UDC 519.87
DOI: 10.15587/1729-4061.2026.365662

DEVELOPMENT OF A CASCADE MATHEMATICAL MODEL FOR DIGITAL IMAGE PROCESSING: A SYSTEMS APPROACH

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The object of this study is to preprocessing digital images degraded by additive Gaussian noise using cascaded Gaussian filtering. The scientific problem is to improve the output signal-to-noise ratio and reduce reference-based residual error of noisy digital images under additive Gaussian noise while preserving the informative structure required for subsequent image analysis. The model represents a noisy image as an additive combination of the reference image and a Gaussian noise component and describes two filtering stages through convolution of Gaussian kernels. The model is tested on the Lena benchmark image at noise levels from –10 dB to +10 dB and compared with median and bilateral filtering using signal-to-noise ratio and root mean square error. The results show that the cascade Gaussian model provides the highest signal-to-noise ratio over the studied range. At –10 dB the model increases SNR to 15.02 dB, whereas median and bilateral filters reach 4.21 dB and 1.26 dB. At +10dB, the cascade model achieves 28.19 dB. The model lowers RMSE at –10 dB to 45.25 pixels, while median and bilateral filtering give 81.95 and 115.16 pixels. This improvement comes from how Gaussian smoothing reduces random noise and how the Gaussian kernel creates a predictable filtering effect. The feature of the research results is that higher denoising accuracy is achieved together with mathematical transparency and simple implementation, without training data or a reference noise channel. Practical application of the model is possible as a preprocessing stage in machine vision, biomedical image analysis, robotic systems, monitoring, and other tasks with Gaussian-like noise

***Keywords:** image processing, cascade model, Gaussian filtering, mathematical model, machine vision*

Received 20.03.2026
Received in revised form 01.06.2026
Accepted 09.06.2026
Published 30.06.2026

***How to Cite:** Rakhmetova, P., Altay, Y. (2026). Development of a cascade mathematical model for digital image processing: a systems approach. Eastern-European Journal of Enterprise Technologies, 3 (9 (141)), 49–60. https://doi.org/10.15587/1729-4061.2026.365662*

1. Introduction

Digital image preprocessing has a vital role in machine vision, biomedical imaging, robotics, remote monitoring and measurement-based systems. In these fields, digital images provide visual and quantitative information for object recognition, defect detection, contour extraction, localization, classification and decision making. Any loss in image quality can therefore directly affect the reliability of the process.

Image denoising has become more and more relevant since the visual data is often captured in unstable or resource-limited environment. Challenges faced by systems in robotics, medical diagnostics, remote sensing, microscopy and monitoring include sensor limits, changing lighting, transmission issues, noise and environmental interference. Recent studies show that image denoising still remains a hot topic as real-world systems not only need better visual effects, but also require the accurate restoration of image data [1, 2]. In medical imaging, denoising is of particular importance, since it should enable the interpretation of the images while preserving the critical diagnostic information [2, 3]. Good signal-to-noise ratio is a requirement for microscopy and high-resolution imaging as low-SNR data can make measurements and analysis less reliable [4, 5]. In remote sensing and observation, filtering helps find weak objects and reduce background noise in complex conditions [6, 7].

Reliable preprocessing is also required for robotic and machine vision systems, as the quality of the image affects the quality of the detection of defects, the location of objects and the automation of inspections. For instance, in the vision-based in-pipe defect detection and classification, image preprocessing is important, since the poor image quality can affect the accuracy of the subsequent recognition steps [8]. This is also the case in hyperspectral image processing, where Gaussian-Laplacian multiscale techniques are used to extract relevant spatial and spectral features [9], in pansharpening, where the choice of filter affects the detail preservation and image fusion quality [10], in acoustic microscopy, where denoising helps to clarify noisy microscopic structures [11] and in low-SNR optoelectronic signal reconstruction, where multi-stage collaborative filtering helps to recover useful signals from strong noise [12].

Even with advances in image denoising, current applications still need methods that can reduce noise, keep important details, be reproducible, run efficiently and be easy to understand. Deep learning-based denoising can produce high-quality results, but these methods often rely on training data, model design, generalization and computing power [13, 14]. Despite the need for additional decision rules, optimization, or system modifications, hybrid and adaptive filters are capable of enhancing images [15, 16]. Hardware-oriented processing in accordance with speed, memory and reliability is emphasized in efficient solutions and practical applications [17, 18].

In such cases, mathematically understandable filtering models are valuable because they can be reproduced and measured without the need for learning methods. Gaussian filtering is a basic tool in image preprocessing, has a clear formula and operates via convolution. This makes it useful both for smoothing images and for constructing understandable mathematical models. When an image contains strong additive noise, a single stage of Gaussian filtering is insufficient. In this case, applying multiple stages of Gaussian filtering helps reduce random noise while maintaining the simplicity and ease of model analysis.

2. Literature review and problem statement

A broad survey of digital image denoising is given in [1], which organizes the field into spatial, transform-domain, non-local, sparse-representation and learning-based families. It is a useful map and it also makes a gap visible: most algorithms are studied in isolation and the combined effect of several mathematically coordinated filtering stages is rarely written down in closed form. That omission matters most for additive Gaussian noise, where the degradation model, the filtering operator and the output metrics can all be described analytically. For the present work [1] sets the methodological starting point for a compact, reproducible and interpretable cascade of Gaussian filters.

A complete review of medical image denoising [2] sets classical filters, transform-based approaches, optimization models and deep-learning solutions side by side. Its main lesson is that the best-performing methods usually demand careful parameter tuning, training data or modality-specific assumptions. For a model meant to run without training data and without a separate reference-noise channel, that is a real constraint and it argues for analytically transparent filtering, which behavior follows from the mathematics of the Gaussian kernel rather than from empirical training.

Denoising for cephalometric X-ray images is examined in [3] with the reminder that medical preprocessing has to cut noise without erasing diagnostically important contours and anatomical structures and that the result should be judged by quantitative quality measures, not only by eye. The methods there, though are tied to X-ray pipelines and stop short of a general convolution model for additive Gaussian noise. The open problem is a simpler preprocessing model that transfers across image classes while keeping the informative structure intact.

Single-image SNR estimation for scanning electron microscopy is reviewed in [4], which shows that SNR is one of the central quantitative indicators for objective image-quality assessment and addresses how to estimate it from SEM images when no clean reference is available.

A wider review of SNR in scanning electron microscopy [5] ties image quality to acquisition conditions, noise behavior and measurement methodology and treats SNR not as an auxiliary number but as a primary measure of how far an image can be trusted. Being SEM-specific, it does not describe how a cascade of Gaussian convolution operators changes SNR in a controlled digital-processing model. It does, however, reinforce the case for SNR-based assessment and for a formal filtering model with predictable output.

An in-pipe defect detection and classification system is built in [8], where preprocessing turns out to be essential for reliable machine vision in robotic inspection. The link it draws between denoising and real monitoring tasks is the relevant part, since image degradation feeds straight into later feature extraction and classification. Its focus, however, is detection and classification rather than the denoising model itself, so improving the preprocessing stage under controlled additive Gaussian noise, with reference-based error evaluation, is left largely untouched.

An iterative Gaussian-Laplacian pyramid network for hyperspectral image classification is proposed in [9] and it shows how a Gaussian-based multiscale representation can preserve spatial information for later analysis. The target there is hyperspectral classification, not the denoising of ordinary grayscale images under additive Gaussian noise and the Gaussian part is folded into a larger multiscale architecture, so the analytical effect of repeated Gaussian smoothing on SNR and residual error is never isolated. That is precisely what a separate study of Gaussian convolution as a cascade denoising operator would have to do.

