Проаналізовано проблеми формування кореляційних матриць в задачах ідентифікації матричних моделей динаміки реальних виробничих об'єктів. Запропоновано узагальнені алгоритми, що дозволяють звести ці матриці до аналогічних матрицям корисних сигналів. При цьому враховані специфіки реальних зашумлених технологічних параметрів, показана можливість застосування даних алгоритмів для випадків відсутності і присутності кореляції між корисним сигналом і перешкодою

Ключові слова: стохастичний процес, технологічний параметр, зашумлений сигнал, кореляційна матриця, модель динаміки

Проанализированы трудности формирования корреляционных матриц в задачах идентификации матричных моделей динамики реальных производственных объектов. Предложены обобщенные алгоритмы, позволяющие свести эти матрицы к аналогичным матрицам полезных сигналов. При этом учтены специфики реальных зашумленных технологических параметров, показана возможность применения данных алгоритмов для случаев отсутствия и присутствия корреляции между полезным сигналом и помехой

Ключевые слова: стохастический процесс, технологический параметр, зашумленный сигнал, корреляционная матрица, модель динамики

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DEVELOPMENT OF THE ALGORITHMS OF CORRECTION OF CORRELATION MATRICES

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1. Introduction

It is known [1–6] that one of the main challenges in solving problems of automated control of industrial objects is establishing the quantitative interrelations between technological parameters characterizing the processes in those objects both in statics and dynamics. Such interrelations are called static and dynamic characteristics, respectively. These characteristics can be determined from differential equations of control objects [1–6]. However, those differential equations are often unknown, which is why statistical methods are widely used – they make it possible to determine dynamic characteristics during normal operation of objects [1–6]. In practice, such dynamic characteristics as impulsive admittance $k(t)$ and transfer functions $\varphi(s)$ of linear systems are determined by applying to their input artificial stimulation of a certain type (impulse, step function, sinusoids) and measuring the response. However, in that case, random uncontrollable disturbances are superimposed on these impacts. As a result, it proves impossible to precisely determine dynamic characteristics based on typical input signals [6–8].

2. Analysis of published data and problem statement

The statistical correlation method for determining these dynamic characteristics is based on the solution of an integral equation that includes the correlation functions $R_{XX}(\tau)$ and $R_{XY}(\tau)$ of the input $X(\tau)$ and output $Y(\tau)$ signals. It allows us to obtain the dynamic characteristics of an object without disturbing its normal operation mode. Therefore, statistical methods are widely used for determining the dynamic characteristics of objects during their normal operation [6–8].

However, the application of statistical methods for building mathematical models of real-life industrial objects presents the following difficulty. Interferences and noises are imposed upon the useful signal (that has to be obtained with the least possible amount of distortion), thus hindering the calculation of the estimates of their static characteristics.

One should take into account that interferences and noises are also represented by random functions $\varepsilon(\tau)$. The reasons behind the formation of interferences and noises can be very diverse [6–9]:

a) thermal noises;

b) noises caused by other machinery and equipment operating nearby;

c) noises caused by power supply sources;

d) noised caused by self-oscillations generated in feedback circuits, etc.

For instance, for deep-water offshore platforms, noises are caused by waves, wind, etc. Another example is the radio detector of an antenna under a wind load, which also represents a random time function.

In view of the above, many algorithms and technologies of filtration have been proposed with the aim of eliminating the effects of the noise on the result of identification of statistical models of the dynamics of control objects over a long period of time [8–10]. The ones that allow for eliminating the error of the noises caused by external factors have found a wide application [10–12]. However, in real-life objects, noises of technological processes form under the influence of various factors. Some of them reflect indirectly certain processes that cause defects in the objects under investigation. For this reason, the range of the noise spectrum frequently overlaps the spectrum of the useful signal. Besides, the spectra of the noise and the useful signal in real-life technological parameters are not strictly stable. Therefore, filtration does not always yield the desired result. Sometimes, the spectrum of the useful signal is even distorted from the filtration [11, 12].

Taking into account the above, the paper considers one possible option of creating alternative digital methods and technologies for eliminating the error induced by noise during the formation of correlation matrices in the process of identification of the dynamic model of industrial objects.

As stated above [6, 7], the main dynamic characteristics of linear objects are their impulsive admittance $k(t)$ and transfer $\varphi(s)$ functions. The differential equations of those objects are often unknown, and the methods based on the application of artificial stimulation are inapplicable, usually due to the following reasons:

– it is undesirable or impossible to apply a special kind of stimulation to the object's input, as it disturbs the normal running of the process;

– random uncontrollable disturbances are imposed on that stimulation, and their effects are impossible to separate from the effect of the artificial stimulation.

In this regard, in creating systems for automated control of continuous stochastic processes, the statistical method is widely used, allowing one to determine the dynamic characteristics of complex objects during their normal operation. In practice, the solving of this problem comes to solving the problem of identification of the mathematical model of object's dynamics by methods of theory of stochastic processes [6–8, 13, 14]. Object's state in the general case is described by matrix equations of the following type:

$$\vec{R}_{XY}(\mu) \approx \vec{R}_{XX}(\mu)\ \vec{W}(\mu),$$

$$\mu = 0, \Delta t, 2\Delta t, \ldots, (N-1)\Delta t \ , \quad (1)$$

where

$$\vec{R}_{XX}(\mu) \approx \begin{Vmatrix} R_{XX}(0) & R_{XX}(\Delta t) & \ldots & R_{XX}[(N-1)\Delta t] \\ R_{XX}(\Delta t) & R_{XX}(0) & \ldots & R_{XX}[(N-2)\Delta t] \\ \ldots & \ldots & \ldots & \ldots \\ R_{XX}[(N-1)\Delta t] & R_{XX}[(N-2)\Delta t] & \ldots & R_{XX}(0) \end{Vmatrix}, \quad (2)$$

$$\vec{R}_{XY}(\mu) \approx \left[R_{XY}(0) \quad R_{XY}(\Delta t) \quad \ldots \quad R_{XY}[(N-1)\Delta t]\right]^T, \quad (3)$$

$$\vec{W}(\mu) \approx \left[W(0) \quad W(\Delta t) \quad \ldots \quad W((N-1)\Delta t)\right]^T,$$

$$R_{XX}(\mu) \approx \frac{1}{N}\sum_{k=1}^{N} X(i\Delta t) X((i+\mu)\Delta t),$$

$$R_{XY}(\mu) \approx \frac{1}{N}\sum_{k=1}^{N} X(i\Delta t) Y((i+\mu)\Delta t).$$

$\vec{R}_{XX}(\mu)$ is the square symmetric matrix of the autocorrelation functions with dimension $N \times N$ of the centered input signal $X(t)$; $\vec{R}_{XY}(\mu)$ is the column vector of the cross-correlation functions between the input $X(t)$ and the output $Y(t)$, $\vec{W}(\mu)$ is the column vector of the impulsive admittance functions.

For equation (1), matrices (2), (3) are formed from the estimates of the useful signals $X(t)$ and $Y(t)$.

As previously stated, the real-life technological parameters $g(\Delta t)$ and $\eta(i\Delta t)$ are the sum of the useful signals $X(t)$, $Y(t)$ and noises $\varepsilon(i\Delta t)$, $\eta(i\Delta t)$, i. e.

$$g(t) = X(t) + \varepsilon(t),$$
$$\eta(t) = Y(t) + \phi(t).$$

Therefore, matrix equation (1) and the correlation matrix of real technological processes can be represented as follows:

$$\vec{R}_{g\eta}(\mu) = \vec{R}_{gg}(\mu)\vec{W}(\mu), \mu = 0, \quad \Delta t, \quad 2\Delta t, \quad \ldots, \quad (N-1)\Delta t$$

$$\vec{R}_{gg}(\mu) \approx \begin{Vmatrix} R_{gg}(0) & R_{gg}(\Delta t) & \ldots & R_{gg}[(N-1)\Delta t] \\ R_{gg}(\Delta t) & R_{gg}(0) & \ldots & R_{gg}[(N-2)\Delta t] \\ \ldots & \ldots & \ldots & \ldots \\ R_{gg}[(N-1)\Delta t] & R_{gg}[(N-2)\Delta t] & \ldots & R_{gg}(0) \end{Vmatrix}, \quad (4)$$

$$\vec{R}_{g\eta}(\mu) \approx \left[R_{g\eta}(0) \quad R_{g\eta}(\Delta t) \quad \ldots \quad R_{g\eta}[(N-1)\Delta t]\right]^T, \quad (5)$$

where

$$\left.\begin{aligned} R_{gg}(\mu) &\approx \frac{1}{N}\sum_{i=1}^{N} g(i\Delta t) g((i+\mu)\Delta t) = \frac{1}{N}\sum_{i=1}^{N}\big(X(i\Delta t)+\varepsilon(i\Delta t)\big)\big(X((i+\mu)\Delta t)+\varepsilon((i+\mu)\Delta t)\big), \\ R_{g\eta}(\mu) &\approx \frac{1}{N}\sum_{i=1}^{N} g(i\Delta t)\eta((i+\mu)\Delta t) = \frac{1}{N}\sum_{i=1}^{N}\big(Y(i\Delta t)+\varepsilon(i\Delta t)\big)\big(Y((i+\mu)\Delta t)+\phi((i+\mu)\Delta t)\big), \end{aligned}\right\} \quad (6)$$

$D_g \approx R_{gg}(0)$, $D_\eta \approx R_{g\eta}(0)$ are the estimates of variances of the signals $g(t)$, $\eta(t)$ at $\mu = 0$; m_g, m_η are the mathematical expectations of $g(t)$, $\eta(t)$.

It is impossible to calculate the estimates of the correlation functions $R_{XX}(\mu)$, $R_{XY}(\mu)$ of the useful signals $X(t)$ and $\eta(t)$ of the technological parameters $g(t)$, $\eta(t)$ in practice. For this reason, correlation matrices (4), (5) are formed based on the estimates of $R_{gg}(\mu)$, $R_{g\eta}(\mu)$ correlation functions of the noisy signals $g(t)$, $\eta(t)$.

