Abstract

The characteristics of the radio waves scattering over the acoustic wave packets created by sophisticated acoustic sounding signals in radio acoustic sounding systems of atmosphere are analyzed. In this case, there is an expansion of the scattering body along spatial frequencies, expanding the wave numbers range of effective interaction acoustic sounding and electromagnetic signals, and a decrease in the duration of the scattered radio signals is also observed. The physical reason signal length decrease is the signal length duration spatial domain decrease of acoustic and electromagnetic waves, due to the presence of modulation parameters in space. The form and parameters of the spatial spectrum, that is embedded in the scattering signal, in a great degree acts on the quality characteristics resulting estimates of signal parameters and atmospheric parameters. The presence of specific errors in the estimation of the Doppler frequency information parameters and radio signals time delay, scattered on the complex acoustic sounding signals is shown. The use of sound oscillations can significantly increase the efficiency of the radio acoustic sounding stations when measuring vertical temperature profiles of the atmosphere

Keywords: *radio acoustic sounding of atmosphere, dispersion bodies, complex acoustic signal*

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USING ADAPTIVE EQUALIZING IN LTE WITH MIMO

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Робота присвячена аналізу використання методу еквалізації в LTE системах з різними конфігураціями MIMO з метою підвищення характеристик якості зв'язку

Ключові слова: LTE, MIMO, просторово-часове кодування, адаптивний еквалайзер, Zero Forcing, метод мінімуму середньоквадратичної помилки

Работа посвящена анализу использования метода эквализации в LTE системах с различными конфигурациями MIMO с целью повышения характеристик качества связи

Ключевые слова: LTE, MIMO, пространственно-временное кодирование, адаптивный эквалайзер, Zero Forcing, метод минимума среднеквадратической ошибки

1.Introduction

Equalization is the process of adjusting the balance between frequency components within an electronic signal. It has important applications in telecommunications. Equalization is used to render the frequency response and prepare data signals for transmission. When a channel has been "equalized" the frequency domain attributes of the signal at the input are faithfully reproduced at the output.

Equalizers are critical to the successful operation of LTE systems. In this application the actual waveform of the transmitted signal must be preserved, not just its frequency content. Adaptive Equalizer filters must cancel out any group delay and phase delay between different frequency components. Especially in broadband applications where Intersymbol Interference (ISI) is a critical factor. Equalizers are employed to reduce such interference. MIMO systems transmits different signals from each transmit element so that the receiving antenna array receives a superposition of all the transmitted signals.

2. Intersymbol Interference and Equalization

The all-pass assumption made in the AWGN (or non-dispersive) channel model is rarely practical. Due to the scarcity of the frequency spectrum, we usually filter the transmitted signal to limit its bandwidth so that efficient sharing of the frequency resource can be achieved. Moreover, many practical channels are bandpass and, in fact, they often respond differently to inputs with different frequency components, i.e., they are *dispersive*. We have to refine the simple AWGN (or non-dispersive) model to accurately represent this type of practical channels. One such commonly employed refinement is the *dispersive* channel model1

$$r(t) = u * h_c(t) + n(t), \tag{1}$$

where $u(t)$ - the transmitted signal, $h_c(t)$ - the impulse response of the channel, $n(t)$ - AWGN with power spectral density $N_0/2$.

In essence, we model the dispersive characteristic of the channel by the linear filter $h_c(t)$. The simplest dispersive channel is the *bandlimited channel* for which the channel impulse response $h_c(t)$ is that of an ideal lowpass filter. This lowpass filtering smears the transmitted signal in time causing the effect of a symbol to spread to adjacent symbols when a sequence of symbols are transmitted. The resulting *intersymbol interference (ISI)* degrades the error performance of the communication system. There are two major ways to mitigate the detrimental effect of ISI. The first method is to design bandlimited transmission pulses which minimize the the effect of ISI. We will describe such a design for the simple case of bandlimited channels. The ISI free pulses obtained are called the *Nyquist pulses*. The second method is to filter the received signal to cancel the ISI introduced by the channel impulse response. This approach is generally known as *equalization*.

