Наведено особливості проектування робастних систем стабілізації рухомих платформ з обладнанням, призначених для функціонування на наземних рухомих об'єктах. Розв'язано проблему проектування робастної системи стабілізації з двома ступенями вільності з урахуванням координатних збурень та завад вимірювань. Розглянуто прикладну реалізацію запропонованого підходу на прикладі системи управління кутовим рухом платформи з апаратурою спостереження, призначеної для експлуатації на наземних транспортних засобах. Наведено результати моделювання, які підтверджують можливість забезпечення високих характеристик системи в складних умовах реальної експлуатації

Ключові слова: робастна стабілізація, системи з двома ступенями вільності, наземні рухомі об'єкти, рухомі платформи з обладнанням, параметричні та координатні збурення

Изложены особенности проектирования робастных систем стабилизации подвижных платформ с оборудованием, предназначенным для функционирования на наземных подвижных объектах. Решена проблема проектирования робастной системы стабилизации с двумя степенями свободы с учетом координатных возмущений и помех измерений. Рассмотрена прикладная реализация предложенного подхода на примере системы управления угловым движением платформы с аппаратурой наблюдения, предназначенной для эксплуатации на наземных транспортных средствах. Приведены результаты моделирования, подтверждающие возможность обеспечения высоких характеристик системы в сложных условиях реальной эксплуатации

Ключевые слова: робастная стабилизация, системы с двумя степенями свободы, наземные подвижные объекты, подвижные платформы с оборудованием, параметрические и координатные возмущения

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DESIGN OF TWO-DEGREE-OF-FREEDOM ROBUST SYSTEM FOR GROUND VEHICLE EQUIPMENT STABILIZATION

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1. Introduction

Now the inertially stabilized platforms are widely used to stabilize and point sensors, cameras, antennas and weapon systems operated at the vehicles of the different type [1]. The further progress of the above listed equipment by accuracy and other operating performances is impossible without stabilization of a base, at which they are mounted. This proves the actuality of the inertially stabilized platforms technology development. It should be noted, that the above stated platforms assigned for operation at the ground vehicles are of great importance for Ukrainian device-building in the area of both civil and military applications.

Operation of the observation equipment at the moving platforms mounted at the ground vehicles is accompanied by change of their parameters in the wide range (±50 %) and action of the intensive and various external (coordinate) disturbances. As a rule, the most changed parameters are the inertia moment and the rigidity of elastic connection between an actuator and a base at which a plant is mounted [1].

Therefore it is expedient to solve the problem of the ground vehicle stabilized platforms design using the principles of robust control. The main goal of the design of the researched robust systems is the search of the control law providing the system accuracy in the give limits in spite of presence uncertainties in the system mathematical description. The uncertainties may be caused by many factors such as the external disturbances, parametrical and structural changes.

One of perspective approaches to the robust system design lies in using the H_2-synthesis [2, 3]. The choice of the method of the robust structural synthesis depends on features of the designed system and conditions of its operation. In designing the inertially stabilized platforms for ground vehicles it is necessary to take into consideration two factors. In the first place, some parameters of the ground vehicle stabilized platforms vary in the wide range during their operation. In the second place, the researched systems operate in the difficult conditions of the external disturbances caused by irregularities of the relief of the road or terrain, by which a ground vehicle is moving.

It should be noted, that creation of the modern inertially stabilized platforms, as a rule, is implemented by means of the MatLab software including special toolboxes for the automated optimal design of the robust systems [4].

2. Analysis of publications data and problem statement

The actuality of the inertially stabilized platforms application in the different industrial areas and the basic trends of their technology progress are represented in [5]. Features of control by platforms with the useful load operated at vehicles, structural schemes of feedback and feedforward control for stabilization and tracking systems of the researched type are given in [5, 6]. The basic design phases of the inertially stabilized platforms are described in [7]. The newest applications and gimbals features are mentioned in [8]. But all these papers do not research such aspect of the inertially stabilized platforms design as their immunity to the internal and external disturbances action. In other words, the robustness aspect is not mentioned in these publications.

There are many papers, which deal with the theory and application of robust systems as a whole, for example, [9, 10]. In these papers problems of the robust systems analysis and synthesis are discussed. Now the application of the H_∞-synthesis becomes widespread for the robust systems design. For example, the basic principles of the H_∞-synthesis procedure applied to an unmanned helicopter are given in [11]. In its turn, loop shaping methods of the H_∞-theory are widely used today. They have such essential advantage as the possibility to design the system with the desired amplitude-frequency characteristics. Such approach to control by unmanned aerial vehicles is given in [12]. In contrast to this paper, publication [13] represents problems more important for the inertially stabilized platforms design, as deals with the robust control of the electric drive based on the H_∞-synthesis [13].

It should be noted, that the H_∞-theory has well known fundamental theoretical achievements but its applications for the different areas require the development of the appropriate methods and procedures. So, the results obtained for the H_∞-synthesis of vehicles and drives, of course, may be useful as a whole, but the design of the robust system for stabilization and tracking of the inertially stabilized platforms requires the development of the special methods taking into consideration the features of their mathematical description and operation conditions.

The standard procedure of the H_∞-synthesis solved by means of (2)–(5) equations is well developed in the H_∞-theory. It is important that this procedure is provided for software developed in MatLab system. The main problems of such procedure creation relative to the concrete system are the development of the mathematical description, forming augmented plant and creation of the optimization functional.

The mathematical model of the system for stabilization and tracking of the observation equipment assigned for operation at the ground vehicles was developed in [14]. An algorithm of the robust system assigned for the platform with observation equipment operated at the ground vehicle is represented in [15]. But the modern requirements to the researched system accuracy define the necessity to use the stabilization and tracking systems with two-degrees-of-freedom [16]. The actuality of this choice is proved by the paper [17], which deals with the design of the rocket with two-degrees-of-freedom.

The newness of the suggested research lies in the introduction of the optimization functional, which takes into consideration the influence of the coordinate disturbances and measurement noise. Respectively the features of the generalized plant forming for the researched system and approach to loop shaping were considered too.

It should be noted, that check of the synthesized system requires using mathematical models of the random disturbances caused by irregularities of the relief of the road and terrain by which the ground vehicle is moving. Such models were developed in [18].

Achievement of the high accuracy of tracking processes requires using control both by the error signal, which represents the difference between the reference and output signals of the system, and by the reference signal. In this case a system must include two controllers, which implement feedback and feedforward control. Configuration of the control by the plant in such systems is implemented by means of the prefilter and feedback controller. The problem must be solved taking into consideration the influence of the external (coordinate) disturbances and measurement noise.

3. Research purpose and tasks

The purpose of the research is the development of the H_∞-synthesis procedure directed to the creation of the robust two-degree-of-freedom system by means of the loop shaping methods. The synthesized system must have immunity to the influence of the coordinate disturbances and measurement noise.