Image filtering for pansharpening is studied in [10], which shows that filter design largely decides the balance between preserving spatial detail and reducing distortion. The emphasis on filtering accuracy as a precondition for downstream quality is what makes it relevant here. But pansharpening is a different inverse problem with different degradation assumptions, it neither attacks additive Gaussian noise through a two-stage Gaussian cascade nor compares output SNR and RMSE against classical median and bilateral filtering. So [10] supports the general importance of filter design without closing the specific gap addressed here.

Acoustic-microscopy image denoising by block-matching and 4D filtering is examined in [11], with strong restoration results in that specialized setting. Such methods are also heavier than classical filters and harder to implement and for preprocessing in machine vision, robotics and monitoring, that extra complexity works against reproducibility and practical use. The paper therefore puts the quality-versus-complexity trade-off plainly, which is itself an argument for a simpler cascade Gaussian model with a clear mathematical meaning.

Low-SNR optoelectronic signal reconstruction by zero-phase multi-stage collaborative filtering is proposed in [12]. It confirms that multi-stage filtering holds up under strong noise and that coordinating the stages improves reconstruction. The signals there are one-dimensional, though, whereas image preprocessing also has to protect spatial structure, contours and informative intensity transitions. Carrying the multi-stage idea over to two-dimensional Gaussian convolution and judging it with image-based SNR and RMSE, is the adaptation that remains to be made.

The deep-learning turn in image denoising is surveyed in [13], which reports the high restoration accuracy neural models reach on many benchmarks, alongside their well-known difficulties with interpretability, generalization, reproducibility and dependence on training data.

A CT denoising technique combining wavelet representation, anisotropic Gaussian filtering and a denoising CNN is proposed in [14]. It is directly relevant because Gaussian filtering sits inside the architecture, but that is also the catch: the Gaussian step is embedded in a larger wavelet-neural pipeline, so the standalone contribution of repeated Gaussian convolution is never isolated and the method is bound to a medical-imaging context. The paper supports the usefulness of Gaussian filtering and, at the same time, shows why a simpler model is needed in which two Gaussian stages are stated explicitly and evaluated on their own.

A hybrid denoising algorithm built from adaptive and modified decision-based filters is developed in [15], showing that classical filtering operations can be combined to good effect when they are coordinated to the noise and the image. The mechanism there is not posed as a Gaussian convolution cascade and is not aimed at additive Gaussian noise, so while it justifies comparing against classical filtering, it does not deliver a mathematically predictable equivalent Gaussian operation.

An adaptive, optimizable method combining Gaussian-process regression and linear least-squares regression for SEM images is proposed in [16], linking filtering to SNR estimation and measurable quality gains. It leans on adaptive optimization and SEM-specific assumptions, but general preprocessing of additive Gaussian degraded images calls for something simpler and more reproducible.

A cascade notch filter with unity feedback and improved transient response is studied in [19]. It belongs to filter theory rather than image denoising, but it makes a point that carries over: the behavior of a filtering system comes not only from its individual stages but from how they interact in a cascade, and the same idea applies to two-dimensional convolution operators. What [19] does not touch is image degradation, Gaussian kernels, SNR, RMSE or the preservation of image structure.

A non-local anisotropic diffusion framework based on Caputo derivatives and Gaussian convolution for the Perona-Malik model is proposed in [20]. It connects Gaussian convolution to mathematically controlled denoising directly, showing that Gaussian smoothing can live inside advanced analytical models for noise suppression and edge preservation. The price is complexity: fractional derivatives, non-local diffusion, and parameter-rich optimization all add to the computational and implementation burden.

Color image denoising under mixed multiplicative and Gaussian noise, using group-sparse representation and spatial vector total-variation regularization, is considered in [21].

The relevance of this work is that it addresses Gaussian noise as part of a more complex degradation process and confirms the importance of preserving image structure during denoising. However, the method involves advanced regularization and is designed for mixed-noise color images.

The analyzed sources show that the most relevant literature for the present study can be grouped into five areas, as general denoising theory [1, 2, 3, 13], SNR-based quality assessment [4, 5, 16], Gaussian-based and multi-stage filtering [9, 12, 14, 19, 20], comparison with classical and hybrid filters [10, 11, 15] and applied preprocessing for machine vision and robotic inspection [8]. These studies confirm the importance of noise suppression before subsequent image analysis, but they don't provide a compact two-stage Gaussian cascade model in which the noisy image is represented as an additive combination of a reference image and Gaussian noise, the filtering process is expressed through convolution of Gaussian kernels, and the output quality is assessed by SNR and RMSE against median and bilateral filters.

In summary, the literature review shows that current image denoising methods improve both visual quality and accuracy, but some challenges remain. Many leading methods need complex parameter tuning, training data or heavy optimization, which makes them hard to use in simple preprocessing systems. Classical filters are usually tested on their own and how they work together in a mathematically defined cascade is not well explained. For images with additive Gaussian noise, there is still a need for a simple model that is easy to understand, improves SNR, lowers residual error and is practical. The main challenge is to create a cascaded Gaussian filtering model where each stage has a clear mathematical purpose and the overall denoising effect is explained by the properties of Gaussian kernel convolution, not just by results from experiments.

3. The aim and objectives of the study

The aim of the study is to develop a two-stage cascaded Gaussian filtering model for preprocessing digital images degraded by additive Gaussian noise.

To achieve this aim, the following objectives are accomplished:

– to justify the choice of the structure and parameters of the cascaded Gaussian filtering model for digital image preprocessing under additive Gaussian noise;

– to formulate an additive Gaussian-noise degradation model and derive the cascaded Gaussian filtering operator through convolution of two Gaussian kernels;

– to compare the proposed cascaded Gaussian model with median and bilateral filters under identical noise conditions and explain the rationale for choosing these baseline spatial filters;

– to analyze the statistical reliability and computational complexity of the obtained results in order to assess the practical applicability of the method for preprocessing tasks in machine vision, robotics, biomedical image analysis and monitoring system.

4. Materials and methods

The objective of this study is the preprocessing of digital images degraded by additive Gaussian noise. The subject of the study is the filtering transformation applied to a noisy digital image in order to evaluate the behavior of a cascaded Gaussian filtering model under controlled noise conditions. In practical terms, the study considers the preprocessing stage that is performed before subsequent image analysis tasks, including recognition, classification, edge detection, segmentation or decision-making procedures in machine vision and related applied systems.

The hypothesis of the study is that a two-stage Gaussian filtering structure can provide analytically interpretable suppression of additive Gaussian noise because successive Gaussian filtering operations can be represented through convolution. This hypothesis is tested by comparing the cascaded Gaussian filtering model with two conventional spatial filtering methods under identical input conditions.

The purpose of the methodological design isn't to obtain an empirical result for one isolated image, but to create controlled conditions for verifying the mathematical behavior of the filtering model and for comparing its output with reproducible baseline methods. Before conducting the computational experiment, several assumptions were accepted. The original benchmark image was considered as a reference image without noise. The noise component added to the image was assumed to be additive, Gaussian-distributed, centered and stationary. Image degradation was assumed to come from Gaussian noise alone. Other factors were deliberately left out: motion blur, impulse and speckle noise, compression artifacts, geometric deformation, nonuniform illumination, sensor saturation and mixed noise. Restricting the model this way isolates the effect of the proposed filtering structure and keeps the comparison with the baseline filters fair.

The test material is the Lena benchmark image, a standard choice for comparing image-denoising methods. It was picked because it contains smooth regions, edge transitions, and textured areas at once, which lets one judge both noise suppression and the preservation of informative structure. The image was converted to grayscale to remove color-channel effects and keep the focus on intensity-based filtering. Noise was modelled as additive Gaussian noise, standing in for the random background component that typically appears during image acquisition in machine-vision systems.

