Розглянуті принципи побудови інваріантних до факторів впливу п'єзорезонансних пристроїв з керованою динамікою, представлених у вигляді адаптивних систем керування з прогнозуючою еталонною моделлю. Сформульовані задачі і критерій термінального керування, приведені результати дослідження динаміки п'єзорезонансної тричастотної коливальної системи на основі узагальнених скорочених диференціальних рівнянь, які описують поведінку амплітуд, фаз коливань та напруг автозміщень в каналах збудження

Ключевые слова: кварцовий резонатор, п'єзорезонансні пристрої, система з керованою динамікою, багаточастотна коливальна система

Рассмотрены принципы построения инвариантных к возмущающим факторам пьезорезонансных устройств с управляемой динамикой, представленных в виде адаптивных систем управления с прогнозирующей эталонной моделью. Сформулированы задачи и критерий терминального управления, приведены результаты исследования динамики пьезорезонансной трёхчастотной колебательной системы на основе обобщённых укороченных дифференциальных уравнений, описывающих поведение амплитуд, фаз колебаний и напряжений автосмещений в каналах возбуждения

Ключевые слова: кварцевый резонатор, пьезорезонансные устройства, система с управляемой динамикой, многочастотная колебательная система

УДК 621.373.5

THE PRINCIPLES OF DEVELOPING INVARIANT PIEZORESONANCE UNITS WITH CONTROLLED DYNAMICS

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1. Introduction

The further improving functionality of piezoresonance units (PRU) being the inseparable parts of the telecommunication and measuring systems, determines conversion to multi-frequency excitation mode of piezoresonance oscillation system PRU.

This allows formulating new algorithmic approach to solving problem of providing PRU invariance to destabilizing disturbing factors (DF) on the base of combining the main function, which is frequency determinant and stabilizing function of quartz resonator (QR) and measuring function, which provides current identification of DF.

The modern PRU functioning is often related to rapid changes of temperature mode and considerable vibrational and mechanical effects, what significantly complicates the problem to provide invariance in parametric non-stationary condition and demands specific solution approaches basing on assuming PRU as the dynamic object. In this case the following mechanisms, determining multi-frequency PRU, should be emphasized:

– conversion to multi-frequency oscillation mode (MOM) of QR caused by competition of oscillations in excitation channels of PRU reduces stability of oscillations and significantly complicates character of transient processes, what brings to extending time for setting MOM as compared with one-frequency mode. That demands additional means to provide stability of oscillations of multi-frequency PRU and reduce their ready time;

– using MOM in condition of temperature DF demands more rigid controlling the summary power of excitation and thermal condition of QR. Presence of rapidly changing thermal flows and considerable temperature gradients leads to irregular heating piezoelectric cell of quartz resonator and causes specific thermodynamic shift of QR frequency, sharply distorting its temperature and frequency characteristics, what altogether essentially reduces longtime stability of PRU oscillations;

– though the quartz resonator is inert to temperature DF, the inertial properties are negligible to mechanical forces (impacts, vibration, acoustic noise). The vibrational and mechanical DF effects cause interference (parasite) frequency modulation of PRU (lowering short-time stability), especially appearing in frequency measuring transducers where QR with enhanced power sensitivity is used [1-3].

2. The objective

Generalizing principles of developing and analyzing dynamic characteristics of invariant multi-frequency piezoresonance units represented as the adaptive control system with predictive reference model.

3. Developing invariant piezoresonance units with controlled dynamics

The architecture of invariant multi-frequency PRU with controlled dynamics (IMPRU/CD) represented as adaptive controlling system with predictive reference model (fig. 1) is considered. Its main component is the PRU core – multi-frequency piezoresonance oscillatory system (MPOS) with supporting control circuits, interfacing, thermal and vibrational stabilization which is exposed to destabilizing disturbing factors. On the base of monitoring signals vector $\mathbf{Y}$ the system of optimal (suboptimal) assessment and identification accomplishes assessment of condition vector $\hat{\mathbf{X}}_\mathbf{c}$ and assessment of parameters' vector $\hat{\mathbf{X}}_\mathbf{P}$ of PRU mathematical model (parametric identification). The optimal control system, basing on reference model and current assessment of PRU condition vector, forms the vector of control signals $\mathbf{u}$, thereby providing the system to move in accordance to specified optimization criteria and limits $\mathbf{L}$ related to distinguished physical implementation of MPOS. The control signal $\mathbf{u}'$ is also used by the optimal assessment and identification system (fig. 1).

