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DEVELOPMENT OF AN APPROACH TO THE NORMALIZED FUNCTIONAL FOR DEVIATION IN EFFICIENCY INDICATOR

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This study examines controlled technological process of heating liquid, which is considered as a cybernetic system for transforming resources into a usable technological product. The work investigates the possibility of constructing a normalized functional of deviation in the efficiency indicator in dynamic models of technological processes, used to synthesize optimal control effects. They provide a quantitative assessment of deviation of the actual efficiency from the required level and allow the application of optimal control methods.

The task addressed is to find a standardized indicator of efficiency deviation E, which would ensure the dimensionlessness of this indicator and its large-scale independence. It is a dimensionless quadratic measure of difference between the total useful result and the total costs. It has been shown that the parameter E has the properties of computational constancy and could be used for analysis, normalization, and comparison of various control modes.

The ELF (Normalized Efficiency Criterion) computing block has been designed, which makes it possible to convert input parameters into cost form, accumulate total costs and useful effect, form indicators of additional benefit and resource intensity, as well as calculate a normalized efficiency criterion. It is shown that ELF is an integrated indicator of efficiency as a ratio of additional benefit to the resource intensity of the permissible mode; the indicator E represents its normalized metric form.

The results make it possible to assess the effectiveness of the process in a quantitative way. And the functional E shows a deviation from the required mode, which gives the opportunity to conduct an analysis and make the right decisions in the management of a technological process.

The research findings could be used in any technological processes

Keywords: cybernetic control, efficiency criterion, ELF, normalized deviation index, mode selection

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1. Introduction

The theoretical basis of modern optimal control is formed within the framework of the Pontryagin maximum principle, which is a strict mathematical apparatus for the analysis of controlled dynamic systems. The principle of Pontryagin maximum is important in the theory of optimal processes as it makes it possible to reduce the task of searching for optimal control to the study of a system of necessary optimality conditions. In contrast to mathematical methods, which are more often suitable for simple processes without serious restrictions, the Pontryagin maximum principle takes into account the presence of restrictions on the control effects and the state of the system. This makes it especially useful for practical tasks of managing technical, economic, and other processes.

Modern technological systems consider the methods of economic predictive control (Economic Model Predictive Control, EMPC), which make it possible to control the technological process taking into account economic benefits. Indeed, such an approach can increase productivity and reduce costs. However, the efficiency criterion in EMPC is set in advance and depends on the selected cost function, which limits its universality [1]. In such systems, management is connected with the transformation of material, energy, and operational resources into a usable result. The EMPC method makes it possible to take into account various options for process evolution and choose a more reliable control. The disadvantage is the complexity of calculations [2].

The classical theory of optimal control is based on the Hamiltonian formalism and Pontryagin maximum principle. However, traditional management techniques are primarily aimed at quality indicators [3]. This does not ensure maximum efficiency, determined by the ratio of usable results to costs. As a result, the criteria of optimality used in the classical theory do not always reflect the actual efficiency of the functioning of technological processes.

In engineering practice, the efficiency of management is often estimated using particular indicators, such as productivity, energy consumption, and specific costs [4]. Despite their practical significance, these indicators do not form a single universal criterion. This does not provide a correct comparison of alternative control modes. All this is due to their dimensional nature, dependence on the scale of the system, as well as the absence of a formal connection with the dynamic model of the process.

The use of fractional efficiency criteria makes it possible to take into account the ratio of results and costs; however, it leads to the complication of setting the task of optimal management [5]. Standard methods of variational calculus are focused on additive functionals. The application of fractional criteria requires a special mathematical apparatus, including Dinkelbach parameterization and methods of non-smooth analysis in the case of the presence of restrictions and discontinuous dependences [6].

A promising way to solve this problem is the construction of a normalized functional of the deviation of the efficiency indicator, which is based on the integrated ratio of the useful result and costs. Such an approach ensures the dimensionlessness of the indicator, its independence from the scale and makes it possible to include the efficiency criterion in the dynamic model of the system. Within this approach, the efficiency indicator is considered to be an internal characteristic of the controlled dynamic system.

In this regard, research aimed at the development of a normalized functional of the deviation of the efficiency indicator, consistent with the Hamiltonian formalism and the Pontryagin maximum principle, is relevant as it allows for the following:

– a strict mathematical statement of the efficiency optimization problem;

– inclusion of efficiency criterion in the dynamic model of the system;

– the possibility of applying optimal control methods;

– large-scale independence and dimensionlessness of the indicator;

– the possibility of algorithmic implementation in digital control systems.

Thus, the relevance of research into this area is due to the expectation that their results could contribute to the development of effective algorithms for controlling technological processes, allowing us to objectively and quantitatively assess the effectiveness of the modes of operation of cybernetic systems.

2. Literature review and problem statement

General methods for designing economic control systems with model-based forecasting (EMPC) for a wide class of nonlinear systems are considered in work [1]. The mathematical foundations of the method, issues of stability, convergence, and practical application in industry are investigated. The proposed methods use a variety of tools, starting from the analysis of nonlinear systems and ending with control methods based on the Lyapunov function. These methods take into account key practical issues, namely, direct optimization of the economic efficiency of the process, time-varying economic functions of costs and computational efficiency. The applicability and effectiveness of the proposed methods are demonstrated on a number of examples of chemical processes. The disadvantage of the work is that the methods are mainly focused on chemical-technological systems.

Paper [2] shows the relationship between economic efficiency and sustainability of EMPC systems. The conditions under which it is possible to simultaneously ensure high economic productivity and stable behavior of a closed system are considered. The paper reviews recent achievements in the optimization of nonlinear systems, which make it possible to optimize transient and stationary economic indicators. The disadvantage of the model is the fulfillment of a number of strict mathematical conditions that are not always met in practice, as well as high computational complexity.

Theoretical calculations of EMPC for nonlinear switched systems using neural networks are carried out in [3].

The accuracy of forecasting of the model is performed on the basis of current data, which makes it possible to improve economic indicators while maintaining the stability of management. The method can be applied to complex industrial objects with changing dynamics. The disadvantage of this method is the presence of large volumes of data, as well as the need for significant computational operations.

In work [4], methods of optimal control, MPC, as well as the tasks of controlling the demand for energy resources are considered, and an overview of modern methods of controlling technological processes from the point of view of their influence on energy efficiency is given. It is shown that improving the quality of regulation makes it possible to simultaneously reduce energy consumption, decrease operational costs, and improve the stability of the system. Energy efficiency should be evaluated together with production indicators. The paper is an overview and does not offer a universal criterion for evaluating efficiency.