Several simplifications were introduced to keep the validation controlled, reproducible and analytically interpretable. The experiment used a single benchmark image, so the numerical values reported here should not be read as universal across biomedical, robotic, industrial, or monitoring images; in practice the absolute figures for signal-to-noise ratio, root-mean-square error, peak SNR, structural similarity, and edge preservation will shift with texture density, contrast, edge complexity, illumination, object structure, and acquisition parameters. The analytical convolution property of cascaded Gaussian filtering, on the other hand, holds for any two-dimensional image degraded by additive Gaussian noise. Degradation was limited to additive Gaussian noise, which removes uncontrolled factors and tests the filter under one clearly defined assumption. Finally, the cascade was kept to two Gaussian stages. Two stages keep the derivation transparent and avoid over-smoothing the detail that matters; more stages would suppress noise further but would also risk blurring contours, flattening local contrast and damaging structural information.

The experiment ran in a numerical image-processing environment: loading the image, converting it to grayscale, applying the filters, computing the quality indicators, and visualizing the output. It was carried out in MATLAB R2021a on a personal computer with an Intel Core i5 processor, 8 GB of RAM, and Windows 10. Since the proposed model does not require neural network training or GPU acceleration, all filtering operations can be reproduced on a standard personal computer using conventional numerical image-processing libraries.

The experimental procedure consisted of the following stages. At the first stage, the reference benchmark image was prepared and normalized for further processing. At the second stage, additive Gaussian noise was generated and added to the reference image. The noise intensity was controlled by changing the input noise level from –10 dB to +10 dB. At the third stage, the noisy image was processed by the proposed cascaded Gaussian filter. At the fourth stage, the same noisy image was processed by the median and bilateral filters under identical input conditions. At the fifth stage, the filtered images were compared with the reference image using quantitative quality indicators. At the final stage, the obtained numerical values were summarized and prepared for graphical and statistical analysis.

The purpose of this comparison wasn't to cover all existing denoising algorithms, but to evaluate the proposed cascaded Gaussian model against simple, reproducible and practically implementable preprocessing filters that can be used in machine vision and robotic systems without training data or a separate reference noise channel. The main tools of the study were systems analysis of filtering operations, controlled computational modelling, comparative numerical evaluation and statistical analysis of quality indicators. Systems analysis was used to define the filtering process as a sequence of interconnected stages. Computational modelling was used to reproduce identical noise conditions for all compared methods.

The quality of filtering was assessed using signal-to-noise ratio and root mean square error as the main quantitative indicators. These metrics were selected because they characterize noise immunity, reference-based restoration accuracy and residual distortion after filtering. Additional structural indicators, such as peak signal-to-noise ratio, structural similarity index measure and edge preservation index, can be used to provide a more complete interpretation of image restoration quality. Visual comparison was used only as an auxiliary qualitative tool and wasn't considered a substitute for numerical assessment.

All compared methods were applied to the same input image and the same controlled noise levels. This ensured comparability of the computational experiment and excluded the influence of different initial conditions. No training dataset, preliminary learning stage or external noise reference channel was used. All analytical derivations, numerical results, graphical dependencies and statistical relationships obtained from the experiment are presented in the corresponding subsections of results.

5. Results of image processing modelling using cascaded Gaussian filtering

5. 1. Justification of the structure and parameters of the cascaded Gaussian filtering model

According to the first objective of the study, the initial result is the substantiation of the structure and principal parameters of the proposed cascaded Gaussian filtering model. Since the aim of the study is the development of a mathematical model, the first result should not be reduced to a numerical comparison of filtering outcomes. Instead, it must establish the internal organization of the model, define the role of each functional block, and explain the rationale for selecting the cascade structure and its governing parameters.

The general structure of the proposed model is presented in Fig. 1. As shown in the Fig. 1, the model is organized as a preprocessing system consisting of three logically connected parts: the input degradation model, the cascaded filtering structure, and the output evaluation block. Such a representation makes it possible to describe the full trajectory of image transformation, beginning with the reference image, continuing through controlled degradation and filtering, and ending with the quantitative assessment of restoration quality.

At the input stage, the Lena benchmark image is used as the reference image $I(x,y)$. This image was chosen because it has smooth areas, edges, and textured regions, making it useful for testing both noise reduction and the preservation of important image details. In this model, the benchmark image acts as the starting point for measuring the effect of filtering.

The second block of the input part is the additive Gaussian noise component $n(x,y)$, which is introduced as a centered random disturbance with zero mean and standard deviation σ_n. Its inclusion reflects the adopted degradation assumption according to which the observed distortion is caused by additive Gaussian noise. This approach is justified because Gaussian noise is a common way to model random interference that occurs during image capture and transmission in machine vision systems.

When the reference image and the noise are combined, they create the noisy image $Z(x,y)$, which is simply the sum of the clean image and the noise. This first part of the model sets up the input degradation and gives a clear starting point for the filtering steps that follow. As shown in Fig. 1, this stage creates the noisy image by adding Gaussian noise to the Lena benchmark image.

At the center of the model is the filtering block, which applies two Gaussian stages in sequence. The first stage convolves the noisy image with a Gaussian kernel of standard deviation σ_1 to produce the intermediate image $I_1(x,y)$; this mainly knocks down the random component and the strongest fluctuations. One stage alone, though, rarely strikes the right balance between removing noise and keeping important detail, particularly when the noise level varies.

A second Gaussian stage, with standard deviation σ_2, then processes the intermediate image $I_2(x, y)$ into the final output. The extra stage deepens the smoothing while keeping the process mathematically clean. Two stages are a practical compromise: they remove more random noise than a single stage, yet stop short of the heavy blurring that longer cascades tend to produce.

One part of the structure carries most of the theoretical weight: the equivalent cascaded operator shown in Fig. 1. It expresses the fact that convolving two Gaussian kernels yields another Gaussian-type operator with transformed parameters. That is what makes the cascade more than a repeated calculation; it is a single mathematical transformation with a predictable result. The arrangement is justified on both counts at once: in practice it improves filtering, and analytically it keeps a compact, interpretable operator form. The governing parameters are the standard deviations σ_1 and σ_2 of the two stages. They set how much smoothing each stage applies and, together, fix the behavior of the whole cascade. Too small a value smooths weakly and can leave much of the noise in place; too large a value smooths hard but can blur contours and wash out local detail. σ_1 and σ_2 are therefore the structurally important quantities and the model's main tuning variables.

The block outputs the filtered image $I'(x, y)$, also shown in Fig. 1. This is the final result of the cascade and the basis for the quantitative evaluation that follows; compared with the noisy input, it should carry a smaller random component while keeping the main visual structures of the reference image.

The final part of the structural model is the quality assessment block, where the filtering result is evaluated using the quantitative indicators SNR and RMSE. As shown in Fig. 1, these indicators are computed with respect to the reference Lena benchmark image $I(x, y)$. Their inclusion in the structural scheme is important because it completes the model logically – the image transformation is linked not only to the degradation and filtering stages, but also to an explicit system of quantitative evaluation. In this way, the model becomes a closed analytical framework that includes the input assumptions, the transformation mechanism and the output criteria of effectiveness.

Thus, the structural justification presented in Fig. 1 defines the functional composition and governing parameters of the proposed model. Therefore, the next subsection proceeds to the analytical derivation of the cascaded Gaussian filtering operator via the additive degradation model and the convolution of two Gaussian kernels.