Let us consider the transmission sequence of symbols $\sum_n b_n u(t-nT)$ [1]. Based on the dispersive channel model, the received signal is given by

$$r(t)=\sum_n b_n v(t-nT)+n(t), \quad (2)$$

where $v(t)=u*h_c(t)$ is the received waveform for a symbol.

If a single symbol, say the symbol b_0, is transmitted, the optimal demodulator is the one that employs the matched filter, i.e., we can pass the received signal through the matched filter $\tilde{v}(t)=v(-t)$, and then sample the matched filter output at time $t=0$ to obtain the decision statistic. When a sequence of symbols are transmitted, we can still employ this matched filter to perform demodulation. A reasonable strategy is to sample the matched filter output at $t=mT$ to obtain the decision statistic for the symbol b_m. At $t=mT$, the output of the matched filter is [1]

$$\begin{aligned} z_m &= \sum_n b_n v*\tilde{v}(mT-nT)+n_m = \\ &= b_m\|v\|^2+\sum_{n\neq m} b_n v*\tilde{v}(mT-nT)+n_m, \end{aligned} \quad (3)$$

where n_m is a zero-mean Gaussian random variable with variance $N_0\|v\|^2/2$.

The first term in (3) is the desired signal contribution due to the symbol b_m and the second term contains contributions from the other symbols. These unwanted contributions from other symbols are called *intersymbol interference (ISI)*.

3. The ISI Channel Model

A model for linear ISI channels is shown in Fig. 1. In this model, x_k is scaled by $\|p\|$ to form $x_{p,k}$ so that $\varepsilon_{x_p}=\varepsilon_x*\|p\|^2$. The additive noise is white Gaussian, although it can be included

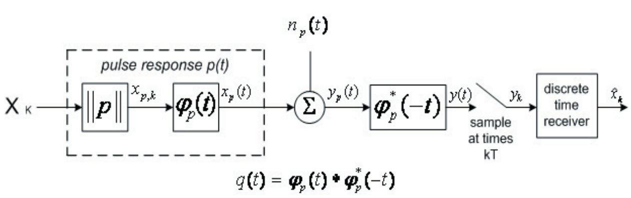


Fig. 1. The ISI-Channel model

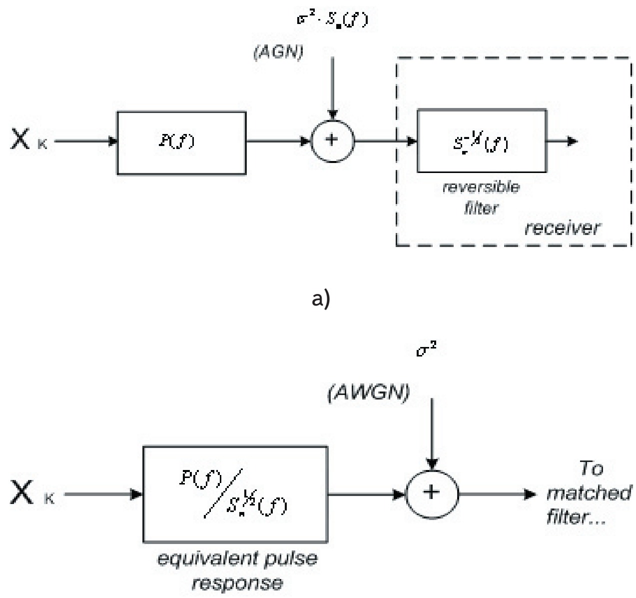
by transforming the correlated-Gaussian-noise channel into an equivalent white Gaussian noise channel using the methods in the previous subsection and illustrated in Fig. 2a,b.

The channel output $y_p(t)$ is passed through a matched filter to generate $y(t)$. Then, $y(t)$ is sampled at the symbol rate and subsequently processed by a discrete time receiver. The following theorem illustrates that there is no loss in performance that is incurred via the matched-filter/sampler combination [2].

