length of the rectilinear generating surface. At $p = 0$, equations (3) taking into account (2) describe a single-cavity hyperboloid of revolution – the surface of revolution onto which the open helicoid bends. At $p = 1/\sin^2\beta$ – an oblique closed helicoid, at $1/\sin^2\beta > p > 0$ – intermediate positions of the surface during continuous bending, among which there is a straight open helicoid (at $p = 1$).

5. Results of investigating a technique for constructing an approximate sweep of a helicoid coil of the same pitch

5. 1. Construction of an approximate sweep of a straight closed helicoid coil

In a straight closed helicoid, all rectilinear generatrixes intersect its axis at a right angle. The coil is characterized by the following design parameters (Fig. 1, *a*): H – helicoid pitch, R – radius of the outer bounding cylinder, r – radius of the inner bounding cylinder (shaft). The approximate sweep takes the form of a flat ring, the dimensions of which are given in Fig. 1, *b*.

The design parameters of the coil were taken as follows: $H = 100$ linear units, $r = 20$ linear units and $R = 60$ linear units. It was constructed according to equations (1), with $\varphi = 0...2\pi$, $u = 0...60$. Based on the dependence $H = 2\pi b$, the value of the helical parameter was determined: $b = 15.9$. Under the condition of the bending parameter $p = b$.

The frontal projection of the coil is shown in Fig. 2, *a*. Another extreme position of the surface after bending (catenoid), obtained at $p = 0$, is shown in Fig. 2, *b*.

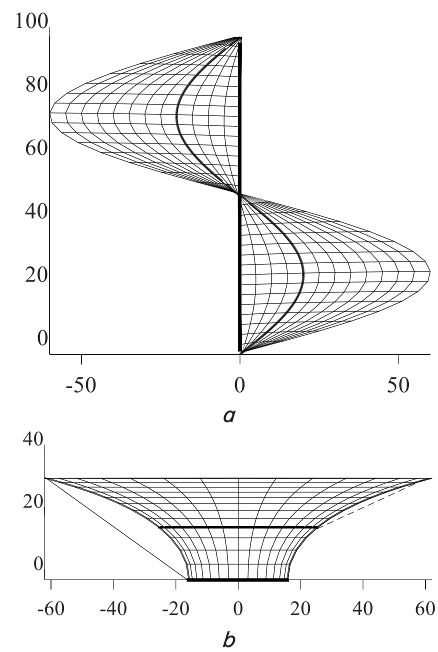
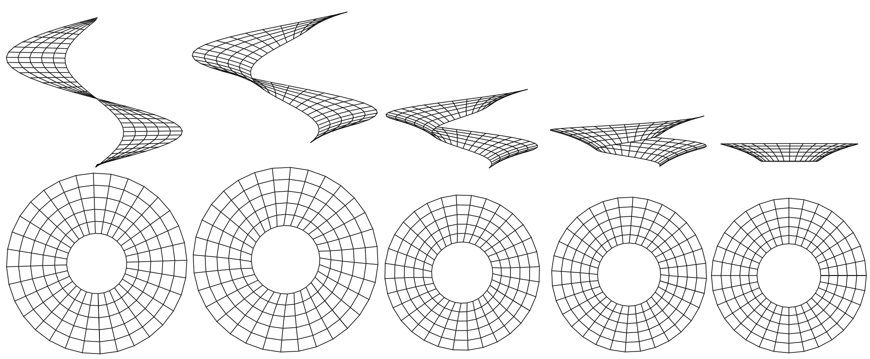


Fig. 2. Frontal projections of the extreme positions of a closed helicoid: *a* — before bending at $p = b$; *b* — after bending to a catenoid at $p = 0$

The straight line (the axis of the surface of the turn), which is marked with a thick line, on the catenoid turns into a circle – its throat line. If the task were to construct an approximate sweep of such a turn, then the catenoid would be approximated by a truncated cone rather roughly. This is evidenced by the large deviation of the generatrix cone in Fig. 2, *b* (solid line) from the meridian of the catenoid. However, part of the surface where the shaft is located must be cut out.

This corresponds to the construction of the surface when u changes within $u = 20...60$. If such a limited section of the surface (from the thick dashed line) on the catenoid is approximated by a truncated cone, then such an approximation is much better (thin dashed line). From this illustrative example, an important conclusion can be drawn: the approximation of the catenoid by a truncated cone improves with an increase in the size of the internal limiting cylinder (shaft). The exact sweep of a truncated cone is an approximate sweep of a closed helicoid. Accordingly, improving the approximation should lead to a decrease in plastic deformations when forming the approximate sweep into a helicoid turn.