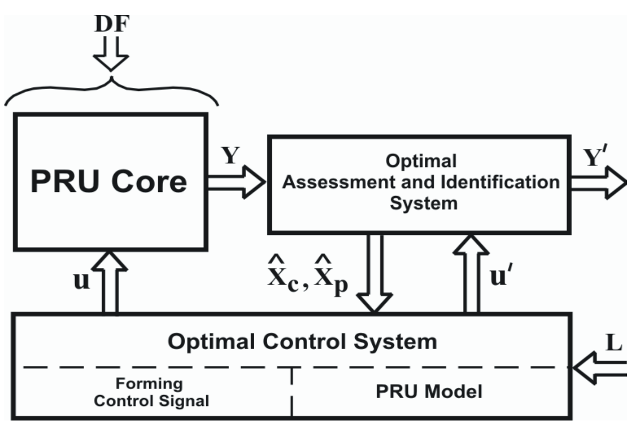


Fig. 1. The Generalized architecture of IMPRU/CD

To identify optimal control criteria in accordance to the offered concept for creating IMPRU/CD the control process will be divided into two stages. At the first stage of the setting oscillation the control signal is formed in each excitation channel, which provides coming onto stationary mode for the shortest reachable time to set stable multi-frequency oscillation mode. At the second stage of the stabilizing oscillation the control signal is formed to support stability of generated oscillations in condition of destabilizing factors (providing technical invariance) [1, 2].

The problem of control is defined as followed. The mathematical model of the system is specified as the differential equation system

$$\dot{\mathbf{x}} = \Phi(\mathbf{x}, \mathbf{u}), \quad (1)$$

at initial condition

$$\mathbf{x}(0) = \mathbf{x}_0, \quad (2)$$

where $\Phi = (\varphi_1, ..., \varphi_n)^T$ – vector function of the right part of the equation (1) of n dimension; $\mathbf{x} = (x_1, ..., x_n)$ – vector of phase coordinate of n dimension; $\mathbf{u} = (u_1, ..., u_k)$ – vector of control signals of k dimension.

The optimal control signal $\mathbf{u}(t)$ is to be found within $t \in [0, T]$ time period for system (1) at initial condition (2), providing for each excited oscillation mode the movement $x_i(t)$ at needed accuracy on the pass $x_i^*(t)$:

$$\left| x_i^*(t) - x_i(t) \right| \le \varepsilon_i; \; i = \overline{1, n}, \quad (3)$$

where ε_i – specified tracking accuracy.

The movement pass $x_i^*(t)$ is defined by PRU reference model of the optimal control system (fig. 1), which allows, on the basis of priori and current information of IMPRU/CD condition, defining the final time T^* and optimal pass for coming onto stationary mode of multi-frequency oscillations in accordance to criteria of time setting minimum [4, 5]:

$$\tau_{stat}^{opt} = \min_{\mathbf{P} \in \mathbf{D}} \max_{1 \le i \le n} \tau_{stat_i}, \quad (4)$$

$$\mathbf{D} = \left\{ \mathbf{P} \in \Re^N : \; p_{j\min} \le p_j \le p_{j\max}, \; 1 \le j \le N \right\},$$

where τ_{stat_i} – time for setting stationary oscillation of mode i, defined by the specified longtime stability of oscillations δ_i, assuming structural specificity of PRU and its functionality; $\mathbf{P}_{min} = \{ p_{1min}, ..., p_{Nmin} \}$, $\mathbf{P}_{max} = \{ p_{1max}, ..., p_{Nmax} \}$ – specified vectors of minimal and maximal acceptable values of IMPRU/CD core parameters.

The problem (1) – (4) may be considered as a problem of the terminal control according to which within the time T^* the system is to transit from initial condition $\mathbf{x}_0$ to terminal one

$$\mathbf{x}(T^*) = \mathbf{x}_{T^*}: \; \mathbf{x}_0 \to \mathbf{x}_{T^*} \quad (5)$$

In this case the optimal control algorithm is defined on the basis of minimizing the functional, which represents energy consumption of acceleration

$$G(\mathbf{u}) = \frac{1}{2} \sum_{i,j}^{n} \left(\ddot{x}_i^*(t) - \ddot{x}_i(t, \mathbf{u}) \right) \left(\ddot{x}_j^*(t) - \ddot{x}_j(t, \mathbf{u}) \right), \quad (6)$$

from condition of reaching proximity to absolute minimum

$$\min_G G(\mathbf{u}) = G(\mathbf{u}^*) = 0 \quad (7)$$

on control signals $\mathbf{u}$ [6].