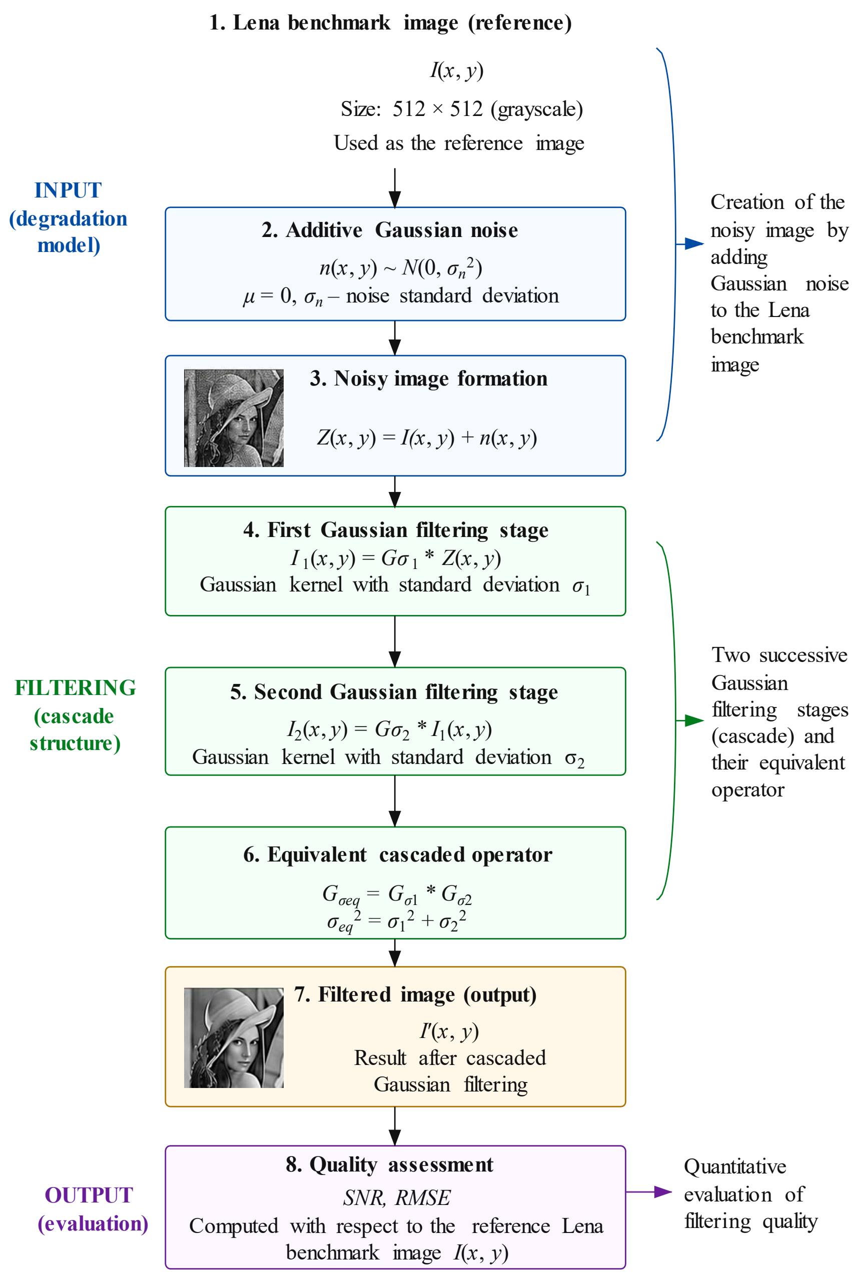

Fig. 1. Structural block diagram of the proposed cascaded Gaussian filtering model

5. 2. Mathematical formulation of the Gaussian-noise degradation model and cascaded filtering operator

An additive Gaussian-noise degradation model was formulated and the cascaded Gaussian filtering operator was derived through convolution of two Gaussian kernels. The analytical transformation shows that two sequential Gaussian stages act as a single smoothing operation with a new effective parameter and a multiplicative coefficient. This is important for the study method because it proves the cascade is a mathematically sound model for reducing noise, not just a repeated calculation. The derived expression also shows that the filtering effect can be controlled through the Gaussian kernel parameter and the number of stages, which creates a basis for further adaptive tuning of the method.

The original Lena benchmark image model and Gaussian noise are represented as an additive mixture and have the following form

$$Z(x, y) = I(x, y) + n(x, y), \tag{1}$$

where x and y – the row and column vectors of the digital image signal I, n – the additive Gaussian measurement noise, which is a centered and stationary process with zero mean μ and standard deviation σ for $N(\mu, \sigma)$, and $Z(x, y)$ is the noisy image.

The research method is primarily based on the theory of systems analysis of linear and nonlinear filters and their convolution to refine the assigned properties and improve the quality of digital processing of noisy images. The application of this method reveals the specific features of filter cascading method and their pre-cascading properties, as presented below in equations using convolution. The Gaussian filter equation is:

$$I'(x,y)=\sum_{s=-k}^{k}\sum_{t=-k}^{k}G_\sigma(s,t)\cdot I(x-s,y-t), \tag{2}$$

$$G_\sigma(s,t)=\frac{1}{2\pi\sigma^2}\cdot\exp\left(-\frac{s^2+t^2}{2\sigma^2}\right), \tag{3}$$

where s, t – the offsets along the coordinate axes, σ – the standard deviation of the kernel. The median filter equation is

$$I'(x,y)=\text{median}\left\{I(x+s,y+t)\right\}, \tag{4}$$

where W – a rectangular window of size at $(s, t)\in W$. The bilateral filter equation is:

$$\begin{cases} I'(x,y)=\dfrac{1}{W_p}\sum_{(s,t)\in\Omega}I(s,t)\cdot f_r\left(\left|I(s,t)-I(x,y)\right|\right)\cdot g_s\left(\left|(s,t)-(x,y)\right|\right),\\ W_p=\sum_{(s,t)\in\Omega}f_r\left(\left|I(s,t)-I(x,y)\right|\right)\cdot g_s\left(\left|(s,t)-(x,y)\right|\right),\\ g_s(r)=\exp\left(-\dfrac{r^2}{2\sigma_r^2}\right),\\ f_r(\Delta)=\exp\left(-\dfrac{r^2}{2\sigma_r^2}\right), \end{cases} \tag{5}$$

where Ω – the neighborhood of the pixel with a diameter.

Filter equations show their differences and the effectiveness of image processing may depend on this. The synthesis and convolution of a Gaussian filter for cascading is presented below

$$I''(x,y)=\sum_k\sum_l G_\sigma(k,l)\cdot I'(x-k,y-l)= \\ =\sum_k\sum_l G_\sigma(k,l)\cdot\left[\sum_s\sum_t G_\sigma(s,t)\cdot I\begin{pmatrix}x-k-s,\\ y-l-t\end{pmatrix}\right]. \tag{6}$$

The convolution of two Gaussian filter kernels is

$$(G_\sigma\cdot G_\sigma)(m,n)= \\ =\exp\left[-\frac{k^2+l^2}{2\sigma^2}\right]\exp\left[-\frac{(m-k)^2+(n-l)^2}{2\sigma^2}\right]- \\ =\exp\left[-\frac{k^2+(m-k)^2+l^2+(n-l)^2}{2\sigma^2}\right]. \tag{7}$$

Let's open and select a full square for k and l:

$$\begin{cases} k^2+(m-k)^2=2k^2-2mk+m^2=2\left(k-\dfrac{m}{2}\right)^2+\dfrac{m^2}{2},\\ l^2+(n-l)^2=2l^2-2nl+n^2=2\left(l-\dfrac{n}{2}\right)^2+\dfrac{n^2}{2}. \end{cases} \tag{8}$$

