У роботі проведено чисельне дослідження мультифрактальних властивостей атракторів нелінійних дискретних систем для різних хаотичних режимів. Проведено мультифрактальний аналіз послідовностей RR-інтервалів кардіорітма людини до та після застосування препаратів

Ключові слова: хаотична динаміка, атрактор системи, мультифрактал, RR-інтервали

В работе проведено численное исследование мультифрактальных свойств аттракторов нелинейных дискретных систем для разных хаотических режимов. Проведен мультифрактальный анализ последовательностей RR-интервалов кардиоритма человека до и после применения медицинских препаратов

Ключевые слова: хаотическая динамика, аттрактор системы, мультифрактал, RR-интервалы

In this paper a numerical investigation of multifractal properties of the attractors of nonlinear discrete systems for diverse chaotic regimes is carried out. Also a multifractal analysis of RR-interval's sequences of human heart rate was conducted before and after using a drugs

Keywors: chaotic dynamic, system's attractor, multifractal, the RR-intervals

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ANALYSIS OF MULTIFRACTAL PROPERTIES OF CHAOTIC MAPS

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Introduction and relevance

It's generally recognized now that many of the informational, biological, physical and manufacturing processes have a complex fractal structure. Fractal analysis is used for modeling, analysis and control a complex system in various areas of science and technology. For example, the fractal analysis of medical and biological signals is used in the identification of irregular heart rate and blood flow's turbulence according to the electrocardiogram; for determination of apoplectic seizure or epileptic seizure on the electroencephalographic signal; for improving the image's quality on mammograms; for optimal compression of medical signals etc. [1-3].

The investigation of bioelectric medical signals can be made by using the methods developed in the theory of dynamical chaos under the assumption that the physiological signal is considered as nonlinear system sensitive to initial conditions. Thus, it's assumed that the bioelectrical activity is described by the implicit chaotic system and despite the absence of the modeling system of equations, it's possible to study the behavior of the system from its output data. [4-7]

Phase trajectories of a dissipative chaotic systems are never closed and seek to fill a certain area that called a strange attractor. The strange attractor is located in the bounded area of phase space of the system that is attracted all sufficiently close trajectories and has a complicated fractal structure.

The aim of the presented work is a numerical investigation of fractal structure of attractors of nonlinear discrete systems with diverse chaotic behaviors and application of the results to the analysis of an experimental medical data.

Chaotic dynamics of nonlinear systems [6-7]

Chaos is a complex form of deterministic system's behavior in a steady state. Although the evolution of this system is uniquely determined by the dynamical laws and it isn't affected by any other random forces, system's dynamic is stochastic. The main property of such systems is sensitive dependence of behavior to arbitrary small changes in initial conditions. If d_0 is initial distance between two points, the distance between paths coming out of these points in a short time t will be $d(t)=d_0e^{\lambda t}$, where λ is Lyapunov exponent. This fact leads to loss of deterministic predictability and the necessity to bring in new probability characteristics to describe dynamic systems with chaotic behavior.

One of the simplest mathematical models that have chaotic behavior are iterated maps $x_{n+1}=f(C,x_n)$, where C – the control parameter. For a wide class of nonlinear functions f sequence of values $\{x_n\}_{n=0}^{\infty}$ is chaotic. In the case of dissipative maps orbits $\{x_n\}_{n=0}^{\infty}$ tend to an attractor that in general has multifractal structure.

While changing the control parameter of dynamic systems its nonlinear properties are shown differently. The complexity of dynamic regime and consequently of attractor structure occurs with the growing of influence of the nonlinearity. Changing of systems dynamic while parameter is changing allows us to observe bifurcation's sequence which resulted in a forming of chaotic attractor. Typical bifurcation sequences are combined by the meaning of scenarios of chaos development. Many systems demonstrate transition to the chaos by a cascade of doubling period bifurcations.