The discrete-time signal samples $yk=y(kT)$ in Fig. 3. are sufficient to represent the continuous-time ISI-model channel output $y(t)$, if $0<\|p\|<\infty$.

$$\phi_{p,k}(t)\overset{\Delta}{=}\phi_p(t-kT), \quad (4)$$

where $\{\phi_{p,k}(t)\}_{k\in(-\infty,\infty)}$ is a linearly independent set of functions.



b)

Fig. 2. White Noise Equivalent Channel

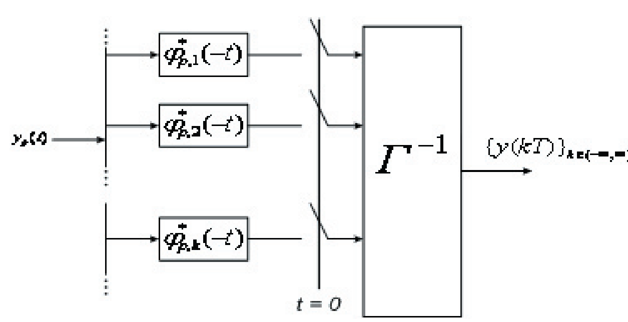


Fig. 3. Equivalent diagram of ISI-channel model matched-filter/sampler

The set $\{\phi_{p,k}(t)\}_{k\in(-\infty,\infty)}$ is related to a set of orthogonal basis functions $\{\phi_{p,k}(t)\}_{k\in(-\infty,\infty)}$ by an invertible transformation Γ (use Gram-Schmidt an infinite number of times). The transformation and its inverse are written:

$$\left\{\phi_{p,k}(t)\right\}_{k\in(-\infty,\infty)} = \Gamma\left(\left\{\phi_{p,k}(t)\right\}_{k\in(-\infty,\infty)}\right);$$

$$\left\{\phi_{p,k}(t)\right\}_{k\in(-\infty,\infty)} = \Gamma^{-1}\left(\left\{\phi_{p,k}(t)\right\}_{k\in(-\infty,\infty)}\right), \quad (5)$$

where Γ is the invertible transformation. In Fig. 3, the transformation outputs are the filter samples $y(kT)$. The infinite set of filters $\left\{\phi^*_{p,k}(-t)\right\}_{k\in(-\infty,\infty)}$ followed by Γ^{-1} is equivalent to an infinite set of matched filters to $\left\{\phi^*_{p,k}(-t)\right\}_{k\in(-\infty,\infty)}$. Equation (4) is equivalent a single matched filter $\phi^*_p(-t)$, whose output is sampled at $t=kT$ to produce $y(kT)$. Since the set $\left\{\phi_{p,k}(-t)\right\}_{k\in(-\infty,\infty)}$ is orthogonal, the set of sampled filter outputs in Fig. 3. are sufficient to represent $y_p(t)$, the sampled matched filter output y(kT) is a sufficient representation of the ISI-channel output yp(t) [2]

$$y(t)=\sum_k \|p\| * x_k q(t-kT)+n_p(t)*\phi^*_p(-t). \quad (6)$$

4. Adaptive Equalizers

Digital communication LTE using MIMO, has recently emerged as one of the most Significant technical breakthroughs in modern wireless communications. The effect of fading and interference always causes an issue for signal recovery in wireless communication. This can be combated with application of an equalizer. Equalization compensates ISI created by multipath signal prorogation within time dispersive channels. Take investigate the performance characteristics of two types of equalizers namely, Zero Forcing (ZF) and Minimum Mean Square Error Estimator (MMSE) equalizers for MIMO wireless receiver.

4.1. Zero Forcing Equalizer Mathematics

Zero Forcing Equalizer is a linear equalization algorithm used in communication systems, which inverts the frequency response of the channel [3]. This equalizer was first proposed by Robert Lucky [4, 5]. The Zero-Forcing Equalizer applies the inverse of the channel to the received signal, to restore the signal before the channel. The name Zero Forcing corresponds to bringing down the ISI to zero in a noise free case. This will be useful when ISI is significant compared to noise. For a channel with frequency response $F(f)$ the zero forcing equalizer $C(f)$ is constructed such that $C(f)=1/F(f)$.Thus the combination of channel and equalizer gives a flat frequency response and linear phase $F(f)*C(f)=1$.