After obtaining (8), the resulting terms are substituted into the exponent and then divided by the corresponding factors:

$$\exp\left[-\frac{m^2+n^2}{4\sigma^2}\right]\cdot\exp\left[-\frac{\left(k-\frac{m}{2}\right)^2+\left(l-\frac{n}{2}\right)^2}{\sigma^2}\right],$$

$$\sum_k\sum_l\exp\left[-\frac{\left(k-\frac{m}{2}\right)^2+\left(l-\frac{n}{2}\right)^2}{\sigma^2}\right], \tag{9}$$

$$a=k-\frac{m}{2},$$

$$b=l-\frac{n}{2}\Rightarrow\sum_x\sum_y\exp\left(-\frac{a^2+b^2}{\sigma^2}\right)=\iint f(a,b)\,da\,db. \tag{10}$$

Let's move on to polar coordinates to integrate the filter components:

$$a=r\cos\theta,$$

$$b=r\sin\theta,$$

$$dadb=rdrd\theta. \tag{11}$$

Then the integral becomes

$$\int_0^{2\pi}\int_0^{\infty}\exp\left(-\frac{r^2}{\sigma^2}\right)r\,dr\,d\theta.$$

Calculate the radial integral and make a substitution:

$$u=\frac{r^2}{\sigma^2},$$

$$du = \frac{2r}{\sigma^2} dr. \quad (12)$$

After substitution of the integration components, the following expression is obtained:

$$\int_0^{\infty} re^{-\frac{r^2}{\sigma^2}} \mathrm{d}r = \frac{\sigma^2}{2} \int_0^{\infty} e^{-u} \mathrm{d}u = \frac{\sigma^2}{2},$$

$$\int_0^{2\pi} \int_0^{\infty} \exp\left(-\frac{r^2}{\sigma^2}\right) r\mathrm{d}r\mathrm{d}\theta = \int_0^{2\pi} d\theta \frac{\sigma^2}{2} = 2\pi \frac{\sigma^2}{2} = \pi\sigma^2,$$

$$\sum_k \sum_l \exp\left(-\frac{\left(k - \frac{m}{2}\right)^2 + \left(e - \frac{n}{2}\right)^2}{\sigma^2}\right) = \pi\sigma^2. \quad (13)$$

Thus, based on the transition to polar coordinates (11) and the calculation of the radial integral (12) lead to expression (13). The inverse transformation (9), (10) to (7), the following result is obtained:

$$\left(G_\sigma \cdot G_\sigma\right)(m,n) = \frac{1}{4\pi\sigma^2} \cdot \exp\left(-\frac{m^2 + n^2}{4\sigma^2}\right),$$

$$\sigma_{image} = 2\sqrt{\sigma}, \quad (14)$$

$$\left(G_\sigma \cdot G_\sigma\right)(m,n) = \frac{1}{2\pi\sigma_{image}} \cdot \exp\left(-\frac{m^2 + n^2}{2\sigma_{image}^2}\right).$$

The final analytical transformation Gaussian filter for cascading is presented below

$$I''(x,y) = \sum_n G_{\sigma_{image}}(m,n) \cdot I(x - m, y - n). \quad (15)$$

The performance of the method in filtering image noise is assess using three well-known quantitative metrics: signal-to-noise ratio and root-mean square error normalized error [1]:

$$\mathrm{SNR} = 10\log_{10}\left(\frac{\sum I_{\mathrm{original}}^2}{\sum\left(I_{\mathrm{noisy}} - I_{\mathrm{original}}\right)^2}\right), \quad (16)$$

$$\mathrm{RMSE} = \sqrt{\frac{1}{\mathrm{MN}} \sum_{x,y} \left(I_{\mathrm{noisy}}(x,y) - I_{\mathrm{original}}(x,y)\right)^2}. \quad (17)$$

Thus, the analytical derivation of the cascaded Gaussian filtering operator and the selected quality metrics establishes a formal basis for evaluating the proposed model. The next subsection presents the numerical and graphical results of image processing obtained for the cascaded Gaussian, median and bilateral filtering methods under identical controlled noise conditions.

5. 3. Quantitative assessment of the effectiveness of the cascaded Gaussian filtering model compared with conventional filtering methods

According to the third objective of the study, the effectiveness of the proposed cascaded Gaussian filtering model was evaluated under controlled additive Gaussian noise conditions and compared with conventional spatial filtering methods. The quantitative assessment was performed using the same reference benchmark image, identical noise levels, and the same reference-based quality indicators for all compared methods. This experimental design ensured the comparability of the obtained results and made it possible to evaluate the behavior of each filtering method under strong, medium, and weak Gaussian noise. The reference Lena benchmark image and the corresponding noisy image are shown in Fig. 2.

a *b*

Fig. 2. Results of noise reduction: *a* – benchmark image; *b* – noisy image model

The Lena image was chosen as a test object because it has smooth areas, clear edges and textured parts. This makes it useful for checking how well denoising works and whether important image details are preserved.

As shown in Fig. 2, additive Gaussian noise significantly changes local intensity values, distorts the visual structure of the benchmark image and reduces the clarity of edge transitions. This confirms the need for a filtering model that suppresses the random component while preserving the useful image information needed for subsequent digital processing.

The visual results of image filtering are presented in Fig. 3. The proposed cascaded Gaussian model was compared with median and bilateral filters. The median filter was used as a representative of nonlinear local order-statistic filtering, whereas the bilateral filter was used as a representative of edge-aware spatial-intensity filtering.

Fig. 3 shows that the cascaded Gaussian filter fixes noisy images more evenly than the other filters. The median filter removes some distortion but leaves visible noise. The bilateral filter keeps some local details but works less well when the noise is strong. This visual comparison shows the need for clear evaluation using numerical quality measures.

Computational modelling was performed for five input noise levels: –10 dB, –5 dB, 0 dB, +5 dB and +10 dB. For each noise level, the same noisy image was processed by the proposed cascaded Gaussian filter, the median filter and the bilateral filter. The simulation results are summarized in Table 1 and graphically interpreted in Fig. 4, 5. The use of several noise levels is methodologically important because it demonstrates not a single successful filtering case, but the stability of the developed model over the full investigated interval.

The results presented in Table 1 show that the proposed cascaded Gaussian model provides the highest signal-to-noise ratio at all investigated noise levels. At –10 dB, the proposed model achieves an output SNR of 15.02 dB, whereas the median and bilateral filters achieve 4.21 dB and 1.26 dB, respectively. Thus, under strong noise conditions, the proposed model provides an SNR gain of 10.81 dB relative to the median filter and 13.76 dB relative to the bilateral filter.