Solving problems (3) – (7) in accordance to characteristics (physical properties) of PRU and its functionality is followed by limitations to oscillation amplitude U_i, initial frequency run-out $\Delta\omega_i$, maximal power of quartz resonator excitation P_{excit_Σ} and its increasing rate on the stage of setting multi-frequency mode for reducing thermal and dynamical instability of oscillation frequency:

$$U_{min} \le U_i \le U_{max};$$
$$\Delta\omega_i \le \Delta\omega_{ultim},\ i = \overline{1,n}; \quad (8)$$
$$P_{excit_\Sigma} \le P_{ultim};\ \frac{dP_{excit_\Sigma}}{dt} \le K_d^{(P)}.$$

Upon reaching terminal condition (5) the system location relative to set condition $\mathbf{x}_{T^*}$ is stabilized. At this stage the minimizing functional (6) becomes:

$$G(\mathbf{u}) = \frac{1}{2}\sum_{i,j}^{n}\left(x_{T_i^*} - x_i(t,\mathbf{u})\right)\left(x_{T_j^*} - x_j(t,\mathbf{u})\right), \quad (9)$$

providing minimizing deviation of the system

$$\Delta x_i = \left|x_{T_i^*} - x_i(t,\mathbf{u})\right|,\ i = \overline{1,n}.$$

4. Mathematical model of multi-frequency piezoresonance oscillation system

High effectiveness of multi-frequency generating mode is provided by condition of high stability in generated oscillations. This is possible only with using filtering schemes in which QR is embedded in feedback circuit (FC) and is excited in the frequencies close to those of its own consequent resonances. At limited quantity of generated oscillations the other generator schemes can be used, for example those of oscillating type [1, 2, 7].

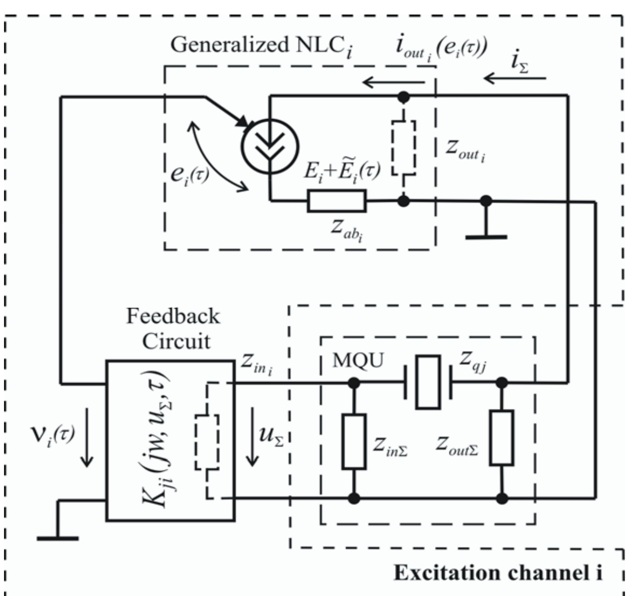


Fig. 2. The basic IMPRU/CD core architecture

The basic IMPRU/CD core architecture represents MPOS to have principles of creating filtering schemes implemented (fig. 2). It incorporates passive multi-frequency quartz quadripole unit (MQU) on the base of quartz resonator with m-frequencies generating z_{q_j} and n excitation channels embedding the generalized non-linear component (NLC) and phasing selective feedback circuit (FBC). The automatic bias circuits with complex equivalent resistance z_{b_i} are used for stabilizing NLC$_i$ operational mode. The selective non-linear circuits FBC with gain $K_{ji}(jw, u_\Sigma, \tau)$, $j = \overline{1,m}$, $i = \overline{1,n}$, except for their function to set required amplitude-phase ratio in excitation channels, provide significant reducing competition in oscillations due to their own selective properties $K_{ji}(jw)$ and also automatic adjustment $K_{ji}(u_\Sigma)$ of oscillation amplitudes for fixing the specified (ultimately acceptable) power dissipation on QR.

Fig. 2 has the following symbols' identifications:

$z_{in_\Sigma} = R_{in_\Sigma} / (1 + jw\tau_{in_\Sigma})$, $R_{in_\Sigma}^{-1} = \sum_i R_{in_i}^{-1}$, $C_{in_\Sigma} = \sum_i C_{in_i}$ – the complex equivalent total resistance of partial FC input circuits;

$z_{out_\Sigma} = R_{out_\Sigma} / (1 + jw\tau_{out_\Sigma})$, $R_{out_\Sigma}^{-1} = \sum_i R_{out_i}^{-1}$, $C_{out_\Sigma} = \sum_i C_{out_i}$ – the complex equivalent total resistance of output NLC$_i$ circuits;

$z_{ab_i} = R_{ab_i} / (1 + jw\tau_{ab_i})$ – complex resistance of auto-bias circuit i;

$\tau_{ab_i} = R_{ab_i} C_{ab_i}$, $\tau_{in\,\Sigma} = R_{in\,\Sigma} C_{in\,\Sigma}$, $\tau_{out\,\Sigma} = R_{out\,\Sigma} C_{out\,\Sigma}$ – time constants;