The classic approach to fractional optimization for nonlinear problems is considered in [5]. The work shows the possibility of applying fractional efficiency criteria in complex optimization problems. However, they make it possible to take into account relative performance indicators. The disadvantage of this approach is the complexity of the calculation procedure and the need to use special settings.

Paper [6] considers dynamic programming and optimal control over complex systems based on Bellman's principle. The disadvantage of the work is the lack of a universal indicator of efficiency, independent of the scale and units of measurement.

In engineering practice, the efficiency of management is often estimated using particular indicators, such as productivity, energy consumption, and specific costs [7]. Despite their practical significance, these indicators do not form a single universal criterion, do not provide a correct comparison of alternative management modes. This is due to their dimensional nature, dependence on the scale of the system, as well as the absence of a formal connection with the dynamic model of the process.

Methods of optimal control and various quality criteria for engineering systems are considered in [8]. Methods for choosing the optimal operating mode of the process are shown. The disadvantage is that the quality criterion is set in advance and is not directly related to the process properties.

In work [9], the relationship between production planning and modern methods of controlling technological processes is considered. Most attention is paid to the complex approach to the management of industrial facilities. It is shown that the integration of various indicators makes it possible to improve the use of resources and increase productivity.

The disadvantage is the lack of a quantitative criterion for direct assessment of the efficiency of the operating modes.

Methods of analysis and design of automatic control systems are considered in [10]. A technique for ensuring the stability and quality of the system's work has been devised. There is no assessment of the ratio of results and costs in the work.

Paper [11] considers feedback control systems and stability research. Methods for building sustainable process management are discussed. The disadvantage is the lack of multiple evaluation of the efficiency of resource use.

In [12], MPC methods for controlling technological processes taking into account constraints are considered. Ways to improve the system's performance when its state is changed are investigated. The disadvantage is the dependence of the result on the cost structure and selected parameters.

Methods for controlling production processes and automation systems are studied in [13]. A model was built that takes into account the dynamics of the technological process and electricity tariffs. Numerous experiments are considered, which show the possibility of reducing energy costs without violating the production plan. The paper does not consider efficiency criteria for a wide class of objects. This reduces the universality of the proposed method.

Study [14] investigates methods of training and optimal management for decision-making. Algorithms that use the accumulated experience of the system are considered. The disadvantage is the dependence of the result on a predetermined evaluation function.

In work [15], a universal integrated criterion of cybernetic control, intended for evaluating the effectiveness of systems of a different nature, is proposed. Efficiency is considered as the ratio of the useful result to the resources, and the possibility of using this method to compare different operating modes of the system is shown. The merit of the work is the formation of an efficiency assessment methodology based on the joint accounting of results and costs. The proposed criterion has a universal character and can be used to analyze various control systems. The disadvantage is that the efficiency criterion is considered primarily as an integrated assessment of the completed operation. The paper does not consider the task of including the efficiency indicator in the dynamic system of the process itself and does not introduce a special function of efficiency deviation relative to the required level. There is no normalized form of such a deviation, necessary for use as a quality functional in optimal control problems.

In [16], the task of devising a global criterion for the effectiveness of target operations is solved. The proposed approach is based on the formation of an integrated assessment of the result and costs of the operation. This makes it possible to quantitatively compare various options for achieving the set goal. Special attention is paid to the concept of target operation, useful result, and resources. The criterion analyzes completed operations and evaluates their effectiveness. The work does not consider the task of forming a dynamic indicator of efficiency that changes over time during the operation. The disadvantage of the work is also the absence of a standardized dimensionless form of the efficiency deviation, suitable for inclusion in the functional quality of optimal control. Within the framework of the proposed approach, the possibility of representing the efficiency as an internal variable of the state of the dynamic system and its use as part of the Hamiltonian formalism is not explored.

In [15, 16], the task of devising a standardized efficiency deviation functional, which makes it possible to take into account the dynamics of efficiency changes over time, comparing actual and required work modes, and applying optimal control methods, remains unsolved.

Our review of the literature [1–16] shows that existing methods do not contain a common normalized functional of efficiency deviation. The criteria used are either set in advance, or depend on the scale and units of measurement, or are not directly related to the dynamics of the process.

This indicates the need to devise a normalized efficiency deviation functional, which:

- does not depend on the scale and units of measurement;
- takes into account the total result and costs;
- is included in the dynamic model of the system;
- makes it possible to solve management task as an optimization problem.

3. The aim and objectives of the study

The purpose of our work is to devise an approach to the formation of the normalized functional of the deviation of the efficiency indicator, which is included in the dynamic model of the technological process and is consistent with the Pontryagin maximum principle. The proposed approach will make it possible to consider the efficiency deviation as an internal characteristic of the system and to formulate the control task as the task of minimizing or maximizing this function, to compare different modes of operation and to calculate the control efficiency.

To achieve this goal, it is necessary:

- to build a dynamic model of the controlled technological process taking into account the flows of costs and results and derive a theorem on the normalized quadratic form of the ELF efficiency criterion;
- to construct a model of the normalized functional of the efficiency deviation and determine the resources and the useful result;
- to include the devised functionality in the dynamic model of the technological process, with the possibility of using it in tasks of optimal control;
- to formulate the management task taking into account the efficiency deviation functional;
- to develop a management calculation algorithm on the example of a technological process.

4. The study materials and methods

4. 1. The object and hypothesis of the study

The object of our study is a controlled technological process of liquid heating, which works in discrete or continuous time at a given interval

$$t \in \left[t_0, T_{op}\right].$$

It was assumed that the efficiency of control over the technological process of liquid heating could be correctly assessed with the help of a universal integrated criterion formed on the basis of the ratio of the accumulated useful result and the total costs of resources.

It was assumed that the following conditions are met:

- costs and the result of the process are represented in a single cost scale;
- integrated indicators are formed recursively in time;

– the mechanism of selection of permissible control modes is used;

– a standardized efficiency criterion is formed, possessing the following properties:

– dimensionlessness;

– scale invariance;

– computational stability;

– applicability in optimal control problems.

The following blocks are considered: transfer of resources into value form; accumulation of costs and results; calculation of efficiency deviations. This is due to the fact that it provided an opportunity to move from the local characteristics of the process to the general assessment of efficiency, to obtain a standardized indicator that does not depend on the scale, to include the deviation of efficiency in the dynamic model, to strictly state the problem of optimal control.

4. 2. Theoretical research methods and the procedure for finding optimal control

Our work applies methods of system analysis, mathematical modeling, integrated evaluation of efficiency on the example of the technological process of liquid heating, considered as a controlled dynamic system of transformation of resources into a useful result.

The state of the liquid heating process is described by the state vector

$$x(t)\in\mathbb{R}^n.$$

Its change in time is given by the equation of state

$$\dot{x}(t)=f\left(x(t),u(t),t\right),\ x(t_0)=x_0,\ u(t)\in U, \tag{1}$$

where u is the controlling action, U is the set of permissible controlled ones.