By gradually reducing the bending parameter p, it is possible to trace the bending of the surface by reducing its pitch. Fig. 3 shows in projections individual positions of the surface during bending, from which a part is removed since the corresponding space must be occupied by the shaft. A characteristic feature of such bending is that one full turn of the surface remains so during bending. In addition, rectilinear generatrixes of surfaces are only at the beginning of bending (Fig. 3, *a*), after which they become curvilinear.



a *b* *c* *d* *e*

Fig. 3. Individual positions of the surface of a closed helicoid at its continuous bending into a catenoid: *a* – $p = 15.9$; *b* – $p = 12$; *c* – $p = 8$; *d* – $p = 4$; *e* – $p = 0$

After substituting $u = \text{const}$ into the surface equation (1), a line was described on the surface. At $p = 0$, a circle was obtained on the surface of the catenoid. For $u = 20$, a circle was obtained, indicated in Fig. 2, *b*, by a dashed line, and at $u = 60$, a circle of a larger base. Their radii are determined from the square root of equations (1) before the trigonometric functions. They are denoted, respectively, as $r_c = 25.5$, and $R_c = 62$. If the values $u = 20$ and $u = 60$ were substituted into the last equation (1), then the difference from the obtained results determined the height of this catenoid compartment would be $h_c = 15.7$. These dimensions can be taken as the dimensions of a truncated cone that approximates the catenoid compartment and whose generatrix in Fig. 2, *b* is depicted by a thin dashed line. Having these dimensions, we can find the radii of the outer R_0 and inner r_0 of the bounding circles of the flat ring (Fig. 1, *b*):

$$R_0 = \frac{R_c\sqrt{h_c^2 + \left(R_c - r_c\right)^2}}{R_c - r_c},$$

$$r_0 = \frac{r_c\sqrt{h_c^2 + \left(R_c - r_c\right)^2}}{R_c - r_c}. \qquad (4)$$

After substituting the dimensions of the truncated cone into (4) and performing calculations, we can obtain: $R_0 = 67.6$, $r_0 = 27.8$. The φ_0 angle is determined provided that the length of the arc of the circle of the truncated cone is equal to the length of the corresponding arc of the circle on the flat ring, i.e., $2\pi R_c = \varphi_0 R_0$. Hence, we can determine $\varphi_0 = 330°$.

5. 2. Construction of an approximate sweep of a turn of a straight open helicoid

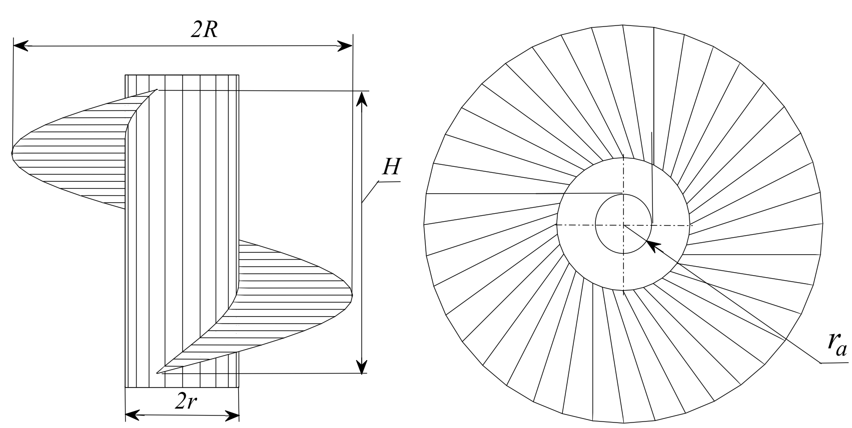
A straight open helicoid (Fig. 4) differs from a closed one in that its rectilinear generatrixes, which also form a right angle with the axis, do not intersect it, but are transverse to it, i.e., tangent to a circle of radius r_a (Fig. 4, *b*). As the radius r_a decreases, the surface approaches a closed helicoid. Inside the space bounded by a cylinder of radius r_a, the surface does not exist. The parameters of the turn are similar to those previously accepted: $H = 100$, $r = 20$ and $R = 60$. In this case, the radius r_a can be taken from the interval $0 < r_a \le r$ (at $r_a = 0$, equations (2), (3) do not work and, in this case, it is necessary to proceed to equations (1)). The average value from this interval can be taken, i.e., $r_a = 10$.