However, obvious inequalities emerge in this case:

$$\left.\begin{aligned} R_{XX}(\mu) &\neq R_{gg}(\mu), \\ R_{XY}(\mu) &\neq R_{g\eta}(\mu), \end{aligned}\right\}$$

due to which the following inequalities take place

$$\left.\begin{aligned} \vec{R}_{XX}(\mu) &\neq \vec{R}_{gg}(\mu), \\ \vec{R}_{XY}(\mu) &\neq \vec{R}_{g\phi}(\mu). \end{aligned}\right\} \quad (7)$$

As a result, in practice, adequacy of identification of the model of the dynamics (1) of technological processes fails in many cases.

At the same time, in many real-life industrial objects, various sensors are used, in which signals often represent various physical quantities (such as temperature, pressure, displacement, vibration, etc.). In such cases, the estimates of correlation function of the signals $X(t)$, $Y(t)$ are reduced to dimensionless values [8]. To that end, the estimates of the normalized auto- and cross-correlation functions of the useful signals $X(t)$, $Y(t)$ are calculated from formulas [4, 6]:

$$\left.\begin{aligned} r_{XX}(\mu) &\approx R_{XX}(\mu)/D_X, \\ r_{XY}(\mu) &\approx R_{XY}(\mu)/\sqrt{D_X D_Y}, \end{aligned}\right\}$$

where $D_X \approx R_{XX}(0)$, $D_Y \approx R_{YY}(0)$ – where $R_{XX}(\mu)$, $R_{XY}(\mu)$ are the estimates of the auto- and cross-correlation functions of the signals $X(t)$, $Y(t)$ at $\mu=0$, $\mu=\Delta t$, $\mu=2\Delta t$, $\mu=3\Delta t$, ...

In this case, the normalized correlation matrices of the useful signals are as follows:

$$\overrightarrow{r_{XX}}(\mu) \approx \left\| \begin{matrix} \frac{R_{XX}(0)}{D_X} & \frac{R_{XX}(\Delta t)}{D_X} & \dots & \frac{R_{XX}[(N-1)\Delta t]}{D_X} \\ \frac{R_{XX}(\Delta t)}{D_X} & \frac{R_{XX}(0)}{D_X} & \dots & \frac{R_{XX}[(N-2)\Delta t]}{D_X} \\ \dots & \dots & \dots & \dots \\ \frac{R_{XX}[(N-1)\Delta t]}{D_X} & \frac{R_{XX}[(N-2)\Delta t]}{D_X} & \dots & \frac{R_{XX}(0)}{D_X} \end{matrix} \right\|, \quad (8)$$

$$\overrightarrow{r_{XY}}(\mu) \approx \left[\frac{R_{XY}(0)}{\left(\sqrt{D_X D_Y}\right)} \quad \frac{R_{XY}(\Delta t)}{\left(\sqrt{D_X D_Y}\right)} \quad \dots \quad \frac{R_{XY}[(N-1)\Delta t]}{\left(\sqrt{D_X D_Y}\right)} \right]^T. \quad (9)$$

Naturally, matrix equation (1) for this case can be represented in the following form:

$$\vec{r}_{XY}(\mu) \approx \vec{r}_{XX}(\mu)\overrightarrow{W}(\mu), \; \mu=0, \; \Delta t, \; 2\Delta t, \; \dots, \; (N-1)\Delta t,$$

where $\vec{r}_{XX}(\mu)$ is the square symmetric matrix of the normalized autocorrelation functions with dimension $N \times N$ of the centered input signal $X(t)$; $\vec{r}_{XY}(\mu)$ is the column vector of the normalized cross-correlation functions between the input $X(t)$ and the output $Y(t)$, $\vec{W}(\mu)$ is the column vector of the impulsive admittance functions.

It is known that the normalized auto- and cross-correlation functions $r_{gg}(\mu)$, $r_{g\eta}(\mu)$ of the noisy signals consisting of the sum of the random useful signals $X(t)$, $Y(t)$ and the corresponding noises $\varepsilon(t)$, $\phi(t)$ are calculated from the following formulas:

$$\left.\begin{aligned} r_{gg}(\mu) &\approx R_{gg}(\mu)/D_g, \\ r_{g\eta}(\mu) &\approx R_{g\eta}(\mu)/\sqrt{D_g D_\eta}. \end{aligned}\right\} \quad (10)$$

The corresponding normalized correlation matrices of the noisy signals $g(t)$, $\eta(t)$ are represented in the following form:

$$\vec{r}_{gg}(\mu) \approx \left\| \begin{matrix} \frac{R_{gg}(0)}{D_g} & \frac{R_{gg}(\Delta t)}{D_g} & \dots & \frac{R_{gg}[(N-1)\Delta t]}{D_g} \\ \frac{R_{gg}(\Delta t)}{D_g} & \frac{R_{gg}(0)}{D_g} & \dots & \frac{R_{gg}[(N-2)\Delta t]}{D_g} \\ \dots & \dots & \dots & \dots \\ \frac{R_{gg}[(N-1)\Delta t]}{D_g} & \frac{R_{gg}[(N-2)\Delta t]}{D_g} & \dots & \frac{R_{gg}(0)}{D_g} \end{matrix} \right\|, \quad (11)$$

$$\vec{r}_{g\phi}(\mu) \approx \left[\frac{R_{g\phi}(0)}{\left(\sqrt{D_g D_\eta}\right)} \quad \frac{R_{g\phi}(\Delta t)}{\left(\sqrt{D_g D_\eta}\right)} \quad \dots \quad \frac{R_{g\phi}[(N-1)\Delta t]}{\left(\sqrt{D_g D_\eta}\right)} \right]^T. \quad (12)$$

Comparing matrices (8) and (11), substantial difference between their respective elements are obvious, i. e.

$$\left.\begin{aligned} r_{gg}(\mu) &\neq r_{XX}(\mu), \\ r_{g\eta}(\mu) &\neq r_{XY}(\mu), \end{aligned}\right\}$$

therefore, the following inequalities take place.

$$\left.\begin{aligned} \overrightarrow{r_{gg}}(\mu) &\neq \overrightarrow{r_{XX}}(\mu), \\ \overrightarrow{r_{g\eta}}(\mu) &\neq \overrightarrow{r_{XY}}(\mu). \end{aligned}\right\} \quad (13)$$

From inequalities (7) and (13), it follows that correlation matrices (4), (5) and (11), (12) differ from original matrices (2), (3) and (8), (9). Therefore, in many cases, ensuring adequacy of identification of the dynamic model of an object by means of these matrices in actual practice is impossible [11]. Accordingly, to ensure adequate identification of matrix models of the dynamics of industrial objects, it is necessary to develop technologies for forming the robust correlation matrices $\overrightarrow{R^R_{gg}}(\mu)$, $\overrightarrow{R^R_{g\eta}}(\mu)$, $\overrightarrow{r^R_{gg}}(\mu)$, $\overrightarrow{r^R_{g\eta}}(\mu)$, ensuring that the following equalities hold:

$$\left.\begin{aligned} \overrightarrow{R^R_{gg}}(\mu) &\approx \overrightarrow{R_{XX}}(\mu), \\ \overrightarrow{R^R_{g\eta}}(\mu) &\approx \overrightarrow{R_{XY}}(\mu), \\ \overrightarrow{r^R_{gg}}(\mu) &\approx \overrightarrow{r_{XX}}(\mu), \\ \overrightarrow{r^R_{g\eta}}(\mu) &\approx \overrightarrow{r_{XY}}(\mu). \end{aligned}\right\} \quad (14)$$

3. Purpose and objectives of the study

The key purpose of this paper is to develop algorithms that allow for correcting the elements of the correlation matrices of technological processes with the purpose of reducing them to the matrices of useful signals.

In accordance with the set goal the following research objectives are identified:

1. To avoid errors in the elements of correlation matrices, which emerge during the application of traditional methods of their formation due to the effects of the noise of technological parameters, and to ensure the robustness of the estimates of the elements of the correlation matrices.

2. To create technologies of noise analysis, with regard to the effects of the noise on the estimates of elements of the correlation matrices as a consequence of the noise emerging in real-life objects at the onset of various faults during operation.

3. To avoid the effects of the additional errors emerging during the normalization of the elements of correlation matrices of dynamics models, because the input and output technological parameters in many real-life industrial objects are physical quantities, such as consumption, pressure, temperature, speed, etc.

4. To create generalized robust technologies that enable one, with regard to all of the above, to reduce the correlation matrices of noise technological processes to the matrices of their useful signal, both in the absence of a correlation between the useful signal and the noise and in the presence of such.

4. Technologies for forming the robust correlation matrices in the absence of a correlation between X(t) and ε(t)

The research in [11] has demonstrated that the conditions of stationarity and normalcy of distribution law hold for technological parameters of many industrial objects.

When the correlation between the useful signals $X(t)$, $Y(t)$ and the noise $\varepsilon(t)$ is zero, i. e.

$$\left.\begin{aligned}&\frac{1}{N}\sum_{i=1}^{N}X(i\Delta t)\varepsilon\big((i+\mu)\Delta t\big)\approx 0,\\&\frac{1}{N}\sum_{i=1}^{N}Y(i\Delta t)\varepsilon\big((i+\mu)\Delta t\big)\approx 0,\end{aligned}\right\}\tag{15}$$

expression (6) for calculating the estimates of the auto- and cross-correlation functions can be represented as follows:

$$R_{gg}(\mu)\approx\frac{1}{N}\sum_{k=1}^{N}g(i\Delta t)g\big((i+\mu)\Delta t\big)\approx\begin{cases}R_{XX}(0)+D_\varepsilon & \text{at } \mu=0,\\ R_{XX}(\mu) & \text{at } \mu\neq 0,\end{cases}\tag{16}$$

$$R_{g\eta}(\mu)\approx\frac{1}{N}\sum_{k=1}^{N}g(i\Delta t)\eta\big((i+\mu)\Delta t\big)\approx\frac{1}{N}\sum_{k=1}^{N}\Big(X(i\Delta t)+\varepsilon(k\Delta t)\Big)\times$$

$$\times\Big(Y\big((i+\mu)\Delta t\big)+\phi\big((i+\mu)\Delta t\big)\Big)\approx R_{XY}(\mu).\tag{17}$$

Taking into account expression (16), the correlation matrix of the noisy signals $g(t)$, $\vec{R}_{\overset{\circ}{g}\overset{\circ}{g}}(\mu)$ from formula (4) can be transformed as follows:

Based on expressions (17), correlation matrix (5) can also be represented as follows:

$$\overrightarrow{R^{R}_{g\eta}}(\mu)\approx\Big[R_{g\eta}(0)\approx R_{XY}(0)\quad R_{g\eta}(\Delta t)\approx R_{XY}(\Delta t)\quad\dots\quad R_{g\eta}\big[(N-1)\Delta t\big]\approx R_{XY}\big[(N-1)\Delta t\big]\Big]^{T}\approx\overrightarrow{R_{XY}}(\mu).\tag{19}$$

Experimental research has demonstrated that for those industrial objects, for which conditions (15) are met by determining the estimates of the elements of $R_{g\eta}(\mu)$ from expression (17), it is possible to form the robust matrices

$$\overrightarrow{R^{R}_{g\eta}}(\mu)$$

from formula (19), which would match the correlation matrix $\vec{R}_{XY}(\mu)$ of the useful signals $X(t)$, $Y(t)$. At the same time, it follows from expression (18) that the correlation matrix $\vec{R}_{gg}(\mu)$ of the noisy input signal $g(t)$ differs from the correlation matrix

$$\overrightarrow{R_{XX}}(\mu)$$

(2) of the useful signal $X(t)$ in the diagonal elements that represent the sum of estimates of the correlation function of the useful signals $R_{XX}(0)$ and the noise variance D_ε.