At each stage of control, various resources are used and a useful result is obtained.

Resource functions are introduced:

$$v(t)\geq 0 \text{ – material resource,} \tag{2}$$

$$e(t)\geq 0 \text{ – energy resource,} \tag{3}$$

$$\theta(t)\geq 0 \text{ – temperature resource,} \tag{4}$$

$$w(t)\geq 0 \text{ – service life (equipment wear),} \tag{5}$$

$$h(t)\geq 0 \text{ – useful result of the process.} \tag{6}$$

The model of the liquid heating process is built on a time interval according to the heat balance equation. The change in liquid temperature at each step is determined using the following expression

$$T_{k+1}=T_k+\Delta t\left(CQ_k-\beta\left(T_k-T_{env}\right)\right), \tag{7}$$

where T_k is the liquid temperature at step k; Q_k – heating capacity; C is the heat capacity coefficient of the system; β is the coefficient of heat transfer to the environment; T_{env} – ambient temperature; Δt is the step of the time interval.

The first term on the right side of the equation describes the increase in temperature due to the supplied thermal energy, and the second takes into account heat losses to the environment. The temperature dynamics are determined by the balance between incoming thermal energy and heat losses occurring in the heating process.

The rate of heating of the liquid is used as the flow of the useful result

$$pe_k=\max\left(0,T_k-T_0\right), \tag{8}$$

where T_0 is the initial temperature of the liquid.

The flow of costs is determined by the consumed heating power

$$re_k=c_eQ_k, \tag{9}$$

where c_e is the cost of a unit of energy.

All these functions are defined and limited to the time interval $[0, T_{op}]$, so that it is possible to compare different resources, price coefficients are introduced

$$p_v(t),\ p_e(t),\ p_\theta(t),\ p_w(t),\ p_h(t), \tag{10}$$

where p_i is the unit cost of the corresponding resource, and p_h is the unit cost of the useful result.

The cost function is defined as

$$re(t)=p_vv+p_ee+p_\theta\theta+p_ww, \tag{11}$$

where $re(t)$ is the cost of resources consumed by the system per unit of time.

The cost of the obtained result is determined as

$$pe(t)=p_hh, \tag{12}$$

where $pe(t)$ is the cost of the useful result received per unit of time.

The total values of costs and results are found with the help of integrals:

– total costs

$$RE=\int_0^{T_{op}}re(t)\mathrm{d}t; \tag{13}$$

– total result

$$PE=\int_0^{T_{op}}pe(t)\mathrm{d}t, \tag{14}$$

where $RE(t)$ is the accumulated cost of resources; $PE(t)$ is the accumulated useful result.

The accumulated useful result and the accumulated costs are calculated recursively:

$$PE_{k+1}=PE_k+pe_k\Delta t, \tag{15}$$

$$RE_{k+1}=RE_k+re_k\Delta t, \tag{16}$$

where RE_k — accumulated costs for step k; PE_k is the accumulated useful result at step k; re_k – costs per step k; pe_k is a useful result on step k.

The resulting values are used for the subsequent calculation of the efficiency indicator and the normalized efficiency deviation functional.

A dimensionless efficiency index is introduced

$$ELF=\frac{\left(PE-RE\right)^2}{PE\cdot RE}\cdot\frac{T_p^2}{T_{op}^2}, \tag{17}$$

where T_p is the planned execution time of the process, T_{op} is the actual execution time.

This indicator has the following properties: it is dimensionless; does not depend on the cost scale; does not depend on the selected time unit.

An efficiency deviation is introduced

$$\Delta E = E - E^{*}, \tag{18}$$

where E^{*} is the required (target) efficiency value.

The normalized deviation functional is given by the following formula

$$J = \left(E - E^{*}\right)^{2}, \tag{19}$$

or in dynamic form

$$J = \int_{0}^{T_{op}} \Phi\left(\Delta E\left(t\right)\right)\mathrm{d}t. \tag{20}$$

The following methods are used in our work:
- system analysis of technological processes;
- mathematical modeling;
- methods of optimal management;
- integrated evaluation of efficiency.

The procedure for finding the optimal control includes modeling the dynamics of heating, calculating the flow of costs and useful results, forming integrated indicators *PE* and *RE*, calculating the *ELF* function.

Such a representation has made it possible to evaluate the efficiency of the heating process, taking into account the simultaneously achieved useful result, the spent resources, and the duration of the operation.

4. 3. Conditions for conducting experiments

The computational experiment is carried out in discrete time with a step Δt.

The following operations are performed on each cycle:
- calculation of the temperature state of the system;
- determination of instantaneous energy and resource costs;
- calculation of cost indicators $re(t)$ and $pe(t)$;
- recursive accumulation of integrated indicators;
- calculation of the ELF efficiency criterion;
- assessment of efficiency deviation.

4. 4. Software for experiments

The software implementation of the experiments is carried out in the Microsoft Excel environment. The Python programming language with the NumPy and pandas libraries is used to implement a dynamic model of the technological process and conduct computational experiments. Calculations are performed in discrete time with a constant integration step Δt.

The program includes the following main blocks:
- transfer of resources into value form;
- calculation of temperature changes over time;
- accumulation of total indicators;
- calculation of the ELF efficiency criterion;
- assessment of efficiency deviation.

The computational experiment includes:
- setting initial parameters;
- modeling of the heating process;
- calculation of *RE*, *PE*, and *ELF* indicators;
- checking the stability of the results.

5. Results of designing a normalized functional of efficiency deviation

5. 1. Construction of a dynamic model of the technological process and formulation of the theorem on the normalized quadratic form of the ELF efficiency criterion

As a result of our study, the technological process of liquid heating is represented in the form of a controlled dynamic system in which the transformation of resources into a useful result is carried out.

The dynamic model of the heating process is shown in Fig. 1.