If the channel response for a particular channel is H(s) then the input signal is multiplied by the reciprocal of its value. This is intended to remove the effect of channel from the received signal, in particular the Intersymbol Interference (ISI). For simplicity let us consider a 2x2 MIMO channel. Model of such system can be represented in matrix form

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix} \quad (7)$$

or

$$Y = HS + N. \quad (8)$$

To solve equation (8) for s, we need to find a matrix W which satisfies $WH = I$. The Zero Forcing (ZF) detector for meeting this constraint is given by

$$W = \left(H^H H\right)^{-1} H^H, \quad (9)$$

where W - Equalization Matrix, H - Channel Matrix.

This matrix is known as the Pseudo inverse for a general m x n matrix, where

$$H^H H = \begin{bmatrix} h^*_{1,1} & h^*_{2,1} \\ h^*_{1,2} & h^*_{2,2} \end{bmatrix} \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix} = \quad (10)$$

$$= \begin{bmatrix} |h_{1,1}|^2 + |h_{2,1}|^2 & h^*_{1,1}h_{1,2} + h^*_{2,1}h_{2,2} \\ h^*_{1,2}h_{1,1} + h^*_{2,2}h_{2,1} & |h_{1,2}|^2 + |h_{2,2}|^2 \end{bmatrix}.$$

Note that the off diagonal elements in the matrix $H^H H$ are not zero, because the off diagonal elements are non-zero in values. Zero forcing equalizer tries to null out the interfering terms when performing the equalization, i.e. when solving for s_1 the interference from s_2 is tried to be nulled and vice versa. While doing so, there can be an amplification of noise. Hence the Zero forcing equalizer is not the best possible equalizer. However, it is simple and reasonably easy to implement. For Binary phase-shift keying (BPSK) Modulation in fading channel, the Bit Error Rate (BER) is defined as

$$P_b = \frac{1}{2}\left[1 - \sqrt{\frac{(E_b/N_0)}{(E_b/N_0)+1}}\right], \quad (11)$$

where P_b - Bit Error Rate, E_b/N_0 - Signal to noise Ratio.

The algorithm ZF can work without knowing the channel in form of the triangular matrix H_u. We identify $G = H_u^H$. For this algorithm on the other hand we have to find the smallest eigenvalue of $\left(H_u H_u^H\right)^{-1} H_u + H_u^H \left(H_u H_u^H\right)^{-1}$ in order to find the upper step-size bound which is practically impossible without knowing the matrix:

$$S_{rel,k} = \frac{E\left\|g_{ZF} - \hat{g}_k\right\|_2^2}{\left\|g_{ZF}\right\|_2^2}; \quad (12)$$

$$S_{rel,k} = \frac{E\left\|H_u^H\left(f_{ZF} - \hat{f}_k\right)\right\|_2^2}{\left\|H_u^H f_{ZF}\right\|_2^2}, \quad (13)$$

where $\hat{f}_k$ is equalizer estimate.

4.2. Minimum Mean Square Error Estimator

A minimum mean square error (MMSE) estimator describes the approach which minimizes the mean square error (MSE), which is a common measure of estimator quality. The classical adaptive MMSE equalizer as it is much easier to analyze than its ZF counterpart. Such algorithm is also known under the name Least-Mean-Square (LMS) algorithm for equalization [6]. The main feature of MMSE equalizer is that it does not usually eliminate ISI completely but, minimizes the total power of the noise and ISI components in the output. Let s be an unknown random variable, and let

r be a known random variable. An estimator $S^\wedge(r)$ is any function of the measurement r, and its mean square error is given by

$$MSE = E\left\{\left(S^\wedge - S^2\right)\right\}, \qquad (14)$$

where the expectation is taken over both s and r.

The MMSE estimator is then defined as the estimator achieving minimal MSE. In many cases, it is not possible to determine a closed form for the MMSE estimator. In these cases, one possibility is to seek the technique minimizing the MSE within a particular class, such as the class of linear estimators. The linear MMSE estimator is the estimator achieving minimum MSE among all estimators of the form $AR+b$. If the measurement R is a random vector, A is a matrix and b is a vector.