It is obvious that by eliminating the errors of noise from the diagonal elements of matrix (18), it can be reduced to the form similar to matrix (2), whose elements contain no noise-induced error. Therefore, to form such matrices for real-life objects, it is necessary to determine the estimates of the noise variance D_ε of the noisy technological parameters [16]. In this case it is possible to form a matrix, for which equalities (13), (14) will hold, i. e.

$$\left.\begin{aligned}&\overrightarrow{R^{R}_{gg}}(\mu)\approx\overrightarrow{R_{XX}}(\mu),\\&\overrightarrow{R^{R}_{g\eta}}(\mu)\approx\overrightarrow{R_{XY}}(\mu).\end{aligned}\right\}$$

However, as discussed, solving identification problems for real-life objects often requires normalizing the estimates of correlation functions. It is clear that given expressions (16), formula (10) for determining the normalized estimate of the autocorrelation function can be transformed as follows:

$$r_{gg}(\mu\neq 0)\approx\frac{R_{gg}(\mu\neq 0)}{D_g-D_\varepsilon}.\tag{20}$$

Naturally, the formula for calculating the estimates of normalized cross-correlation functions can also be represented as follows:

$$r_{g\eta}(\mu)\approx\frac{R_{g\eta}(\mu)}{\sqrt{\big(D_g-D_{\varepsilon\varepsilon}\big)\big(D_\eta-D_{\phi\phi}\big)}}.\tag{21}$$

$$\overrightarrow{R^{R}_{gg}}(\mu)\approx\left\|\begin{matrix}R_{gg}(0)-D_\varepsilon\approx R_{XX}(0) & R_{gg}(\Delta t)\approx R_{XX}(\Delta t) & \dots & R_{gg}\big[(N-1)\Delta t\big]\approx R_{XX}\big[(N-1)\Delta t\big]\\ R_{gg}(\Delta t)\approx R_{XX}(\Delta t) & R_{gg}(0)-D_\varepsilon\approx R_{XX}(0) & \dots & R_{gg}\big[(N-2)\Delta t\big]\approx R_{XX}\big[(N-2)\Delta t\big]\\ \dots & \dots & \dots & \dots\\ R_{gg}\big[(N-1)\Delta t\big]\approx R_{XX}\big[(N-1)\Delta t\big] & R_{gg}\big[(N-2)\Delta t\big]\approx R_{XX}\big[(N-2)\Delta t\big] & \dots & R_{gg}(0)-D_\varepsilon\approx R_{XX}(0)\end{matrix}\right\|.\tag{18}$$

Therefore, normalized correlation matrix (11) of the noisy signals $g(i\Delta t)$ can be represented as follows:

$$\overrightarrow{r_{gg}}(\mu)\approx$$
$$\approx\begin{Vmatrix} 1 & \frac{R_{gg}(\Delta t)\approx R_{XX}(\Delta t)}{D_g-D_\varepsilon} & \dots & \frac{R_{gg}\left[(N-1)\Delta t\right]\approx R_{XX}\left[(N-1)\Delta t\right]}{D_g-D_\varepsilon} \\ \frac{R_{gg}(\Delta t)\approx R_{XX}(\Delta t)}{D_g-D_\varepsilon} & 1 & \dots & \frac{R_{gg}\left[(N-2)\Delta t\right]\approx R_{XX}\left[(N-2)\Delta t\right]}{D_g-D_\varepsilon} \\ \dots & \dots & \dots & \dots \\ \frac{R_{gg}\left[(N-1)\Delta t\right]\approx R_{XX}\left[(N-1)\Delta t\right]}{D_g-D_\varepsilon} & \frac{R_{gg}\left[(N-2)\Delta t\right]\approx R_{XX}\left[(N-2)\Delta t\right]}{D_g-D_\varepsilon} & \dots & 1 \end{Vmatrix}. \quad (22)$$

The matrix of normalized cross-correlation function can be formed in a similar manner:

$$\overrightarrow{r_{g\eta}}(\mu)\approx$$
$$\approx\left[\frac{R_{g\eta}(0)\approx R_{XY}(0)}{\sqrt{(D_g-D_\varepsilon)(D_\eta-D_\phi)}} \quad \frac{R_{g\eta}(\Delta t)\approx R_{XY}(\Delta t)}{\sqrt{(D_g-D_\varepsilon)(D_\eta-D_\phi)}} \quad \dots \quad \frac{R_{g\eta}\left[(N-1)\Delta t\right]\approx R_{XY}\left[(N-1)\Delta t\right]}{\sqrt{(D_g-D_\varepsilon)(D_\eta-D_\phi)}}\right]^T. \quad (23)$$

Thus, after the correction of errors of the noise, the diagonal elements of the normalized correlation matrix $\overrightarrow{r_{gg}}(\mu)$ of the noisy signals $g(t)$ match the diagonal elements of the normalized correlation matrix $\overrightarrow{r_{XX}}(\mu)$ of the useful signals $X(t)$ and are equal to one. However, the other elements of the normalized correlation matrix $\overrightarrow{r_{gg}}(\mu)$ of the input signal, as well as all elements of the normalized cross-correlation matrix $\overrightarrow{r_{g\eta}}(\mu)$ of the noisy input and output signals contain in the radical expression of the denominator the values of variances D_X, D_Y of the useful signals $X(t)$, $Y(t)$ and the values of variances D_ε, D_ϕ of the noises $\varepsilon(t)$, $\phi(t)$. It follows that normalization leads to additional errors in the elements of correlation matrices. It is obvious that by eliminating said errors with the use of formulas (20), (21), normalized correlation matrices (22), (23) equivalent to matrices (8), (9) of the useful signals [15] can be formed. However, that requires determining the estimates of the noise variances D_ε and D_ϕ of the technological parameters $g(t)$, $\eta(t)$. The research has demonstrated that it is appropriate to use expressions [11, 12] for that purpose

$$D_\varepsilon\approx\frac{1}{N}\sum_{i=1}^{N}\Big[g(i\Delta t)g(i\Delta t)-2g(i\Delta t)g\big((i+1)\Delta t\big)+g(i\Delta t)g\big((i+2)\Delta t\big)\Big], \quad (24)$$

$$D_\phi\approx\frac{1}{N}\sum_{i=1}^{N}\Big[\eta(i\Delta t)\eta(i\Delta t)-2\eta(i\Delta t)\eta\big((i+1)\Delta t\big)+\eta(i\Delta t)\eta\big((i+2)\Delta t\big)\Big], \quad (25)$$

which allow for calculating the estimates D_ε, D_ϕ of the variances of the noises $\varepsilon(t)$, $\phi(t)$ of the noisy input $g(t)$ and output $\eta(t)$ signals [11, 12, 16]. At that, taking into account formula (16) and using the obtained estimates $R_{gg}(\Delta t)\approx R_{XX}(\Delta t)$, $R_{gg}(2\Delta t)\approx R_{XX}(2\Delta t),\dots,R_{gg}[(N-1)\Delta t]\approx R_{XX}[(N-1)\Delta t]$ robust normalized correlation matrices:

$$\overrightarrow{r_{gg}}(\mu)\approx\begin{Vmatrix} 1 & \frac{R_{gg}(\Delta t)\approx R_{XX}(\Delta t)}{D_g-D_\varepsilon} & \dots & \frac{R_{gg}\left[(N-1)\Delta t\right]\approx R_{XX}\left[(N-1)\Delta t\right]}{D_g-D_\varepsilon} \\ \frac{R_{gg}(\Delta t)\approx R_{XX}(\Delta t)}{D_g-D_\varepsilon} & 1 & \dots & \frac{R_{gg}\left[(N-2)\Delta t\right]\approx R_{XX}\left[(N-2)\Delta t\right]}{D_g-D_\varepsilon} \\ \dots & \dots\dots & \dots & \dots \\ \frac{R_{gg}\left[(N-1)\Delta t\right]\approx R_{XX}\left[(N-1)\Delta t\right]}{D_g-D_\varepsilon} & \frac{R_{gg}\left[(N-2)\Delta t\right]\approx R_{XX}\left[(N-2)\Delta t\right]}{D_g-D_\varepsilon} & \dots & 1 \end{Vmatrix}, \quad (26)$$

$$\overrightarrow{r_{g\eta}^{R}}(\mu)\approx\left[\frac{R_{g\eta}(0)\approx R_{XY}(0)}{\sqrt{(D_g-D_\varepsilon)(D_\eta-D_\varphi)}} \quad \frac{R_{g\eta}(\Delta t)\approx R_{XY}(\Delta t)}{\sqrt{(D_g-D_\varepsilon)(D_\eta-D_\varphi)}} \quad \dots \quad \frac{R_{g\eta}\left[(N-1)\Delta t\right]\approx R_{XY}\left[(N-1)\Delta t\right]}{\sqrt{(D_g-D_\varepsilon)(D_\eta-D_\varphi)}}\right]^T \quad (27)$$

can be formed.