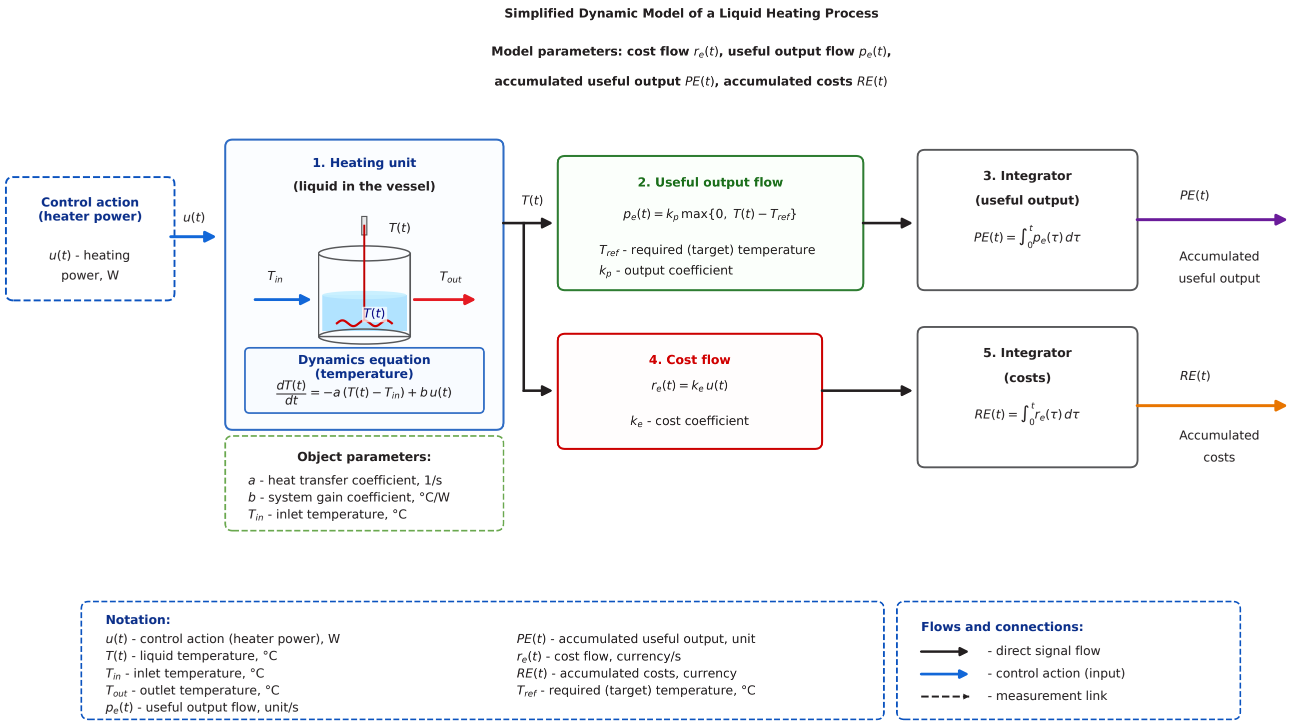


Fig. 1. Dynamic model of the liquid heating process

The process is described through cost streams:

- $r_e(t)$ – rate of resource consumption;
- $p_e(t)$ – speed of formation of a useful result.

Such a representation allowed us to connect the foll:

- resource costs;
- the result obtained;
- process execution time.

The use of a dynamic model has made it possible to analyze the effectiveness of management over time (Fig. 2).

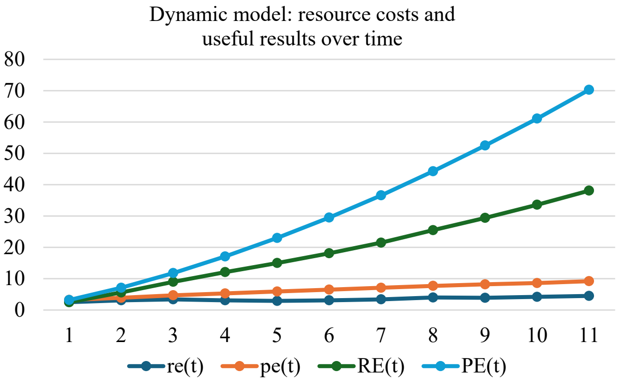


Fig. 2. Costs of resources and useful results in time

Fig. 2 shows that the cost flow of the useful result $pe(t)$ is higher than the cost flow $re(t)$. Therefore, the accumulated result PE grows faster than the accumulated costs RE. This confirms that the dynamic model has made it possible to analyze the effectiveness of management over time and compare the modes of operation of the process.

Based on the above, a theorem on the normalized quadratic form of the ELF efficiency criterion has been derived.

Theorem.

The normalized criterion of the efficiency of the controlled technological process can be represented in the form of a dimensionless quadratic measure of the deviation of the integrated useful result from the integrated costs, taking into account the time efficiency of the process

$$ELF = \frac{(PE - RE)^2}{PE \cdot RE} \cdot \frac{T_p^2}{T_{op}^2}, \tag{21}$$

where the ELF value is a dimensionless normalized indicator characterizing the degree of deviation of the integrated useful result from the integrated costs, taking into account the time efficiency of the process.

Proof.

To prove this theorem, the difference between the total useful result and the total costs is considered

$$\varDelta = PE - RE. \tag{22}$$

The value $\varDelta$ characterizes the absolute discrepancy between the useful result and costs; however, it is dimensional and depends on the scale of the cost scale.

To eliminate dimensionality and ensure scale invariance, we normalize this difference to the geometric mean of integrated indicators:

$$\delta = \frac{PE - RE}{\sqrt{PE \cdot RE}}. \tag{23}$$

Then δ becomes a dimensionless quantity. Here, the numerator and denominator have the same value dimensionality, so their ratio has no dimension.

Since the sign of the $PE - RE$ value depends on whether the useful result exceeds the cost, to construct a non-negative criterion characterizing the degree of deviation, the square of the normalized difference is considered

$$\delta^2 = \frac{(PE - RE)^2}{PE \cdot RE}. \tag{24}$$

This value is non-negative, dimensionless, and resistant to proportional changes in cost coefficients.

To account for the time component of efficiency, an additional dimensionless multiplier is introduced

$$\theta = \frac{T_p^2}{T_{op}^2}, \tag{25}$$

which reflects the deviation of the actual process execution time from the planned one.

Then

$$ELF = \delta^2 \theta. \tag{26}$$

Substituting (24) and (25) into (26), we obtain

$$ELF = \delta^2 \left(\frac{T_p}{T_{op}} \right)^2 = \frac{(PE - RE)^2}{PE \cdot RE} \cdot \frac{T_p^2}{T_{op}^2}. \tag{27}$$

The ELF criterion is a normalized quadratic measure of disagreement between the integrated useful result and the integrated costs, taking into account the time efficiency of the process.

Based on the above, the theorem has been proven.

As a result, it has been proven that the criterion

$$ELF = \frac{(PE - RE)^2}{PE \cdot RE} \cdot \frac{T_p^2}{T_{op}^2} \tag{28}$$

is a normalized quadratic measure of the deviation between the integrated useful result PE and the integrated costs RE.

The obtained criterion has the following properties:

- it is dimensionless;
- it does not depend on the scale of cost coefficients;
- it accepts only non-negative values;
- it takes into account the difference between the result and costs;
- it takes into account the deviation of the actual process time from the planned time.