$i_\Sigma = \sum_i i_{out_i}(e_i)$ – total current of NLC$_i$;

$e_i(\tau) = \sum_j v_i(\tau) + E_i + \tilde{E}_i(\tau)$ – control voltage in NLC$_i$ input, where E_i, $\tilde{E}_i(\tau)$ – constant and variable components of auto-bias voltage;

$u_\Sigma(\tau) = U_0(\tau) + \sum_j U_j(\tau)\cdot\cos[w_j t + \phi_j(\tau)]$ – the total voltage in FC circuit input, where $U_j(\tau)$, w_j and $\phi_j(\tau)$ – envelope, frequency and phase of oscillation j correspondently, $\tau = t - t_0$ – time interval from initial moment t_0 (MPOS startup moment) [7].

MPOS operation (fig. 2) is described by the system of differential equations:

$$u_\Sigma = Z(p)\cdot\sum_i i_{out_i}\left(\sum_j K_{ji}\cdot u_\Sigma,\ \tilde{E}_i,\ E_i\right), \quad (10)$$

$$E_i = -z_{ab_i}(p)\cdot i_{out_i}\left(\sum_j K_{ji}\cdot u_\Sigma,\ \tilde{E}_i,\ E_i\right), \quad (11)$$

where

$$Z(p) = (z_{in_\Sigma}(p)\, z_{out_\Sigma}(p)) / (z_{in_\Sigma}(p) + z_{out_\Sigma}(p) + z_q(p)) \quad (12)$$

– symbolic gain of MQU, representing control resistance of MPOS; $p \equiv d/dt$.

Representing control resistance $Z(p)$ (12) as the ratio

$$Z(p) = \frac{\delta\cdot P(p,\delta)}{Q(p,\delta)}, \quad (13)$$

where $P(p,\delta)$, $Q(p,\delta)$ – polynomials of p; δ – low parameter, physically determined for resonance systems as damping ratio, and taking into account that in the general case solving system (10) is as sum of oscillations with frequencies close to resonance ones of QR ω_{q_j}.

$$u_\Sigma = \sum_{j=1}^{m} U_j(\tau)\cos(\omega_{q_j} t + \phi_j(\tau)), \quad (14)$$

where $U_j(\tau)$ and $\phi_j(\tau)$ – slowly changing parameters, the output current of NLCi with low parameter δ accuracy can be written as:

$$i_{out_i}(e_i) = I_{oi}(V_{ji}, E_i, \tilde{E}_i) + \sum_{j=1}^{m} I_{ji}(V_{ji}, E_i, \tilde{E}_i)\cos[\omega_{q_j} t + \psi_{ji}(t)], \quad (15)$$

where I_{oi} – constant component;

$$I_{ji} = \bar{S}_{ji}(V_{ji}, E_i, \tilde{E}_i)\cdot V_{ji} = \bar{S}_{ji}(V_{ji}, E_i, \tilde{E}_i)\cdot U_j \cdot K_{ji};$$

$$\psi_{ji} = \phi_j + \Delta\phi_{ji}; \ K_{ji},$$

$\Delta\phi_{ji}$ – gain and phase shift of circuit i of FC for resonance frequency j of QR;

$\bar{S}_{ji}(V_{ji}, E_i, \tilde{E}_i) = \frac{1}{V_{ji}} L_j\left[i_{out_i}\left(V_{ji}, E_i, \tilde{E}_i\right)\right]$ – mean slope for oscillation j; $L_j[\bullet]$ – mean operator.

Having entered complex amplitudes $\dot{U}_j = U_j \exp(j\phi_j)$, $\dot{I}_{ji} = I_{ji}\exp(j\psi_{ji})$ and complex FC gain $\dot{K}_{ji} = K_{ji}\exp(j\Delta\phi_{ji})$, and split real and imaginary units the reduced equations for amplitudes, MPOS oscillation phases and auto-bias voltages become [1, 2, 5]:

$$T_j \frac{dU_j}{dt} = \left[R_{e_j} K_{\varphi_j} \sum_{i=1}^{n} \bar{S}_{ji}(V_{ji}, \tilde{E}_i, E_i)\cdot K_{ji}\cos(\Delta\phi_{ji} + \Delta_{\varphi_j}) - 1\right]\cdot U_j; \quad (16)$$

$$T_j \frac{d\phi_j}{dt} = R_{e_j} K_{\varphi_j} \sum_{i=1}^{n} \bar{S}_{ji}(V_{ji}, \tilde{E}_i, E_i)\cdot K_{ji}\sin(\Delta\phi_{ji} + \Delta_{\varphi_j}) - \Delta\omega_j T_j; \quad (17)$$