In many cases one-dimensional maps occur in describing of complex multidimensional processes and allow to significantly simplify dynamic system's analysis. One way to obtain such map is to construct map by using local maxima of system's paths. Let first maxima M_1 is culminated at time t_1, second M_2 - in time t_2 etc. Sequence of values $\{M_n\}$ is orbit of one-dimensional map that allows to construct simple models of learning facts.

Characteristics of attractors with multifractal structure. [2,3,8,9]

Self-similarity of fractal objects is confined in saving object's structure of zooming. Let consider main characteristics of multifractal set. Suppose that, in general, multifractal attractor occupies some bounded region in d-dimensional Euclidean space and defines set of $N \to \infty$ points. Let divide the entire region into box of side ε and volume ε^d. Let consider the partition function $Z(q,\varepsilon)$ characterized by an exponent q ($-\infty < q < +\infty$):

$$Z(q,\varepsilon) = \sum_{i=1}^{N(\varepsilon)} p_i^q(\varepsilon), \qquad (1)$$

where $p_i(\varepsilon) = \lim\limits_{N\to\infty} \frac{n_i(\varepsilon)}{N}$, $n_i(\varepsilon)$ - number of points get into the box with number i, $N(\varepsilon)$ - total number of occupied cells that depends from the size of the box ε. Probabilities p_i characterize relative population of the box.

In general multifractal set is characterized of nonlinear function $\tau(q)$, that determine behavior of partition function $Z(q,\varepsilon)$ with $\varepsilon \to 0$:

$$Z(q,\varepsilon) \propto \varepsilon^{\tau(q)}. \qquad (2)$$

Function $\tau(q)$ usually is called scaling exponent and defined as

$$\tau(q) = \lim_{\varepsilon\to 0} \frac{\ln Z(q,\varepsilon)}{\ln\varepsilon}. \qquad (3)$$

In the case of homogeneous fractal set with fractal dimension D all busy boxes have the same number of points, that mean $p_i(\varepsilon) = p(\varepsilon) = 1/N(\varepsilon)$ and partition function is

$$Z(q,\varepsilon) = N^{1-q}(\varepsilon) = \varepsilon^{-D(1-q)}$$

and function $\tau(q) = (q-1)D$ is linear. If the distribution of points in the boxes isn't the same, the fractal set is heterogeneous, i.e. multifractal, and is $\tau(q)$ is a nonlinear function. If $q \to +\infty$, the main contribution to the partition function is made by the boxes that contain the greatest number of particles n_i and, consequently, most likely characterized by the filling p_i. Conversely, if $q \to -\infty$, the main contribution to the partition function is made by the most sparse boxes with small values p_i. Thus, the function $\tau(q)$ shows how heterogeneous set of points is investigated.

Along with the scale exponent $\tau(q)$ for the multifractal characteristics of the set the function of multifractal spectrum (the spectrum of singularities) $f(\alpha)$ is used. The dependence of the probability from the box size $p_i(\varepsilon)$ has an exponential character

$$p_i(\varepsilon) \propto \varepsilon^{\alpha_i},$$

where α_i is some exponent, in general various for the diverse boxes (a measure of the singularity). For the homogeneous fractal all of the exponents α_i are the same and equal to the fractal dimension D.

Function of multifractal spectrum $f(\alpha)$ characterize a probability distribution for the diverse values α_i. If value $n(\alpha)d\alpha$ is probability of the fact that α_i is in the interval (α, $\alpha + d\alpha$), i.e. the number of the boxes i that have the same measure $p_i(\varepsilon)$ with $\alpha_i \in (\alpha, \alpha + d\alpha)$, then

$$n(\alpha) \approx \varepsilon^{-f(\alpha)}. \qquad (5)$$

So function $f(\alpha)$ is fractal dimension of the some homogeneous fractal subset ξ_α from the original set ξ that is characterized by the same probabilities of the box filling $p_i(\varepsilon) \approx \varepsilon^\alpha$.