The ELF criterion makes it possible to evaluate the efficiency of the technological process by the ratio of results and costs. This makes it suitable for comparing different control modes and for further use in the task of optimal control.

Corollary 1. Connection of the ELF criterion with the deviation functional.

The target efficiency value E^* is introduced, then the efficiency deviation is determined as

$$\varDelta E = ELF - E^*. \tag{29}$$

Then the normalized deviation functional takes the form

$$J = \left(ELF - E^* \right)^2. \tag{30}$$

Consequently, the functional J is the quadratic form of the deviation of the actual value of the standardized efficiency criterion from the required value.

Corollary 2. A particular case of target zero deviation.

If the following target value is accepted

$$E^* = 0, \tag{31}$$

then the deviation functional coincides with the square of the criterion

$$J = ELF^2. \tag{32}$$

In this case, the minimization of functional J is equivalent to the minimization of the normalized quadratic deviation between the total useful result and the total costs.

The ELF criterion characterizes the normalized intensity of the deviation between the integrated effect and costs. Therefore, it is represented as a criterion of agreed efficiency, and not as a simple economic ratio of the result to the costs.

5. 2. Construction of a model of the normalized functional of efficiency deviation and defining resources and useful result

A model of the normalized functional of efficiency deviation has been built (Fig. 3).

The model describes the process of heating the liquid as a controllable dynamic system, in which the efficiency indicator is formed on the basis of the accumulated useful result, the accumulated cost of resources, and the time of the operation. The structure of the model includes twelve interconnected blocks:

Block 1. The physical system is a heating device, where a cold liquid with a temperature of T_{in} is supplied. Heating is carried out by control action $u(t)$ (heating power), the outlet temperature of the liquid is denoted by $T(t)$.

Block 2. The dynamic model of the process shows the change in temperature T_k in discrete time and is described by the heat balance equation

$$T_{k+1} = T_k + \Delta t\left(CQ_k - \beta\left(T_k - T_{env}\right)\right), \tag{33}$$

where Q_k is the heating power, C is the heat capacity, β is the heat transfer coefficient, T_{env} is the medium temperature, and Δt is the discretization step.

Block 3. Momentary indicators – at this stage, the momentary characteristics of the process are determined:

– the flow of the useful result $pe_k = \max\,(0,\, T_k - T_0)$ reflects the increase in temperature relative to initial T_0;

– the cost flow $re_k = c_e Q_k$ expresses the current energy costs at cost factor c_e.

In this way, elementary indicators of the result, costs, and time are formed.

Block 4. Integrated indicators – in this block, the indicators are integrated:

– accumulated useful results and costs are defined as:

$$PE(t) = \int_0^t pe(\tau)\,\mathrm{d}\tau,$$

$$RE(t) = \int_0^t re(\tau)\,\mathrm{d}\tau, \tag{34}$$

or in discrete form $PE_{k+1} = PE_k + pe_k\Delta t$, $RE_{k+1} = RE_k + re_k\Delta t$.

Block 5. Efficiency indicator – an integrated efficiency indicator is calculated based on the obtained data

$$ELF = \frac{\left(PE - RE\right)^2}{PE \cdot RE} \cdot \frac{T_p^2}{T_{op}^2}, \tag{35}$$

where T_p is the planned time, T_{op} is the actual time of the operation.

Block 6. Required efficiency – the required efficiency level $E^*(t)$ is set, which is determined by technological or regulatory values.

Block 7. Deviation of efficiency – the deviation of actual efficiency from the required efficiency is formed

$$\Delta E(t) = E(t) - E^*(t). \tag{36}$$

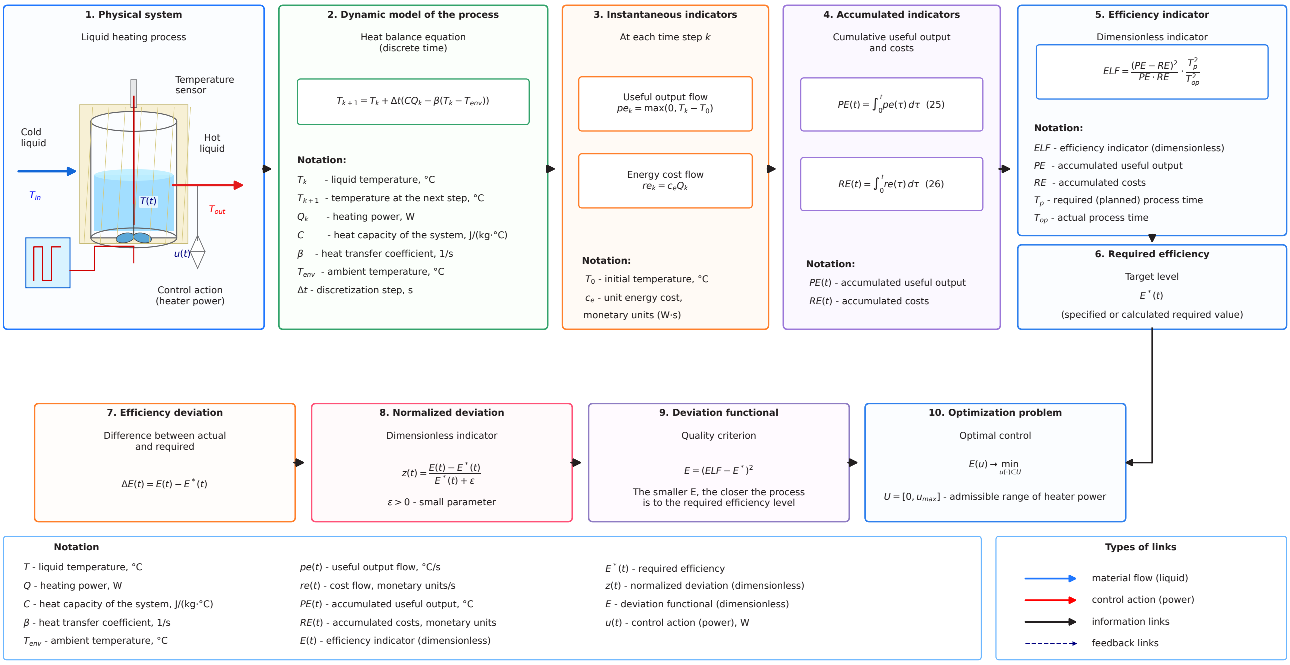


Fig. 3. Model of the normalized functional of efficiency deviation

Block 8. Standardized deviation – in this block, a dimensionless characteristic is introduced to eliminate the influence of dimensionality

$$z(t)=\frac{E(t)-E^{*}(t)}{E^{*}(t)+\varepsilon}, \tag{37}$$

where $\varepsilon > 0$ excludes division by zero.

The value represents a normalized functional of the efficiency deviation and shows a relative control error.