$$T_{ab_i}\frac{d\tilde{E}}{dt} = -\left[R_{ab_i} I_{oi}(V_{ji}, \tilde{E}_i, E_i) + \tilde{E}_i\right]; \ j = \overline{1,m}; \ i = \overline{1,n}, \quad (18)$$

or as generalized matrix

$$\mathbf{T}\frac{d\mathbf{U}}{dt} = [\mathbf{G_R}\cdot\mathbf{R} - \mathbf{E}_{(mm)}]\cdot\mathbf{U}; \quad (19)$$

$$\mathbf{T}\frac{d\Phi}{dt} = [\mathbf{G_I}\cdot\mathbf{R} - "\,]\cdot\mathbf{E}_{(m1)}; \quad (20)$$

$$\mathbf{T_{ab}}\frac{d\tilde{\mathbf{E}}}{dt} = -(\mathbf{R_{ab}}\cdot\mathbf{I_0} + \tilde{\mathbf{E}}), \quad (21)$$

where $\mathbf{T} = \mathrm{diag}(T_1,...,T_m)$, $\mathbf{T_{ab}} = \mathrm{diag}(\tau_{ab_1},...,\tau_{ab_n})$ – time constant matrixes of partial oscillation system $T_j = 2/\omega_{q_j}\delta_j$ of $m\times m$ dimensions and auto-bias circuits NLCi of $n\times n$ dimensions; $\mathbf{U} = (U_1,...,U_m)^T$, $\Phi = (\phi_1,...,\phi_m)^T$ and $\tilde{\mathbf{E}} = (\tilde{E}_1,...,\tilde{E}_n)^T$ – vectors of oscillation amplitudes and phases of m dimension and vector of auto-bias voltage of n dimension;

$\mathbf{G_R} = \mathrm{Re}\,\dot{\mathbf{G}} = \left|\dot{\mathbf{G}}\right|\cos\Delta\phi^T$, $\mathbf{G_I} = \mathrm{Im}\,\dot{\mathbf{G}} = \left|\dot{\mathbf{G}}\right|\sin\Delta\phi^T$ – matrixes of real and imaginary units of equivalent complex conductance of active part of MPOS $\left|\dot{\mathbf{G}}\right| = \left(\bar{S}_{ji}K_{ji}\right)_{j=1,i=1}^{m,n}$ of $m\times n$ dimensions;

$\mathbf{R} = \mathbf{R_e}\cdot\mathbf{K_\Phi}$ – matrix of reduced resistances; $\mathbf{R_e} = \mathrm{diag}\left(R_{e_1},...,R_{e_m}\right)$ – matrix of equivalent resistances $R_{e_j} = R_{in_\Sigma}\cdot R_{out_\Sigma}/(R_{in_\Sigma} + R_{out_\Sigma} + R_{q_j})$ of $m\times m$ dimensions, $\mathbf{K_\Phi} = \mathrm{diag}\left(K_{\varphi_1},...,K_{\varphi_m}\right)$ – matrix of equivalent phase link gains $K_{\varphi_j} = 1/\sqrt{1 + (\omega_{q_j}\cdot T_{\varphi_j})^2}$, $T_{\varphi_j} \approx R_{in_\Sigma}(C_0 + C_{in_\Sigma}) + R_{out_\Sigma}(C_0 + C_{out_\Sigma})$ – time constant of equivalent phase link, providing phase shift $\Delta\phi_{\varphi_j} = -\mathrm{arctg}(\omega_{q_j}\cdot T_{\varphi_j})$ between immediate values of voltage $u(t)$ and current $i(t)$; ω_{q_j}, C_0 – resonance frequency and parallel capacity of QR;

$\Delta\phi = \left(\Delta\phi_{ji} + \Delta\phi_{\varphi_j}\right)_{j=1,i=1}^{m,n}$ – matrix of $m\times n$ dimensions, which provides phase relations in excitation channels of MPOS; $\Delta = \mathrm{diag}(\Delta\omega_1 T_1,...,\Delta\omega_m T_m)$ – matrix of generalized detuning, $\Delta\omega_j = \omega_{q_j}^2\left(R_{in_\Sigma}\tau_{in_\Sigma} + R_{out_\Sigma}\tau_{out_\Sigma}\right)/2Q_{q_j}R_{q_j}$ – frequency adjustment to oscillation j explained by the fact that reduction was produced relative to natural frequencies of QR ω_{q_j} with no respect to phase-shift in FC circuits; Q_{q_j}, R_{q_j} – quality factor and dynamic resistance of QR for oscillation frequency j;

$\mathbf{R_{ab}} = \mathrm{diag}(R_{ab_1},...,R_{ab_n})$ – matrix of auto-bias resistance of $n\times n$ dimensions; $\mathbf{I_0} = (I_{0_1},...,I_{0_n})^T$ – vector of constant components of output currents $i_{out_i}(e_i)$; $\mathbf{E_{(mm)}}$, $\mathbf{E_{(m1)}}$ – unit matrix of $m\times m$ dimensions and unit column vector $m\times 1$; m – quantity of generating frequencies, n – quantity of excitation channels.