At $p = 1$, a straight open helicoid passes through the helical line (2). In this case, $a = r_a$ and $b = r_a \text{tg}\beta$. The helical parameter $b = 15.9$, as in the previous case. Hence, we can determine the angle β: $\beta = \text{arctg}(b/r_a) = 57{,}8°$. To construct the surface using equations (3), we need to know the limits of parameter u. For a closed helicoid, its reference was from the axis, i.e., it was within $u = r...R$. For an open helicoid, such reference does not work, since the straight-line generatrix does not intersect the axis. The distance ρ from the axis of the surface to a point on it is given by formula $\rho = \sqrt{X^2 + Y^2}$, where X and Y are equations (3) for the horizontal projection of the surface. After algebraic transformations, we find

$$\rho = \sqrt{a^2 + \frac{u^2}{a^2 + b^2}\left(a\cos\beta + b\sin\beta\right)^2}. \qquad (5)$$

At $p = 1$ based on (5) we can derive

$$\rho = \sqrt{u^2 + r_a^2}, \text{ hence } u = \sqrt{\rho^2 - r_{a2}}. \qquad (6)$$



a *b*

Fig. 4. Projections of a straight closed helicoid with an indication of its design parameters: *a* – frontal projection; *b* – horizontal projection

In order to find the required section of the open helicoid, which is located between cylinders $r = 20$ and $R = 60$, these values must be substituted in turn into (6) and find the limits of change in parameter u: $u = 17.3...59.2$. It is advisable to construct a full turn of the oblique closed helicoid at $p = 1/\sin^2\beta$. In this case, as in the case of $p = 1$, according to (2) we can find: $b = r_a \text{tg}\beta$. The pitch of the surface $H = 2\pi b = 2\pi r_a \text{tg}\beta$. This value is the length of the straight line into which the arc of the circle of radius r_a is transformed. Therefore, the limits of change in arc length s were as follows: $s = 0...2\pi r_a \text{tg}\beta = 0...100$. In Fig. 5, *a*, according to equations (2), (3), a full turn of the oblique closed helicoid is constructed. The parameter u varied within the range of $u = 0...59.2$, and for $u = 17.3$, the thick dashed line shows the boundary of the inner part of the surface that needs to be removed for the shaft.

Fig. 5, *b* shows a single-cavity hyperboloid of revolution, onto which an oblique closed helicoid is bent. It should be noted that the straight open helicoid, which is the target and which is depicted in Fig. 4, is an intermediate surface during bending when $p = 1$. As can be seen from Fig. 5, *b*, the approximation of the hyperboloid by a truncated cone also improves as one moves away from the thickened throat line or from the thickened straight line in Fig. 5, *a*. The arc of the circle r_a on the original surface (Fig. 4, *b*) transforms into these lines. For this reason, the r_a value was taken midway between $r_a = 0$ and $r_a = r$.

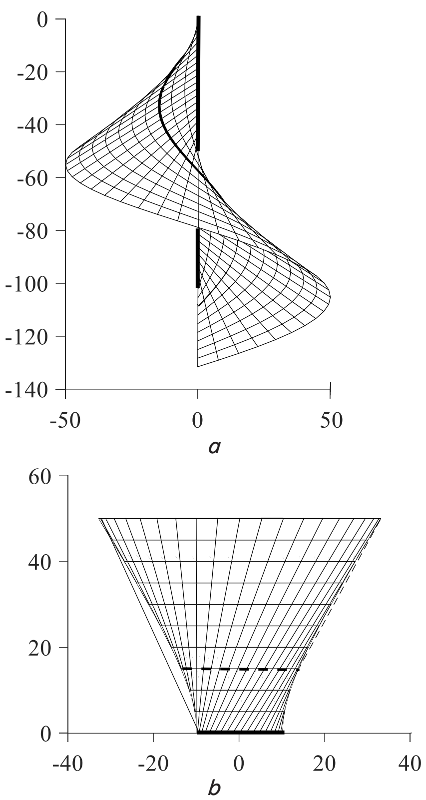


Fig. 5. Frontal projections of the extreme positions of the open helicoid: *a* — before bending at $p = 1/\sin^2\beta$; *b* — after bending onto a hyperboloid of revolution at $p = 0$

Fig. 6 shows, in projections, individual positions of the surface during bending, in which the inner part of the surface, starting from the thickened dashed line, is removed. At $p = 1$ it becomes a tangent line to the shaft of radius $r_a = 20$.