To achieve this research purpose it is necessary to solve the following tasks such as:

– creation of the plant mathematical description;

– choice of the loop shaping method and appropriate weighting transfer functions;

– forming the augmented plant;

– introduction of the sensitivity functions by the coordinate disturbance and the measurement noise into the optimization functional;

– checking the results of the designed system efficiency by means of modeling in the MatLab system.

4. Research materials and methods

4. 1. Standard problem of H_∞-control

In accordance with the formulation [19] the standard problem of the H_∞-control may be represented in the following form (Fig. 1).

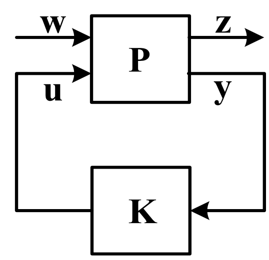


Fig. 1. Structural scheme of the standard H_∞-control

The system represented in Fig. 1 consists of the plant and the controller described by the matrix transfer functions $\mathbf{P}(s)$, $\mathbf{K}(s)$, which are fractional-rational and proper. In the general case the input vector $\mathbf{w}$ includes disturbances, measurement noise and reference signals. The input vector $\mathbf{u}$ represents control systems. The output vector $\mathbf{z}$ defines the quality of control processes, for example, the reference signal error. The output vector $\mathbf{y}$ consists of observed signals, which may be used for feedback implementation. The plant $\mathbf{P}$ in Fig. 1 represents the generalized plant and the controller $\mathbf{K}$ is the general controller respectively. In the case of augmentation the generalized plant is augmented by the weighting transfer functions.

The control system shown in Fig. may be represented in the state space in the following way [3, 4]:

$$\begin{bmatrix}\dot{\mathbf{x}}(t)\\ \mathbf{z}(t)\\ \mathbf{y}(t)\end{bmatrix}=\left[\begin{array}{c|c|c}\mathbf{A} & \mathbf{B}_1 & \mathbf{B}_2\\ \hline \mathbf{C}_1 & \mathbf{D}_{11} & \mathbf{D}_{12}\\ \hline \mathbf{C}_2 & \mathbf{D}_{21} & \mathbf{D}_{22}\end{array}\right]\begin{bmatrix}\mathbf{x}(t)\\ \mathbf{w}(t)\\ \mathbf{u}(t)\end{bmatrix}. \quad (1)$$

The first usage of the H_∞-synthesis for the design of control systems was represented in [20]. Usage of such approach ensures the robust performance and the robust stabilization of the designed system. The advantage of the method is the possibility to synthesize a system with the desired amplitude-frequency characteristics. The disadvantage of the method is the determinative influence of the mathematical description adequacy and the necessity of the heuristic choice of the weighting transfer functions.

In practical applications it is necessary to carry out a search of the suboptimal controller, when the H_∞-norm of the closed loop system transfer function will not exceed some small given number γ [3].

The algorithm of the search of the H_∞-suboptimal controller is represented in many books [3, 4]. In accordance with the algorithm for the control system described by the matrix equation (1) for the search of the suboptimal controller $\mathbf{K}(s)$ it is necessary to carry out the following steps [3].

1. To solve the algebraic Riccati equation relative to $\mathbf{X}_\infty$

$$\mathbf{A}^T\mathbf{X}_\infty+\mathbf{X}_\infty\mathbf{A}+\mathbf{C}_1^T\mathbf{C}_1+ \\ +\mathbf{X}_\infty(\gamma^{-2}\mathbf{B}_1\mathbf{B}_1^T-\mathbf{B}_2\mathbf{B}_2^T)\mathbf{X}_\infty=0. \quad (2)$$

2. To solve the algebraic Riccati equation relative to $\mathbf{Y}_\infty$

$$\mathbf{A}\mathbf{Y}_\infty+\mathbf{Y}_\infty\mathbf{A}^T+\mathbf{B}_1\mathbf{B}_1^T+ \\ +\mathbf{Y}_\infty(\gamma^{-2}\mathbf{C}_1^T\mathbf{C}_1-\mathbf{C}_2^T\mathbf{C}_2)\mathbf{Y}_\infty=0. \quad (3)$$

3. To check conditions

$$\mathrm{Re}\lambda_i[\mathbf{A}+(\gamma^{-2}\mathbf{B}_1\mathbf{B}_1^T-\mathbf{B}_2\mathbf{B}_2^T)\mathbf{X}_\infty]<0,\forall i\ ,$$

$$\mathrm{Re}\lambda_i[\mathbf{A}+\mathbf{Y}_\infty(\gamma^{-2}\mathbf{C}_1^T\mathbf{C}_1-\mathbf{C}_2^T\mathbf{C}_2)]<0,\forall i\ ,$$

$$\rho(X_\infty Y_\infty)<\gamma^2 . \quad (4)$$

It is necessary to take into consideration that the control system described by the equation (1) must satisfy the following limitations [3, 4].

1. Pairs of the matrices $\mathbf{A},\mathbf{B}_1$, $\mathbf{A},\mathbf{B}_2$ must be stabilized and pairs of the matrices $\mathbf{A},\mathbf{C}_1$, $\mathbf{A},\mathbf{C}_2$ – detected.

2. $\mathbf{D}_{12}^T\begin{bmatrix}\mathbf{C}_1 & \mathbf{D}_{12}\end{bmatrix}=\begin{bmatrix}\mathbf{0} & \mathbf{I}\end{bmatrix}$.

3. $\begin{bmatrix}\mathbf{B}_1\\ \mathbf{D}_{21}\end{bmatrix}\mathbf{D}_{21}^T=\begin{bmatrix}\mathbf{0}\\ \mathbf{I}\end{bmatrix}$.

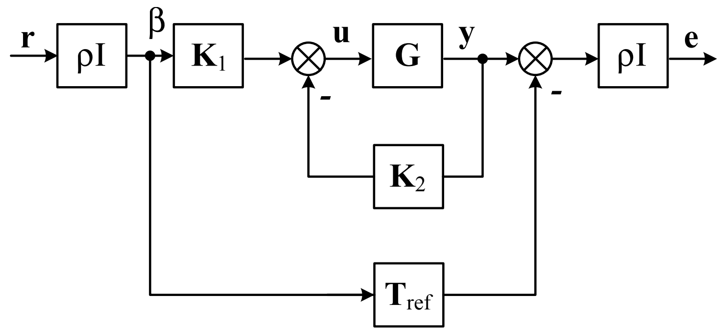
4. 2. Classification of methods for the robust structural synthesis

Methods of the H_∞-synthesis (robust structural synthesis) may be classified by different signs such as quantity of degrees of freedom, ways of loop shaping and augmented plant forming.