5. Dynamics of multi-frequency piezoresonance oscillation system

The case considers three-frequency excitation of the main mode of oscillations of multi-frequency QR and two additional ones – informative: temperature and vibration ($m = n = 3$) [6, 7]. At polynomial approximation the output current of NLCi (fig. 2) can be represented as

$$i_{out_i}(e_i) = I_s\left[\hat{a}_0 + \sum_{r=1}^{k}\hat{a}_r(\hat{v}_i + X_{b_i})^r\right], \ i = \overline{1,n}, \ k = 3,5, \quad (22)$$

where $\hat{v}_i = v_i\cdot S_0/I_s$ – normalized control voltage; $X_{b_i} = \bar{x}_{b_i} + \tilde{x}_{b_i}$, $\bar{x}_{b_i} = (E_i - E_s)\cdot S_0/I_s$, $\tilde{x}_{b_i} = \tilde{E}_i\cdot S_0/I_s$ – constant and variable components of normalized bias; $\hat{a}_0 = a_0/I_s$, $\hat{a}_r = a_r\cdot I_s^{(r-1)}/S_0^r$ – normalized coefficients of approximating polynomial; I_s, E_s – the coordinates of the point in the center of volt-amps diagram (VAD) with maximal gain slope S_0.

Having used trigonometry formulas of multiple argument and averaging accordantly to (15), the spectral components in NLCi input at approximation by polynomial of the third degree will be written as

$$I_{ji} = V_{ji}\cdot\bar{S}_{ji} = V_{ji}\cdot S_0\left[A_{1i} + 0{,}75\cdot\hat{a}_3 B_{1i}\right]; \quad (23)$$

$$I_{0_i} = I_s\left[A_{2i} + 0{,}5(\hat{a}_2 + 3\hat{a}_3 X_{b_i})B_{2i}\right]. \quad (24)$$

Having substituted the expressions (23), (24) into reduced equations (19) – (21) and assuming $K_{\varphi_j}\approx 1$ and $\Delta\phi_{\varphi_j}\approx 0$, the three-frequency MPOS ($m = 3$) motion equation with three non-linear components ($n = 3$) will be obtained:

$$\xi_j\frac{d\hat{U}_j}{d\hat{t}} = S_0 R_{e_j} K_{jj}\cdot\hat{U}_j\left(\sum_{i=1}^{3}\left(A_{1i} + 0{,}75\hat{a}_3 B_{1i}\right)\cdot\gamma_{ji}^2\frac{K_{ii}}{K_{jj}} - \frac{1}{S_0 R_{e_j} K_{jj}}\right); \quad (25)$$

$$\xi_j \frac{d\phi_j}{d\hat{t}} = S_0 R_{e_j} K_{jj} \cdot \sum_{i=1}^{3}\left(A_{1i} + 0{,}75\hat{a}_3 B_{1i}\right)\cdot \gamma_{ji}\sqrt{1-\gamma_{ji}^2}\,\frac{K_{ii}}{K_{jj}} - \xi_j \Delta\omega_j; \quad (26)$$

$$\mu_i \frac{dX_{b_i}}{d\hat{t}} = -S_0 R_{ab_i}\cdot\left(A_{2i} + \frac{X_{b_i}}{S_0 R_{ab_i}} + 0{,}5(\underline{a}_2 + 3\underline{a}_3 X_{b_i})B_{2i}\right) + \bar{x}_{b_i}. \quad (27)$$

The correspondent equations (25) – (27) at approximating transfer characteristics by polynomial of the fifth degree (22) are given in [1]. The following symbols used in expressions (23) – (27) stand for:

$$A_{1i} = \sum_{r=1}^{3} r\hat{a}_r \cdot X_{b_i}^{(r-1)}, \quad A_{2i} = \hat{a}_0 + \sum_{r=1}^{3}\hat{a}_r \cdot X_{b_i}^{r},$$

$$B_{1i} = K_{ii}^2 \sum_{k=1}^{3} b_{kij}\cdot \hat{U}_k^2, \quad B_{2i} = K_{ii}^2 \sum_{j=1}^{3}\gamma_{ji}^2 \cdot \hat{U}_j^2;$$

$b_{kij} = \gamma_{ki}^2$, for $k = j$ and $b_{kij} = 2\gamma_{ki}^2$, for $k \neq j$; $\gamma_{ji} = \frac{K_{ji}}{K_{ii}} = \cos\Delta\phi_{ji}$ – coefficients of suppressing interchannel interference (parasite) oscillations $(j \neq i)$; $\hat{U}_j = S_0 U_j / I_s$ – normalized voltage in MQU output; $\xi_j = \frac{T_j}{\max\limits_{1\le j\le 3} T_j}$, $\mu_i = \frac{T_{ab_i}}{\max\limits_{1\le j\le 3} T_j}$ – normalized time constants of partial oscillation system j and bias circuit i; $\hat{t} = \frac{t}{\max\limits_{1\le j\le 3} T_j}$ – normalized time; $j = \overline{1,3}$, $i = \overline{1,3}$.