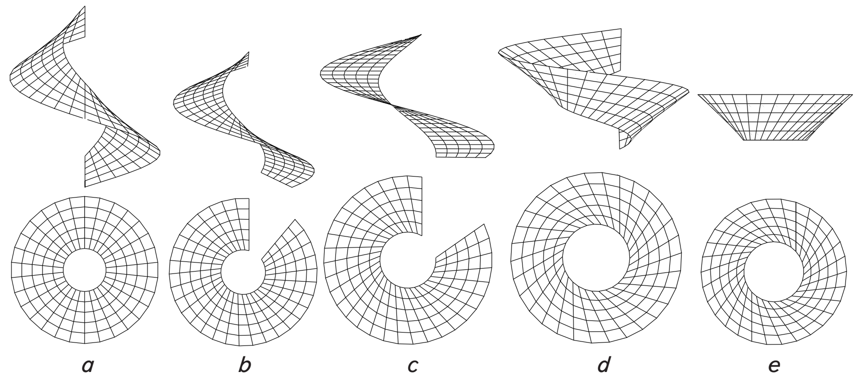


Fig. 6. Individual positions of the helicoid surface during its continuous bending into a hyperboloid: *a* — $p = 1/\sin^2\beta$; *b* — $p = 1.2$; *c* — $p = 1$; *d* — $p = 0.6$; *e* — $p = 0$

A characteristic feature of such a bending is that only the initial turn is a closed helicoid while the intermediate positions are open helicoids. With such a bending, the straight generatrixes remain straight, and a full turn can be transformed into an incomplete one, as, for example, in Fig. 6, *c* at $p = 1$.

It is required to find an approximate sweep of one full turn of a straight open helicoid, that is, of the intermediate surface at $p = 1$. At $p = 0$, the straight open helicoid bends into a single-cavity hyperboloid of revolution (Fig. 5, *b*, 6, *e*). The radii of its larger and smaller base can be determined. For this purpose, formula (5) is used at $p = 0$ in expressions a and b, by alternately substituting instead of u its value $u = 59.2$ for the larger base (outer edge of the helicoid) and $u = 17.3$ (inner edge, which should contact the shaft). The result of our calculations is $R_c = 33$ and $r_c = 13.6$.

From the last equation (3), it is possible to determine the difference of coordinate Z, which is height h_c of the hyperboloid section. As a result, it is possible to derive: $h_c = 35.5$. These dimensions are taken as the dimensions of the truncated cone, which quite accurately approximates the section of the hyperboloid of revolution (this can be seen from Fig. 5, *b*, on which the straight-line generatrix of the cone is depicted by a thin dashed line). According to formulae (4), it is possible to find the radii of the circles that limit the flat ring: $R_0 = 68.3$, $r_0 = 27.7$. The next step was to determine the angle φ_0 (Fig. 1, *b*).

In the initial state, the projection of the internal helix of the helicoid, which is in contact with the shaft, has a length of $2\pi r$. The height of one turn $H = 100$. Hence, we can find the length of the internal helix of the helicoid: $s = \sqrt{(2\pi r)^2 + H^2} = 160.6$. During the bending process, its length does not change, respectively, the inner circle of the flat workpiece had such a length. On the other hand, its length is determined from the product $s = \varphi_0 r_0$. By equating these lengths, we can find $\varphi_0 = 332°$.

5. 3. Comparative analysis of the process of bending and construction of approximate sweeps of straight helicoids

The approximate sweeps for closed and open straight helicoids with the same design parameters are practically the same. But the bending process and the dimensions of the truncated approximating cones differ significantly. For example, the height of the truncated cone that approximates the section of the hyperboloid is more than twice the height of the similar cone for the section of the catenoid. It is important that the approximation of the hyperboloid is better. This is explained by the fact that the meridian of the hyperboloid, i.e., the branch of the hyperbola, as it moves away from the throat line asymptotically approaches the straight line – the generatrix of the approximating cone. Unlike the closed helicoid, when bending the open straight generatrixes always remain straight. To some extent, this creates an analogy with the bending of unfolded surfaces.

If during the bending of a closed helicoid the turn remains complete all the time, then in the second case this does not happen. Fig. 7 plots the change in radius ρ of the inner line of the turn, which in the initial state is in contact with the shaft, as the bending parameter p decreases to zero.

