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DETERMINING TRANSIENT PATTERNS IN PARALLEL-CONNECTED TRANSFORMERS OF DIFFERENT CAPACITIES

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This study investigates transient processes and current distribution in parallel-connected transformers on a common load. The task addressed relates to the fact that the mathematical description of processes is performed in instantaneous current values, in which the parameters used are widely known on the one hand while rarely used in practice on the other. This approach allows a single system of differential equations to describe most operating modes of transformers, as well as transitions between them at the same time. That, in turn, makes it possible to obtain separate instantaneous values of currents for each transformer and the balancing current between them. That is, to control the load distribution between transformers, energy losses, both under steady-state and transient modes. It is almost impossible to acquire such data with control and measuring equipment.

This work proves that the ratio of active and reactive components of the windings should also be taken into account. Short-circuit voltages can be the same while the ratio between active and reactive resistances of the windings can be different. And this affects the transient processes and the balancing current. Such results were achieved by compiling differential equations of parallel-connected transformers and the load.

It was established that the total duration of transients decreases when transformers are connected in parallel. At the same time, the duration of transients of the balancing current does not depend on the load at all but only on the parameters of the transformer and on the moment of switching.

The studies were conducted on transformers TM-180/10 and TM-530/10; it was confirmed that the balancing currents take small values, compared to the load currents, but they are on the same order of magnitude as the magnetizing currents and can distort the magnetizing curve, which provokes higher harmonics. For the specified transformers, the duration of transients on the balancing currents, under certain conditions, was more than 1 second, and when a short circuit occurs on the common buses, more than 2 seconds.

This study showed that it is possible to measure most of the parameters for the specified system of equations on a real transformer. It also becomes possible to calculate them according to the specifications and compare the operation of the models under different modes. This could allow a more objective assessment of the technical condition of the transformer

Keywords: transformer inductance, transient processes, current distribution, balancing current, operating modes

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1. Introduction

The use of power transformers in parallel connection is widely used in the energy industry due to objective reasons. In particular, the load of consumers is of a probabilistic nature and varies within wide limits. The maximum efficiency of each transformer is its own and has a fixed value according to the load. A significant deviation of the power of consumers from the maximum point leads to no less significant losses of electrical energy over the years of operation. Therefore, energy structures are forced to install 2–3 transformers in parallel connection, in order to coordinate them with the capacities of the transformers as the load of consumers changes. Installing identical transformers of the same capacity narrows the maneuver in terms of capacities. Achieving a proportional distribution of capacities between transformers is half the battle. The other half is not to provoke transient currents during their switching, to which the relay protection would react as emergency.

The requirements for the conditions of their parallel operation are known, as well as disadvantages and advantages.

Considerable efforts are made to minimize energy losses during parallel operation and to achieve proportional distribution of power. In [1], this goal is sought to be achieved by aligning the power distribution between them with the variation of the taps for regulating the voltage of transformers. This path requires, in addition to the construction of models, further refinements on specific groups of transformers. In practice, such refinements are extremely difficult to carry out. In addition, such a factor as the influence of the balancing current on the additional costs of the transformer is not properly taken into account. And these losses, like idle losses, are always present and, in fact, do not depend on the degree of transformer loading.

The issue of parallel operation of transformers, under the condition of an allowable deviation of the transformation ratios, is addressed in work [2]. It is rightly emphasized that the short-circuit impedance of transformers is quite small, and therefore, with a slight deviation of the transformation ratios, significant balancing currents arise. This is true but the fact that not only the impedance value is important for power distribution but also the ratio of the active and reactive components of the impedance is ignored. This is especially evident under the transient modes of transformers operating in parallel.

It is also considered permissible to operate transformers in parallel with certain deviations in the short-circuit voltage [3]: the studies were based on the use of the SCADA program. The permissible limits of deviations in the short-circuit voltage were established taking into account the load. The fact is that the operation of a transformer is always accompanied by transient processes and they occur differently in separate and paired transformers. And the fulfillment of the conditions in the steady state does not guarantee their trouble-free course in the transient.

In engineering calculations of power supply systems, which include transformers, their various models are widely used. The most widespread are the "G", "T", "P"-like equivalent circuits. Models for linear and nonlinear transformers are built on their basis (taking into account hysteresis). For stationary modes, these models give quite satisfactory results. The situation changes somewhat if it is necessary to describe the entire transformer operation cycle with one model: start-up under no-load conditions, loading, parallel operation, unloading, and short-circuit occurrence. For each case of the transient process, specific additional corrective elements are introduced. This makes research into this area a relevant task.

2. Literature review and problem statement

A traditional system of differential equations describing the operation of a transformer in dynamics is given in [4]. But this expression remained purely theoretical, a number of the given coefficients are not used in practice. At the same time, they are available for measurements and are reasonably informative for assessing the state of the transformer as a whole. This applies to the no-load inductances from the high and low sides, the mutual inductance coefficient, leakage inductances, and active resistances of the windings. The reason is probably the historical course of circumstances associated with the evolution of digital technologies. The logical transition of the description of a single-phase transformer to a three-phase, and then to a parallel connection, with a load, complicated the procedure for their solution. An option for using this type of equations is to build an accompanying mathematical model that describes the steady-state and transient processes of a combination of substations and power lines in sequential time. However, in [5–7], partial issues in this direction are resolved.