Comparing matrices (26), (27) with matrices (8), (9), one can see that the effects of the noise-induced errors on the elements have been eliminated and matrices (26), (27) can be regarded as equivalent to matrices (8), (9) of the useful signals. Therefore, in the absence of a correlation between $X(t)$ and $\varepsilon(t)$, $Y(t)$ and $\phi(t)$ one can assume that the following equalities take place between those matrices:

$$\left.\begin{array}{l}\overrightarrow{r_{gg}^{R}}(\mu)\approx\overrightarrow{r_{XX}}(\mu),\\ \overrightarrow{r_{g\eta}^{R}}(\mu)\approx\overrightarrow{r_{XY}}(\mu).\end{array}\right\}$$

5. Technology for forming the correlation matrix in the presence of a correlation between the useful signal and the noise

It should be noted that it is characteristic of real-life industrial objects to go into the latent period of origin of various defects, such as wear, microcracks, carbon deposition, fatigue strain, etc. [12, 15–18]. It usually affects the signals received from the corresponding sensors as noise, which in most cases correlates with the useful signal $X(t)$ [15–19]. For this reason, the sum noise in such cases forms from the noise $\varepsilon_1(t)$, which is caused by the external factors and the noise $\varepsilon_2(t)$ that emerge as a result of origin of various defects. The variance of the noisy signal in that case takes the following form [12, 16, 19]:

$$R_{gg}(0)\approx\frac{1}{N}\sum_{i=1}^{N}g^2(i\Delta t)\approx$$
$$\approx\frac{1}{N}\sum_{i=1}^{N}X^2(i\Delta t)+2\frac{1}{N}\sum_{i=1}^{N}X(i\Delta t)\varepsilon(i\Delta t)+$$
$$+\frac{1}{N}\sum_{i=1}^{N}\varepsilon^2(i\Delta t)\approx R_{XX}(0)+2R_{X\varepsilon}(0)+D_{\varepsilon\varepsilon}.$$

The sum noise

$$\varepsilon(i\Delta t)=\varepsilon_1(i\Delta t)+\varepsilon_2(i\Delta t)$$

has a correlation with the useful signal $X(t)$ and its variance D_ε is determined from the expression

$$D_\varepsilon=2R_{X\varepsilon}(0)+D_{\varepsilon\varepsilon},$$

where $R_{X\varepsilon}(0)$ is the cross-correlation function between the useful signal $X(t)$ and the noise $\varepsilon(i\Delta t)$, $D_{\varepsilon\varepsilon}$ is the estimate of the variance of the noise $\varepsilon_1(i\Delta t)$.

Therefore, in that case, the variance of the sum noise D_ε represents the sum of the variance $D_{\varepsilon\varepsilon}$ of the noise $\varepsilon_1(i\Delta t)$, which is caused by external factors and the cross-correlation function $R_{X\varepsilon}(0)$ between the useful signal $X(t)$ and the noise $\varepsilon_2(i\Delta t)$, which is caused by various processes originating in the object itself [12, 16, 19].

In view of the above, the formula for determining the estimate $R_{gg}(\mu)$ can be represented as follows

$$R_{gg}(\mu)\approx\frac{1}{N}\sum_{i=1}^{N}g(i\Delta t)g((i+\mu)\Delta t)\approx$$
$$\approx\begin{cases}R_{XX}(0)+D_\varepsilon & \text{at}\,\mu=0,\\ R_{XX}(\mu)+R_{X\varepsilon}(\mu) & \text{at}\,\mu\neq0.\end{cases}$$

It is essential to account for the correlation between $X(t)$ and $\varepsilon(t)$ when forming the correlation matrices, because in real-life industrial objects a correlation between $X(t)$ and $\varepsilon(i\Delta t)$ often takes place even during several sampling intervals, i. e. at $\mu=\Delta t$, $\mu=2\Delta t$, $\mu=3\Delta t$, ... [16, 17].

Therefore, it is necessary to develop technologies for determining the estimates of the cross-correlation functions $R_{X\varepsilon}(0)$, $R_{X\varepsilon}(\Delta t)$, $R_{X\varepsilon}(2\Delta t)$, $R_{X\varepsilon}(3\Delta t)$.... During forming the correlation matrices, this will allow for ensuring that they are equivalent to the matrix of the useful signals by compensating for the errors of the elements $R_{gg}(0)$, $R_{gg}(\Delta t)$, $R_{gg}(2\Delta t)$, $R_{gg}(3\Delta t)$,... in the corresponding lines and columns of the correlation matrices (18), (22). Thus, to ensure that the correlation matrices are equivalent to the matrices of the useful signals, it is necessary to subtract the value of D_ε from the estimates of $R_{gg}(0)$, and the value of $R_{X\varepsilon}(\mu)$ from the values of the estimates of $R_{gg}(\mu)$, i. e.

$$\overrightarrow{R_{gg}^{R}}(\mu)\approx\overrightarrow{R_{XX}^{R}}(\mu)\approx$$
$$\approx\left\|\begin{array}{cccc}R_{gg}(0)-D_\varepsilon & R_{gg}(\Delta t)-R_{X\varepsilon}(\Delta t) & \dots & R_{gg}[(N-1)\Delta t]-R_{X\varepsilon}[(N-1)\Delta t]\\ R_{gg}(\Delta t)-R_{X\varepsilon}(\Delta t) & R_{gg}(0)-D_\varepsilon & \dots & R_{gg}[(N-2)\Delta t]-R_{X\varepsilon}[(N-2)\Delta t]\\ \dots & \dots & \dots & \dots\\ R_{gg}[(N-1)\Delta t]-R_{X\varepsilon}[(N-1)\Delta t] & R_{gg}[(N-2)\Delta t]-R_{X\varepsilon}[(N-2)\Delta t] & \dots & R_{gg}(0)-D_\varepsilon\end{array}\right\|,$$

$$\overrightarrow{r_{gg}}(\mu)\approx\overrightarrow{r_{XX}}(\mu)\approx$$
$$\approx\left\|\begin{array}{cccc}1 & \frac{R_{gg}(\Delta t)-R_{X\varepsilon}(\Delta t)}{D_g-D_\varepsilon} & \dots & \frac{R_{gg}[(N-1)\Delta t]-R_{X\varepsilon}[(N-1)\Delta t]}{D_g-D_\varepsilon}\\ \frac{R_{gg}(\Delta t)-R_{X\varepsilon}(\Delta t)}{D_g-D_\varepsilon} & 1 & \dots & \frac{R_{gg}[(N-2)\Delta t]-R_{X\varepsilon}[(N-2)\Delta t]}{D_g-D_\varepsilon}\\ \dots & \dots & \dots & \dots\\ \frac{R_{gg}[(N-1)\Delta t]-R_{X\varepsilon}[(N-1)\Delta t]}{D_g-D_\varepsilon} & \frac{R_{gg}[(N-2)\Delta t]-R_{X\varepsilon}[(N-2)\Delta t]}{D_g-D_\varepsilon} & \dots & 1\end{array}\right\|.$$

In view of the above, alongside with determining the estimate D_ε, it is also necessary to develop technologies for determining the estimate $R_{X\varepsilon}(\mu \neq 0)$. To that end, let us first consider one of the possible ways to determine the estimate $R_{X\varepsilon}(\mu)$ at $\mu = 0$, $\mu = \Delta t$, $\mu = 2\Delta t, \ldots$ by means of the estimates of the relay correlation functions $R^*_{gg}(0)$ of the technological parameter $g(i\Delta t)$. With this in mind, assuming the following notation

$$\operatorname{sgn} g(i\Delta t) = \operatorname{sgn} X(i\Delta t) = \begin{Bmatrix} +1 \text{ at } g(i\Delta t) > 0 \\ 0 \text{ at } g(i\Delta t) = 0 \\ -1 \text{ at } g(i\Delta t) < 0 \end{Bmatrix},$$

the formula for determining the estimates of the relay correlation function $R^*_{gg}(0)$ of the noisy signal $g(i\Delta t)$ is represented as follows:

$$\begin{aligned} R^*_{gg}(0) &\approx \frac{1}{N}\sum_{i=1}^{N} \operatorname{sgn} g(i\Delta t) g(i\Delta t) \approx \\ &\approx \frac{1}{N}\sum_{i=1}^{N} \operatorname{sgn} g(i\Delta t) \cdot \left[X(i\Delta t) + \varepsilon(i\Delta t)\right] \approx \\ &\approx \frac{1}{N}\sum_{i=1}^{N} \left[\left[\operatorname{sgn} g(i\Delta t) \cdot X(i\Delta t)\right] + \left[\operatorname{sgn} g(i\Delta t) \cdot \varepsilon(i\Delta t)\right]\right] \approx \\ &\approx \frac{1}{N}\sum_{i=1}^{N} \operatorname{sgn} g(i\Delta t) X(i\Delta t) + \frac{1}{N}\sum_{i=1}^{N} \operatorname{sgn} g(i\Delta t) \varepsilon(i\Delta t) \approx \\ &\approx \frac{1}{N}\sum_{i=1}^{N} \operatorname{sgn} X(i\Delta t) X(i\Delta t) + \\ &+ \frac{1}{N}\sum_{i=1}^{N} \operatorname{sgn} X(i\Delta t) \varepsilon(i\Delta t) \approx R^*_{XX}(0) + R^*_{X\varepsilon}(0), \end{aligned}$$

$$R^*_{gg}(0) \approx R^*_{XX}(0) + R^*_{X\varepsilon}(0). \tag{28}$$

It is known from [16–19] that the estimate of $R^*_{X\varepsilon}(0)$ can be determined from the expression

$$\begin{aligned} R^*_{X\varepsilon}(0) &\approx \frac{1}{N}\sum_{i=1}^{N} \left[\operatorname{sgn} g(i\Delta t) g(i\Delta t) - \right. \\ &\left. -2 \operatorname{sgn} g(i\Delta t) g((i+1)\Delta t) + \operatorname{sgn} g(i\Delta t) g((i+2)\Delta t)\right]. \end{aligned} \tag{29}$$

Expanding the right-hand side of the formula with an allowance for expression (28), one can get

$$\begin{aligned} &\frac{1}{N}\sum_{i=1}^{N} \left[\operatorname{sgn} g(i\Delta t) g(i\Delta t)\right] - \frac{1}{N}\sum_{i=1}^{N} \left[2 \operatorname{sgn} g(i\Delta t) g((i+1)\Delta t)\right] + \\ &+ \frac{1}{N}\sum_{i=1}^{N} \left[\operatorname{sgn} g(i\Delta t) g((i+2)\Delta t)\right] \approx \\ &\approx R^*_{gg}(0) - 2R^*_{gg}(\Delta t) + R^*_{gg}(2\Delta t) = \\ &= R^*_{X\varepsilon}(0) + R^*_{XX}(0) - 2R^*_{XX}(\Delta t) + R^*_{XX}(2\Delta t) \approx R^*_{X\varepsilon}(0). \end{aligned}$$

Considering that the following equality holds for stationary technological parameters with the normal distribution law

$$R^*_{XX}(0) + R^*_{XX}(2\Delta t) - 2R^*_{XX}(\Delta t) \approx 0,$$

it can be assumed that the result of the calculations in formula (29) can be regarded as the estimate $R^*_{X\varepsilon}(0)$ [19].