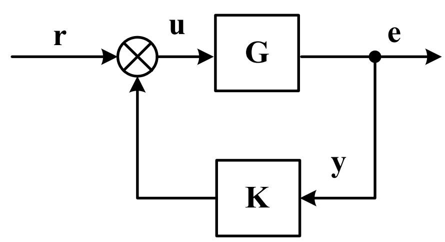
Classification of the H_∞-synthesis methods by quantity of degrees of freedom

Methods for the design of the robust systems with various quantities of degrees of freedom differ by an approach to the controller configuration.

The structures of controllers with the different quantity of degrees of freedom [2, 3] are represented in Fig. 2.



a



b

Fig. 2. Control system structures: *a* — two-degree-of-freedom; *b* — one-degree-of-freedom: **r**, **u**, **y**, **e** represent the reference, control, measured output signals and the error signal respectively; $\mathbf{K}_1$ is the pre-filter or the forward controller; $\mathbf{K}_2$ is the feedback controller

Classification of the H_∞-synthesis methods by loop shaping ways

There are such control loop shaping ways as forming the closed loop system frequency characteristics based on analysis of the transfer function and analysis of the frequency characteristics of the system signals [3].

In the first case, H_∞-optimization provides forming the desired frequency characteristics of the closed loop systems due to determination of the transfer function singular values. The maximum singular values may be formed by means of the given upper bounds. Such approach may ensure the desired bandwidth and slope of the amplitude-frequency characteristic.

In the second case, the given set of the input signals is used and some their errors are minimized. Usually, such input signals as external disturbances, measurement noise and reference signals are believed to be components of this set.

So, the H_∞-synthesis may be used for the design of the feedback control systems based on shaping the desired frequency characteristics of the closed loop system. In the most cases, designing robust systems it is possible to estimate only the upper bound of deviation of the desired frequency characteristics from nominal ones. There are different approaches to the determination of the bounded characteristics. One of the possible approaches uses the results of the experimental tests for plotting bounded characteristics [3]. But in the most cases, the information about frequency characteristics of the real system is absent before the design process. The most widespread method of the bounded characteristics determination is based on the frequency requirements to the designed system.

There are methods of the robust systems design grounded on shaping the transfer function of the open loop system. And the goal of these methods is improvement of the frequency characteristics of the closed loop system. Such procedure of the H_∞-synthesis based on the robust stabilization and taking into consideration the parametrical disturbances given by means of coprime factorization is represented in [3, 22].

The H_∞-synthesis methods based on analysis of the system signals features are well agreed with the multi-purpose problems solution. But implementation of these methods requires using the complicated mathematical description, namely, mathematical description of the plant, models of uncertainties, determination of the class of the system input signals and norms of the signal errors [3]. For the latter methods the signal values and not the transfer functions characteristics are of great importance. Moreover, using these methods it is desirable to use the weighting functions for description of the desired or known spectra of the external signals frequencies and desired frequency spectra of the error signals. The weighting functions are used also if the uncertainty model represents the normalized disturbance ($\|\Delta\|_\infty < 1$). The successful implementation of the H_∞-synthesis requires stable and proper weighting functions [3, 4].

Classification of the H_∞-synthesis methods by a way to apply the weighting transfer function for extension of a plant

There are different methods to form a plant by means of the weighting transfer functions such as shaping the required transfer function of the closed loop system; using of pre- and post-compensators for extension of the open loop system and weighting functions of the system signals. It should be noticed, that designed systems become extended after application of the matrix weighting transfer functions. Therefore they are called augmented [3, 4].

The structural schemes of the augmented systems obtained by means of all the above stated ways are represented in Fig. 3.

Augmentation of the plant by means of the weighting transfer functions in accordance with Fig. 3, *a*, where the augmented plant with the additional outputs $\mathbf{z}_1, \mathbf{z}_2, \mathbf{z}_3$ is represented, is typical for the method of the mixed sensitivity. The weighting transfer functions $\mathbf{W}_1, \mathbf{W}_2, \mathbf{W}_3$ are used for decreasing the reference signal error, limitation of the control power and ensuring system robustness.

For the H_∞-synthesis procedures, the influence of all uncertainties acting on the plant is estimated by the single multiplicative disturbance Δ_M. Then the requirements to the designed system may be determined in the following way [3, 4, 22]

$$\bar{\sigma}(\mathbf{S}(j\omega)) \le |\mathbf{W}_1^{-1}(j\omega)|;$$

$$\bar{\sigma}(\mathbf{S}(j\omega)) \le |\mathbf{W}_2^{-1}(j\omega)|;$$

$$\bar{\sigma}(\mathbf{T}(j\omega)) \le |\mathbf{W}_3^{-1}(j\omega)|, \quad (5)$$

where $\bar{\sigma}$ is the maximum singular value.

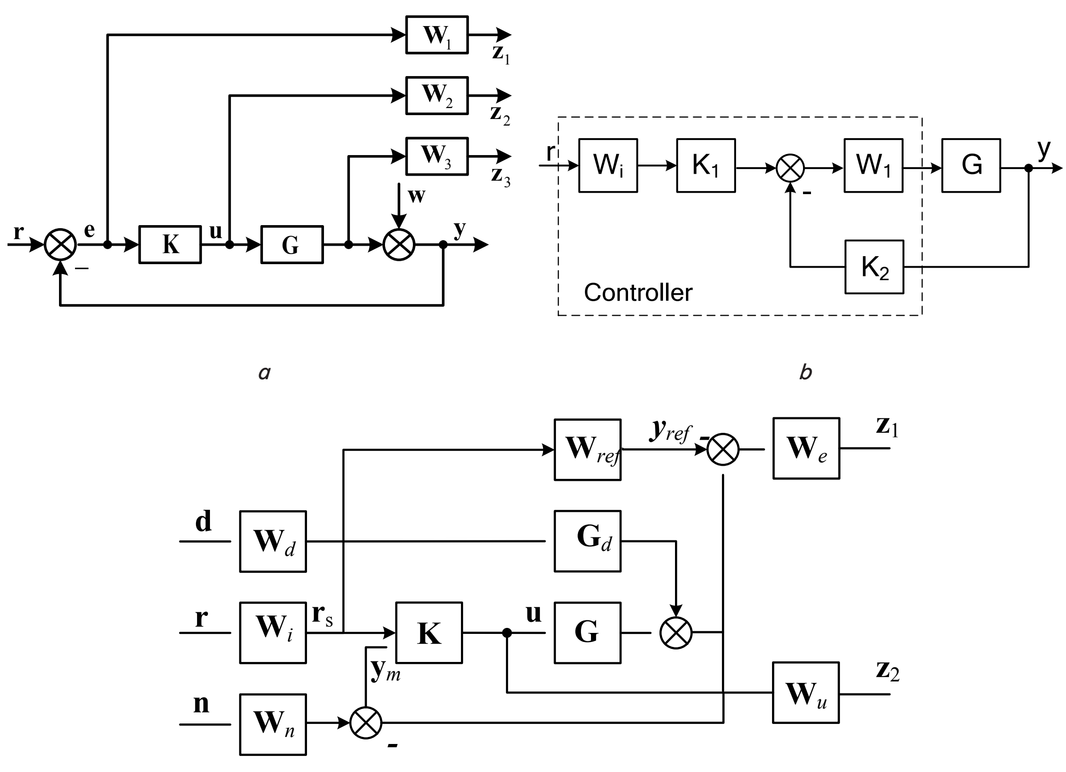


Fig. 3. Ways to use the weighting transfer functions: *a* — for augmentation of the closed loop transfer function; *b* — usage of the pre- and post-compensators; *c* — usage of the weighting functions for system signals