In particular, in work [5], a specific brand of power transformer is given, and it is proposed to calculate the values of inductances and active resistances according to the conventional equivalent circuit. But these data are conditional, they cannot be directly measured on the transformer, for comparison and assessment. The mathematical description of transient processes in a transformer due to flux linkage using the Runge-Kutta method is covered in [6]; however, the resulting plot of the transient process is not given, so it is difficult to assess how the results correspond to actual processes.

An important factor that has a decisive influence on the course of the transient process is the moment of its occurrence, in relation to the external supply voltage, as indicated in work [7]. The EMTP/ATP modeling system is based on the use of standard, previously mathematically described, components of power systems. This approach, of course, simplifies the analysis process itself but makes it impossible, for example, to find the surge current in connection with the moment of occurrence of a short circuit and the nature of the prehistory before its occurrence. Paper [8] highlights the possibilities of modeling transformer transient processes in the MATLAB SIMULINK environment, confirms the reliability of the results. Both the EMTP/ATP [7] and MATLAB SIMULINK [8] environments make it possible to construct effective models that successfully solve engineering problems. Modeling involves objects, it is relatively simple but in certain circumstances complications arise. In particular, this applies to cases when not one transient process is considered but their combination is described – switching on under idle mode, load introduction, load discharge, short circuit, operation of transformers with different capacities in parallel, in addition, taking into account the circumstances under which they arose. In this sense, MATLAB allows one to compose systems of differential equations that describe the sequence of processes. But the program requires bringing the systems of differential equations to the standard form with respect to the senior derivative. And then determine the initial conditions of all derivatives, except the senior. For differential equations above the second order, this becomes burdensome.

Parallel operation of power transformers that have non-identical characteristics is a common practice; the limits of permissible deviations are established. In work [9], using the SCADA system, a methodology for assessing the economic consequences of parallel operation of transformers with certain deviations in indicators is devised. But this equipment does not make it possible to make a preliminary forecast of what consequences this or that combination of ratios of permissible deviations in transformer parameters and unknown ratios of active and reactive power of common consumers may lead to. The admissibility of parallel operation of transformers with different indicators is given in [2], in which analysis of models confirmed the possibility of their parallel operation. Although the influence of transient processes during transformer operation on the common load was not analyzed. The analysis of optimal modes of parallel operation of transformers was carried out in [10]; attention is drawn to the expediency of taking into account reactive power overflows. At the same time, there is no algorithm on how to take them into account and under what circumstances it is advisable to do so.

The features in determining the short-circuit voltage taking into account the asymmetry of the magnetic field are described in [11]. It is indicated how this affects the accuracy of determining this parameter and the subsequent distribution of the load. The aspect of the significance of this influence on the redistribution of power was left out of consideration. Work [12] considers the transient processes that occur when switching on parallel-connected unloaded transformers. Proposals are made regarding the limitation of the inrush current and the possible course of events, but the phenomenon is considered in isolation from other processes related to the operation of transformers, such as starting under partial load. Perhaps 2–5% of the load could solve the inrush current problems but how the nature of the limited load affects these and other issues have not been fully clarified.

Thus, papers [1–4, 9] do not consider the issue of current distribution between parallel-operating transformers, depending on the ratios of the active and reactive components of the windings. All studies are conducted around generally accepted parameters, short-circuit voltage, and load factor with subsequent optimization of transformer operation. Transient process models [5, 8] concern only a single transformer, also in [6], but for the no-load mode, similar to work [7].

So, in the papers considered above, transient processes of a single transformer and steady-state modes when they are connected in parallel are studied. But issues related to the description of the transformer operation in such a way that the ideology of the model works both for a single transformer and for the group were left out of attention. In this case, the parameters of the model could be measured on transformers themselves and then compared with the calculated ones. Also, there is an urgent desire for one and the same model to describe the entire transformer operation cycle: power-up, changes in both the magnitude and nature of the load, and the occurrence of a short circuit, all within a single operating cycle.

Therefore, most transformer calculations are carried out using conventional equivalent schemes. Each individual transformer operating mode requires a specific method, specifically, steady-state, transient, and short-circuit conditions. A graphical method is generally used to determine short-circuit currents. Inrush currents are determined without taking into account their moment of occurrence and the nature of the load. All these problems pile up in importance when transformers are connected in parallel.

3. The aim and objectives of the study

The purpose of our study is to determine patterns of current distribution between parallel operating transformers of different power under transient and steady-state modes. This will make it possible to avoid false triggering of relay protection and minimize energy losses.

To achieve the goal, the following tasks were set:

– to establish correspondence of parameters from the specified system of differential equations and specifications of typical transformers;

– to investigate the ability of the model to describe transient processes occurring when two typical transformers with different rated power and different load characteristics are connected in parallel;

– to determine parameters of the balancing current under transient and steady-state modes.

4. The study materials and methods

The object of our study is transient processes and current distribution in parallel-connected transformers on a common load. The mathematical description of the processes is performed in instantaneous current values where the parameters used are, on the one hand, widely known, and on the other, rarely used in practice. This applies to the inductance of the primary and secondary windings of the transformer, the dissipation factors, and the coupling factor, which are included in the following system of differential equations of the transformer:

$$\begin{cases} U_0 \sin\left(\omega t + \varphi\right) = L_1 i_1' + i_1 r_1 - M i_2'; \\ M i_1' = L_2 i_2' + i_2\left(r_2 + R\right), \end{cases} \quad (1)$$

where r_1, r_2, R – active resistance of the primary, secondary winding of the transformer and the load; L_1, L_2, M – inductances of the primary, secondary, and mutual inductance of the transformer windings; i_1, i_2, i_1', i_2' – currents and their derivatives of the primary and secondary windings; ω – angular frequency; φ – switching phase; U_0 – amplitude value of the supply voltage.