An analysis of expression (29) has demonstrated that considering the specifics of determining the estimate $R^*_{X\varepsilon}(\mu)$ of the cross-correlation function between $X(t)$ and $\varepsilon(t)$ can also be represented as follows:

$$\begin{aligned} R'_{X\varepsilon}(\Delta t) &\approx \frac{1}{N}\sum_{i=1}^{N} \operatorname{sgn}\left[g(i\Delta t) g((i+1)\Delta t)\right] - \\ &- \frac{1}{N}\sum_{i=1}^{N} 2\operatorname{sgn}\left[g(i\Delta t) g((i+2)\Delta t)\right] + \\ &+ \frac{1}{N}\sum_{i=1}^{N} \operatorname{sgn}\left[g(i\Delta t)(g(i+3)\Delta t)\right] \approx \\ &\approx \frac{1}{N}\sum_{i=1}^{N} \left[\operatorname{sgn}\left[X(i\Delta t) + \varepsilon(i\Delta t)\right]\left[X((i+1)\Delta t) + \varepsilon((i+1)\Delta t)\right] - \right. \\ &- \frac{1}{N}\sum_{i=1}^{N} 2\operatorname{sgn}\left[X(i\Delta t) + \varepsilon(i\Delta t)\right]\left[X((i+2)\Delta t) + \varepsilon((i+2)\Delta t)\right] + \\ &+ \left. \frac{1}{N}\sum_{i=1}^{N} \left[X(i\Delta t) + \varepsilon(i\Delta t)\right]\left[X((i+3)\Delta t) + \varepsilon((i+3)\Delta t)\right]\right] \approx \\ &\approx R^*_{XX}(\Delta t) + R^*_{X\varepsilon}(\Delta t) + R^*_{\varepsilon X}(\Delta t) + R^*_{\varepsilon\varepsilon}(\Delta t) - 2R^*_{XX}(2\Delta t) - \\ &- 2R^*_{X\varepsilon}(2\Delta t) - 2R^*_{\varepsilon X}(2\Delta t) - 2R^*_{\varepsilon\varepsilon}(2\Delta t) + R^*_{XX}(3\Delta t) + \\ &+ R^*_{X\varepsilon}(3\Delta t) + R^*_{\varepsilon X}(3\Delta t) + R^*_{\varepsilon\varepsilon}(3\Delta t). \end{aligned}$$

Considering that when

$$R^*_{X\varepsilon}(\Delta t) > 0, R^*_{X\varepsilon}(2\Delta t) \approx 0,\ R_{X\varepsilon}(3\Delta t) \approx 0$$

and the conditions of stationarity and normalcy of distribution law hold, the following equalities can be regarded as true:

$$\begin{aligned} &R^*_{XX}(\Delta t) + R^*_{XX}(3\Delta t) - 2R^*_{XX}(2\Delta t) \approx 0, \\ &R^*_{\varepsilon\varepsilon}(\Delta t) + R^*_{\varepsilon\varepsilon}(3\Delta t) - 2R^*_{\varepsilon\varepsilon}(3\Delta t) \approx 0, \\ &R^*_{X\varepsilon}(2\Delta t) \approx 0,\ R^*_{X\varepsilon}(3\Delta t) \approx 0, \\ &R^*_{\varepsilon X}(2\Delta t) \approx 0,\ R^*_{\varepsilon X}(3\Delta t) \approx 0, \end{aligned}$$

in the right-hand side there is

$$R'_{X\varepsilon}(\Delta t) \approx R^*_{x\varepsilon}(\Delta t) + R^*_{\varepsilon X}(\Delta t) \approx 2R^*_{X\varepsilon}(\Delta t),$$

$$R^*_{X\varepsilon}(\Delta t) \approx \frac{1}{2} R'_{X\varepsilon}(\Delta t). \tag{30}$$

It can be shown that the formula for determining the estimate $R^*_{x\varepsilon}(2\Delta t)$ can also be represented in a similar form, i. e.

$$\begin{aligned} R'_{X\varepsilon}(2\Delta t) &\approx \frac{1}{N}\sum_{i=1}^{N} \left[\operatorname{sgn} g(i\Delta t) g((i+1)\Delta t) - \right. \\ &\left. -2 \operatorname{sgn} g(i\Delta t) g((i+2)\Delta t) + \operatorname{sgn} g(i\Delta t) g((i+3)\Delta t)\right] \end{aligned}$$

and the estimate $R^*_{X\varepsilon}(2\Delta t)$ in that case will equal

$$R^*_{X\varepsilon}(2\Delta t) = \frac{1}{2} R'_{X\varepsilon}(2\Delta t). \tag{31}$$

Our analysis of literatures [15–19] and research have demonstrated that the following equalities take place between $R_{X\varepsilon}(0)$, $\Delta R_{gg}(0)$ and $R^*_{X\varepsilon}(0)$, $\Delta R^*_{gg}(0)$; $R_{X\varepsilon}(\Delta t)$, $\Delta R_{gg}(\Delta t)$ and $R^*_{X\varepsilon}(\Delta t)$, $\Delta R^*_{gg}(\Delta t)$; $R_{X\varepsilon}(2\Delta t)$, $\Delta R_{gg}(2\Delta t)$ and $R^*_{X\varepsilon}(2\Delta t)$, $\Delta R^*_{gg}(2\Delta t)$, respectively:

$$\left.\begin{aligned}&\frac{R_{X\varepsilon}(0)}{\Delta R_{gg}(0)}\approx\frac{R^*_{X\varepsilon}(0)}{\Delta R^*_{gg}(0)},\\&\frac{R_{X\varepsilon}(\Delta t)}{\Delta R_{gg}(\Delta t)}\approx\frac{R^*_{X\varepsilon}(\Delta t)}{\Delta R^*_{gg}(\Delta t)},\\&\frac{R_{X\varepsilon}(2\Delta t)}{\Delta R_{gg}(2\Delta t)}\approx\frac{R^*_{X\varepsilon}(2\Delta t)}{\Delta R^*_{gg}(2\Delta t)},\end{aligned}\right\}$$

from which, using the formulas

$$\left.\begin{aligned}&R_{X\varepsilon}(0)\approx\frac{\Delta R_{gg}(0)R^*_{X\varepsilon}(0)}{\Delta R^*_{gg}(0)},\\&R_{X\varepsilon}(\Delta t)\approx\frac{\Delta R_{gg}(\Delta t)R^*_{X\varepsilon}(\Delta t)}{\Delta R^*_{gg}(\Delta t)},\\&R_{X\varepsilon}(2\Delta t)\approx\frac{\Delta R_{gg}(2\Delta t)R^*_{X\varepsilon}(2\Delta t)}{\Delta R^*_{gg}(2\Delta t)},\end{aligned}\right\}\qquad(32)$$

the estimates $R_{X\varepsilon}(0)$, $R_{X\varepsilon}(\Delta t)$, $R_{X\varepsilon}(2\Delta t)$, ... are determined.

Thus, as determining the estimates D_ε and $R_{X\varepsilon}(0)$, $R_{X\varepsilon}(\Delta t)$, $R_{X\varepsilon}(2\Delta t)$,..., $R^*_{X\varepsilon}(0)$, $R^*_{X\varepsilon}(\Delta t)$, $R^*_{X\varepsilon}(2\Delta t)$,..., it becomes possible to analyze the errors of the estimates of the correlation functions and the results of formation of the robust correlation matrices. It also becomes possible, depending on the presence or absence of a correlation between $X(t)$ and $\varepsilon(i\Delta t)$, to make a decision on the appropriate choice of a technology for identifying the models of control objects [12, 25, 28]. It should be noted that when $R_{X\varepsilon}(0)>0$, $R_{X\varepsilon}(\Delta t)\approx 0$, $R_{X\varepsilon}(2\Delta t)\approx 0$ take place, the correlation matrix is formed in a similar way as in the absence of a correlation between $X(t)$ and $\varepsilon(i\Delta t)$. At the same time, if a correlation is detected between $X(t)$ and $\varepsilon(i\Delta t)$ at time shifts $\mu\Delta t=\Delta t$, $\mu=2\Delta t$,..., the estimates $R_{X\varepsilon}(\Delta t)$, $R_{X\varepsilon}(2\Delta t)$ are determined, using expressions (32), and they are subtracted from the estimates of the elements in the respective lines and columns of correlation matrices (18), (22).

Since it is essential to ensure the robustness of the correlation matrices and adequacy of identification of the dynamics model, an alternative way to correct the errors of the corresponding elements of the correlation matrices is proposed below [19]. In this way, the estimates D_ε, $R_{X\varepsilon}(0)$, $R_{X\varepsilon}(\Delta t)$, $R_{X\varepsilon}(2\Delta t)$, etc. of the technological parameters $g(i\Delta t)$ are determined by means of the expressions developed on the basis of expressions (24), (25).