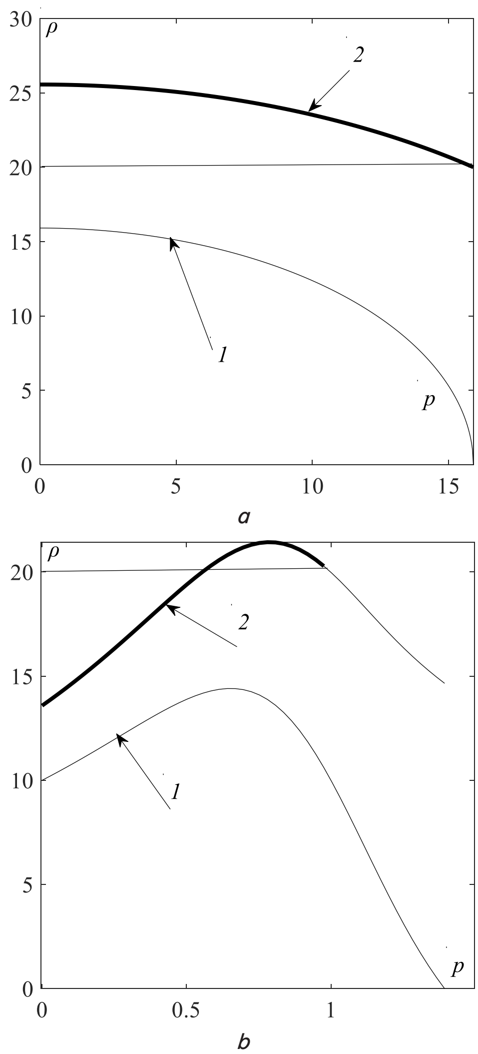


Fig. 7. Plots of change in the helical lines of a surface when it is bent into a surface of revolution, i.e., as the parameter p decreases (1 – a line that is initially straight and coincides with the axis of the surface; 2 – a line that is initially in contact with the shaft): *a* – closed straight helicoid; *b* – open straight helicoid

When we consider closed or open straight helicoids, we mean that they are such only in the initial state (closed – at $p = 15.9$ and open – at $p = 1$). At other values of the bending parameter p, other helical surfaces were obtained, and at $p = 0$ – surfaces of revolution. If in a closed helicoid, when its pitch decreases, the space for the shaft increases from $\rho = 20$ to $\rho = 25.6$ (Fig. 7, *a*, curve 2), then in an open helicoid it initially increases and then decreases (Fig. 7, *b*, curve 2). This corresponds to the fact that the surface first "untwists" and then "twists" (Fig. 6). In the final state, when it becomes a section of a hyperboloid, its surface overlaps in a certain sector, the value of which can be found if necessary.

6. Discussion of results based on constructing approximate helicoid sweeps

Our results are explained by the application of the classical theory of bending of helical non-sweep surfaces in the surface of revolution. Despite the similarity of the turns of closed (Fig. 1, *a*) and open (Fig. 4, *a*) straight helicoids, the surfaces of revolution on which they bend differ significantly in both shape and size (Fig. 2, 5, *b*). However, the sweeps of approximating truncated cones, which are approximate sweeps for helicoids, are the same with minor deviations. As can be seen from the figures (Fig. 2, 5, *b*), the approximation of the open helicoid is more accurate. This means that when forming the same flat workpiece (approximate sweep) into an open helicoid, the plastic deformations will be smaller than when forming it into a closed helicoid.

The bending of helical surfaces in the surface of revolution is known in differential geometry. For example, in Fig. 3, a number of intermediate positions are constructed for continuous bending of the surface of a closed helicoid into a catenoid according to equations (1). In [14], the practical implementation of such bending is considered using the example of a straight open helicoid. However, other parametric equations are given there that describe its continuous bending. The use of new equations has made it possible to approach the bending of an open helicoid in a new way and obtain new results. The intermediate positions of such bending are given in Fig. 6. There is a significant difference between the bending of these surfaces.

In the first case (straight closed helicoid), the full turn of the surface remains so until it is transformed into a catenoid (Fig. 3). In the second case, the surface first "untwists" around the axis and turns into an incomplete turn. Then it begins to "twist" and becomes a full turn at $p = 0.6$ (Fig. 6). With a further decrease in the pitch, further "twisting" occurs with the overlap of the turns in a certain sector in the top view. The transformation of the inner edge during bending for both surfaces, which at the beginning of bending adjoins the shaft, is demonstrated by plots in Fig. 7.