Taking into account the fact that the control over transient and balancing currents in transformers connected in parallel by means of control and measuring instruments is extremely burdensome, the following hypothesis was proposed. Having found the parameters of the transformer in accordance with (1), one can verify the reliability of the obtained currents compared to specifications. This gives reason to believe that the currents obtained with parallel connection will also be reliable. Thus, one can find instantaneous currents for each transformer separately, as well as their common balancing current, for different transient modes.

In this case, it was assumed that the magnetic system of the transformer does not enter saturation, as in the case of switching on at idle. The processes occurred on the linear part of the hysteresis loop.

The L_1, L_2, M coefficients are quite complex parameters and may depend on many factors. Our work adopted a simplification – to consider them constant. Temperature dependences were also not taken into account.

Currently, various transformer equivalent schemes used in engineering calculations and in various models have become widespread. Each model has its specific simplifications and tolerances adjusted for specific tasks. Certain physical processes in the transformer are not reflected by these models. This is especially true for transient processes in the transformer. It is also difficult to represent a single process using existing models, which includes switching on at idle, subsequent processes of changing loads. Our paper proposes using a single approach to describe the processes of transformer operation under different operating modes. The proposed method allows us to describe their operation by a system of differential equations of type (1). That is, to mathematically describe the accompanying process in real time, which corresponds to the actual operation of transformers.

The research method is as follows. We take two typical transformers from Table 1, and we conduct the research when they are connected star-star in one phase.

The parameters were calculated using the conventional equivalent circuit. The next step was to calculate parameters for the system of differential equations (1) based on the data obtained. The inductances of the primary and secondary windings, the active resistance of the primary and secondary windings, the load resistance, the dissipation factor, the coupling factor, and the mutual inductance factor – L_1, L_2, r_1, r_2, R, σ, k, M. The system of differential equations was solved for the separate operation of the transformers. According to our results, it was found out how much they differ from specifications, for example, by analyzing deviations in no-load current and deviations under the transformer's rated load, we can assess the nature of transient processes. Subsequently, a system of differential equations was compiled for two parallel-connected transformers TM-160/10, TM-530/10 on a common load.

Table 1

Transformer specifications

Transformer type	U_{HV}, kV	U_{LV}, kV	U_{SC}, %	ΔP_{SC}, kW	ΔP_{NL}, kW	I_{NL}, %
No. 1. TM-180/10	10	0.4	4.5	2.6	0.45	2.5
No. 2. TM-530/10	10	0.4	4.5	6.8	1.1	2

The transformer parameters were determined from the "G"-like equivalent circuit, separately from the high side L_1, r_1 and the low side L_2, r_2, R. The line voltage on the high side of the transformers is $U_1 = 10$ kV, taking into account the star-star connection, the phase voltage on the high side is $U_{ph1} = 5774$ V, on the low side $U_{ph1} = 230$ V. The rated phase currents on the high and low sides for the transformer No. 1, $I_{ph1}/I_{ph2} = 10.4/259.8$ A; No. 2, $I_{ph1}/I_{ph2} = 30.6/765$ A. Further calculations were carried out for transformers No. 1 and No. 2.

The next stage was to establish the reliability of the resulting parameters for expression (1). Transformer No. 1 was used: TM-180/10. For it, on the high side, the supply voltage is linear 10 kV, for the star – phase 5774 V, the amplitude value $U_0 = 8165$ V, $w = 2\pi f = 100\pi$, $L_1 = 70.68$ H, $L_2 = 0.112$ H, $M = 2.81$ H, $r_1 = 4.01$ Ohm, $r_2 = 0.0064$ Ohm. So, the following equation is derived:

$$\begin{cases} 8165 \cdot \sin\left(100\pi t + \varphi\right) = 70.68 \cdot i_1' + 4.01 \cdot i_1 - 2.81 \cdot i_2'; \\ 2.81 \cdot i_1' = 0.112 \cdot i_2' + \left(0.0064 + R_n\right) i_2. \end{cases} \tag{2}$$

The R_n parameter is the resistance of the active load of the transformer, corresponding to its rated power of 180 kVA and 530 kVA:

$$\text{Tr.No.1.}\, R_{n1} = \frac{U_{ph2}^2}{\frac{1}{3} \cdot S_n} = \frac{230^2}{\frac{1}{3} \cdot 180 \cdot 1000} = 0.8817 \text{ Ohm}, \tag{3}$$

$$\text{Tr.No2.}\, R_{n1} = \frac{U_{ph2}^2}{\frac{1}{3} \cdot S_n} = \frac{230^2}{\frac{1}{3} \cdot 530 \cdot 1000} = 0.299 \text{ Ohm}. \tag{4}$$

Substituting the value of $R_n = 0.8817$ Ohm into expression (38), the differential equation was solved numerically. Solving such equations in programs like MATLAB is accompanied by certain difficulties, in particular, their representation in the classical form with respect to the senior derivative. This means that the system of equations of type (1), (38) must be solved with respect to the senior derivative; we shall obtain two fourth-order differential equations, then we shall have to find the initial conditions for the current and for all derivatives, except for the senior. That is, quite a lot of manual labor. Therefore, the Maple system was chosen, which solves such issues.

