Запропоновано бажану перехідну функцію системи автоматичного керування представляти у вигляді набору квантованих значень регульованої координати. Продемонстровано можливість застосування квантованих перехідних функцій, представлених у вигляді суми зсунутих у часі функцій Гевісайда, для синтезу регуляторів у розімкнених системах керування. Розроблено математичний апарат для аналітичного визначення операторних зображень бажаних квантованих перехідних функцій кінцевої тривалості

Ключові слова: квантована перехідна функція, функція Гевісайда, перехідна функція кінцевої тривалості

Предложено желаемую переходную функцию системы автоматического управления представлять в виде набора квантованных значений регулируемой координаты. Продемонстрирована возможность применения квантованных переходных функций, представленных в виде суммы смещённых во времени функций Хевисайда, для синтеза регуляторов в разомкнутых системах управления. Разработан математический аппарат для аналитического определения операторных изображений желаемых квантованных переходных функций конечной длительности

Ключевые слова: квантованная переходная функция, функция Хевисайда, переходная функция конечной длительности

UDC 681.51:519.71
DOI: 10.15587/1729-4061.2015.65477

DEVELOPMENT OF A MATHEMATICAL APPARATUS FOR DETERMINING OPERATOR IMAGES OF THE DESIRED QUANTIZED TRANSITION FUNCTIONS OF FINITE DURATION

O. Sheremet
PhD, Associate Professor
Department of Electrical Engineering and Machinery*
E-mail: sheremet-a@mail.ru

O. Sadovoy
Doctor of Technical Sciences, Professor, Vice-Rector for Research*
E-mail: alsadovoy@mail.ru

*Dniprodzerzhynsk State Technical University
Dniprobudivska str., 2, Dniprodzerzhynsk, Ukraine, 51918

1. Introduction

An important part of modern methods for automated control systems synthesis is selection of the desired characteristic polynomial. The desired polynomial is usually selected from a known standard set. There are standard Butterworth polynomials [1], polynomials with binomial coefficients [2], Bessel polynomials [3], Graham and Lathrop polynomials [4], and so on.

In most cases, the roots locations of characteristic equations for closed-loop systems by standard polynomials have semi-empirical nature: they cannot be considered optimal according to some optimization criterion.

For example, when using the binomial characteristic polynomial, all roots of a characteristic equation are selected as identical, negative and real with a value of the module ω_0, which determines the processing rate of the synthesized system. The Butterworth polynomial puts the roots on the half-circle with a radius ω_0 at equal angular distances (Fig. 1). The coefficients of Graham and Lathrop polynomials are defined by mathematical modelling.

In this way, standard polynomials allow us to specify the necessary dynamic properties of the system only roughly. In doing so, almost identical transient processes with various ways of the pole locations can be obtained.

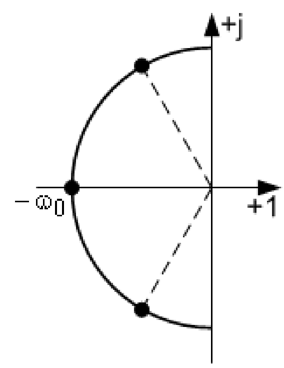

Fig. 1. Roots location by the Butterworth polynomial for a third-order system

The existing problematic issues in using standard characteristic polynomials for the regulators' synthesis require finding innovative approaches to solving the problem of assigning desired dynamics to the automatic control systems. A mathematical apparatus should be suggested and be devoid of the disadvantages of standard characteristic polynomials. This determines the direction and relevance of the research that is presented in the study.

2. Analysis of published data and statement of the problem

So far there have been no general guidelines developed for selecting a trajectory at which the poles should be placed for any automated control system.

The study [5] suggests carrying out the synthesis of characteristic polynomials according to certain quality principles (for example, by an overshoot given with a uniform poles distribution on the circle). However, desirable polynomials are considered to be known in advance.

In general, during the automated control system synthesis by a standard characteristic polynomial, dynamic properties of the system are determined by its coefficients that remain unchanged in operating activities. Standard polynomials cannot reflect dynamic features of real automated control systems because their behaviour is predetermined by poles distribution – that is, polynomial coefficients.