Taking into account the expressions (1) and (5), the generalized partition function $Z(q,\varepsilon)$ can be written by using function of multifractal spectrum $f(\alpha)$ the next way:

$$Z(q,\varepsilon) = \sum_{i=1}^{N(\varepsilon)} p_i^q(\varepsilon) \approx \int d\alpha\, n(\alpha)\varepsilon^{q\alpha} \approx \int d\alpha\, \varepsilon^{q\alpha - f(\alpha)}.$$

Formally, the transition of variables $\{q,\tau(q)\}$ to the variables $\{\alpha, f(\alpha)\}$ can be made with the help of the next Legendre transformations:

$$\begin{cases} \alpha = \dfrac{d\tau}{dq} \\ f(\alpha) = q\dfrac{d\tau}{dq} - \tau \end{cases} \quad \text{and} \quad \begin{cases} q = \dfrac{df}{d\alpha} \\ \tau(q) = \alpha\dfrac{df}{d\alpha} - f \end{cases} \qquad (6)$$

The method of modulus maxima of wavelet transform [10-12]

One of the most popular tools in multifractal analysis is the method of modulus maxima of wavelet transform (WTMM). It's based on wavelet analysis that is called "mathematical microscope" because of its ability to maintain a good resolution at diverse scales. Because wavelet functions are localized in time and frequency, method WTMM is a powerful tool for the statistical description of non-stationary processes.

The wavelet transform of one-dimensional function – is its representation as generalized series or integral in the system of basic functions $\psi_{ab}(t) = \frac{1}{\sqrt{a}}\psi\left(\frac{t-b}{a}\right)$ derived from the mother's wavelet $\psi(t)$, that has certain properties due to the shift of operations in time b and changing the time scale a.

Continuous wavelet-transform of function $X(t)$ can be written as $W(a,b) = \frac{1}{\sqrt{a}}\int_{-\infty}^{\infty} X(t)\psi_{ab}(t)dt$. Function $W(a,b)$ is called wavelet-spectrum and can be presented as the surface of wavelet coefficients in three-dimensional space. The most important information is contained in lines of local extremes of the surface $W(a,x)$, which search is conducted on every scale a.

Method WTMM is allowed to get numerically the partition function:

$$Z(q,a)=\sum_{l\in L(a)}\left(\sup_{a'\le a}\left|W(a',x_l(a'))\right|\right)^q ,$$

where L(a) – is the set of all lines l of maxima of wavelet coefficient's modulus on the scale a; $x_l(a)$ - is the location of maximum on this scale.

To calculate $Z(q,a)$ the maximum absolute value of wavelet-coefficients is selected along each line on the scales that smaller than a given value of scale a.

In this case the dependence is executed:

$$Z(q,a)\approx a^{\tau(q)} ,$$

where $\tau(q)$ is scale exponent that is defined for each value q by calculating the slope of $\ln Z(q,a)$ from ln a.

The investigation of the model chaotic realization's characteristics

In the paper was investigated using of WTMM method realizations of diverse one-dimensional maps obtained for different chaotic regimes. Multifractal analysis has showed the presence of multifractal structure for the majority of attractors of chaotic sequences: scaling exponent $\tau(q)$ is non-linear function. For every considering chaotic regime defined by a control parameter, we calculated the Lyapunov exponent λ and multifractal characteristics $\tau(q)$ and $f(\alpha)$.

One of the most famous examples of chaotic maps is logistic map [6,7]. It's one-dimensional square map defined as:

$$x_{n+1}=Ax_n(1-x_n) ,$$

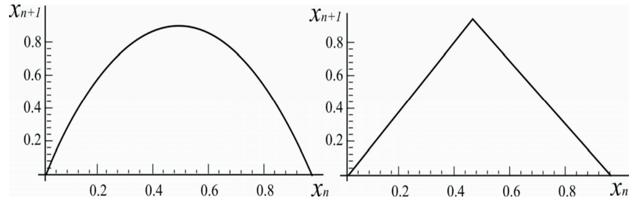
where A - is control parameter, $A\in(0,4]$ and values $x_n\in[0,1]$. Graphic presentation of logistic function for the value $A=4$ is showed at the Fig. 1(a). When $A>3$ fixed point loses its stability and two-cycle's period stands out: steady state is an alternating sequence of two numbers. Then a stable cycle of period 4 appears, further in the same way there are losing of stability to infinity. The corresponding cascade of doubling bifurcations is showed in Fig. 2(a) above, that shows the bifurcation diagram of logistic map. The abscissa represents the values of control parameter A, and the vertical axis – values of orbits $\{x_n\}$ in the steady state. The chaos is observed, when values $\lambda>\lambda^*=3.569...$. Areas of chaos alternate with "windows of stability" – narrow areas where dynamic becomes periodic.

As the next example let consider a triangle map [7]:

$$x_{n+1}=r\left(1-2\left|\frac{1}{2}-x_n\right|\right).$$

where r - is control parameter, $r\in(0,1]$, values $x_n\in[0,1]$. Graphic of this function for parameter $r=1$ is showed on fig. 1(b).

If the value of parameter $r<\frac{1}{2}$, then attractor is a fixed point 0. When $r>1/2$ this function generates a chaotic sequence. In this case the transition to the chaos occurs immediately without a cascade of bifurcations. The corresponding bifurcation diagram is showed in Fig. 2(b) above. In general for the triangle map the Lyapunov exponent is equaled $\lambda=\ln 2r$.



а) б)

Fig. 1. Logistic function (a) and triangle (б) maps

Fig. 2(a) shows a time series of logistic map with parameters $A_1=3.79$ and $A_2=3.94$. The bifurcation diagram shows attractors for given values of control parameter. The corresponding Lyapunov exponent equals $\lambda_1=0.433$ and $\lambda_2=0.563$. Obviously, in the second case chaotic regime is more developed because initially close trajectories diverge at a faster rate. Multifractal analysis conducted by the WTMM showed the presence of multifractal structure for the attractors of logistic map with the values of control parameter A_1 and A_2. The corresponding functions of multifractal spectrum are showed in fig. 2(a) below. The values of multifractal spectrum for more developed chaotic regime shifted to the right and have a larger range that indicating a more heterogeneous structure of the strange attractor.

Fig. 2(b) shows trajectories of triangle map with parameter's values $r_1=0.77$ and $r_1=0.98$. The corresponding attractors are showed in the bifurcation diagram. The Lyapunov exponents for these chaotic regimes equal $\lambda_1=0.318$ and $\lambda_2=0.665$. In the second case chaotic regime is more developed. The corresponding functions of multifractal spectrum are showed in fig. 2(b) below. As the case of logistic map multifractal spectrum for more developed chaotic regime possess a larger range, i.e. the corresponding strange attractor is more nonhomogeneous.