The next step was to study the operation of two transformers No. 1. TM-180/10, No. 2. TM-530/10 in parallel connection to a common load. The scheme of their connection for one phase is shown in Fig. 1. The input voltages U_1 and U_2 are the same, supplied from a common power source, phase 5774 V. The output, per phase, is 230 V.

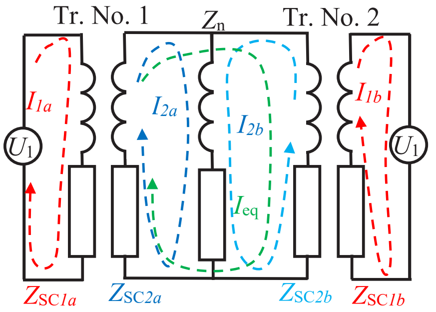


Fig. 1. Parallel connection of transformers to a common load

The diagram in Fig. 1 shows the primary and secondary windings of transformers, their L_{SC}, R_{SC} specifications, active loads and inductive component L_n, R_n. Also shown are independent bypass circuits, two for the primary – red, two for the secondary – blue, and the balancing current circuit – green. Two equations for the primary circuits were derived:

$$U_0 \cdot \sin\left(\omega \cdot t + \varphi\right) = L_{1a} \cdot i_{1a}' + r_{1a} \cdot i_{1a} - M_a \cdot i_{2a}' + M_a \cdot i_{eq}';$$

$$U_0 \cdot \sin\left(\omega \cdot t + \varphi\right) = L_{1b} \cdot i_{1b}' + r_{1b} \cdot i_{1b} - M_b \cdot i_{2b}' - M_b \cdot i_{eq}'.$$

The equations do not differ from (1) except that an additional term from the secondary circuit of the balancing current has appeared, between the secondary windings of the transformers. According to the chosen direction of the balancing current for Tr. No. 1, it has a positive sign, and for transformer Tr. No. 2 it has a negative sign. This is due to the fact that it was agreed that the primary current should be considered in antiphase with the secondary. Thus, we obtain:

$$M_a \cdot i_{1a}' = \left(L_{2a} + L_n\right) \cdot i_{2a}' + \left(r_{2a} + R_n\right) \cdot i_{2a};$$

$$M_b \cdot i_{1b}' = \left(L_{2b} + L_n\right) \cdot i_{2b}' + \left(r_{2b} + R_n\right) \cdot i_{2b}.$$

This is the equation of the secondary circuits of transformers, taking into account the inductances of the windings, their active resistances, and the active and reactive components of the load.

The equation of the balancing current takes the following form

$$M_a \cdot i_{1a}' - M_b \cdot i_{1b}' = L_{2a} \cdot i_{eq}' + L_{2b} \cdot i_{eq}' = \left(r_{2a} + r_{2b}\right) \cdot i_{eq}.$$

The direction of the bypass circuit is taken into account, and the fact that the voltages of the secondary windings are directed in opposite directions.

Since transient processes were considered at the moment of switching on the transformers, the initial conditions for the currents are zero, that is:

$$i_{1a}\left(0\right) = 0;\ i_{2a}\left(0\right) = 0;\ i_{1b}\left(0\right) = 0;\ i_{2b}\left(0\right) = 0;\ i_{eq}\left(0\right) = 0.$$

The general form of the system of differential equations of two transformers switched on for parallel operation is as follows:

$$\begin{cases} U_0 \cdot \sin\left(\omega \cdot t + \varphi\right) = L_{1a} \cdot i'_{1a} + r_{1a} \cdot i_{1a} - \\ - M_a \cdot i'_{2a} + M_a \cdot i'_{eq}; \\ M_a \cdot i'_{1a} = \left(L_{2a} + L_n\right) \cdot i'_{2a} + \left(r_{2a} + R_n\right) \cdot i_{2a}; \\ U_0 \cdot \sin\left(\omega \cdot t + \varphi\right) = L_{1b} \cdot i'_{1b} + r_{1b} \cdot i_{1b} - \\ - M_b \cdot i'_{2b} - M_b \cdot i'_{eq}; \\ M_b \cdot i'_{1b} = \left(L_{2b} + L_n\right) \cdot i'_{2b} + \left(r_{2b} + R_n\right) \cdot i_{2b}; \\ M_a \cdot i'_{1a} - M_b \cdot i'_{1b} = L_{2a} \cdot i'_{eq} + \\ + L_{2b} \cdot i'_{eq} = \left(r_{2a} + r_{2b}\right) \cdot i_{eq}; \\ i_{1a}\left(0\right) = 0; i_{2a}\left(0\right) = 0; i_{1b}\left(0\right) = 0; \\ i_{2b}\left(0\right) = 0; i_{eq}\left(0\right) = 0, \end{cases} \tag{5}$$

where U_0 is the amplitude value of the input voltage; φ is the switching angle of the transformer in relation to the voltage sinusoid; i_{1a}, i_{2a} – primary and secondary (loop) current of transformer #1; L_{1a}, L_{2a} – dissipation inductances of the primary and secondary windings of transformer #1; R_H, L_H – active resistance and load inductance; M_a, M_b – mutual inductance of transformers #1, #2; i_{1b}, i_{2b} – primary and secondary (loop) current of transformer #2; L_{1b}, L_{2b} – dissipation inductances of the primary and secondary windings of transformer No. 2; I_{eq} is the balancing current between the secondary windings of transformers; $i_{1a}(0)$, $i_{2a}(0)$, $i_{1b}(0)$, $i_{2b}(0)$, $i_{eq}(0)$ are the initial conditions for the currents at the moment of switching on.