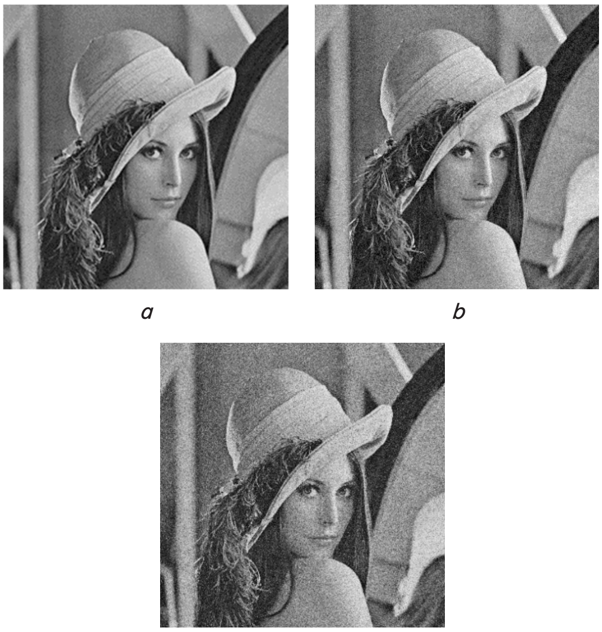

Fig. 3. Results of image noise filtering: *a* – proposed cascade method; *b* – median filter; *c* – bilateral filter

At −5 dB the cascaded Gaussian model reaches 16.67 dB, while the median and bilateral filters yield 7.66 dB and 2.71 dB, respectively. At 0 dB, the proposed model achieves 19.66 dB, compared with 12.03 dB for median filtering and 5.80 dB for bilateral filtering. At +5 dB the proposed model achieves 23.99 dB, whereas the median and bilateral filters achieve 16.63 dB and 11.77 dB, respectively. At +10 dB the proposed model achieves 28.19 dB, while the median and bilateral filters achieve 20.95 dB and 20.08 dB, respectively.

The average SNR over the investigated interval is 20.71 dB for the proposed cascaded Gaussian model, 12.30 dB for the median filter and 8.32 dB for the bilateral filter. Therefore, the average SNR advantage of the proposed model is 8.41 dB relative to median filtering and 12.39 dB relative to bilateral filtering.

Fig. 4 confirms that the proposed cascaded Gaussian model provides the highest output SNR over the entire investigated range. The difference between the compared methods is especially pronounced at low input SNR levels, where additive Gaussian noise strongly distorts the image structure. This indicates that the cascade structure is more effective under conditions of strong random degradation.

The RMSE values also confirm the advantage of the proposed model. At −10 dB, the RMSE of the cascaded Gaussian filter is 45.25 pixels, while the median and bilateral filters produce 81.95 pixels and 115.16 pixels, respectively. This means that under the strongest noise condition, the proposed model reduces RMSE by 44.78% relative to the median filter and by 60.71% relative to the bilateral filter.

At −5 dB, the proposed model gives an RMSE of 37.40 pixels, compared with 55.13 pixels for median filtering and 97.42 pixels for bilateral filtering. At 0 dB, the RMSE values are 26.52 pixels for the proposed model, 33.31 pixels for the median filter and 68.30 pixels for the bilateral filter. At +5 dB, the proposed model reaches 16.11 pixels, whereas median and bilateral filtering reach 19.62 pixels and 34.31 pixels. At +10 dB, the proposed model provides the lowest RMSE value of 9.93 pixels, compared with 11.93 pixels for the median filter and 13.18 pixels for the bilateral filter.

The average RMSE over the investigated interval is 27.04 pixels for the proposed cascaded Gaussian model, 40.39 pixels for the median filter and 65.67 pixels for the bilateral filter. Therefore, the proposed model reduces average RMSE by 33.05% relative to median filtering and by 58.82% relative to bilateral filtering. This confirms that the increase in SNR is accompanied by a consistent reduction in reference-based residual error. The integral comparison of the average advantage of the proposed model is presented in Fig. 5.

The integral comparison in Fig. 5 summarizes the quantitative advantage of the developed model. The cascaded Gaussian filter demonstrates the best combined behavior because it simultaneously provides the highest average SNR and the lowest average RMSE. This confirms that the improvement is not limited to one isolated metric but reflects a general enhancement of noisy image preprocessing quality.

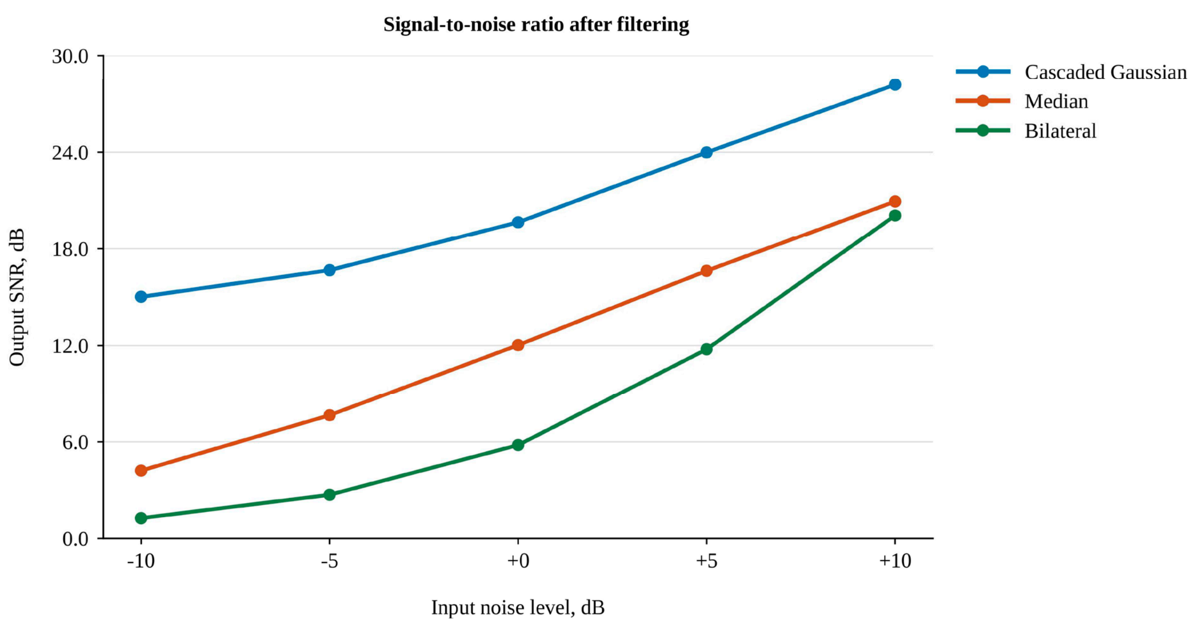

Fig. 4. Dependence of signal-to-noise ratio on input noise level for the compared filtering methods

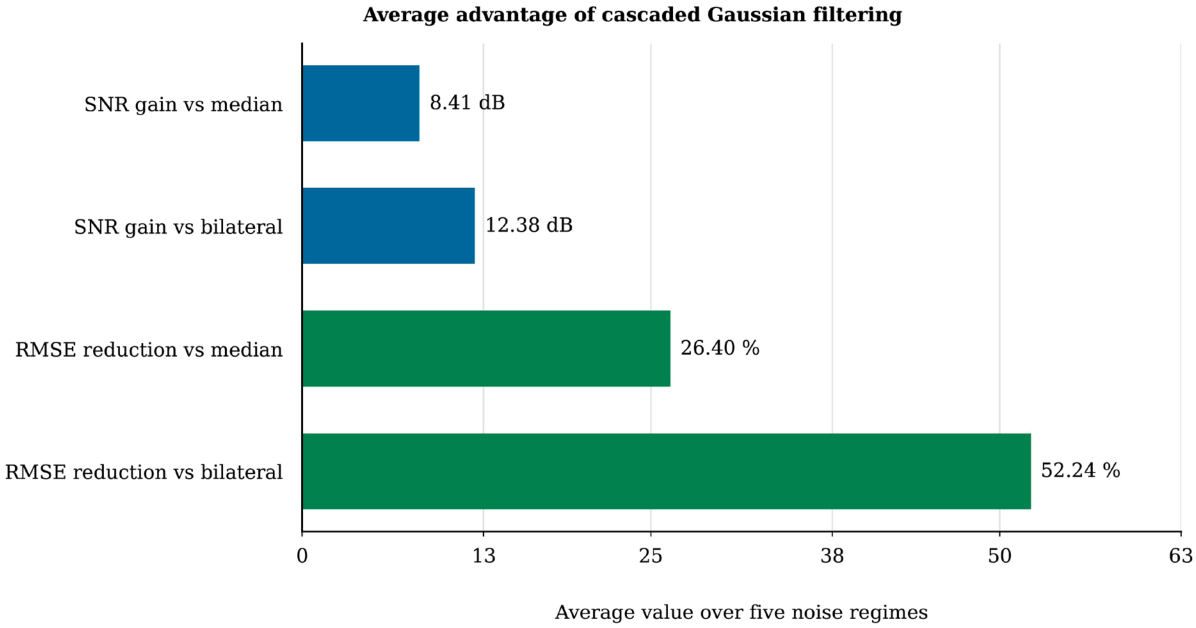

Fig. 5. Integral graphical comparison of the average advantage of the filtering methods