In this case, the following condition must be satisfied [3, 4, 22]

$$\bar{\sigma}(\mathbf{W}_1^{-1}(j\omega))+\bar{\sigma}(\mathbf{W}_3^{-1}(j\omega))>1 \text{ for } \forall\,\omega. \qquad (6)$$

The choice of the weighting transfer functions satisfying conditions (5), (6) is an ambiguous problem, which requires for its solution heuristic methods, for example, cut and try method taking into consideration experience of the system designer.

The approach to robust systems design based on determination of the singular values of the system transfer function and minimization of the H_∞-norm may be implemented by means of Robust Control Toolbox of MatLab software [22].

If $\mathbf{W}_1$, $\mathbf{W}_2$ represent pre- and post-compensators respectively, the augmented plant with the bounded frequency characteristics $\mathbf{G}_s$ will be determined by the expression [3]

$$\mathbf{G}_s=\mathbf{W}_2\mathbf{G}\mathbf{W}_1. \qquad (7)$$

Usage of the pre- and post-compensators ensures limitation of the singular values of the open loop system transfer function and shaping desired frequency characteristics of the closed loop system transfer function. It should be noticed, that the pre-compensator provides the feedforward by the reference signal.

Usage of the weighting functions in the statement of the H_∞-synthesis problem for the method based on analysis of the system signals is shown in Fig. 3, *c*. In this scheme $\mathbf{G}$ and $\mathbf{G}_d$ represent the plant and disturbance nominal models and $\mathbf{K}$ is the designed controller. The weighting functions of such external signals as disturbances, reference signal and measurement noise $\mathbf{W}_d$, $\mathbf{W}_i$, $\mathbf{W}_n$ may be constants or transfer functions [3]. The weighting function $\mathbf{W}_{ref}$ represents a desired closed loop system transfer function between the weighting reference signal $\mathbf{r}_s$ and the system output signal $\mathbf{y}$. The weighting functions $\mathbf{W}_e$ and $\mathbf{W}_d$ characterize the desired frequency spectrum of the error $\mathbf{z}_1=\mathbf{y}-\mathbf{y}_{ref}$ and control signal $\mathbf{z}_2=\mathbf{u}$ respectively.

For the system represented in Fig. 3, *c* it is desired to minimize the H_∞-norm of the transfer function from the input signals $\mathbf{r},\mathbf{d},\mathbf{n}$ to the output signals $\mathbf{z}_1$, $\mathbf{z}_2$ [3]

$$\begin{bmatrix}\mathbf{z}_1\\ \mathbf{z}_2\end{bmatrix}=\begin{bmatrix}\mathbf{W}_e\mathbf{S} & -\mathbf{W}_e\mathbf{S}\mathbf{W}_d & \mathbf{W}_e\mathbf{T}\mathbf{W}_n\\ \mathbf{W}_b\mathbf{K}\mathbf{S} & -\mathbf{W}_b\mathbf{K}\mathbf{S}\mathbf{W}_d & -\mathbf{W}_b\mathbf{K}\mathbf{S}\mathbf{W}_n\end{bmatrix}\begin{bmatrix}\mathbf{r}\\ \mathbf{d}\\ \mathbf{n}\end{bmatrix}. \qquad (8)$$

Finally, this problem may be formulated as the H_∞-synthesis problem, for which the signals $\mathbf{w}$, $\mathbf{z}$, $\mathbf{y}$, $\mathbf{u}$ may be determined in the following way

$$\mathbf{w}=\begin{bmatrix}\mathbf{r}\\ \mathbf{d}\\ \mathbf{n}\end{bmatrix};\ \mathbf{z}=\begin{bmatrix}\mathbf{z}_1\\ \mathbf{z}_2\end{bmatrix};\ \mathbf{y}=\begin{bmatrix}\mathbf{r}_s\\ \mathbf{y}_s\end{bmatrix};\ \mathbf{u}=\mathbf{u}. \qquad (9)$$

4. 3. H_∞-synthesis of robust two-degree-of-freedom stabilization and tracking system

There are different approaches to the design of robust two-degree-of-freedom systems. The approach represented in [21] is based on the robust stabilization and definition of the parametrical disturbances by means of the normalized coprime factorization. It is based on the H_∞-synthesis procedure taking into consideration forming the desired amplitude-frequency characteristics by means of augmentation of the open loop system transfer function. Forming the augmented plant is implemented by means of the pre- and post-compensators.

But in designing systems for control by the motion of the platforms with the payload operated at the vehicles, it is necessary to take into consideration the influence of the coordinate disturbances. For the stabilization systems operated at the ground vehicles the unbalance moment and the moment of friction in gimbals bearings, and the moment caused by the vehicle angular rate due to irregularities of the relief of the road or terrain are of great importance. In designing the system it is necessary to take into consideration the influence of the measurement noise too. The structural scheme of the two-degree-of-freedom system, which in contrast to [3] takes into consideration the coordinate disturbances and measurement noise are represented in Fig. 4.

Here the feedback controller $\mathbf{K}_2$ provides the robust stability and the prefilter $\mathbf{K}_1$ ensures correspondence between responses of the closed loop system and the reference model [3].