In this way, a system of five differential equations and five unknown currents was constructed.

5. Results of investigating transient processes in transformers

5. 1. Determining the alignment between resulting parameters and specifications for typical transformers

Our study on the operability of power transformers, models TM-180/10 and TM-530/10, based on differential equations (1) and (5), was carried out as follows. First, the necessary coefficients included in the specified equations were calculated; the results of calculations are given in Table 2.

Table 2

Transformer parameters for system (41)

Transformer type	$R_{SC}1$	σ	$L_{SC}1$	$R_{SC}2$	$L_{SC}2$	M
TM-180/10	4.012	0.0011	0.0785	0.0064	0.00012	2.81
TM-530/10	1.210	0.0008	0.0267	0.0019	0.00004	1.19

In order to verify the reliability of the resulting parameters and the operability of the Tr. No. 1 model, one was turned on at a rated active load $R_n = 0.8817$ Ohm. The simulation results are shown in Fig. 2, where the red curve (Cure 1) depicts the current of the primary circuit of the transformer, the blue curve (Cure 2) – the secondary current, at rated load. The rated phase currents of the primary and secondary windings of the 180 kVA transformer are 10.4 A and 259.8 A, respectively, their amplitude values are $I_{\max 1} = 10.4 \cdot \sqrt{2} = 14.7$ A, $I_{\max 1} = 259.8 \cdot \sqrt{2} = 367.4$ A. The indicated amplitude values, in a first approximation, correspond to the amplitude values on the plot in Fig. 1. The selected system of signs in equation (1) corresponds to the placement of the primary current to the secondary in antiphase. The switching on occurred at an angle of 0°, but with a previously switched on load, therefore no transient processes are observed.

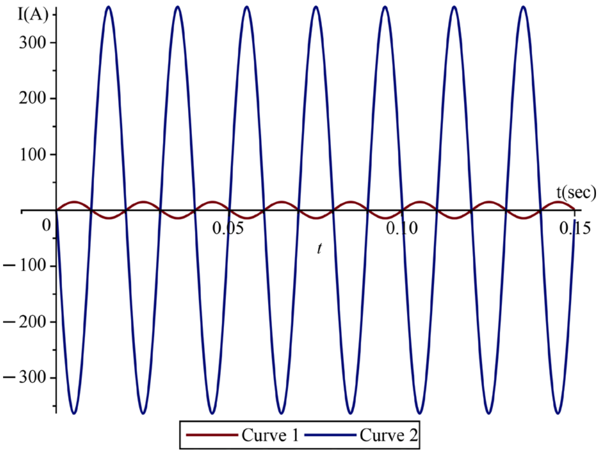


Fig. 2. Currents at rated active load Tr. No. 1 180 kW — red color: primary winding; blue color — secondary winding

The nature of the current in the primary and secondary windings corresponds to the classical case of active load, when the primary and secondary currents are oriented in opposite phase.

5. 2. Investigating the adequacy of model's performance when connecting two transformers in parallel with different rated power and different load characteristics

Next, studies were conducted with two transformers switched on according to the scheme in Fig. 2, with a common active load of 180 kW, which is 25% of the total power of the two transformers. The red curves refer to transformer No. 1 – 180 kVA, the blue curves to transformer No. 2 – 530 kVA in Fig. 3. To highlight transient processes, switching is carried out at angles of 0°.

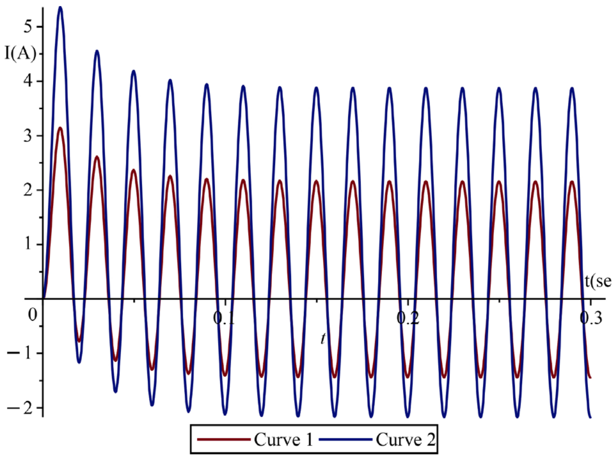


Fig. 3. Currents of the primary windings of transformers Tr. No. 1 and Tr. No. 2: total load power — 180 kW

Fig. 3 shows the distribution of the primary current between Tr. No. 1 (red curve 1) and Tr. No. 2 – blue curve 2. Since the currents are distributed proportionally to the transformer capacities, the correspondence is confirmed under steady-state modes. A transient process emerges, its duration increases significantly; Fig. 4 shows a fragment at the time interval 25–25.3 sec. Compared with Fig. 3, the sinusoids of Fig. 4 have not yet taken a symmetrical position relative to the time axis.

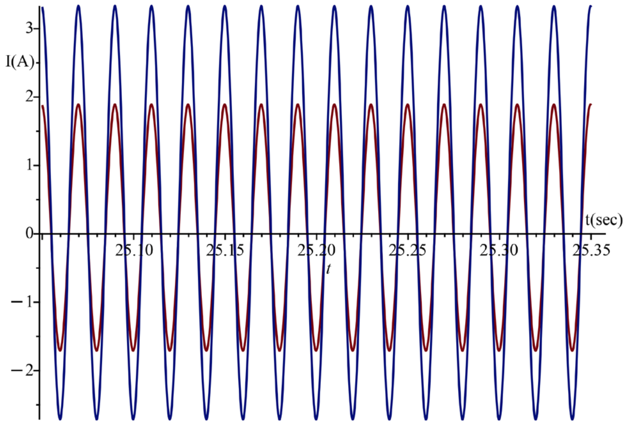


Fig. 4. Primary winding current in the time interval 25–25.3 sec

The currents along the positive coordinate axis are ≈ 3.2 A, along the negative axis are ≈ 2.6 A, which indicates the presence of asymmetry.