Block 9. Deviation functional – in the block, the deviation functional is formed to evaluate the quality of the process

$$E=\left(ELF-E^{*}\right)^{2}, \tag{38}$$

which characterizes the difference between real and required efficiency.

Block 10. The optimization task – on the basis of the deviation functional, the task of optimal control $E \to \min u(\cdot) \in U$ is performed by choosing the admissible controlled $u(t)$.

The optimization task is to find such a change in the heating power $u(t)$, which ensures the minimum deviation of the actual efficiency from the required one.

The model built provides a transition from the physical process of heating the liquid to the problem of optimal control based on the efficiency deviation criterion. The main result of the model is the formation of a normalized functional of efficiency deviation, which made it possible to consider efficiency as an internal characteristic of a dynamic system for the search of the optimal control mode.

Within the framework of the model, the results are obtained that make it possible to correctly take into account their contribution to the overall process and compare them (Fig. 4).

Fig. 4 demonstrates that different resources can be combined into a single system of indicators and is the basis for the subsequent calculation of integrated characteristics and efficiency criteria.

Fig. 4 shows basic types of resources: energy, material, temperature, and equipment resources, as well as their total cost $re(t)$ and the useful result of the process $pe(t)$.

The results reveal the following:

– each resource affects the total costs of the process;

– total costs $re(t)$ consist of all types of resources;

– useful result $pe(t)$ increases with time and shows the heating process;

– the transfer of all resources into a single monetary form makes it possible to correctly compare their influence.

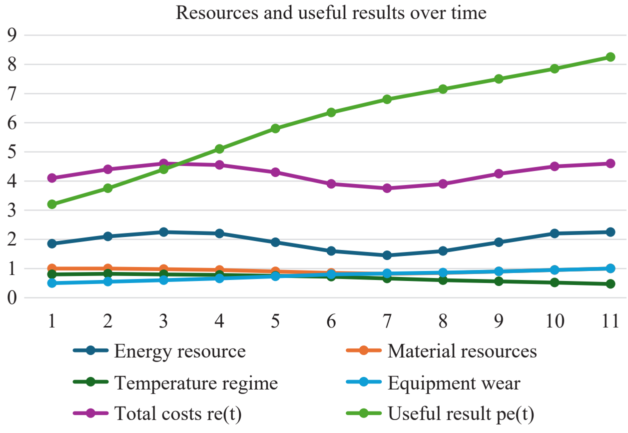


Fig. 4. Resources and useful result in time

Fig. 4 shows that different types of resources can be combined into one system of indicators. This serves as the basis for further calculation of general characteristics and evaluation of efficiency.

Below is a diagram of the shares of various resources in the total costs of the technological process (Fig. 5).

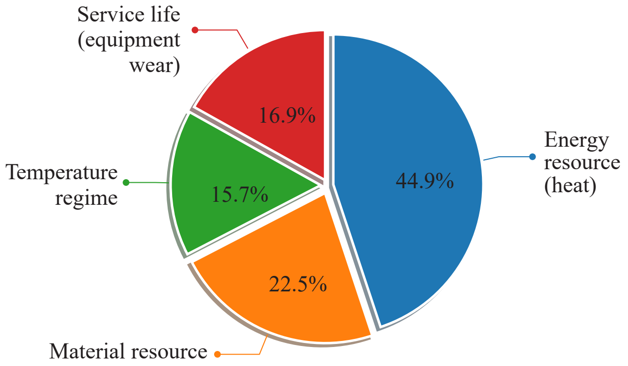


Fig. 5. Shares of resources in total costs (%)

Fig. 5 shows the share of each resource in total costs. The results indicate that the main part of the costs is related to energy consumption and the optimization of processes must be directed to the reduction of energy resources.

5. 3. Inclusion of the normalized deviation functional in the dynamic model of a technological process

On the basis of integrated indicators of costs and result

$$RE=\int_{0}^{T_{op}} re(t)\,\mathrm{d}t,\quad PE=\int_{0}^{T_{op}} pe(t)\,\mathrm{d}t, \tag{39}$$

a standardized efficiency criterion is introduced

$$ELF=\frac{\left(PE-RE\right)^{2}}{PE\cdot RE}\cdot\frac{T_{p}^{2}}{T_{op}^{2}}. \tag{40}$$

This criterion:

– is dimensionless;

– does not depend on the cost scale;

– takes into account the time efficiency of the process.

The deviation function is introduced on its basis

$$E=\left(ELF-E^{*}\right)^{2}. \tag{41}$$

This functionality makes it possible to quantitatively estimate the deviation of the current mode from the required one.

5. 4. Use of functionality in the control problem

It is shown that the efficiency deviation functional can be directly used in the control problem.

The task is reduced to the following

$$E \to \min, \tag{42}$$

that is, make the efficiency deviation as small as possible.

This is equivalent to increasing the value of the ELF criterion.

Using Dinkelbach method, the problem is simplified and written in the form

$$\max=\left(PE-\lambda RE\right). \tag{43}$$

Such a problem can be conveniently solved with the help of well-known methods of optimal control.

As a result, the efficiency criterion is used to select the best operating mode of the system.

5. 5. Algorithm of control calculation on the example of a technological process

A simple control calculation algorithm has been developed, represented below in the form of a block diagram (Fig. 6).

The algorithm allows for the following:

– calculating the efficiency of the process;