In some cases, the problem of control system synthesis is solved locally, only for specific equipment. Thus, the roots of the characteristic equation are located, taking into account the features and indeterminacies of a separate technical object [6, 7].

An expansion opportunity that is offered by the standard theory of characteristic polynomials is a generalized characteristic polynomial method [8, 9]. This method allows an integrated approach to the location of zeros and poles of the transfer functions for automated electromechanical systems and provides an implementation of transition functions in a standard form.

In particular, synthesis of subordinated regulation systems by using a generalized characteristic polynomial can extend their dynamic properties because regulators in loops can be configured to perform not only a modular or symmetric optimum but other standard forms of transition functions that are defined by standard characteristic polynomials.

However, the generalized characteristic polynomial method is still used as a reference to standard characteristic polynomials and, in this sense, does not bring any innovation. Consequently, no matter how progressive a synthesis method is when it is based on standard polynomials, the possibility of the automated control system will be limited by a selected standard polynomial.

It is not necessary that the trajectories of roots movement should be continuous functions. They can be piecewise monotonic functions or discrete argument functions (lattice functions). For example, the study [10] demonstrates a system with a toggle property between two possible positions of poles, one of which is required in the acceleration mode, and the other – in operation. Discontinuous control in systems with a variable structure is connected with a similar approach. These systems have multiple regulators implementations that can change during the control process [11, 12]. The structure, moments and durations of regulators switching are selected so as to provide the desired control quality [13]. The resulting variation in output coordinates will be some time-expanded superposition of individual components influencing the regulator with a variable structure.

A convenient and clear alternative for imparting the desirable properties to systems in static and dynamic modes (without using standard characteristic polynomials) is not the desired trajectory of roots movement definition but the desired transition function determination. This function is not selected by the designer from some list of standard forms during regulators synthesis but is given solely on the basis of technological requirements and possibilities of technical implementation for a defined type of equipment. Such a transition function can be changed in the operation time of the machine, mechanism or process complex, providing quantitatively and qualitatively new properties for an automated electromechanical system [14].

To improve opportunities for software implementation of the regulator, which allows to control the electromechanical system in that way, it is advisable for the desired transition function to be represented in a numerical form, i. e. as a set of operated coordinate values that change with a period T, which is relatively small with regard to the duration of the transition process (Fig. 2).

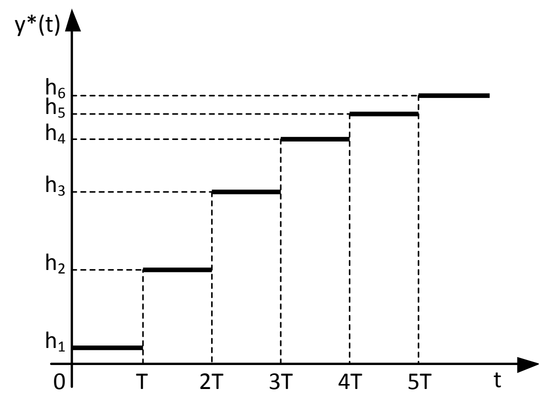

Fig. 2. An example of the desired transition function presentation

Thus, avoiding the use of standard characteristic polynomials in the synthesis of automated control systems requires their replacement by another mathematical apparatus, which would be the basis for creating regulators with desired quality ratings in static and dynamic modes. The desired transition function in a numerical (discrete) form can be considered as such basis. This approach requires development of an appropriate method for performing synthesis of automated electromechanical systems.

3. The purpose and objectives of the study

The purpose of the study is to prove the possibility of analytical synthesis of regulators for a quantized form of desired transition functions and develop a mathematical apparatus for composing quantized transition functions of finite duration as entities that can replace standard characteristic polynomials.

The objectives of the study are as follows:

– to analyse the possibility of using quantized transition functions of finite duration for discrete regulators synthesis;

– to suggest an analytical method for regulators synthesis in open-loop automated control systems by representing the desired quantized transition functions as the amount of time for shifted Heaviside functions;

– to develop a mathematical apparatus for analytical determination of operator images for desired quantized transition functions of finite duration, relying only on the values of the signal levels at quantization points and a quantization period value.