Table 1

Quantitative evaluation

Noise level	Proposed cascade method		Median filtering method [24]		Bilateral filtering method [1]	
	SNR	RMSE (root mean square error)	SNR	RMSE (root mean square error)	SNR	RMSE(root mean square error)
–10 dB	15.02	45.25	4.21	81.95	1.26	115.16
–5 dB	16.67	37.40	7.66	55.13	2.71	97.42
–0 dB	19.66	26.52	12.03	33.31	5.80	68.30
+5 dB	23.99	16.11	16.63	19.62	11.77	34.31
+10 dB	28.19	9.93	20.95	11.93	20.08	13.18

Thus, the quantitative assessment confirms that the proposed cascaded Gaussian filtering model improves digital image preprocessing quality compared with the selected conventional filtering methods. The improvement is expressed in a stable increase in signal-to-noise ratio and a consistent reduction in root mean square error over all investigated noise levels. These results provide an experimental basis for the subsequent statistical analysis of the reliability of the obtained image processing indicators.

5. 4. Statistical and probabilistic analysis of the reliability of image processing results

This section presents the results of an evaluation of the relationships between digital image processing system metrics that characterize the effectiveness of the methods under consideration. The results of the discovered relationships between metrics are presented below in Table 2.

The evaluation results revealed that under the influence of noise, the SNR value increases and leads to a decrease in RMSE at the output of the Gaussian filter cascade, namely $r = -0.996$, $R^2 = 0.992$, $p = 3 \cdot 10^{-4}$; median filter [24] $r = -0.975$, $R^2 = 0.952$, $p = 0.004$; bilateral filter [1] $r = -0.995$, $R^2 = 0.990$, $p = 4 \cdot 10^{-4}$. At the same time, despite the decrease in the RMSE indicator at the output of the methods and the presence of a high close non-random correlation according to the SNRb (SNR before filtering) and RMSE data after filtering, distinguishable significant relationships are traced between SNRa (SNR after filtering) and RMSE. This is mainly due to the fact that the Gaussian filter cascade, in comparison with the bilateral and median filtering methods, provides a high SNR estimate at the method output and increases the processing accuracy, which is easily detected from the established linear non-random and very high correlation relationship. In particular, for the Gaussian filter cascade, $r = -0.980$, $R^2 = 0.961$, $p = 3 \cdot 10^{-3}$; median filter $r = -0.966$, $R^2 = 0.933$, $p = 0.007$; for bilateral filter $r = -0.962$, $R^2 = 0.926$, $p = 0.008$. Moreover, the calculated correlation coefficient values also indicate that the major contribution of external factors to the established linear relationship is 3.9% for the Gaussian filter cascade, 6.7% for the median filter, and 7.4% for the bilateral filter, confirming the closeness of the detected relationship between the SNRa and RMSE indices. The influence of external factors for the cascade filtering method does not exceed 5% compared to the evaluated and previously proposed methods for filtering digital image noise.

Table 2

Relationship between digital image processing system indicators

Indicators	Digital image processing methods		
	Proposed method	Median filtering method [24]	Bilateral filtering method [1]
SNRb: RMSE	$r = -0.996$	$r = -0.975$	$r = -0.995$
	$R^2 = 0.992$	$R^2 = 0.952$	$R^2 = 0.990$
	$p = 3 \cdot 10^{-4}$	$p = 0.004$	$p = 4 \cdot 10^{-4}$
SNRa: RMSE	$r = -0.980$	$r = -0.966$	$r = -0.962$
	$R^2 = 0.961$	$R^2 = 0.933$	$R^2 = 0.926$
	$p = 3 \cdot 10^{-3}$	$p = 0.007$	$p = 0.008$
SNRb: SNRa	$r = 0.986$	$r = 0.998$	$r = 0.957$
	$R^2 = 0.972$	$R^2 = 0.997$	$R^2 = 0.916$
	$p = 0.002$	$p = 5 \cdot 10^{-5}$	$p = 0.010$

The cascade filtering method, with high image noise (–10 dB), ensures stable digital processing with an image signal-to-noise ratio of 15.02 dB. With a minor noise contribution, namely, +10 dB, the system assigns a score of 28.19 dB. The high score for this indicator with a minor noise contribution is explained by the excess of image intensity relative to the processed image signal. In this regard, there is a positive and very high significant correlation between the SNR values before and after processing, which characterizes the increase in the quantitative SNR indicator after noise filtering. The calculated values of the correlation coefficient for the cascade filtering method were $r = 0.986$, $R^2 = 0.972$, $p = 0.002$; median filter $r = 0.998$, $R^2 = 0.997$, $p = 5 \cdot 10^{-5}$; bilateral filter $r = 0.957$, $R^2 = 0.916$, $p = 0.010$, respectively. However, despite the high and highly significant correlation established for the median and bilateral filters, these noise filtering methods produce digital processing with significant distortions of the noise-free image, which is significantly different from the RMSE of the Gaussian filter cascade at $r = -0.980$, $R^2 = 0.961$, $p = 3 \cdot 10^{-3}$, respectively. Thus, the results of a comprehensive quantitative evaluation of the methods revealed that the cascade filtering method simultaneously improves the quantitative indicators characterizing the increased effectiveness of digital processing of noisy images in terms of accuracy and noise immunity.

6. Discussion of the results of digital image processing using cascaded Gaussian filtering

The obtained results are explained by the direct correspondence between the adopted noise model and the mathematical structure of the proposed filter. In formula (1), the noisy image is defined as an additive mixture of the original image and Gaussian noise. This assumption justifies the use of the Gaussian kernel in formulas (2) and (3), since the filtering operation is consistent with the probabilistic nature of the distortion. The main effect of the proposed method is determined by the cascade convolution of Gaussian kernels. Equations (6) and (15) show that applying two Gaussian filters in a row creates a single smoothing effect with a changed parameter and a scaling factor. This means the cascade is more than just repeated smoothing and it is a well-founded model for step-by-step noise reduction.

This mechanism explains the quantitative advantage presented in Table 1. At the strongest noise level of –10 dB the proposed cascade increases SNR to 15.02 dB, whereas the median and bilateral filters provide only 4.21 dB and 1.26 dB. At +10 dB, the cascade reaches 28.19 dB, exceeding both comparison methods. The same tendency is observed for RMSE – at –10 dB, the proposed method reduces the error to 45.25 pixels, while the median and bilateral filters give 81.95 and 115.16 pixels. In short, the model improves image quality on two fronts at once: it raises SNR and lowers residual error.