To that end, the results of decomposing the right-hand side of expression (24) in the presence of a correlation between $X(t)$ and $\varepsilon(t)$ can be considered.

$$\begin{aligned}D_\varepsilon&\approx\frac{1}{N}\sum_{i=1}^{N}\Big[g(i\Delta t)g(i\Delta t)-2g(i\Delta t)g\big((i+1)\Delta t\big)+\\&+g(i\Delta t)g\big((i+2)\Delta t\big)\Big]\approx\\&\approx\frac{1}{N}\sum_{i=1}^{N}\big[X(i\Delta t)+\varepsilon(i\Delta t)\big]\big[X(i\Delta t)+\varepsilon(i\Delta t)\big]-\\&-\frac{1}{N}\sum_{i=1}^{N}2\big[X(i\Delta t)+\varepsilon(i\Delta t)\big]\big[X\big((i+1)\Delta t\big)\varepsilon\big((i+1)\Delta t\big)\big]+\\&+\frac{1}{N}\sum_{i=1}^{N}\big[X(i\Delta t)+\varepsilon(i\Delta t)\big]\big[X\big((i+2)\Delta t\big)+\varepsilon\big((i+2)\Delta t\big)\big]=\\&=R_{XX}(0)+R_{X\varepsilon}(0)+R_{\varepsilon X}(0)+R_{\varepsilon\varepsilon}(0)-2R_{XX}(\Delta t)-\\&-2R_{X\varepsilon}(\Delta t)-2R_{\varepsilon X}(\Delta t)-2R_{\varepsilon\varepsilon}(\Delta t)+\\&+R_{XX}(2\Delta t)+R_{X\varepsilon}(2\Delta t)+R_{\varepsilon X}(2\Delta t)+R_{\varepsilon\varepsilon}(2\Delta t).\end{aligned}\qquad(33)$$

Considering that when

$$R_{X\varepsilon}(0)>0, R_{X\varepsilon}(\Delta t)\approx 0, R_{X\varepsilon}(2\Delta t)\approx 0$$

and the conditions of stationarity and normalcy of distribution of the technological parameters of the objects under investigation hold, the following equalities can be regarded as true

$$R_{XX}(0)+R_{XX}(2\Delta t)-2R_{XX}(\Delta t)\approx 0,$$
$$R_{\varepsilon\varepsilon}(2\Delta t)\approx 0,\quad R_{\varepsilon\varepsilon}(\Delta t)\approx 0,$$
$$R_{X\varepsilon}(\Delta t)\approx 0,\ R_{X\varepsilon}(2\Delta t)\approx 0,\ R_{\varepsilon X}(\Delta t)\approx 0,\ R_{\varepsilon X}(2\Delta t)\approx 0.$$

Therefore, in the right-hand side of formula (33) is

$$R_{\varepsilon\varepsilon}(0)+R_{X\varepsilon}(0)+R_{\varepsilon X}(0)\approx 2R_{X\varepsilon}(0)+D_{\varepsilon\varepsilon}\approx D_\varepsilon.$$

This demonstrates that the estimate obtained from formula (33) actually is the estimate of the variance D_ε of the sum noise.

Now the possibility of calculating the estimate $R_{X\varepsilon}(\Delta t)$ in the presence of a correlation between $X(t)$ and $\varepsilon(t)$ at $\mu=\Delta t$ can be considered from the following expression:

$$\begin{aligned}R''_{X\varepsilon}(\mu)&\approx\frac{1}{N}\sum_{i=1}^{N}\Big[g(i\Delta t)g\big((i+1)\Delta t\big)-\\&-\frac{1}{N}\sum_{i=1}^{N}2\Big[g(i\Delta t)g\big((i+2)\Delta t\big)\Big]+\\&+\frac{1}{N}\sum_{i=1}^{N}\Big[g(i\Delta t)g\big((i+3)\Delta t\big)\Big]\approx\\&\approx\frac{1}{N}\sum_{i=1}^{N}\big[X(i\Delta t)+\varepsilon(i\Delta t)\big]\big[X\big((i+1)\Delta t\big)+\varepsilon\big((i+1)\Delta t\big)\big]-\\&-\frac{1}{N}\sum_{i=1}^{N}2\big[X(i\Delta t)+\varepsilon(i\Delta t)\big]\big[X\big((i+2)\Delta t\big)+\varepsilon\big((i+2)\Delta t\big)+\big]\\&+\frac{1}{N}\sum_{i=1}^{N}[X(i\Delta t)+\varepsilon(i\Delta t)][X((i+3)\Delta t)+\varepsilon((i+3)\Delta t)]\approx\\&\approx R_{XX}(\Delta t)+R_{X\varepsilon}(\Delta t)+R_{\varepsilon X}(\Delta t)+R_{\varepsilon\varepsilon}(\Delta t)-2R_{XX}(2\Delta t)-\\&-2R_{X\varepsilon}(2\Delta t)-2R_{\varepsilon X}(2\Delta t)-2R_{\varepsilon\varepsilon}(2\Delta t)+\\&+R_{XX}(3\Delta t)+R_{X\varepsilon}(3\Delta t)+R_{\varepsilon X}(3\Delta t)+R_{\varepsilon\varepsilon}(3\Delta t).\end{aligned}$$

Considering that when the conditions of stationarity and normalcy of distribution law hold at

$$R_{X\varepsilon}(\Delta t)>0, R_{X\varepsilon}(2\Delta t)\approx 0, R_{X\varepsilon}(3\Delta t)\approx 0,$$

the following equalities can be regarded as true

$$R_{XX}(\Delta t)+R_{XX}(3\Delta t)-2R_{XX}(2\Delta t)\approx 0,$$
$$R_{\varepsilon\varepsilon}(\Delta t)+R_{\varepsilon\varepsilon}(3\Delta t)-2R_{\varepsilon\varepsilon}(2\Delta t)\approx 0,$$
$$R_{X\varepsilon}(2\Delta t)\approx 0,\ R_{X\varepsilon}(3\Delta t)\approx 0,\ R_{\varepsilon X}(2\Delta t)\approx 0,\ R_{\varepsilon X}(3\Delta t)\approx 0.$$

So it is obvious that

$$R''_{X\varepsilon}(\mu)\approx R_{X\varepsilon}(\Delta t)+\ R_{\varepsilon X}(\Delta t)\approx 2R_{X\varepsilon}(\Delta t).$$

Therefore, the estimate $R_{X\varepsilon}(\Delta t)$ can be determined from the expression

$$R_{X\varepsilon}(\Delta t) \approx \frac{1}{2} R''_{X\varepsilon}(\Delta t). \quad (34)$$

It is possible to show that in the presence of a correlation between $X(t)$ and $\varepsilon(t)$ at $\mu = 2\Delta t$, the estimate $R_{X\varepsilon}(2\Delta t)$ can be determined in a similar way, using the expression

$$R''_{X\varepsilon}(2\Delta t) \approx \frac{1}{N}\sum_{i=1}^{N}\Big[g(i\Delta t) g\big((i+2)\Delta t\big) - -2g(i\Delta t) g\big((i+3)\Delta t\big) + g(i\Delta t) g\big((i+4)\Delta t\big)\Big], \quad (35)$$

$$R''_{X\varepsilon}(2\Delta t) \approx 2R_{X\varepsilon}(2\Delta t), \quad (36)$$

$$R_{X\varepsilon}(2\Delta t) \approx \frac{1}{2} R''_{X\varepsilon}(2\Delta t). \quad (37)$$

In the presence of a correlation between $X(t)$ and $\varepsilon(t)$ at $\mu = 3\Delta t$, $\mu = 4\Delta t$, … the formulas for determining $R_{X\varepsilon}(\mu)$ can be similarly represented as follows:

$$R_{X\varepsilon}(3\Delta t) \approx \frac{1}{2} R''_{X\varepsilon}(3\Delta t)_{\varepsilon}, \quad (38)$$

$$R_{X\varepsilon}(4\Delta t) \approx \frac{1}{2} R''_{X\varepsilon}(4\Delta t), \text{ etc.} \quad (39)$$

However, according to the experimental research, in that case the accuracy of the estimate $R_{X\varepsilon}(\mu)$ changes depending on the duration of the time shift μ between $X(t)$ and $\varepsilon(t)$. For instance, when $R_{X\varepsilon}(\Delta t) > 0$, $R_{X\varepsilon}(2\Delta t) > 0$, $R_{X\varepsilon}(3\Delta t) \approx 0$, the estimate $R_{X\varepsilon}(2\Delta t)$ has a lesser error than $R_{X\varepsilon}(\Delta t)$, because the error of the estimate $R_{X\varepsilon}(\Delta t)$ is affected by the correlation between $X(t)$ and $\varepsilon(t)$ at $\mu = 2\Delta t$.

To eliminate this shortcoming, generalized expressions eliminating the impact of length of the distance of correlation between $X(t)$ and $\varepsilon(t)$ on the errors of the sought-for estimates $R_{X\varepsilon}(\mu)$ are proposed below.

$$R''_{X\varepsilon}(\mu) \approx \frac{1}{N}\sum_{i=1}^{N} g(i\Delta t)[g((i+\mu+1)\Delta t) - g((i+\mu)\Delta t) - -3g((i+\mu+\lambda+1)\Delta t) + +2g((i+\mu+\lambda)\Delta t) + g((i+\mu+\lambda+2)\Delta t)], \quad (40)$$

where λ is the length of the distance of correlation between $X(t)$ and $\varepsilon(t)$.

In that case, after the estimate $R''_{X\varepsilon}(\mu)$ has been determined, using the formula

$$R_{X\varepsilon}(\mu) \approx \frac{1}{2} R''_{X\varepsilon}(\mu) \quad (41)$$

it is possible to determine the sought-for estimate similar to expressions (34)–(39).

For instance, when

$$R_{X\varepsilon}(\Delta t) > 0, R_{X\varepsilon}(2\Delta t) > 0, R_{X\varepsilon}(3\Delta t) > 0, R_{X\varepsilon}(4\Delta t) \approx 0,$$

in determining the estimate $R_{X\varepsilon}(\mu\Delta t)$, it can be considered that $\lambda = 3$.