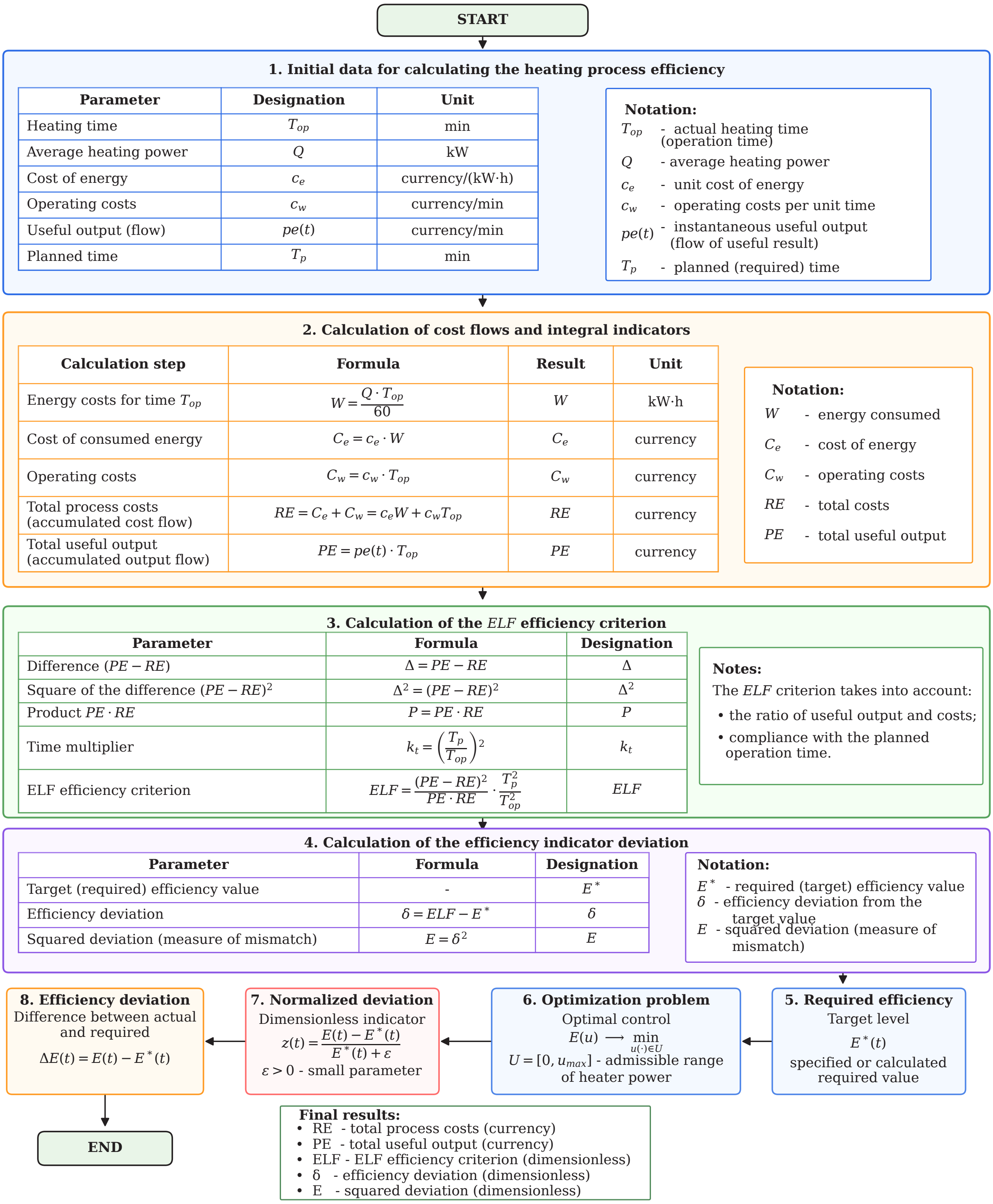
– obtaining stable results;

– comparing different modes of operation.

For illustration, a simple example of calculating the efficiency of the heating process is considered. The initial data are summarized in Table 1.

Cost flows and integrated indicators are calculated (Table 2).

The ELF efficiency criterion is calculated (Table 3).



1. Initial data for calculating the heating process efficiency

Parameter	Designation	Unit
Heating time	T_{op}	min
Average heating power	Q	kW
Cost of energy	c_e	currency/(kW·h)
Operating costs	c_w	currency/min
Useful output (flow)	$pe(t)$	currency/min
Planned time	T_p	min

Notation:

T_{op} - actual heating time (operation time)

Q - average heating power

c_e - unit cost of energy

c_w - operating costs per unit time

$pe(t)$ - instantaneous useful output (flow of useful result)

T_p - planned (required) time

2. Calculation of cost flows and integral indicators

Calculation step	Formula	Result	Unit
Energy costs for time T_{op}	$W = \frac{Q \cdot T_{op}}{60}$	W	kW·h
Cost of consumed energy	$C_e = c_e \cdot W$	C_e	currency
Operating costs	$C_w = c_w \cdot T_{op}$	C_w	currency
Total process costs (accumulated cost flow)	$RE = C_e + C_w = c_e W + c_w T_{op}$	RE	currency
Total useful output (accumulated output flow)	$PE = pe(t) \cdot T_{op}$	PE	currency

Notation:

W - energy consumed

C_e - cost of energy

C_w - operating costs

RE - total costs

PE - total useful output

3. Calculation of the *ELF* efficiency criterion

Parameter	Formula	Designation
Difference $(PE - RE)$	$\Delta = PE - RE$	Δ
Square of the difference $(PE - RE)^2$	$\Delta^2 = (PE - RE)^2$	Δ^2
Product $PE \cdot RE$	$P = PE \cdot RE$	P
Time multiplier	$k_t = \left(\frac{T_p}{T_{op}}\right)^2$	k_t
ELF efficiency criterion	$ELF = \frac{(PE - RE)^2}{PE \cdot RE} \cdot \frac{T_p^2}{T_{op}^2}$	ELF

Notes:

The *ELF* criterion takes into account:

- the ratio of useful output and costs;
- compliance with the planned operation time.

4. Calculation of the efficiency indicator deviation

Parameter	Formula	Designation
Target (required) efficiency value	-	E^*
Efficiency deviation	$\delta = ELF - E^*$	δ
Squared deviation (measure of mismatch)	$E = \delta^2$	E

Notation:

E^* - required (target) efficiency value

δ - efficiency deviation from the target value

E - squared deviation (measure of mismatch)

5. Required efficiency
Target level
$E^*(t)$
specified or calculated required value

6. Optimization problem
Optimal control
$E(u) \longrightarrow \min_{u(\cdot)\in U}$
$U = [0, u_{max}]$ - admissible range of heater power

7. Normalized deviation
Dimensionless indicator
$z(t) = \frac{E(t) - E^*(t)}{E^*(t) + \varepsilon}$
$\varepsilon > 0$ - small parameter

8. Efficiency deviation
Difference between actual and required
$\Delta E(t) = E(t) - E^*(t)$

Final results:

- RE - total process costs (currency)
- PE - total useful output (currency)
- ELF - ELF efficiency criterion (dimensionless)
- δ - efficiency deviation (dimensionless)
- E - squared deviation (dimensionless)

Fig. 6. Block diagram of control algorithm

Table 1

Initial data for calculating the efficiency of the heating process

Parameter	Designation	Value	Measuring unit
Heating time	T_{op}	10	min
Average heating power	Q	2	kW
Energy cost	c_e	5	UAH/(kWh)
Operating costs	c_w	2	UAH/min
Useful result	$pe(t)$	8	UAH/min
Estimated time	T_p	8	min

Table 2

Results of calculating cost flows and integrated indicators

Calculation stage	Formula	Result
Energy costs during heating	$W = Q \cdot \frac{T_{op}}{60}$	0.33 kWh
Cost of consumed energy	$C_e = W \cdot c_e$	1.65 UAH
Operating costs	$C_w = c_w \cdot T_{op}$	20.00 UAH
Total process costs	$RE = C_e + C_w$	21.65 UAH
Total useful result	$PE = pe\left(t\right) \cdot T_{op}$	80.00 UAH

Table 3

Calculation of the efficiency criterion

Parameter	Formula	Value
Difference between results and costs	$PE - RE$	58.35
Squared difference	$(PE - RE)^2$	3404.72
Multiplying $PE \cdot RE$	$PE \cdot RE$	1732.00
Time multiplier	$\theta = \frac{T_p^2}{T_{op}^2}$	0.64
Efficiency criteria	$ELF = \frac{\left(PE - RE\right)^2}{PE \cdot RE} \cdot \frac{T_p^2}{T_{op}^2}$	1.26

The efficiency deviation is calculated (Table 4).