The visual results in Fig. 2, 3 back up the numbers. Fig. 2 shows how badly additive Gaussian noise degrades the benchmark image, while Fig. 2 shows the cascaded Gaussian filter suppressing that noise more evenly than the median and bilateral filters. The curves in Fig. 4 confirm that the advantage holds across the whole tested range, from –10 dB to +10 dB, and Fig. 5 gathers it into integral terms: average SNR gain and RMSE reduction. Table 2 adds a strong, statistically significant link between output SNR and RMSE for the proposed method, with $r = -0.996$, $R^2 = 0.992$, and $p = 3 \cdot 10^{-4}$, which means the improvement is systematic, not accidental.

Against existing approaches, what sets this method apart is analytical transparency and low methodological complexity. Modern denoising spans spatial, transform-based, adaptive and learning-based techniques [1]. Adaptive methods [11, 12] often need coefficient tuning or an extra reference-noise channel and neural or hybrid approaches [16] depend on training data, network architecture and computing resources. The cascade here uses only deterministic convolution and needs none of that: no training, no reference-noise channel, no complex optimization. That makes it a good fit for reproducible preprocessing in machine vision, robotics, biomedical image analysis and monitoring.

A comparison with median and bilateral filtering helps explain why cascade filtering is superior. The median, based on local order statistics, copes well with impulse noise. However, the noise here is additive Gaussian noise, distributed throughout the image, which it cannot completely remove. Bilateral filtering preserves edges by weighing spatial and intensity similarities, however, with strong Gaussian noise, the process limits averaging and allows some noise to pass. Cascade filtering better matches the noise model used in this study, resulting in a higher signal-to-noise ratio and lower mean square error. Therefore, the two baseline models are intended as a simple benchmark for preprocessing, not for a comprehensive comparison with all modern noise reduction algorithms.

Computational cost depends on the image and kernel sizes. A single stage of direct two-dimensional convolution is $O(MNk^2)$, where M and N are the image dimensions and k the kernel size, so the direct two-stage version is about $O(2MNk^2)$. Because the Gaussian kernel is separable, each two-dimensional convolution splits into two one-dimensional ones, dropping each stage to $O(2MNk)$; and since two Gaussian filters in series collapse into one equivalent filter, the cost stays manageable. In robotics, monitoring and machine vision it can be cut further by filtering only the region of interest, using separable convolution, fixing a small kernel, invoking the cascade only when noise is high and running it on GPUs, FPGAs or other parallel hardware. The cascade thus need not add processing time in proportion to its stages and it can serve real-time preprocessing as long as kernel size, image resolution, and region of interest are chosen with care.

The limitations are tied to the experimental conditions. The results come from the Lena benchmark image under additive Gaussian noise from –10 dB to +10 dB, so the numbers should not be carried over directly to impulse, speckle or Poisson noise, motion blur, compression artifacts or mixed non-stationary interference. A second limit is inherent to Gaussian smoothing: pushed too far, a cascade erodes fine detail if the kernel parameter or the number of stages is chosen badly.

The main limitations lie in the methods. This comparison only considers median and bilateral filters. Comparisons of other filtering methods, such as the Wiener filter, wavelet thresholding, nonlocal means, anisotropic diffusion, and new neural models, should be explored in future work. The evaluation primarily uses SNR and RMSE. So it would be useful to include structural metrics such as SSIM, PSNR, edge preservation index and gradient-based metrics. These additional metrics provide a clearer picture, as they complement the energy-based and pixel-based approaches to SNR and RMSE. For example, PSNR measures the degree of correspondence between the reconstructed image and the reference image, SSIM indicates the degree of preservation of the image structure and the edge preservation index verifies important edges after filtering.

Therefore, the effectiveness of the cascaded Gaussian model can be assessed not only by noise suppression, but also by the preservation of visually significant image information. Third, computational time and memory consumption were not analyzed. Although these parameters are important for real-time machine vision and robotic systems.

Further development of this study should focus on adaptive selection of the Gaussian kernel parameter and the number of cascade stages according to the noise level and local image structure. This can improve the balance between noise suppression and edge preservation. The main mathematical difficulty will be to preserve analytical interpretability, when local adaptive parameters are introduced. The main methodological difficulty will be to ensure fair comparison with algorithms that require different tuning strategies or training procedures. The main experimental difficulty will be to verify the model on broader benchmark and real-world datasets. Nevertheless, the obtained results show that cascaded Gaussian filtering is a mathematically justified and practically promising preprocessing method for improving digital image quality under additive Gaussian noise.

7. Conclusions

1. The structure and parameters of the cascaded Gaussian filtering model were justified. The proposed model consists of an input additive Gaussian-noise degradation block, a two-stage Gaussian filtering operator, and an output quality evaluation block. The two-stage structure was selected as a balance between noise suppression and preservation of informative image details. The main parameter of the model is the Gaussian kernel standard deviation, which determines the degree of smoothing. The choice of the Gaussian cascade is justified by the correspondence between the assumed additive Gaussian noise model and the Gaussian kernel, by the analytical transformability of Gaussian convolutions, and by the reproducibility of the filtering procedure without training data.

2. The additive Gaussian-noise degradation model was formulated by representing the noisy digital image as a superposition of the reference image and a Gaussian noise component. The cascaded filtering operator was then derived as the convolution of two Gaussian kernels corresponding to two successive filtering stages. The derivation showed that sequential Gaussian filtering is mathematically reducible to an equivalent Gaussian convolution operator with transformed parameters. Thus, the result of solving this objective is the analytical formulation of the degradation model and the cascaded Gaussian filtering operator, which provides the mathematical basis for evaluating SNR improvement and reference-based residual error reduction.

3. The effectiveness of the developed model was confirmed by a quantitative comparison with median and bilateral filtering for additive Gaussian noise in the range of –10 dB to +10 dB. The cascaded Gaussian filter yielded the highest average signal-to-noise ratio (SNR) of 20.71 dB, compared to 12.30 dB for the median filter and 8.32 dB for the bilateral filter. The lowest average RMSE was also achieved – 27.04 pixels, compared to 40.39 pixels and 65.67 pixels for the compared methods. At the highest noise level of –10 dB, the proposed cascade achieved a signal-to-noise ratio of 15.02 dB and reduced the RMSE to 45.25 pixels, confirming its advantage in high-noise conditions.

4. Statistical and probabilistic analysis confirmed the robustness of the observed improvement. For the cascaded Gaussian filter, a strong and statistically significant relationship was found between signal-to-noise ratio and RMSE, with $r = -0.980$, $R^2 = 0.961$, and $p = 3 \cdot 10^{-3}$. The influence of external factors did not exceed 5%, indicating a stable relationship between noise immunity and processing accuracy. Thus, the proposed cascaded Gaussian filtering model can be considered as an efficient, computationally simple and analytically interpretable pre-processing method for digital image processing systems.

Conflicts of interest

The authors declare that they have no conflict of interest in relation to this study, whether financial, personal, authorship or otherwise, that could affect the study, and its results presented in this paper.

Financing

This study was funded by the Science Committee of the Ministry of Education and Science of the Republic of Kazakhstan (Grant No. AP22685781 Development of a manipulator control system using computer vision to enhance flexibility and automate processes).

Data availability

All data are available in the main text of the manuscript.

Use of artificial intelligence

The authors declare that generative artificial intelligence tools were used exclusively for language editing, grammar checking, and technical formatting of the manuscript under the full control of the authors.

Artificial intelligence was not used to create, process, or interpret scientific data, draw conclusions, or other elements of scientific results of the work.

Tool used: ChatGPT (OpenAI GPT-5, version 2025).

The authors bear full responsibility for the content, reliability, and scientific correctness of the submitted material.

Authors' contributions

Perizat Rakhmetova: Conceptualization, Methodology, Project administration, Writing – review & editing; **Yeldos Altay**: Methodology, Formal analysis, Data Curation, Writing – original draft.

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