In that case, the expressions for determining $R''_{X\varepsilon}(\Delta t)$ and $R_{X\varepsilon}(\Delta t)$ will have the following form:

$$R''_{X\varepsilon}(\Delta t) \approx \frac{1}{N}\sum_{i=1}^{N} g(i\Delta t)[g((i+1+1)\Delta t) - g((i+1)\Delta t) - -3g((i+1+3+1)\Delta t) + +2g((i+1+3)\Delta t) + g((i+1+3+2)\Delta t)],$$

$$R_{X\varepsilon}(\Delta t) \approx \frac{1}{2} R''_{X\varepsilon}(\Delta t).$$

It is natural that in determining the estimates of the relay cross-correlation functions $R^*_{X\varepsilon}(0), R^*_{X\varepsilon}(\Delta t), R^*_{X\varepsilon}(2\Delta t), \ldots$, errors related to the length of the correlation between $X(t)$ and $\varepsilon(t)$ also emerge. To eliminate them, it is also appropriate to use similar generalized expressions that can be represented as follows:

$$R'_{X\varepsilon}(\mu) \approx \frac{1}{N}\sum_{i=1}^{N} \operatorname{sgn} g(i\Delta t)[g((i+\mu+1)\Delta t) - g((i+\mu)\Delta t) - -3g((i+\mu+\lambda+1)\Delta t) + +2g((i+\mu+\lambda)\Delta t) + g((i+\mu+\lambda+2)\Delta t)]. \quad (42)$$

Taking into account formulas (30), (31), one can get:

$$R^*_{X\varepsilon}(\mu) \approx \frac{1}{2} R'_{X\varepsilon}(\mu). \quad (43)$$

Therefore, expression (32) can also be represented as follows:

$$\left.\begin{aligned} R_{X\varepsilon}(0) &\approx \frac{\Delta R_{gg}(0) R^*_{X\varepsilon}(0)}{\Delta R^*_{gg}(0)}, \\ R_{X\varepsilon}(\Delta t) &\approx \frac{\Delta R_{gg}(\Delta t) R^*_{X\varepsilon}(\Delta t)}{\Delta R^*_{gg}(\Delta t)}, \\ R_{X\varepsilon}(2\Delta t) &\approx \frac{\Delta R_{gg}(2\Delta t) R^*_{X\varepsilon}(2\Delta t)}{\Delta R^*_{gg}(2\Delta t)}, \\ &\ldots\ldots\ldots\ldots\ldots\ldots \\ R_{X\varepsilon}(\mu) &\approx \frac{\Delta R_{gg}(\mu) R^*_{X\varepsilon}(\mu)}{\Delta R^*_{gg}(\mu)}. \end{aligned}\right\} \quad (44)$$

It should be noted that the value λ is determined on the basis of the estimate $R'_{X\varepsilon}(\mu)$, at which $R'_{X\varepsilon}(\mu) \approx 0$. It is easy to implement by alternatively determining the estimates $R'_{X\varepsilon}(\mu)$ by means of expression (42) at $\lambda = 0, 1, 2, 3, 4, \ldots$. For instance, if $R'_{X\varepsilon}(3\Delta t) \approx 0$, then $\lambda = 3$.

The use of generalized expressions (40)–(44) makes it possible to correct the corresponding elements of the correlation matrices by determining the estimates $R_{X\varepsilon}(0), R_{X\varepsilon}(\Delta t)$, $R_{X\varepsilon}(2\Delta t)$, $R_{X\varepsilon}(3\Delta t)$, etc. To that end, first of all determination of the presence or absence of a correlation between $X(t)$ and $\varepsilon(t)$ in the elements of the matrix from expression (42) using the estimate $R'_{X\varepsilon}(\mu)$ take place. After that, for the elements with a correlation, the estimates $R_{X\varepsilon}(\mu)$ are determined from expressions (40)–(44) and they are corrected. For instance, in the presence of a correlation between $X(t)$ and $\varepsilon(t)$ in the elements $R_{gg}(\Delta t), R_{gg}(2\Delta t), R_{gg}(3\Delta t), \ldots$, they are corrected by subtracting from them the corresponding estimates $R_{X\varepsilon}(\Delta t), R_{X\varepsilon}(2\Delta t), R_{X\varepsilon}(3\Delta t), \ldots$ and the value D_ε in the columns and lines of the correlation matrices, in which they are located. For clarity the correction procedure at $R_{X\varepsilon}(\Delta t) > 0, R_{X\varepsilon}(2\Delta t) \approx 0$, $R_{X\varepsilon}(3\Delta t) \approx 0, \ldots$, is demonstrated below. Here the estimate $R_{X\varepsilon}(\Delta t) > 0$ is used to correct the second column of the first line and the second line of the first column of matrices (18) and (26)

$$\overrightarrow{R^{R}_{gg}}(\mu) \approx \overrightarrow{R^{R}_{XX}}(\mu) \approx$$

$$\approx \left\| \begin{matrix} R_{gg}(0) - D_{\varepsilon} \approx R_{XX}(0) & R_{gg}(\Delta t) - R_{X\varepsilon}(\Delta t) \approx R_{XX}(\Delta t) & \dots & R_{gg}\left[(N-1)\Delta t\right] \approx R_{XX}\left[(N-1)\Delta t\right] \\ R_{gg}(\Delta t) - R_{X\varepsilon}(\Delta t) \approx R_{XX}(\Delta t) & R_{gg}(0) - D_{\varepsilon} \approx R_{XX}(0) & \dots & R_{gg}\left[(N-2)\Delta t\right] \approx R_{XX}\left[(N-2)\Delta t\right] \\ \dots & \dots & \dots & \dots \\ R_{gg}\left[(N-1)\Delta t\right] \approx R_{XX}\left[(N-2)\Delta t\right] & R_{gg}\left[(N-2)\Delta t\right] \approx R_{XX}\left[(N-2)\Delta t\right] & \dots & R_{gg}(0) - D_{\varepsilon} \approx R_{XX}(0) \end{matrix} \right\| \approx$$

$$\approx \left\| \begin{matrix} 1 & R_{gg}(\Delta t) - R_{X\varepsilon}(\Delta t) \approx R_{XX}(\Delta t) & \dots & R_{gg}\left[(N-1)\Delta t\right] \approx R_{XX}\left[(N-1)\Delta t\right] \\ R_{gg}(\Delta t) - R_{X\varepsilon}(\Delta t) \approx R_{XX}(\Delta t) & 1 & \dots & R_{gg}\left[(N-2)\Delta t\right] \approx R_{XX}\left[(N-2)\Delta t\right] \\ \dots & \dots & \dots & \dots \\ R_{gg}\left[(N-1)\Delta t\right] \approx R_{XX}\left[(N-1)\Delta t\right] & R_{gg}\left[(N-2)\Delta t\right] \approx R_{XX}\left[(N-2)\Delta t\right] & \dots & 1 \end{matrix} \right\|,$$

$$\overrightarrow{r_{gg}}(\mu) \approx \overrightarrow{r_{XX}}(\mu) \approx$$

$$\approx \left\| \begin{matrix} 1 & \dfrac{R_{gg}(\Delta t) - R_{X\varepsilon}(\Delta t) \approx R_{XX}(\Delta t)}{D_g - D_{\varepsilon}} & \dots & \dfrac{R_{gg}\left[(N-1)\Delta t\right] \approx R_{XX}\left[(N-1)\Delta t\right]}{D_g - D_{\varepsilon}} \\ \dfrac{R_{gg}(\Delta t) - R_{X\varepsilon}(\Delta t) \approx R_{XX}(\Delta t)}{D_g - D_{\varepsilon}} & 1 & \dots & \dfrac{R_{gg}\left[(N-2)\Delta t\right] \approx R_{XX}\left[(N-2)\Delta t\right]}{D_g - D_{\varepsilon}} \\ \dots & \dots & \dots & \dots \\ \dfrac{R_{gg}\left[(N-1)\Delta t\right] \approx R_{XX}\left[(N-1)\Delta t\right]}{D_g - D_{\varepsilon}} & \dfrac{R_{gg}\left[(N-2)\Delta t\right] \approx R_{XX}\left[(N-2)\Delta t\right]}{D_g - D_{\varepsilon}} & \dots & 1 \end{matrix} \right\|.$$

In this case, the result of formation of correlation matrices is regarded as valid only when the estimates $R_{X\varepsilon}(\mu)$ at $\mu = 0, \mu = \Delta t, \mu = 2\Delta t, \mu = 3\Delta t\dots$ obtained from expressions (40)–(44) match, i.e. adequacy of the obtained results is achieved by their duplication. Therefore, after such correction, the obtained matrix can be considered equivalent to the matrix of the useful signals.

6. The robust technology for eliminating the errors of calculation of the estimates of correlation functions

An analysis of the specifics of forming correlation matrices shows that during determining the estimates $R_{gg}(\mu)$, $R_{g\eta}(\mu)$, errors emerge in the calculations, which affect validity of the robustness conditions [11, 12]. For instance, during calculating the estimate $R_{gg}(0)$, all paired products $g(i\Delta t)$ and $g((i+\mu)\Delta t)$ have the positive sign. Therefore, the errors of these products are combined and the error of the calculation turns out to be maximum. However, as the time shift μ between $g(i\Delta t)$ and $g((i+\mu)\Delta t)$, as well as between $g_{\eta}(i\Delta t)$ and $g_{\eta}((i+\mu)\Delta t)$ increases, the obtained estimates turn out to be equal to zero at some point. In this case, the sums of errors of the products $g(i\Delta t)g((i+\mu)\Delta t)$ with the positive and negative signs in the amount of N^{+}, N^{-}, from which the sum error $R_{gg}(\mu)$ forms, turn out equal and the equality $N^{+} = N^{-}$ takes place. As a result, the positive and negative errors of the products practically balance each other. Therefore, in determining the estimates $R_{gg}(\mu)$, the calculation errors depend on the difference in the number of the paired products $N^{+} - N^{-}$ with the positive and negative signs. That difference changes depending on the change of the time shift μ between them. Therefore, to ensure equalities (38), there is a need to eliminate the errors of calculating the estimates of elements of matrices (27), (28) and (36), (37). This issue is considered in detail in [11], and the following expressions are recommended to compensate the error from the difference of the positive and negative products of the estimates of the auto- and cross-correlation functions:

$$R^{R}_{gg}(\mu) \approx \frac{1}{N}\sum_{i=1}^{N} g(i\Delta t)g((i+\mu)\Delta t) - \\ -[N^{+}(\mu) - N^{-}(\mu)]\langle \Delta\psi(0) \rangle; \tag{45}$$

$$R^{R}_{g\eta}(\mu) \approx \frac{1}{N}\sum_{i=1}^{N} g(i\Delta t)\eta((i+\mu)\Delta t) - \\ -[N^{+}(\mu) - N^{-}(\mu)]\langle \Delta\psi(\Delta t) \rangle. \tag{46}$$

In that case, the error from the difference of the product $\psi(\Delta t)$ is determined from the expressions

$$\left. \begin{aligned} \left| R_{gg}(\Delta t) - R^{*}_{gg}(\Delta t) \right| \approx \psi(\Delta t), \\ \left| R_{g\eta}(\Delta t) - R^{*}_{g\eta}(\Delta t) \right| \approx \psi(\Delta t), \end{aligned} \right\} \tag{47}$$

where

$$\langle \Delta\psi(\Delta t) \rangle = [1/n^{-}(\Delta t)]\psi(\Delta t). \tag{48}$$

Here $R_{gg}(\Delta t)$, $R^{*}_{gg}(\Delta t)$, $R_{g\eta}(\Delta t)$, $R^{*}_{g\eta}(\Delta t)$ are the estimates of the auto- and cross-correlation functions of the centered and non-centered signals $g(i\Delta t)$, $\eta(i\Delta t)$, respectively; n^{-} is the number of negative products that emerges from the difference of the number of the products $g(i\Delta t)g(i+\mu)\Delta t$ or $g(i\Delta t)\ \eta(i\Delta t)$ with the positive and negative signs, respectively, $N^{+} - N^{-}$. It is obvious from expressions (41), (42) that when expressions (39), (40) are applied, the errors that arise due to the difference of the number of the paired products $g(i\Delta t)g((i+\mu)\Delta t)$ with the positive N^+ and negative N^- signs compensate one another. Therefore, when expressions (39), (40) are applied, the condition of robustness of the elements of correlation matrices [11] is ensured by eliminating the effects of the error on the calculations.