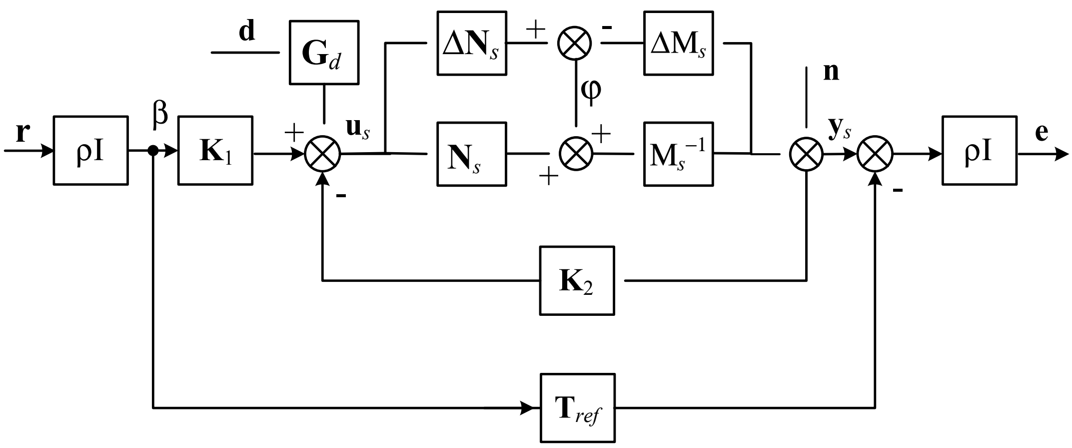


Fig. 4. The structural scheme of the robust two-degree-of-freedom system taking into consideration the coordinate disturbances and measurement noise

The problem lies in determination of the controller $\mathbf{K}=\begin{bmatrix}\mathbf{K}_1 & \mathbf{K}_2\end{bmatrix}$, which provides stabilization of the plant augmented by the pre- and post-compensators in accordance with the expression (7) $\mathbf{G}_s=\mathbf{W}_2\mathbf{G}\mathbf{W}_1$ and represented as a result of the normalized coprime factorization $\mathbf{G}_s=\mathbf{M}_s^{-1}\mathbf{N}_s$ [3]. During such controller design it is necessary to ensure minimization of the H_∞-norms of the transfer functions between signals $\mathbf{r}$, $\mathbf{d}$, ϕ, $\mathbf{n}$ and $\mathbf{u}_s$, $\mathbf{y}_s$, $\mathbf{e}$ in accordance with Fig. 4.

The problem of synthesis of the two-degree-of-freedom system represented in Fig. 4 may be represented as the standard problem of the H_∞-synthesis (Fig. 1) and solved by means of Robust Control Toolbox.

The control of the augmented plant may be defined in the following way [3]

$$\mathbf{u}_s=[\mathbf{K}_1\,\mathbf{K}_2]\begin{bmatrix}\beta\\ \mathbf{y}_s\end{bmatrix}, \tag{10}$$

where $\mathbf{K}_1$ is the prefilter transfer function, $\mathbf{K}_2$ is the feedback controller transfer function; β is the reference scale signal; $\mathbf{y}_s$ is the measured output signal; $\mathbf{T}_{ref}$ is the reference transfer function chosen by a designer with the purpose to form the desired amplitude-frequency characteristics; ρ is the scalar value.

In contrast to the known problem statements the influence of the coordinate disturbance $\mathbf{d}$ and measurement noise $\mathbf{n}$ is taken into consideration in this paper. The goal of the robust structural procedure is to ensure minimization of the H_∞-norm of the transfer function of the system with input and output signals, which in accordance with Fig. 4 may be represented by vectors

$$\mathbf{w}=\begin{bmatrix}\mathbf{r}\\ \mathbf{d}\\ \mathbf{n}\\ \phi\end{bmatrix};\ \mathbf{z}=\begin{bmatrix}\mathbf{u}_s\\ \mathbf{y}_s\\ \mathbf{e}\end{bmatrix}. \tag{11}$$

Connection between these vectors may be defined in the following way

$$\begin{bmatrix}\mathbf{u}_s & \mathbf{y}_s & \mathbf{e}\end{bmatrix}^T=\mathbf{\Phi}\begin{bmatrix}\mathbf{r} & \mathbf{d} & \mathbf{n} & \phi\end{bmatrix}^T$$

or

$$\begin{bmatrix}\mathbf{u}_s\\ \mathbf{y}_s\\ \mathbf{e}\end{bmatrix}=\begin{bmatrix}\rho\mathbf{W}_{s1}K_1 & \mathbf{K}_2\mathbf{W}_{s1}\mathbf{G}_d\mathbf{G}_s & \mathbf{K}_2\mathbf{W}_{s1} & \mathbf{K}_2\mathbf{W}_{s1}\mathbf{M}_s^{-1}\\ \rho\mathbf{W}_{s2}\mathbf{G}_s\mathbf{K}_1 & \mathbf{W}_{s1}\mathbf{G}_d\mathbf{G}_s & \mathbf{W}_{s1}\mathbf{G}_s & \mathbf{W}_{s1}\mathbf{M}_s^{-1}\\ \rho^2[\mathbf{W}_{s2}\mathbf{G}_s\mathbf{K}_1-\mathbf{T}_{ref}] & \rho\mathbf{W}_{s1}\mathbf{G}_d\mathbf{G}_s & \rho\mathbf{W}_{s1}\mathbf{G}_s & \rho\mathbf{W}_{s1}\mathbf{M}_s^{-1}\end{bmatrix}\begin{bmatrix}\mathbf{r}\\ \mathbf{d}\\ \mathbf{n}\\ \phi\end{bmatrix}, \tag{12}$$

where $\mathbf{\Phi}$ is the matrix transfer function of the closed loop system. The H_∞-norm of this transfer function represents the optimization functional because its elements represent sensitivity functions which define the system accuracy, robustness and control costs. Here $\mathbf{W}_{s1}=(\mathbf{I}-\mathbf{K}_2\mathbf{G}_s)^{-1}$, $\mathbf{W}_{s2}=(\mathbf{I}-\mathbf{G}_s\mathbf{K}_2)^{-1}$.

For transition to the standard statement of the H_∞-synthesis problem (Fig. 1) it is necessary to take into account the control signal $\mathbf{u}=\mathbf{u}_s$, the output signal $\mathbf{y}=\begin{bmatrix}\beta\\ \mathbf{y}_s\end{bmatrix}$ and introduce into consideration the matrix transfer function of the generalized plant $\mathbf{P}$. Based on relationships (10), (11) the equation of connection between the inputs and outputs of the interconnected system becomes

$$\begin{bmatrix}\mathbf{u}_s & \mathbf{y}_s & \mathbf{e} & \beta & \mathbf{y}_s\end{bmatrix}^T=\mathbf{P}\begin{bmatrix}\mathbf{r} & \mathbf{d} & \mathbf{n} & \phi & \mathbf{u}_s\end{bmatrix}^T. \tag{13}$$

The generalized plant transfer function in the expression (13) may be represented in the following way

$$\mathbf{P}=\left[\begin{array}{cccc|c}\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I}\\ \mathbf{0} & \mathbf{G}_d\mathbf{G}_s & \mathbf{I} & \mathbf{M}_s^{-1} & \mathbf{G}_s\\ -\rho^2\mathbf{T}_{ref} & \rho\mathbf{G}_d\mathbf{G}_s & \mathbf{0} & \rho\mathbf{M}_s^{-1} & \rho\mathbf{G}_s\\ \hline \rho\mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{G}_d\mathbf{G}_s & \mathbf{I} & \mathbf{M}_s^{-1} & \mathbf{G}_s\end{array}\right]. \tag{14}$$