Table 4

Calculation of deviation in the efficiency indicator

Parameter	Formula	Value
Target performance value	E^*	1.50
Performance deviation	$\delta = ELF - E^*$	–0.24
Square deviation	$E = \delta^2$	0.058

The obtained values show the following:

– the efficiency of the process is lower than required ($ELF < E^*$);

– the deviation is small, but the regime needs to be improved;

– the main reason for the decrease in efficiency is too long heating time.

6. Discussion of results based on the development of the normalized functional of efficiency deviation

In conventional approaches to efficiency analysis (for example, [1–8]), the assessment is usually carried out by separate indicators (resources, costs, duration). In contrast to this, the proposed model initially forms a single dynamic representation of the process through cost flows $re(t)$ and $pe(t)$, which makes it possible to take into account resources, results, and time of the technological process at the same time (Fig. 1). As can be seen from Fig. 1, the dynamic model makes it possible to analyze the effectiveness of control over time and compare the modes of operation of the process. Thus, the advantage here is provided precisely by the transition from static/separate indicators to a dynamic flow model.

Our results are due to the key feature of the proposed approach – the introduction of integrated value flows $re(t)$ and $pe(t)$ and the corresponding aggregated indices RE and PE. It is this design that determines the differences and advantages compared to known solutions.

Using the integrated RE and PE indicators makes it possible to evaluate the process over the entire operational period. The ELF efficiency criterion, developed on their basis, takes into account not only the ratio of output to costs but also the degree of deviation from each other, as well as the time factor through the T_p / T_{op} ratio. Therefore, in the example considered, it is clear that the decrease in efficiency is primarily due to an increase in process execution time.

The proposed approach differs from classical approaches in optimal control theory [1, 2], in which performance criteria are predetermined. In such models, efficiency is determined through weighting coefficients, making it dependent on the choice of scale. In contrast, the proposed ELF criterion is formed directly based on the internal characteristics of the process and is dimensionless.

Unlike the results reported in [6], where efficiency depends on the structure of the cost function, in this paper the criterion is not predetermined but is derived from the structure of the process itself. In our study, the ELF criterion is defined through the ratio of integrated flows, rather than through a predetermined function. This method preserves the correct representation of efficiency when the resource structure changes. It also becomes possible to compare different modes without reconfiguring the criterion.

This enables an objective assessment of efficiency and simplifies the comparison of different control modes. The limitations of this method are related to the fact that all resources must be converted into monetary value (Fig. 4), and the process parameters must be correctly defined.

One advantage is the consideration of the time factor through the T_p / T_{op} ratio. In most similar models, time is either fixed or indirectly considered (e.g., through penalties).

In the proposed methodology, time is explicitly included in the criterion, which allows for the identification of efficiency losses due to increased process duration. It also allows for the evaluation of modes with identical costs and results, but different execution speeds. Our results confirm the provisions of the proven theorem on the normalized quadratic form of the ELF efficiency criterion.

All this ensures a more accurate representation of efficiency in dynamic systems.

The advantages of the proposed approach over known solutions are ensured by the following properties:

– the technological process combines resource, result, and time considerations;

– integrated RE and PE indicators are introduced to evaluate the entire work cycle;

– the ELF criterion is independent of weighting factors;

– scale constancy ensures a change in the cost scale;

– time consideration allows for the accurate comparison of modes of different durations;

– structural versatility makes it possible to apply to various technological processes.

The combination of these features ensures a qualitative difference and practical significance of the results compared to existing approaches.

Our results are valid within the permissible operating conditions of the system. The accuracy of results depends on the choice of the sampling step and the correctness of the initial data.

Among the shortcomings of the study is the simplified nature of the model of the technological process under consideration. It does not take into account nonlinear parameters or their changes over time. Resource cost coefficients are assumed to be constant values, although under real-world conditions they may vary depending on economic conditions. The model could be improved by considering all parameters as variable.

Our work may continue for more complex systems using this method in control algorithms. However, difficulties may arise due to the increased complexity of the model and the increased computational effort.

7. Conclusions

1. A dynamic model of a controlled technological process has been constructed, taking into account the flows of costs $re(t)$ and useful results $pe(t)$. It is shown that integrating these flows makes it possible to determine the integrated indicators of costs RE and result PE, which characterize the process as a whole. A theorem on the normalized quadratic form of the efficiency criterion (ELF) has been proven, thereby establishing a relationship between the integrated useful result, costs, and the execution time of the technological process.

2. The resources and useful results of the process have been determined, including material, energy, temperature, and operational resources. Their cost reduction was performed, which ensured the formation of comparable integrated indicators of the system's performance.

3. A normalized performance indicator deviation functional has been developed, possessing the properties of dimensionlessness and scale invariance, independent of units of measurement and price level. This indicator allows for a quantitative assessment of deviation in actual efficiency from the required value and for comparing different process operating modes.

4. The normalized deviation functional has been integrated into the optimal control model and aligned with Pontryagin maximum principle. This allowed the control problem to be stated as one of minimizing the efficiency deviation and also provided a link between the dynamic model of the system and its performance assessment.

5. An algorithm for calculating optimal control based on parameterization and an iterative procedure has been developed. It provides a computational implementation of the proposed performance indicator deviation functional. This enables comparison of different operating modes and calculation of control effectiveness.

Conflicts of interest

The authors declare that they have no conflicts of interest in relation to the current study, including financial, personal, authorship, or any other, that could affect the study and the results reported in this paper.

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Data availability

All data are available in the main text of the manuscript.

Use of artificial intelligence

The authors confirm that they did not use artificial intelligence technologies when creating the current work.

Authors' contributions

Igor Lutsenko: Conceptualization, Formal analysis, Software, Validation, Visualization, Writing – original draft, Writing – review & editing; **Oksanych Iryna**: Methodology, Conceptualization, Formal analysis, Visualization; **Maksim Drachko**: Conceptualization, Formal analysis, Software, Validation, Visualization, Writing – original draft; **Evgeniia Burdilna**: Methodology, Formal analysis.

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