There is a practice of manufacturing screws in workshops that do not have the necessary machines to form the desired helical surface. The technique involves welding together the required number of flat rings (approximate reamers), which correspond to the number of turns, and their subsequent stretching. Stretching occurs along a shaft, onto which welded rings are previously strung. One end of the welded rings is welded to the shaft, and the other is stretched using a winch along its axis. In principle, with such a technology, it would be necessary to simultaneously stretch and "tighten" the ring to an angle of $360° - \varphi_0$, which corresponds to one turn. With such simultaneous stretching and "tightening", the ring will turn into a helical surface, which will be closer in shape to an open helicoid than to a closed one. This is explained by the fact that the formation of an open helicoid is associated with smaller plastic deformations, therefore requiring less energy consumption. This is a theoretical study on the process of bending a flat workpiece into a coil, which helps understand its essence and take it into account in the practical manufacture of a coil.

The limitation of our research is that it concerns helical surfaces only of constant pitch, limited only by cylindrical surfaces. The disadvantage is that after forming a flat ring into a finished coil, its geometry may differ from the theoretical form. This is due to the fact that after the termination of the action of deforming forces, partial unbending of the coil may occur. Future studies could find approximate helicoid sweeps limited not by cylindrical but by conical casings.

The proposed technique makes it possible to abandon labor-intensive methods of physical prototyping. Our results could be implemented in agricultural engineering, chemical and construction industries, transport, and logistics systems, etc. The economic and technological effect of the implementation of the proposed technique is expected to be due to the reduction of material consumption of production (minimization of allowances for cutting the edge of the coil), reduction of energy consumption of the processes of stamping or stretching the rings, as well as an increase in the operational life of screw surfaces through the elimination of zones of concentration of residual plastic stresses.

7. Conclusions

1. A straight closed helicoid is bent by gradually decreasing the pitch onto a known surface of revolution – a catenoid. A certain catenoid section corresponds to the helicoid section between the cylindrical casings, the dimensions of which are found through the design parameters of the helicoid coil. This section is approximated by a truncated cone, the exact sweep of which is an approximate sweep of the helicoid coil. As a result of theoretical bending, it has been shown that the approximation accuracy increases as the radius of the internal limiting cylinder increases. An approximate sweep of the helicoid coil with a pitch $H = 100$ and limiting cylinders of radii $r = 20$ and $R = 60$ has been constructed.

2. A straight open helicoid is also bent by gradually decreasing the pitch onto a known surface – a single-cavity hyperboloid of revolution. The approximation accuracy also improves as the radius of the internal bounding cylinder increases as the meridian of the hyperboloid, i.e., the hyperbola, approaches the straight line – the generatrix of the approximating cone. Similarly to the first case, its dimensions were found and an approximate sweep of the helicoid with the same design parameters was constructed.

3. Closed and open straight helicoids with the same dimensions practically do not differ from each other. However, they bend into different surfaces of revolution, which are approximated by truncated cones. The dimensions of these cones differ significantly in size from each other, although their sweeps are almost the same. It is characteristic that for an open helicoid, the approximation of its surface of revolution by a truncated cone is much better than for a closed one. This means that from a flat blank that has almost the same dimensions for both helicoids, both closed and open helicoids can be formed. However, when forming an open helicoid, the plastic deformations will be smaller.

Conflicts of interest

The authors declare that they have no conflicts of interest in relation to the current study, including financial, personal, authorship, or any other, that could affect the study and the results reported in this paper.

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Data availability

All data are available in the main text of the manuscript.

Use of artificial intelligence

The authors confirm that they did not use artificial intelligence technologies when creating the current work.

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Authors' contributions

Andrii Nesvidomin: Methodology, Software; **Serhii Pylypaka**: Conceptualization, Supervision; **Tetiana Volina**: Writing – original draft, Project administration; **Tetiana Kresan**: Formal analysis, Data Curation; **Oleksandr Savoiskyi**: Methodology, Writing – review & editing; **Oksana Yurchenko**: Resources, Writing – review & editing; **Oleksandr Savchenko**: Validation, Writing – review & editing; **Serhii Borodai**: Investigation, Data Curation; **Olha Zalevska**: Validation, Visualization; **Olena Nalobina**: Writing – review & editing, Visualization.

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