Respectively the generalized plant model in the state space may be derived based on the equation (14) and represented as

$$\mathbf{P}_{ss}=\begin{bmatrix}\mathbf{A}_s & \mathbf{0} & \mathbf{0} & \mathbf{B}_d & \mathbf{0} & (\mathbf{B}_s\mathbf{D}_s^T+\mathbf{Z}_s\mathbf{C}_s^T)\mathbf{D}_{M_s} & \mathbf{B}_s\\ \mathbf{0} & \mathbf{A}_r & \mathbf{B}_r & \mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I}\\ \mathbf{C}_s & \mathbf{0} & \mathbf{0} & \mathbf{D}_d & \mathbf{0} & \mathbf{D}_{M_s^{-1}} & \mathbf{D}_s\\ \rho\mathbf{C}_s & -\rho^2\mathbf{C}_r & -\rho^2\mathbf{D}_r & \rho\mathbf{D}_d & \mathbf{0} & \rho\mathbf{D}_{M_s^{-1}} & \rho\mathbf{D}_s\\ \mathbf{0} & \mathbf{0} & \rho\mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{C}_s & \mathbf{0} & \mathbf{0} & \mathbf{D}_d & \mathbf{I} & \mathbf{D}_{M_s^{-1}} & \mathbf{D}_s\end{bmatrix}. \tag{15}$$

4. Application of robust structural method for design of stabilized platforms with observation equipment operated at ground vehicles

Design of the robust two-degree-of-freedom system for stabilization of a platform with the payload operated at the grounded vehicles may be implemented by the H_∞-synthesis method based on robust stabilization, loop shaping by means of pre- and post-compensators and the new optimization functional (12) taking into consideration the sensitivity functions by the coordinate disturbance and measurement noise.

This method consists of some steps including creation of the full and linearized mathematical descriptions, representation of the linear model in the state space (15), choice of the reference model, designed parameter ρ, and weighting transfer functions, which form the system frequency characteristics, implementation of the H_∞-synthesis standard procedure, checking of synthesized system operation by means of the full model with non-linearities inherent to a real system and repetition of the synthesis procedure in the case of need if necessary.

In the modern problems of control laws synthesis the plant is considered with the actuator and measuring system as a single object. Obviously, that the concrete application may increase or decrease the list of the necessary components.

The considered system assigned for operation at the ground vehicle consists of the apparatus of measurement and observation as the stabilization plant, direct current motor and gyro device as the measuring system. The signal from the controller enters to the motor through the power-width-modulator forming the set of pulses, the width of which is determined by the controller signal [14]. Stabilization of the platform is carried out in the vertical and horizontal plane separately.

The basic features of the synthesized system mathematical description are the necessity to use the integrated model of the stabilization plant and motor due to the presence of the elastic connection between the motor and the platform, at which the apparatus is mounted; linearization of the power-width-modulator and dry friction moments [14].

Taking these assumptions into consideration it is possible to represent the system vector of the state variables in the following form

$$\mathbf{x}^{\mathrm{T}} = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 \end{bmatrix},$$

where x_1 is the voltage at the gyro sensor output; x_2 is the angle of turn of the platform with the observation equipment; x_3 is the angle of turn of the motor shaft; x_4 is the voltage entering to the width-pulse-modulator; x_5 is the derivative of the voltage at the gyro sensor output; x_6 is the angular rate of the platform with the observation equipment; x_7 is the angular rate of the motor shaft.

Then the state space model of the system for stabilization of the platform with the apparatus operated at the ground vehicles may be defined by the following set of equations

$$\dot{x}_1 = x_5;$$

$$\dot{x}_2 = x_6;$$

$$\dot{x}_3 = x_7;$$

$$\dot{x}_4 = -\frac{1}{T_a}x_4 - \frac{c_e}{T_a}x_6 + 4{,}2\frac{U_{con}}{T_a};$$

$$\dot{x}_5 = -\frac{1}{T_0^2}x_1 - \frac{2\xi T_0}{T_0^2}x_5 + \frac{k_g}{T_0^2};$$

$$\dot{x}_6 = -\frac{c_r}{J_p}x_2 + \frac{c_r}{n_r J_p}x_3 - \frac{f_p}{J_p}x_5;$$

$$\dot{x}_7 = \frac{c_r}{n_r J_m}x_2 - \frac{c_r}{n_r^2 J_m}x_3 + \frac{c_m}{R_w J_m}x_4 - \frac{f_m}{J_m}x_6, \tag{16}$$

where k_g is the transfer constant of the gyro sensor; T_a is the time constant of the motor armature circuit; c_e is the electromotive force; U_{con} is control signal, which enters to the width-pulse-modulator and further to the motor; J_p is the moment of inertia of the platform with installed on it payload; c_r is the reducer rigidity; $п_r$ is the reducer gear ratio; f_p is the coefficient of the linearized moment of the friction in the platform gimbals; J_m is the inertia moment of the motor; f_m is the coefficient of the linearized moment of the friction in the motor bearings; c_m is the constant of the load moment; R_w is the resistance of the motor armature circuit winding; T_0 is the time constant of the gyro device; ξ is the damping coefficient.

For the represented set of equations (16) the matrices of the state, control, observation and disturbance (quadruple of the matrices in the state space) may be represented in the following form

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 0 & 0 & k_g & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -\frac{1}{T_a} & 0 & -\frac{c_e}{T_a} \\ 0 & -\frac{c_r}{J_p} & \frac{c_r}{n_r J_p} & 0 & -\frac{f_p}{J_p} & 0 \\ 0 & 0 & -\frac{c_r}{n_r^2 J_m} & \frac{c_m}{R_w J_m} & 0 & -\frac{f_m}{J_m} \end{bmatrix};$$

$$\mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ U_{con} \\ 0 \\ 0 \end{bmatrix};$$

$$\mathbf{C}^{\mathrm{T}} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix};$$

$$\mathbf{D} = 0.$$

The important step of the robust two-degree-of-freedom structural synthesis is the choice of the reference model. The simplest model may be represented by the transfer function $k / Ts+1$. It should be noticed that the choice of the reference model is defined by the requirements to the transient response. This process is not simple and ambiguous and requires the heuristic approaches. For the synthesized system it is expedient to choose the reference model of the following type

$$k / (T^2 s + 2\xi Ts + 1),$$

where

$$k = 1;\ T = 0{,}2;\ \xi = 0{,}7.$$

The representation of the reference model in the state space looks like

$$A_r = [0,1; -1/T, -2\xi/T],$$

$$B_r = [0;1],$$

$$C_r = [1,0],\ D_r = 0.$$

The choice of the transfer functions of the pre- and post-compensators is of great importance for the solved problem too. Using the generally accepted recommendations the post-compensator may be described by the single gain $W_2 = 1$. The transfer function of the pre-compensator W_1

may be defined in the following way. In the first place, it is necessary to take into account the transfer function, which describes the amplifier by the voltage. In the second place, the pre-compensator gain must be equal to the ratio of the gains of the transfer functions by the disturbance and by the plant. A feature of the H_∞-synthesis procedure for the researched system is the necessity to carry out the minimal realization of the plant G. After this step execution the augmented plant may be defined taking into consideration the experimentally determined transfer functions of the pre- and post-compensators

$$G_s = W_2 G W_1,$$

where

$$W_2 = 1,$$

$$W_1 = W_p W_a W_g;$$

$$W_p = \frac{0,15}{0,1s+1};$$

$$W_a = 10\frac{0,4s+25}{s+25};$$

$$W_g = 1.$$

Further the transfer function of the generalized stabilization plant is determined and the standard algorithm of the H_∞-synthesis is applied [3].