4. The features of discrete control systems

Control systems with signals discretization in some form are called discrete. The realization of the idea of splitting the desired transition function into parts should be solved directly by its application not to analogue systems with continuous signals but to discrete systems that have at one or more points a sequence of impulses or a digital code.

It should be noted that in addition to the term "discrete system" the other terms used are "digital system" and "impulse system". Digital systems are mostly known as systems in which the coded control signals are generated by computer equipment. Impulse systems provide using impulse-amplitude modulation and signal quantization by time. The term "discrete systems" is the most integrant; it describes systems that can have impulse signals as well as digital codes.

In research literature, the term "discrete signal" means a signal sampled in time (Fig. 3, *a*), whereas the term "digital signal" refers to a signal that is sampled both in time and level (Fig. 3, *b*).

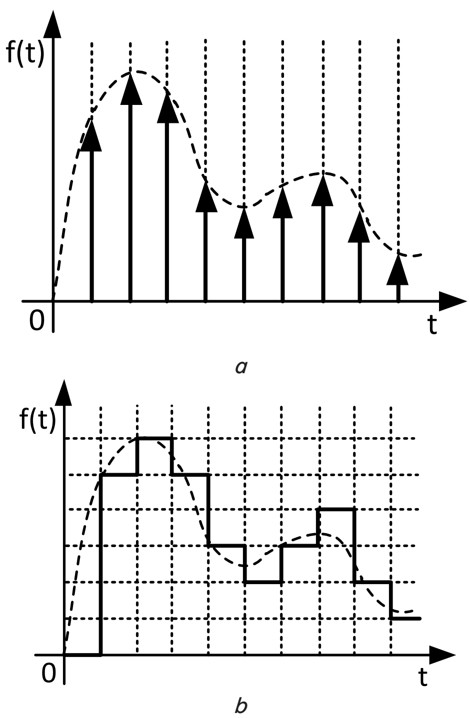

Fig. 3. Comparison of discrete and digital signals: *a* — discrete signal; *b* — digital signal

Let us consider the concept of "discrete" and "digital" signal capabilities within the context of modern microprocessor technology and software. It can be noted that the quantization signal level almost loses its meaning because of significant bit width for the data types that are implemented programmatically. For example, the data type double, which is used in the programming language C++, allows recording the minimal positive value of $1.7{\cdot}10^{-308}$ and the maximum positive value of $1.7{\cdot}10^{-308}$ with 15 significant digits after the decimal point [15]. Such capabilities are enough to solve most technical problems without taking into account quantization by its level.

Specialized computer engineering and general-purpose computer equipment become more used to control technical objects. Such equipment is used at low levels (for local projects) as well as at higher levels (to control the technological lines or workshops) [16]. Microprocessor engineering is often used in control systems as discrete regulators or devices that perform direct digital control of power converters.

One of the most significant advantages of digital regulators is their much greater flexibility in comparison with analogue regulators. The program of a digital regulator that is implemented by using a microcontroller can be changed according to new technological requirements or parameters of a control object at any time, without changing the hardware of the control system.

It can be concluded that discrete regulators provide significantly more opportunities regarding synthesis of control systems for technical objects. Thus, standard polynomials can be abandoned, and it requires searching for new approaches in sampling the desired transition functions and performing synthesis problem solving in a digital form.

5. Analytical regulators synthesis by a quantized form of the desired transition function

Output transition functions of technical systems, at changing the input actions or external disturbances, should satisfy certain requirements that are presented as factors of quality. These factors are determined by the shape of the transition function. In most methods of synthesis, regulators are selected so as to provide the desired level of two or three quality factors. The whole transition function is not covered.

The regulator transfer function can be obtained for the desired transition function by doing the reverse dynamic conversion; however, if the system is nonlinear, or if the transition function has no expression in elementary functions, direct inverse mathematical transformation is impossible. In this case, one can perform quantization, i. e. separation of the desired transition function on a set of Heaviside functions [17] with a rising time shift.

The study [14] is devoted to proving a possibility of analytical regulators synthesis by the desired transition function represented as a set of time-shifted Heaviside functions.