Let us now try to understand the math for extracting the two symbols which interfered with each other. The equation can be represented in matrix notation as follows

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix} \qquad (15)$$

or

$$y = Hs + n. \qquad (16)$$

The Minimum Mean Square Error (MMSE) approach tries to find a coefficient W which minimizes the Criterion

$$E\left\{[W_{r-s}][W_{r-s}]^H\right\},$$

where W - Equalization Matrix, H - Channel Matrix, n - Channel noise, y - Received signal.

To solve (16) for s, we need to find a matrix W which satisfies $WH = I$. The Minimum Mean Square Error (MMSE) detector for meeting this constraint is given by

$$W = \left(H^H H + N_0 I\right)^{-1H^H}. \qquad (17)$$

This matrix is known as the pseudo inverse for a general m x n matrix

$$H^H H = \begin{bmatrix} h_{1,1}^* & h_{2,1}^* \\ h_{1,2}^* & h_{2,2}^* \end{bmatrix} \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix} = \qquad (18)$$

$$= \begin{bmatrix} |h_{1,1}|^2 + |h_{2,1}|^2 & h_{1,1}^* h_{1,2} + h_{2,1}^* h_{2,2} \\ h_{1,2}^* h_{1,1} + h_{2,2}^* h_{2,1} & |h_{1,2}|^2 + |h_{2,2}|^2 \end{bmatrix}$$

When comparing the (18) to the (19) in Zero Forcing equalizer, apart from $N_0 I$ the term in both equations are comparable. In fact, when the noise term is zero, the MMSE equalizer reduces to Zero Forcing equalizer.

In first experiment it is investigated the classical adaptive MMSE equalizer for channel h [6]. Fig. 8. depicts the l_2 norm of the parameter error vector (relative system distance) over iteration numbers for various normalized step-sizes $\alpha = \mu_k \|r_k\|_2^2$ for a time-variant step-size μ_k and for the condition below

$$0 < \mu_k < \frac{2}{\|r_k\|_2^2}. \qquad (19)$$

As expected the stability is guaranteed for $0 < \alpha < 2$. The learning behavior is very much as expected. If we compare the following theoretically derived values:

$$S_{rel,k} = \frac{E\left\|f_{MMSE} - \hat{f}_k\right\|_2^2}{\|f_{MMSE}\|_2^2} = \frac{\alpha \gamma_s \sigma_{vk}^2}{2-\alpha} \frac{1}{\|f_{MMSE}\|_2^2}; \qquad (20)$$

$$S_{rel,\infty} = \frac{\alpha \gamma_s}{2-\alpha} \left(\frac{\|v_{MMSE}\|_2^2}{\|f_{MMSE}\|_2^2} + N_0 \right), \qquad (21)$$

where f_{MMSE} is MMSE solution, $\hat{f}_k$ is equalizer estimate, V_{MMSE} is MMSE modeling noise.

5. The Simulation Results

The results of simulated the Bit Error Rate (BER) for Binary phase-shift keying (BPSK) Modulation in fading channel with 2x2 MIMO and ZF equalizer is shown on Fig. 4.

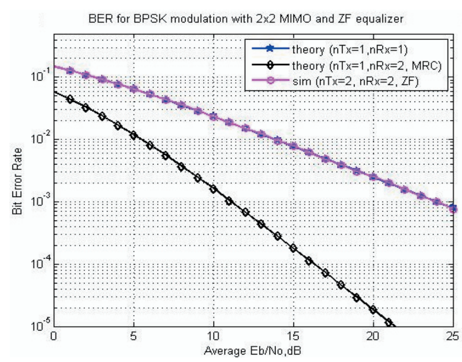


Fig. 4. BER for BPSK modulation with 2x2 MIMO and ZF equalizer

In the simulation result of the adaptive ZF equalizer it again employed the normalized step-size $\alpha = \mu_k / \bar{\mu}_k$ with $\bar{\mu}_k = 1/\|S_k\|_2^2$ to speed up convergence. Note that due to the QPSK symbols for S_k the norm is constant $\bar{\mu} = 1/M$ and the algorithm can also be interpreted as a fixed step-size algorithm [6] it's shown on Fig. 5.