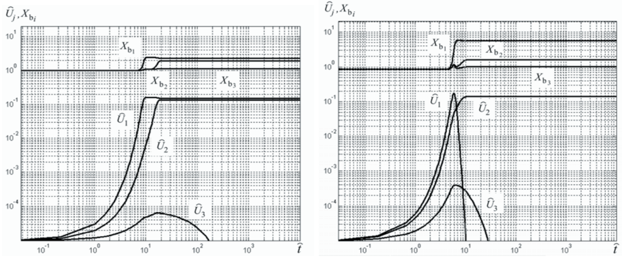
Activity of excited oscillations is determined by regeneration parameter to be found from (25) when $\hat{U}_j = 0$:

$$K_j^{reg} = R_{e_j} K_{jj} S_{s_j} = R_{e_j} K_{jj} S_0 \times \sum_{i=1}^{3}\left(\sum_{r=1}^{3,5} r\hat{a}_r X_{s_j}^{(r-1)}\right)\gamma_{ji}\frac{K_{ii}}{K_{jj}}, \quad (28)$$

where S_{s_j} – gain slope of volt-amps diagram in still point for oscillation j; X_{s_j} – bias in the still point for.NLC$_i$, being the solution for non-linear equation:

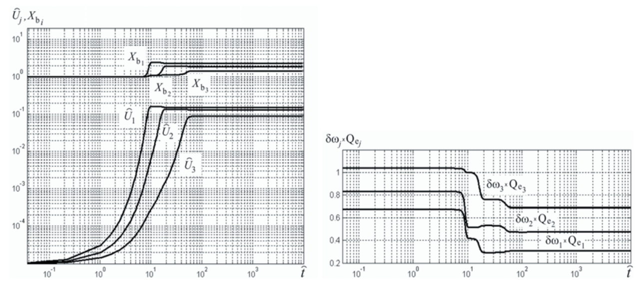
$$S_0 \cdot R_{ab_i}\left(\hat{a}_0 + \frac{X_{b_i}}{S_0 \cdot R_{ab_i}} + \sum_{r=1}^{3}\hat{a}_r \cdot X_{b_i}^{r}\right) - \bar{x}_{b_i} = 0. \quad (29)$$

Fig. 3 and 4 demonstrate distinguished case of stabilizing oscillations in three-frequency MPOS, as result of numerical integrating equations (25) – (27) in MATLAB system. The bipolar transistors KT368 (2SC9018) are used as active components (NLC$_i$), which have average statistical values of VAD parameters: $S_0 = 0{,}1$ A/B; $\hat{a}_0 = 0{,}95$; $\hat{a}_1 = 0{,}55$; $\hat{a}_2 = 0{,}051$; $\hat{a}_3 = -0{,}054$, when approximation error $\varepsilon \le 5\%$. Initial values of low-varying parameters made: $T_j = 0{,}1$ c; $T_{ab_i} = 1\cdot 10^{-4}$ c; $R_{ab_i} = 1$ кОм; $K_{jj} = -20$ дБ; $\bar{x}_{b_i} = 30$.



a) b)

Fig. 3. Distinguished case of appearing competition of oscillations in excitation channels of MPOS: suppressing energetically less active oscillations (a); suppressing energetically more active oscillation by less active one (b)



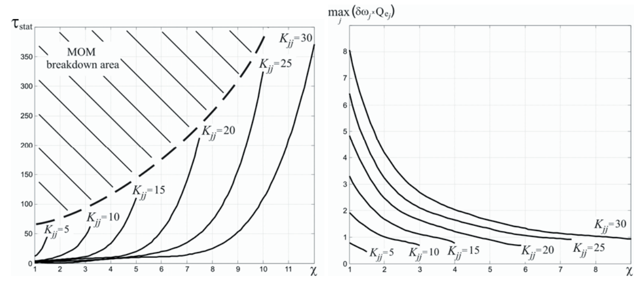
a) b)

Fig. 4. Stable three-frequency generation mode: setting oscillation amplitudes and auto-bias voltages (a); initial run-out of oscillation frequencies, caused by amplitude and phase conversion in excitation channels (b)