5. Results of design of the two-degree-of-freedom stabilization system including mathematical representation of the obtained controller and modeling

As a result of the developed H_∞-synthesis procedure execution the optimal H_∞-controller for the robust two-degree-of-freedom system has been obtained. After maximum possible reduction of the obtained controller (from 10th order to 7th respectively) the structure of the controller may be described by the following quadruple in the state space

$$\mathbf{A}_s = \begin{bmatrix} 1,032 & 0,086 & 0,013 & -0,109 & -0,009 & 0,037 & 0,008 \\ -0,028 & 0,627 & -0,048 & 0,669 & -0,197 & 0,025 & -0,009 \\ 0,044 & 0,044 & 1,007 & -0,052 & -0,021 & 0,035 & 0,007 \\ -0,256 & 0,245 & -0,083 & 0,215 & 0,196 & -0,252 & -0,001 \\ 0,333 & 0,344 & 0,185 & -0,321 & 1,033 & 0,166 & 0,049 \\ -0,097 & -0,203 & -0,052 & 0,247 & -0,046 & 0,828 & -0,013 \\ 0,193 & 0,376 & 0,106 & -0,072 & 0,165 & 0,398 & 0,769 \end{bmatrix};$$

$$\mathbf{B}_s^{T} = \begin{bmatrix} 0,324 & 0,246 & -0,272 & -0,517 & 0,621 & -0,287 & 0,7 \\ -23,28 & -8,63 & -20,75 & 108,3 & -111,7 & 51,9 & -107,4 \end{bmatrix};$$

$$\mathbf{C}_s = \begin{bmatrix} 0,006 & 0,008 & 0,003 & -0,01 & 0,004 & 0,002 & 0,001 \end{bmatrix};$$

$$\mathbf{D}_s = [0,009 \quad -1,659].$$

Results of modeling of the two-degree-of-freedom system with the synthesized H_∞-controller for the nominal and disturbed systems are represented in Fig. 5, 6. Here the constant unbalance moment equal to 100 Nm was considered. The friction moments changes and additional external disturbances were taken into account too.

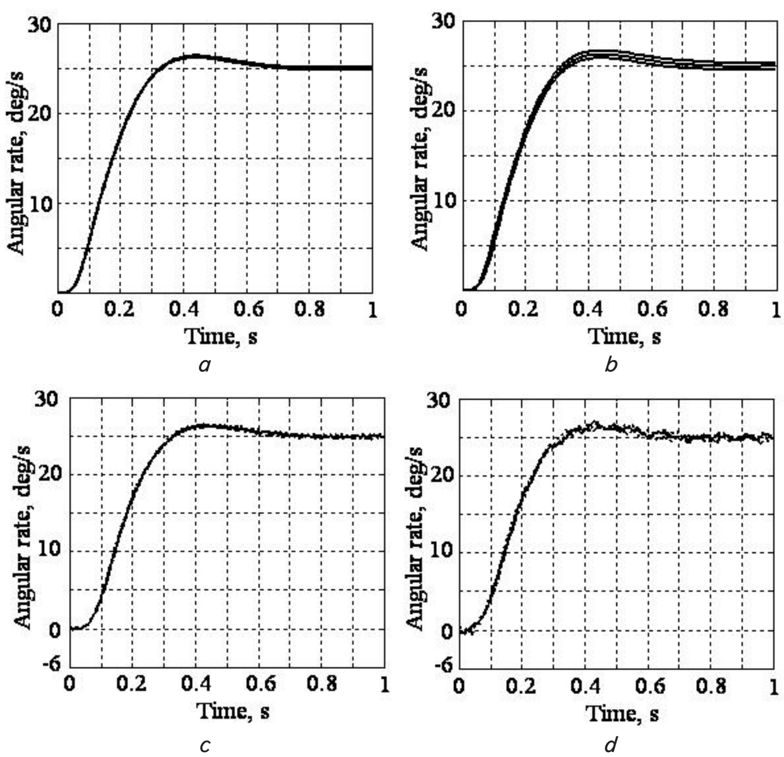
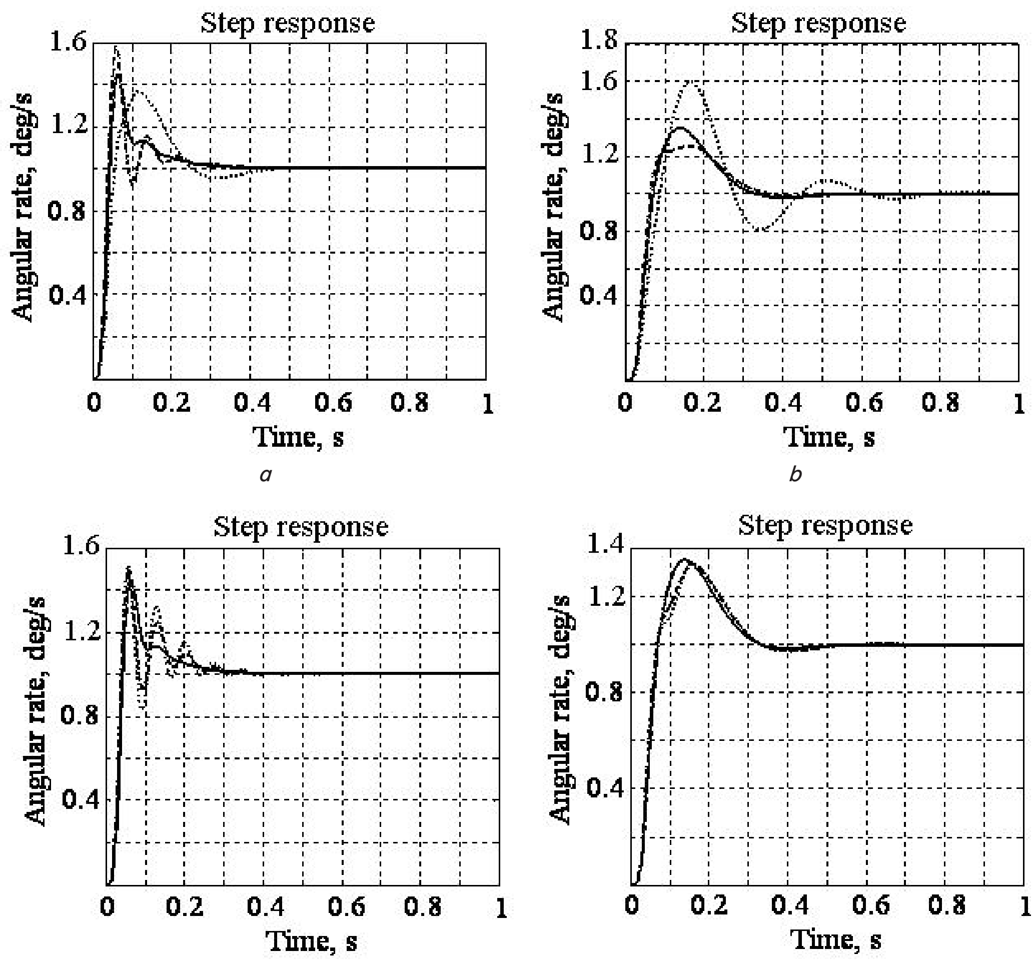