Let us consider the output coordinate $y(t)$ of the automatic control system as a set of discrete values each of which exists during some time T_0. The quantized transition function $y^*(t)$ can be represented as a step function, which is the sum of Heaviside functions $\sigma(t)$ that are delayed relative to the zero point by the whole number of periods T_0.

$$\begin{aligned} y^*(t) &= h_1\big(\sigma(t-T_0)-\sigma(t-2T_0)\big)+ \\ &+h_2\big(\sigma(t-2T_0)-\sigma(t-3T_0)\big)+\ldots\rightarrow \\ &\rightarrow \ldots + h_{n-1}\big(\sigma(t-(n-1)T_0)-\sigma(t-nT_0)\big)+h_n\sigma(t-nT_0)= \\ &=\sum_{i=1}^{n-1} h_i\big(\sigma(t-iT_0)-\sigma(t-(i+1)T_0)\big)+h_n\sigma(t-nT_0). \qquad (1) \end{aligned}$$

The inverse Laplace transform of the expression (1) gives the image $Y^*(p)$ for the signal $y^*(t)$:

$$Y^*(p)=L^{-1}\left\{\sum_{i=1}^{n-1} h_i\big(\sigma(t-iT_0)-\sigma(t-(i+1)T_0)\big)+h_n\sigma(t-nT_0)\right\}=$$

$$=\frac{1}{p}\left(\sum_{i=1}^{n-1} h_i\left(e^{-iT_0p}-e^{-(i+1)T_0p}\right)+h_n e^{-nT_0p}\right).$$

We represent an open-loop system as a constant part $W_{cp}(p)$ and a regulator $W_r(p)$ that should provide changing of the output coordinates by the desired quantized transition function $y^*(t)$, $t \geq 0$ with the input action $u(t)$ (Fig. 4).

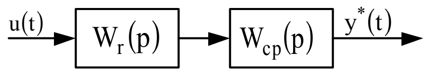

Fig. 4. The open-loop system consisting of a constant part and a regulator

The transfer function of this system:

$$W(p) = W_r(p) \cdot W_{cp}(p). \quad (2)$$

It is possible to attain the necessary dynamics in the open-loop system (Fig. 4). For this purpose, the transfer function $W_{cp}(p)$ and the image $Y^*(p)$ should have the following connection:

$$Y^*(p) = W_r(p) \cdot W_{cp}(p) \cdot U(p). \quad (3)$$

Thus, the transfer function $W_{cp}(p)$ is considered as constant in the dynamic mode, and the image $U(p)$ corresponds to one of the typical input actions, which are used in studying automatic control systems (including the step function and the ramp function).

Based on the formula (3), the regulator can build as follows:

$$W_r(p) = \frac{Y^*(p)}{W_{cp}(p) \cdot U(p)}. \quad (4)$$

With the unit step action and the input of the automatic control system that have an image $U(p) = \frac{1}{p}$, the transfer function for the regulator is converted to the form

$$W_r(p) = \frac{\sum_{i=1}^{n-1} h_i \left(e^{-iT_0 p} - e^{-(i+1)T_0 p} \right) + h_n e^{-nT_0 p}}{W_{cp}(p)}. \quad (5)$$

Exponential functions from the formula (5) can be represented as expansion in a Maclaurin series:

$$\left.\begin{aligned}
e^{-iT_0 p} &= \frac{1}{1 + iT_0 p + \frac{i^2 T_0^2 p^2}{2!} + \frac{i^3 T_0^3 p^3}{3!} + \ldots + \frac{i^k T_0^k p^k}{k!} + \ldots}, \\
e^{-(i+1)T_0 p} &= \frac{1}{1 + (i+1)T_0 p + \frac{(i+1)^2 T_0^2 p^2}{2!} + \frac{(i+1)^3 T_0^3 p^3}{3!} + \ldots + \frac{(i+1)^k T_0^k p^k}{k!} + \ldots}, \\
e^{-nT_0 p} &= \frac{1}{1 + nT_0 p + \frac{n^2 T_0^2 p^2}{2!} + \frac{n^3 T_0^3 p^3}{3!} + \ldots + \frac{n^k T_0^k p^k}{k!} + \ldots},
\end{aligned}\right\} \quad (6)$$

where k is an integer number that reflects the order of the Maclaurin series component.