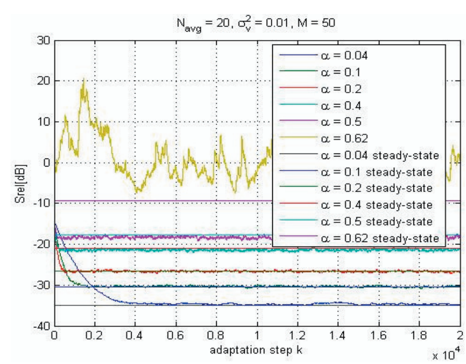


Fig. 5. ZF equalizer with normalized step-size on channel h

The result is shown in Fig. 6. The stability bound varies with the noise and in our example for noise variances larger than one, the algorithm indeed became unstable.

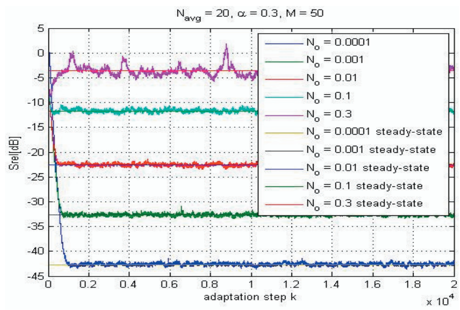


Fig. 6. ZF equalizer with fixed normalized step-size on channel when varying additive noise variance N_o

The results of simulation Bit Error Rate of BPSK Modulation in fading channel with 2x2 MIMO and MMSE equalizer is shown on Fig. 7.

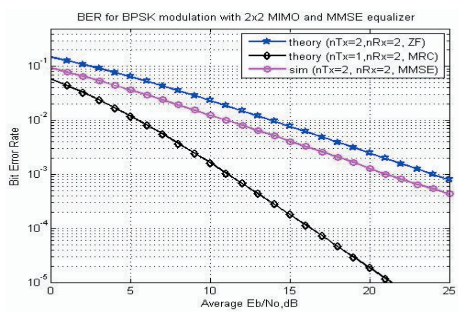


Fig. 7. BER for BPSK modulation with 2x2 MIMO and MMSE equalizer

The result of simulations MMSE equalizer does not differentiate between various channels and behaves perfectly robust.

The step-size bound for α = 2 is tight and holds for various sequences independent of the channel and the noise, it's shown in Fig. 8.

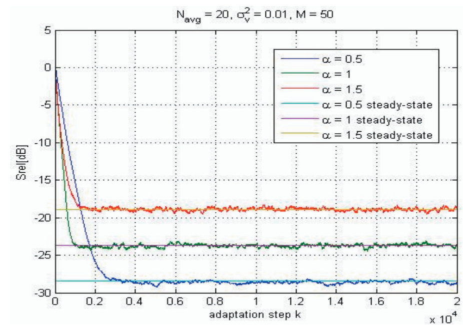


Fig. 8. MMSE equalizer with normalized step-size on channel h

Conclusions

The adaptive MMSE equalizer has shown to be robust, guaranteeing stability for a fixed range of step-sizes independent of additive noise or the channel itself. Novel criteria have been found to ensure convergence of a well-known adaptive ZF receiver.

Different to the general belief these criteria strongly depend on the channel that is to be equalized as well as on the additive noise that is present. The feedback delay and the time variability of the channel are factors that need to be considered both in the design of a rate adaptive system and in the choice of MIMO.

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Abstract

The given work is devoted to the analysis of using method of equalization in LTE systems with different MIMO configuration for purpose increasing characteristics quality of communication. Although equalizers promise to improve the signal to noise energy ratio, zero forcing (ZF) equalizers are derived classically in a deterministic setting minimizing intersymbol interference, while minimum mean square error (MMSE) equalizer solutions are derived in a stochastic context based on quadratic Wiener cost functions. The problem of InterSymbol Interference (ISI) and the basic concept of transversal equalizers are introduced followed by a simplified description of some practical adaptive equalizer structures and their properties. Adaptive equalizers compensate for signal distortion attributed to intersymbol interference, which is caused by multipath within time-dispersive channels

Keywords: *LTE, MIMO, Space time coding, Intersymbol Interference, adaptive equalizer, Zero Forcing, Minimum Mean Square Error*