To sum up, the procedure for eliminating the error of calculation of the estimates $R_{gg}(\mu)$ is presented below

1. The estimate $R_{gg}(\Delta t)$ is determined from the expression

$$R_{gg}(\mu)=\frac{1}{N}\sum_{i=1}^{N} g(i\Delta t)g((i+\mu)\Delta t).$$

2. The error of the estimate at the unit time shift $\mu\Delta t=1\Delta t$ is determined:

$$\psi(\Delta t)=\left|R_{gg}(\Delta t)-R^{*}_{gg}(\Delta t)\right|,$$

$$\langle\Delta\psi(\Delta t)\rangle=[1/n^{-}(\Delta t)]\psi(\Delta t),$$

where n^- is the number of the negative products at $\mu\Delta t=1\Delta t$ due to the difference of N^+-N^-.

3. The error is determined:

$$\psi^{R}_{XX}(\mu)\approx\left[n^{+}(\mu)-n^{-}(\mu)\right]\langle\Delta\psi(\Delta t)\rangle.$$

4. The variance is determined:

$$D_{\varepsilon}=\frac{1}{N}\sum_{i=1}^{N}\big(g(i\Delta t)g(i\Delta t)+g(i\Delta t)\times$$
$$\times\big(g(i+2)\Delta t\big)-2g(i\Delta t)\big(g(i+1)\Delta t\big)\big).$$

5. Finally, the robust estimates are determined:

$$R^{R}_{gg}(\mu)=\begin{cases}R_{gg}(\mu)-\left[\psi^{R}_{XX}(\mu)+D_{\varepsilon}\right] & \text{at}\,\mu=0,\\ R_{gg}(\mu)-\psi^{R}_{XX}(\mu) & \text{at}\,\mu\neq 0.\end{cases}\qquad(49)$$

Thus the formula (49) can be used to eliminate the errors occurring in the process of calculating and ensure fulfilment of the condition of robustness.

6. Conclusion

The paper considers the problems related to identification of the model of dynamics of real-life industrial objects. When traditional methods of formation of the correlation matrix are used, because of substantial errors of the estimates of its elements, the conditions of robustness are violated from the effects of the noise in the technological parameters; therefore, adequacy of the obtained results is not achieved in most cases. It is well known there are many filtration methods that eliminate various errors caused by effects of the noise. However, in real-life objects, noises of technological processes are caused by various faults during operation and affect the signals in the form of noise. The range of their spectrum often overlaps the spectrum of the useful signal. Moreover, their spectra are not strictly stable. For these reasons, filtration does not always yield the desired result. Filtration even causes distortion of the spectrum of the useful signal sometimes.

Besides in many real-life industrial objects, the input and output technological parameters are usually represented by such physical quantities as consumption, pressure, temperature, velocity, etc. Therefore, in identifying mathematical models of dynamics, in forming the correlation matrices, it is necessary to apply the procedure of normalization of their elements. This leads to an additional error, which also leads to the disruption of adequacy of the results. That is why methods and technologies for eliminating that error, which can also be widely used in systems of control and management of technological processes in various industries are proposed.

Taking into account above-mentioned problems two alternative robust generalized technologies that enable one to reduce the correlation matrices of noisy technological processes to the matrices of their useful signals both in the absence of a correlation between the useful signal and the noise and in the presence of such are proposed. The validity of the result is achieved through duplication of the obtained estimates of the elements of matrices by both methods.

References

1. Khinchin, A. Y. Teorija korreljacii stacionarnyh sluchajnyh processov (Theory of correlation of stationary stochastic processes) [Text] / A. Y. Khinchin // Uspekhi matematicheskikh nauk. – 1938. – № 5.– P. 42–51.
2. Kolmogorov, A. N. Interpoljacija i jekstrapoljacija stacionarnyh sluchajnyh posledovatel'nostej (Interpolation and extrapolation of stationary random series) [Text] / A. N. Kolmogorov // Izvestiya AN SSSR. – 1941. – № 5. – P. 3–14.
3. Kolmogorov, A. N. Statisticheskaja teorija kolebanij s nepreryvnym spektrom (Statistical oscillation theory with continuous spectrum) [Text] / A. N. Kolmogorov // Jubilejnyj sbornik AN SSSR. – M. : AN SSSR, 1947. – P. 1. – P. 242–254.
4. Wiener, N. Extrapolation, interpolation and smoothing of stationary time series [Text] / N. Wiener. – New York : John Wiley & Sons, 1949.
5. Solodovnikov, V. V. O dinamicheskoj tochnosti i optimal'nyh harakteristikah sledjashhih sistem i preobrazujushhih ustrojstv (On the dynamic precision and optimal characteristics of tracking systems and converter units) [Text] / V. V. Solodovnikov // DAN SSSR. – 1951. – Vol. XXVII, № 2. – P. 269–272.
6. Solodovnikov, V. V. Vvedenie v statisticheskuju dinamiku sistem avtomaticheskogo upravlenija (Introduction to statistical dynamics of automatic control systems) [Text] / V. V. Solodovnikov. – M. : Gosudarstvennoe izdatel'stvo tehniko-teoreticheskoj literatury, 1952. – 163 p.
7. Solodovnikov, V. V. Statisticheskaja dinamika linejnyh sistem avtomaticheskogo upravlenija (Statistical dynamics of linear automatic control systems) [Text] / V. V. Solodovnikov. – M. : Fizmatgiz, 1960. – 665 p.
8. Bendat, J. Primenenie korreljacionnogo i spektral'nogo analiza (Applications of correlation and spectral analysis) [Text] / J. Bendat, A. Pirsol. – M. : Mir, 1983. – 31 p.

9. Li, G. A novel digital filter structure with minimum roundoff noise [Text] / G. Li, L. Meng, Z. Xu, J. Hua // Digital Signal Processing. – 2010. – Vol. 20, Issue 4. – P. 1000–1009. doi: 10.1016/j.dsp.2009.10.018
10. Yaseen, M. Robust and direct design for highpass ladder wave digital filters exhibiting equiripple characteristics [Text] / M. Yaseen // Digital Signal Processing. – 2013. – Vol. 23, Issue 3. – P. 1059–1064. doi: 10.1016/j.dsp.2012.12.017
11. Aliev, T. A. Robust Technology with Analysis of Interference in Signal Processing [Text] / T. A. Aliev. – New-York : Kluwer Acad./Plen. Publishers, 2003. – 199 p. doi: 10.1007/978-1-4615-0093-3
12. Aliev, T. A. Digital Noise Monitoring of Defect Origin [Text] / T. A. Aliev. – London : Springer, 2007. – 235 p. doi: 10.1007/978-0-387-71754-8
13. Filimonov, A. B. Robastnaja korrekcija v sistemah upravlenija s bol'shim kojefficientom usilenija (Robust correction in control systems with large gain coefficient) [Text] / A. B. Filimonov, N. B. Filimonov // Mekhatronika, Avtomatizatsiya, Upravleniye. – 2014. – № 12. – P. 3–10.
14. Tararykin, S. V. Issledovanie vlijanija polozhitel'nyh obratnyh svjazej na robastnye svojstva sistem avtomaticheskogo upravlenija s reguljatorami sostojanija (A study of the effects of positive feedbacks on the robust properties of automatic control systems with state regulators) [Text] / S. V. Tararykin, V. V. Apolonskiy, A. I. Terekhov // Mekhatronika, Avtomatizatsiya, Upravleniye. – 2013. – № 3. – P. 9–15.
15. Aliev, T. A. Noise technologies for minimization of damages caused by earthquake [Text] / T. A. Aliev. – Saarbrucken : Lambert Academic Publishing, 2012. – 202 p.
16. Aliev, T. A. Noise monitoring technology for objects in transition to the emergency state [Text] / T. A. Aliev, G. A. Guluyev, F. H. Pashayev, A. B. Sadygov // Mechanical Systems and Signal Processing. – 2012. – Vol. 27. – P. 755–762. doi: 10.1016/j.ymssp.2011.09.005
17. Aliev, T. A. System of robust noise monitoring of anomalous seismic processes [Text] / T. A. Aliev, A. M. Abbasov, Q. A. Guluyev, F. H. Pashaev, U. E. Sattarova // Soil Dynamics and Earthquake Engineering. – 2013. – Vol. 32, Issue 1. – P. 11–25. doi: 10.1016/j.soildyn.2012.12.013
18. Aliev, T. A. Analiticheskoe predstavlenie funkcii plotnosti normal'nogo rasprede-lenija shuma (Analytical representation of the density function of normal distribution of noise) [Text] / T. A. Aliev, N. F. Musaeva, M. T. Suleymanova, B. I. Gazyizade // Problemi upravleniya i informatiki. – 2015. – № 4. – P. 104–118.
19. Aliev, T. A. Tehnologii pomehomonitoringa skrytogo perioda izmenenija sejsmostojkosti morskih sooruzhenij (Technologies for the noise monitoring of the latent period of seismic stability of offshore structures) [Text] / T. A. Aliev, E. R. Aliyev, T. A. Alizada // Mekhatronika, Avtomatizatsiya, Upravleniye. – 2012. – № 12 (141). – P. 15–22.