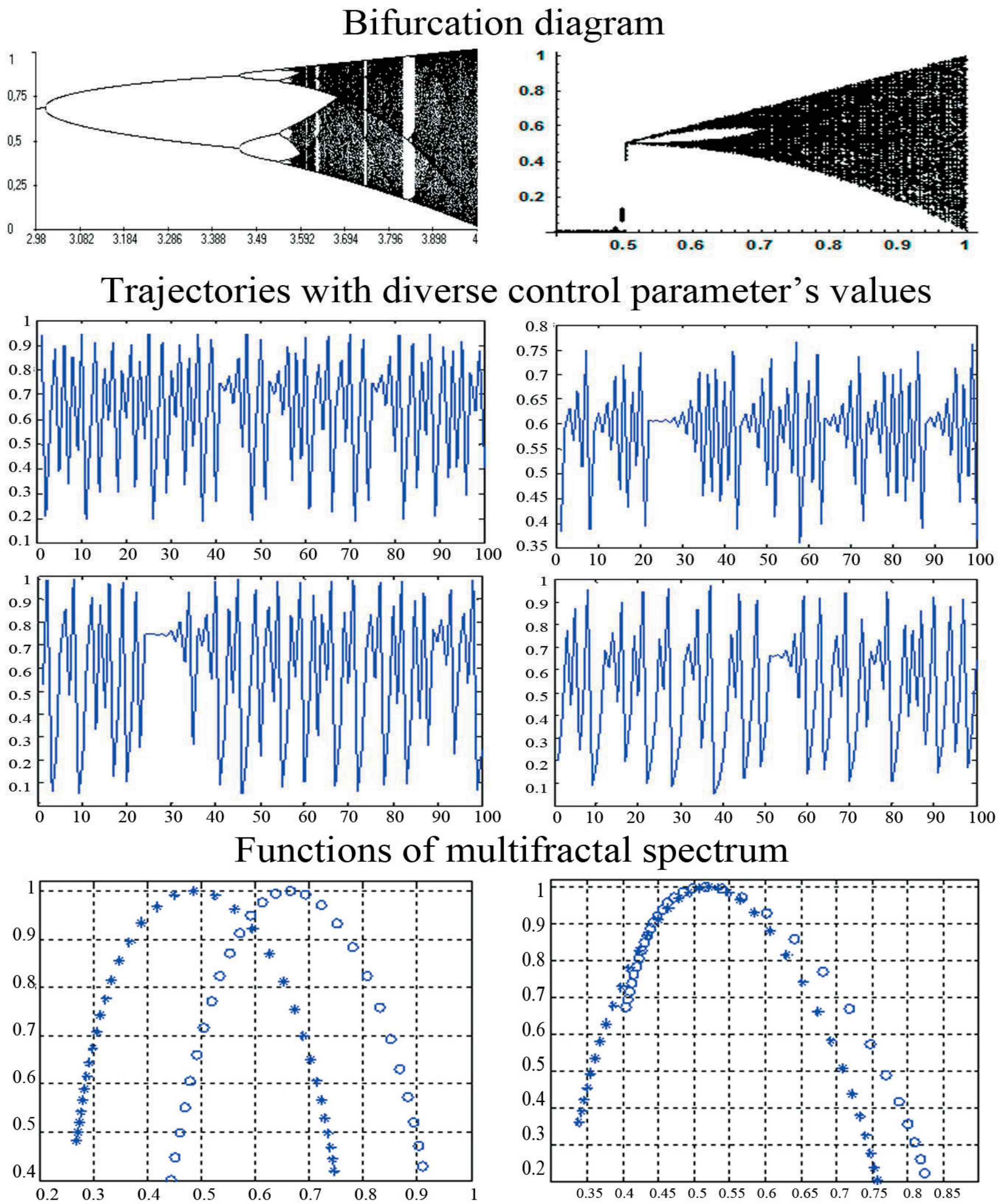
Investigation of characteristics of RR-interval's realizations

It's known that for the diagnosis and detection of diverse heart's diseases analysis of the electrocardiogram (ECG) has an important place. ECG is a recording of electrical heart's activity.

The slightest deviation from the norm may indicates a violation of the cardiac rhythm and also of the presence of diverse diseases. One of the methods of diagnosing heart diseases is an analysis of the series constructed by the RR-intervals.

RR-interval is the time interval between adjacent teeth of electrocardiogram and it equals to the duration of the cardiac cycle. These intervals are very important in determining the heart rate and diagnosis of diverse types of cardiac arrhythmias. fig. 3 shows the construction of the series by using RR-intervals. It's known, that these types of series have chaotic structure [5,7], so it's possible to analyze them by using multifractal methods.