It's obvious that high competition in excitation channels ($\gamma_{ji} = -6$ dB) can bring to situations when oscillations with low energetic activity (oscillation $\hat{U}_3$, fig. 3,a,b) can be suppressed by more active oscillations (with high regeneration coefficient K_j^{reg}). At the same time the overly high increasing regeneration parameter K_j^{reg} can cause a breakdown of more energetically active oscillations (oscillation $\hat{U}_1$, fig. 3,b) in excitation channels of MPOS. Reducing competition of oscillations due to increasing selective properties of FC circuits down to $\gamma_{ji} = -12$ dB provides stable three-frequency mode of oscillation (fig. 4,a) even when regeneration coefficient values $K_1^{reg} = 1{,}25K_2^{reg} = 1{,}5K_3^{reg}$ are growing significantly [8].

In this case it assumes using the term of low or high active oscillation (oscillations) in relation to the other oscillations of MPOS, which, being energetically all-sufficient in one-frequency mode, may fade on transiting to multi-frequency mode of oscillations in the certain conditions.

Fig. 5, a represents approximated relationships of oscillation stabilizing time $\tau_{stat} = \max\limits_{1\le j\le m}\tau_{stat_j}$ at various partial coefficients of FC. It also has bifurcation curve that determines a range of stable MOM. Evidently, at high values of variations $\chi = \max\limits_{1\le j\le m} K_j^{reg} / \min\limits_{1\le j\le m} K_j^{reg} = \max\limits_{1\le j\le m} R_{e_j} / \min\limits_{1\le j\le m} R_{e_j}$ of regeneration coefficients the oscillation stabilizing time τ_{stat} is significantly increasing (by an order and more!). Compensation of this events when having the great scatter of equivalent resistances R_{e_j} is possible by selecting appropriate coefficients K_{jj} of partial FC (increasing energy of oscillations), what it's not only improving dynamic properties of MPOS, but also brings to extending range of stable MOM (fig. 5,a).



a) b)

Fig. 5. Dependences of normalized oscillation stabilizing time (a) and initial frequency run-out on MPOS parameters

The quite important characteristic of MPOS dynamics is group run-out of oscillation frequencies $\delta\omega_j Q_{e_j}$, which is defined according to (26). The modeling results indicate that amplitude-phase conversion slightly exhibits for energetically "weak" oscillations (fig. 4,b). At joint control of oscillation amplitude and voltage of auto-bias the relative run-out of frequencies $\delta\omega_j = \left(\omega_{q_j}\right)^{-1} \cdot \Delta\omega_j(\hat{t}) = \left(\omega_{q_j}\right)^{-1} \cdot \frac{d\phi_j}{d\hat{t}}$ may reach values from 10^{-8} to 10^{-6}, and significantly reduce on increasing coefficients K_{jj} of partial circuits of FC (fig. 5,b).

6. Conclusion

The offered approach to developing piezoresonance units with controlled dynamics, which are represented as the adaptive control system with predictive reference model, allows creating the new class of PRU to be invariant to disturbing destabilizing factors. This approach grounds on the principle of using natural redundancy in multi-frequency basis (multi-frequency) of PRU core – multi-frequency piezoresonance oscillation system what allows on the base of invariance theory not only synthesizing system with current identification of disturbing factors, but also adapting PRU relative to their effect.

Using the developed mathematical model MPOS having in its base reduced differential equations for amplitudes, phases and voltages of auto-bias as predictive reference model of PRU core allows accomplishing effective controlling IMPRU/CD in accordance to the criteria of minimum of energy consumption as at the stage of setting MOM so at the stage of stabilizing multi-frequency oscillations.

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Abstract

The paper represents principles of development of invariant to disturbing factors of piezoresonance units with controlled dynamics. The architecture of invariant multi-frequency piezoresonance units with controlled dynamics (IMFRU/CD) is represented as adaptive control system with predictive reference model. Its main component is the piezoresonance units' core – multi-frequency piezoresonance oscillatory system (MPOS), with embedded supporting circuits of control, thermal and vibrational compensation, which is exposed to destabilize disturbing factors.

The objectives and criteria for terminal control were formulated in accordance to which the control process is divided into two stages: setting and stabilizing oscillation. The mathematical model of MPOS has been developed having in its base reduced by differential equations for amplitudes, phases of oscillations and voltages of auto-bias of active components in excitation channels.

The offered approach to development of piezoresonance units with controlled dynamics has allowed creating the new class of PRU to be invariant to disturbing destabilizing factors. This approach grounds on the principle of using natural redundancy (multi-frequency) of the piezoresonance units' core that allows on the base of invariance theory not only synthesis of the system with current identification of disturbing factors, but also adaptation of the piezoresonance units relatively to their effects.

Keywords: *quartz resonator, piezoresonance units, system with controlled dynamics, multi-frequency oscillation system*