To obtain a complete picture of current distribution along the secondary windings, the following details should be taken into account.

According to the diagram in Fig. 1 and the system of differential equations (5), the load current of transformer No. 1 is found by adding the loop current I_{2a} and the balancing current I_{eq}

$$I_{n2a} = I_{2a} + I_{eq}; \tag{6}$$

for transformer No. 2, on the contrary, in accordance with the direction of the bypass circuit

$$I_{n2b} = I_{2b} - I_{eq}. \tag{7}$$

The solution for the secondary currents of transformers and the distribution of these currents between them is shown in Fig. 5.

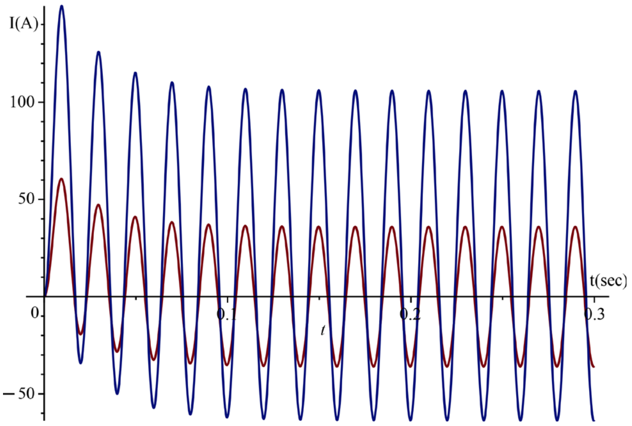


Fig. 5. Current distribution between parallel-connected transformers No. 1 and No. 2

An approximate distribution of currents (powers) between transformers was obtained 1:3. The current flowing to the load is equal to the sum of the loop currents according to Fig. 1

$$I_n = I_{2a} + I_{2b}. \tag{8}$$

The consumer load current chart for this case is shown in Fig. 6. It is slightly less than the total of both transformers since the balancing current does not circulate through the load.

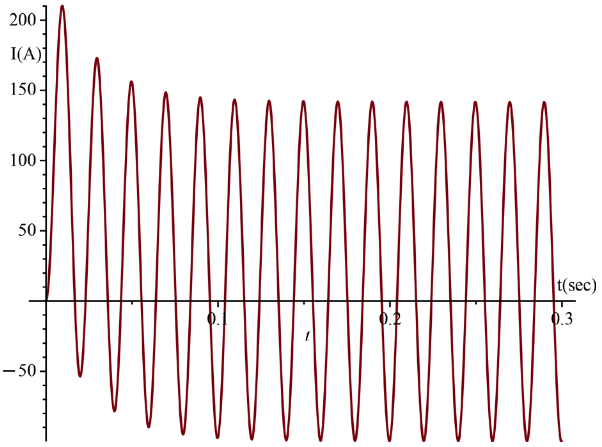


Fig. 6. Total current circulating through the common load of the transformers

5. 3. Determining the parameters of balancing current under transient and steady-state modes

Fig. 7 shows the balancing current circulating through the secondary windings of transformers and independent of the total load of the consumer.

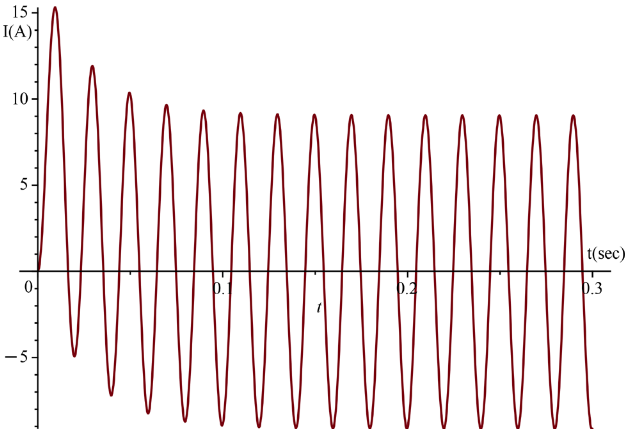


Fig. 7. Balancing current between transformers 180 kVA and 530 kVA, with a total active load of 180 kW

The next step was to investigate the operation of transformers on a common inductive load of ≈ 180 kVA; the transient process has a longer duration, as can be judged from the right fragment between 3 and 3.2 sec., Fig. 8, at 3.5 sec, the current is still symmetrical with respect to the time axis.