Initial data for the research in this paper were obtained on a dedicated website [13] containing an extensive medical database. Fig. 4 shows scaling exponent and multifractal spectrum typical for RR-intervals of the person, who has no heart diseases.



a) б)

Fig. 2. Bifurcation diagrams, trajectories, multifractal spectrum for logistic (a) and triangle (b) maps

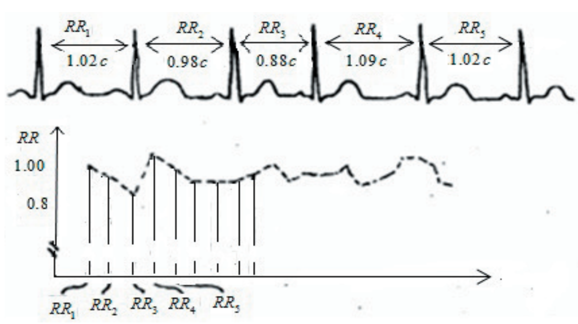
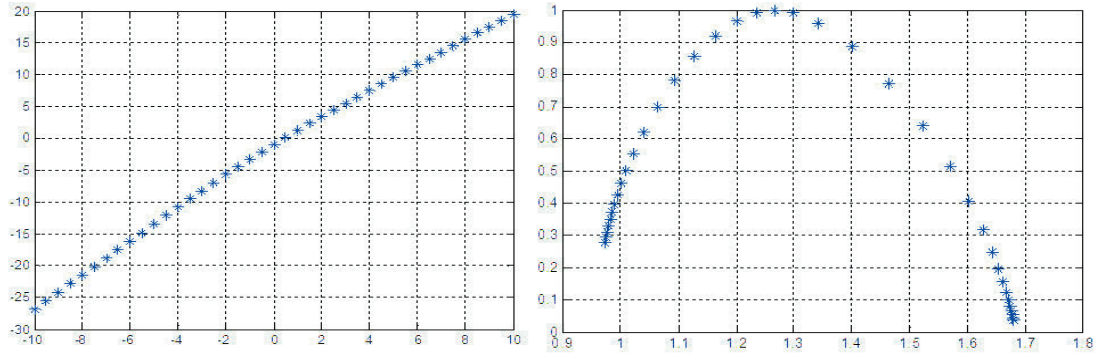


Fig. 3. The image of normal ECG-signal with RR-intervals and the constructing of RR-interval's sequence

The database contains cardiogram records of the patients involved in medication trials. In a medical investigation were involved patients belonging to the age group from 45 to 69 years and have a heart arrhythmia The data of RR-intervals before and after taking medication used to treat and prevent tachycardia by increasing heart rate, are showed in this paper. Fig. 5 shows the values of scaling exponent and multifractal spectrum of two patients that was a typical for the majority of patients before and after drug application.

Researches have shown that drug's application causes changes of multifractal characteristics of RR-interval's sequence.



a) b)

Fig. 4. Scaling exponent (a) and multifractal spectrum (b) for RR — intervals of healthy person