Fig. 5. Step responses by the reference signals of the two-degree-of-freedom robust system (the horizontal channel): *a* — influence of the changed friction moment; *b* — influence of the changed unbalance moment; *c* — influence of the irregularities of the relief of the road with the long undulations; *d* — influence of the irregularities of the relief of the terrain with hummocks



a *b* *c* *d*

Fig. 6. Step responses by the reference signals for the parametrical disturbances: *a* — for the changed inertia moment (the horizontal channel); *b* — for the changed coefficient of rigidity between the actuator and the plant (the horizontal channel); *c* — for the changed inertia moment (the vertical channel); *d* — for the changed coefficient of rigidity between the actuator and the plant (the vertical channel)

The controller for the robust two-degree-of-freedom stabilization and tracking system operated at the ground vehicles was designed and represented by means of quadruple matrices in the state space.

Represented results of the synthesized system modeling prove the possibility of fulfillment of the rigid requirements to accuracy performances to the system for stabilization of the platform with the observation equipment mounted at the ground vehicles and functioned in the difficult conditions of real operation under the action of the coordinate disturbances and parametrical changes in the wide range.

6. Discussing the possibility of creation of the system able to function in difficult conditions of real operation at ground vehicles

The synthesized robust controller requires the application of the reduction procedure. This will make easier its practical implementation.

The synthesized system is characterized by norms $H_2 = 0{,}526$ and $H_\infty = 0{,}258$ respectively. Such values prove the possibility to achieve a compromise between the system accuracy and robustness.

The modeling results of the stabilization system in the tracking mode prove the possibility to use the platform with installed payload by its application in difficult conditions of real operations. The system satisfied the requirements given to accuracy in conditions of the significant coordinate disturbances:

– ± 50 % for the friction moment, Fig. 5, *a*;

– ± 50 % for unbalance moment, Fig. 5, *b*;

– operation in conditions of the road with long undulations;

– operation in conditions of the terrain with hummocks.

Simulation results also prove the execution of requirements to the speed of operation and overshoots, which are given to the transient responses of the researched systems. It should be noted, that these parameters are of great importance for the stabilized platforms with observation equipment operated at the ground vehicles.

The results of the synthesized system modeling represented in Fig. 6 prove the possibility of the system to operate in conditions of the parametrical disturbances. Such parameters as the inertia moment and rigidity of the elastic connections between the actuator and moving platform were changed in the wide range. Fig. 6 shows the results of modeling of the nominal and parametrically disturbed systems (±150 %) by each of the above mentioned parameters.

Comparison of the results obtained for the horizontal and vertical channels respectively shows, that the system of the vertical channel is characterized by the larger rigidity. Such situation corresponds to real system operation.

7. Conclusions

As a result of the carried out researches the following results were obtained.

1. The space state model of the system for stabilization of the platform with the payload (observation equipment) operated at the ground vehicles was obtained.

2. The H_∞-synthesis procedure for the researched system design was developed using the loop shaping method by means of the pre- and post-compensators. The appropriate weighting transfer functions were determined.

3. The augmented plant model in the state space was derived in the form adapted to the standard H_∞-synthesis problem statement. This allows using the developed software in the MatLab system for the robust controller design.

4. The optimization functional of the H_∞-synthesis procedure taking into consideration the sensitivity functions by the coordinate disturbance and measurement noise was obtained.

5. Functioning of the synthesized controller was checked and the modeling results were obtained. Analysis of these results proves the possibility to ensure the high accuracy of the observation equipment on a moving base in difficult conditions of the ground vehicle operation.

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ВИКОРИСТАННЯ МОДИФІКОВАНОГО ПРИНЦИПУ СИМЕТРІЇ СТРУКТУРНИХ СХЕМ ДЛЯ СИНТЕЗУ СИСТЕМ АВТОМАТИЧНОГО КЕРУВАННЯ

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Шляхом математичного моделювання підтверджено, що у реальних системах ідеальне відпрацювання вхідного сигналу за умови повної компенсації динамічних властивостей об'єкта керування є неможливим. Сформульовано правила, за якими можна обернути структурну схему системи у першій канонічній формі керованості. Розроблено модифікований принцип симетрії структурних схем, що забезпечує формування досяжних бажаних динамічних характеристик замкненої системи при обмеженому значенні коефіцієнта підсилення

Ключові слова: зворотні задачі динаміки, модифікований принцип симетрії, симетрія структурних схем

Путем математического моделирования подтверждено, что в реальных системах невозможна идеальная отработка входного сигнала при условии полной компенсации динамических свойств объекта управления. Сформулированы правила, в соответствии с которыми можно выполнить обращение структурной схемы системы в первой канонической форме управляемости. Разработан модифицированный принцип симметрии структурных схем, который обеспечивает формирование достижимых желаемых динамических характеристик замкнутой системы при ограниченном значении коэффициента усиления

Ключевые слова: обратные задачи динамики, модифицированный принцип симметрии, симметрия структурных схем

1. Вступ

Зворотні задачі динаміки від самого початку свого визначення становили один із розділів аналітичної механіки. В результаті розв'язання зворотних задач динаміки визначаються сили, під дією яких система рухається за заданою траєкторією. Якщо вважати рушійні сили керуючими, то в математичному сенсі розв'язання зворотних задач динаміки являє собою синтез алгоритму керування, що забезпечує потрібні динамічні показники системи [1–3].

Основною ідеєю, що лежить в основі перетворення структурних схем, є симетрія, як один із засадничих принципів методології наукового пізнання. Принцип