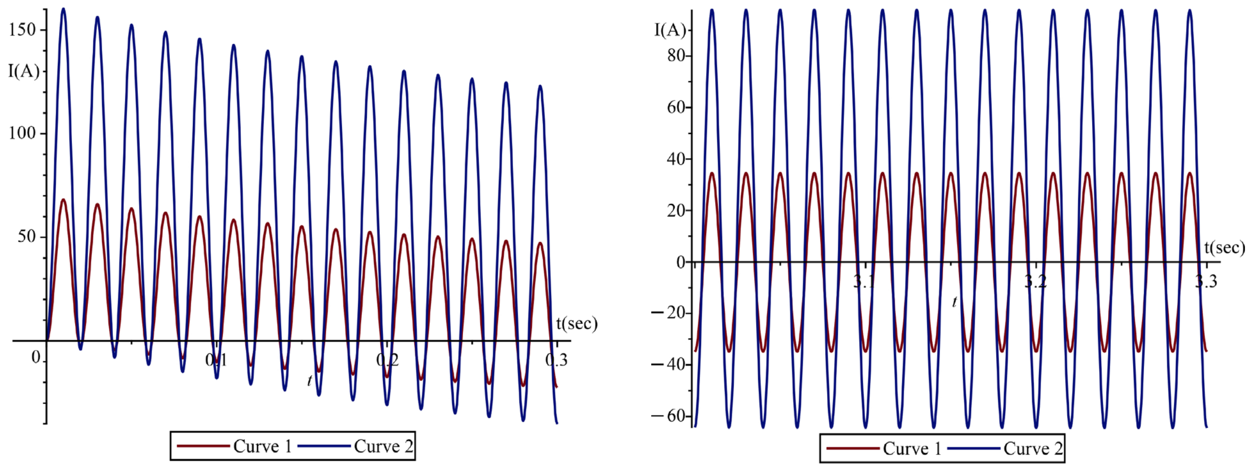
This means that at low loads, transformers have a significant resulting inductance, and the duration of transients can last more than minutes.

Also important is the short-circuit mode on low-voltage buses, near the transformer. For this purpose, in the model, instead of the active and reactive load parameters, we set $R_n = 0$ and $L_n = 0$. In such cases, the short-circuit current is limited only by the inductive dissipation resistances of the transformers. Fig. 9 shows the distribution of the short-circuit current, on the low side, between two transformers; it is obvious that the duration of the transient with a pronounced aperiodic component can last more than several minutes.

The nature and magnitude of the current flowing at this time at the short-circuit point of the common load is shown in Fig. 10; it almost corresponds to the sum of the currents of each transformer, with the exception of the balancing current.

The balancing current at the moment of a short circuit on the load is shown in Fig. 11.

Since on 0.3 sec. the current is almost completely in the first quadrant of Fig. 11, the duration of the entire transient process along the balancing current can last tens of seconds.



a *b*

Fig. 8. Transients under inductive load: *a* – 0–0.3 sec; *b* – 3–3.3 sec

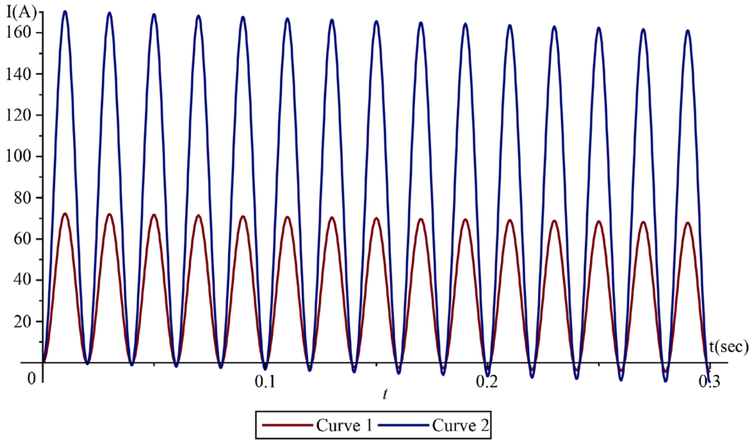


Fig. 9. Distribution of short-circuit currents between secondary windings of transformers

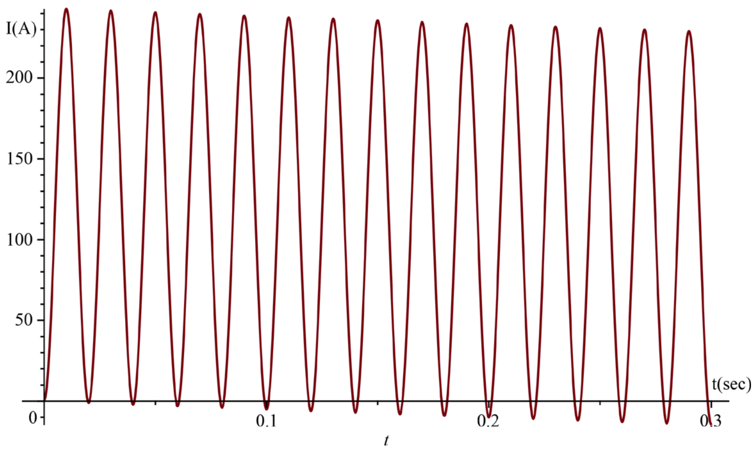


Fig. 10. Current value at the short circuit point

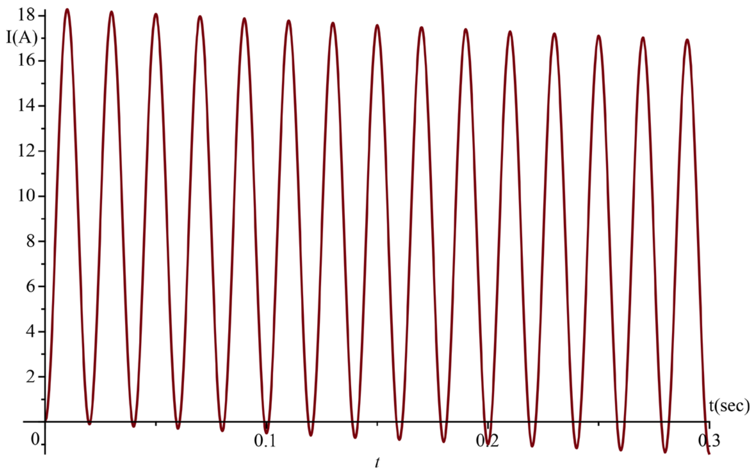


Fig. 11. Character of the balancing current during a short circuit of transformers operating in parallel