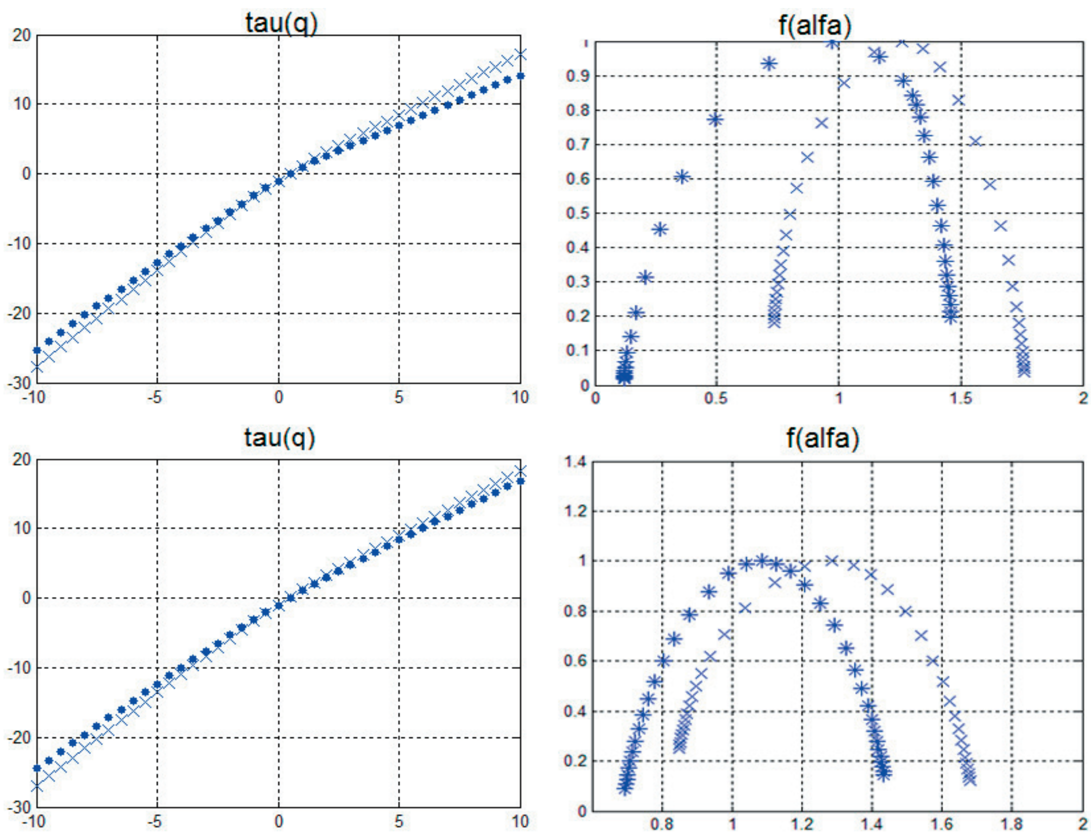


Fig. 5. Scale exponents (a) and multifractal spectrum (b) before drug's application (points / stars) and after it's application (crosses) before drug's application (points /) and after it's application (line 2)

The most evident characteristic that distinguishes time series before and after medication is function of multifractal spectrum $f(\alpha)$. Almost all patients have shifted to the right function $f(\alpha)$ after medication, i.e. values of α has increased and became closer to the characteristics of healthy person. From the viewpoint of nonlinear dynamics, changing of multifractal properties of the trajectories indicates a change in the functional regime of the system.

Conclusions and prospectives for further research

In this paper we investigate multifractal characteristics of attractors of discrete-time chaotic maps for diverse values of bifurcation parameter. It's shown that increasing chaos of the system defined by the Lyapunov exponent, causes a complication of the attractor's structure that appears as enhancing of multifractal properties.

The analysis of multifractal characteristics of RR-interval's sequence of the person was carried out before and after medication.

Research has shown that drug's using causes a changes of multifractal characteristics of RR-interval's sequence that is expressed in the shift of the function of multifractal spectrum to the values of spectrum typical for normal cardiac rhythm.

These results suggest that fractal methods can be used in the analysis of electrocardiological signals and allow to fix functional changes in heart's activity. Multifractal analysis of ECG can be the basis for statistical studies that will allow to formulate such methods of ECG analysis that will be important for clinical practice.

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УДК 681.3

ДУКАСКОПИЯ, КАК МЕТОД НОРМАЛИЗАЦИИ НЕЙРОННЫХ СЕТЕЙ

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У статті подається нове рішення проблеми нормалізації вхідних векторів для нейронних мереж за допомогою дукаскопії, зокрема для прогнозування часових рядів

Ключові слова: нормалізація, нейронні мережі, дукаскопія, прогнозування

В статье представляется новое решение проблемы нормализации входных векторов для нейронных сетей посредством дукаскопии, в частности для прогнозирования временных рядов

Ключевые слова: нормализация, нейронные сети, дукаскопия, прогнозирование

The paper presents a new solution of the problem of normalization of the input vectors for neural networks through dukascopy, particularly for time series prediction

Keywords: normalization, neural networks, dukascopy, prediction

1. Введение

Предсказание финансовых временных рядов - необходимый элемент любой инвестиционной деятельности.

Сама идея инвестиций - вложения денег сейчас с целью получения дохода в будущем - основывается на идее прогнозирования будущего. Соответственно, предсказание финансовых временных рядов лежит в основе деятельности всей индустрии инве-