6. Discussion of results based on using a model to determine the current distribution between parallel-connected transformers

The reliability of the results obtained from the model is confirmed by the correspondence of the values of instantaneous winding currents in Fig. 2 and their rated amplitude values. The distribution of the values of the currents of the primary windings of transformers No. 1 and No. 2 in Fig. 3 approximately corresponds to the ratios of their rated capacities of 180 and 530 kVA.

The plots of transient processes when switching on 0° in Fig. 3–6 indicate a characteristic current surge that corresponds to the accepted theory. However, the currents in Fig. 3 almost formed a steady state on 0.1 s. At the same time, there is a clear asymmetry along the time axis, which disappears only on 25 s in Fig. 4. This phenomenon is due to the total effect of the inductance of the transformers connected in parallel. Transient processes in the secondary circuits also have a characteristic jump at the beginning of the process and the subsequent long-term asymmetry along the time axis. That is, we have two transient processes, one caused by load currents, the second by the balancing current – Fig. 7. The introduction of an actively inductive load in Fig. 8 significantly increases the duration of the transient process, which is explained by the combined action of inductances. Our study of the short-circuit process is illustrated in Fig. 9–11. We observe a sharp increase in the duration of the transient process, this is explained by a sharp decrease in the active resistance, the actual increase in the *Q* factor of the circuit. The current distribution between the windings remains the previous one in proportion to the powers. Therefore, our studies have shown that equation (41) covers most of the processes occurring during the operation of transformers connected in parallel.

In [1–3, 5–12], and elsewhere, conventional schemes of transformer substitution "P", "G", "T" are used. They are convenient for steady processes, as well as for calculation of a single transformer. A feature of the proposed method is that equations (1), (41) actually describe all operating modes of transformers switched on in parallel. The coefficients included in them can be both calculated and measured. This makes it possible to study the model according to specifications and actual data, and to draw a conclusion about the technical condition of the transformer.

However, our method has a significant limitation, equations (1), (41) do not work to describe the transient process when the transformer is switched on at idle. Also, complications arise when the duration of transient processes exceeds 1 min. Such processes require significant computer time.

The disadvantages of the proposed method include the fact that the non-linearity of the hysteresis loop is not taken into account. At loads less than rated, this is permissible, but during overloads significant errors appear.

For a complete description of the transformer, one step is missing – introducing nonlinearity into the system of equations (41). There are many solutions for describing the hysteresis loop. But in transient processes, there are many of these loops and they must automatically change, depending on the change in the transformer operating mode. In the future, this issue needs to be resolved.

7. Conclusions

1. Our study has confirmed the correspondence of the rated currents of the transformer, at rated load, through the instantaneous currents of the model. In particular, for Tr. No. 1, at rated active load, $R_n = 0.8817$ Ohm, which corresponds to currents of 10.4 A and 259.8 A, converted to amplitude values of 14.7 A and 367.4.

2. The nature of the transients captured by a model does not contradict generally accepted standards. In particular, the distribution of currents between transformers is proportional to their capacities. However, generally accepted practice takes into account only the short-circuit voltage. Our study has proven that the ratio of active and reactive resistances of the transformer windings is of decisive importance. This is due to the fact that at the same short-circuit voltages, these ratios may be different, and therefore, different current redistributions. The model allows one to determine the value of the shock current not only from the moment of the short circuit but also from the history of the circuit. That is, from the magnitude of the load and its nature, which preceded the short circuit.

3. It was found that the value of the balancing current, when the relevant requirements are met, takes a small value. But the duration of the transient process caused by it exceeds several seconds. Such an aperiodic component could lead to incorrect operation of the relay protection and the transformer itself. In other words, both transformers operate (for the period of the transient process of the balancing current) with direct current bias. The direction of bias in them is opposite. Considering the fact that the magnetization current is 1.5–4% of

the rated, of the same order as the balancing current, all this together distorts the hysteresis loop. The result is incorrect operation of the automation and additional electricity losses.

Conflicts of interest

The authors declare that they have no conflicts of interest in relation to the current study, including financial, personal, authorship, or any other, that could affect the study and the results reported in this paper.

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Data availability

All data are available in the main text of the manuscript.

Use of artificial intelligence

The authors confirm that they did not use artificial intelligence technologies when creating the current work.

Authors' contributions

Anatoly Panchenko: Conceptualization, Methodology, Software, Validation, Formal analysis, Investigation, Resources, Data Curation, Writing – original draft, Writing – review & editing, Visualization, Supervision, Project administration, Funding acquisition; **Iryna Boklah**: Resources, Data Curation, Writing – review & editing, Visualization, Supervision, Project administration; **Yulia Musairova**: Writing – review & editing, Visualization, Supervision, Project administration**; Volodymyr Honcharov**: Data Curation, Writing – review & editing, Visualization, Supervision, Project administration; **Nadia Kuravska**: Resources, Data Curation, Writing – review & editing, Visualization, Supervision, Project administration.

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