The sampling period T_0 is a very small value, so to simplify the calculations by the formula (6), the components, including the second and higher powers of T_0, i. e.

$$e^{-iT_0 p} = \frac{1}{1 + iT_0 p}, \quad e^{-(i+1)T_0 p} = \frac{1}{1 + (i+1)T_0 p}, \quad e^{-nT_0 p} = \frac{1}{1 + nT_0 p},$$

should be disregarded. With these expressions, the formula (5) takes the following form:

$$\begin{aligned}
W_r(p) &= \frac{1}{W_{cp}(p)} \left(\sum_{i=1}^{n-1} h_i \left(\frac{1}{1 + iT_0 p} - \frac{1}{1 + (i+1)T_0 p} \right) + h_n \frac{1}{1 + nT_0 p} \right) = \\
&= \frac{1}{W_{cp}(p)} \left(\sum_{i=1}^{n-1} h_i \left(\frac{1 + (i+1)T_0 p - 1 - iT_0 p}{(1 + iT_0 p)(1 + (i+1)T_0 p)} \right) + h_n \frac{1}{1 + nT_0 p} \right) = \\
&= \frac{1}{W_{cp}(p)} \left(\sum_{i=1}^{n-1} h_i \left(\frac{T_0 p}{(1 + iT_0 p)(1 + (i+1)T_0 p)} \right) + h_n \frac{1}{1 + nT_0 p} \right) = \\
&= \frac{T_0 p}{W_{cp}(p)} \left(\sum_{i=1}^{n-1} h_i \left(\frac{1}{(1 + iT_0 p)(1 + (i+1)T_0 p)} \right) + h_n \frac{1}{(1 + nT_0 p) T_0 p} \right). \quad (7)
\end{aligned}$$

In this way, the regulator synthesis is performed by the formula (7). The levels h_i and h_n are defined by the desired transition function, but $W_{cp}(p)$ is defined by the constant part of the system. The parameter T_0 is selected from the technical characteristics of the control object, and it affects the locations of regulator poles in the complex plane.

Thus, for the considered open-loop system, the desired quantized transition function can be used in the regulators synthesis instead of the standard characteristic polynomial. A similar synthesis for a closed-loop system can be performed by setting only levels h_i and quantization time T_0.

However, the scope of the proposed method is limited by Heaviside transformation capabilities, i.e. transfer functions should be presented in a balanced ratio of two polynomials that have simple (aliquant) roots [18].

The theory of transient processes of finite duration can provide one of the possible approaches to forming quantized transition functions that has no faults of the Heaviside transformation.

6. Quantized transition functions with finite duration

In analogue systems, transition functions are theoretically completed when time $t \to \infty$, so in practice some trust zones are added. The transition functions are considered as finished when entering into such zones.

Transition functions of finite duration can be achieved in closed-loop discrete systems, unlike in analogue systems. Such processes are completed during the final number of quantization periods.

If the transfer function of a discrete system is in Z-form – $W(z)$ and the input signal is a unit step action, the image of the output signal is represented by the following formula:

$$Y(z) = \frac{z}{z-1} W(z), \quad (8)$$

where z is the operator of Z-transform associated with the Laplace operator p by the equation (9):

$$z = e^{T_0 p}, \qquad (9)$$

where T_0 is the quantization period in a discrete system.

The transitional function for a closed-loop system in a general case will be a ratio of polynomials:

$$W(z) = \frac{a_k z^k + a_{k-1} z^{k-1} + a_{k-2} z^{k-2} + \ldots + a_1 z + a_0}{z^k + b_{k-1} z^{k-1} + b_{k-2} z^{k-2} + \ldots + b_1 z + b_0}, \qquad (10)$$

where a_k, a_{k-1},..., a_1, a_0 are numerator coefficients of the transfer function; b_{k-1}, b_{k-2},..., b_1, b_0 are denominator coefficients of the transfer function; k is the order of the characteristic polynomial.

It should be noted that a system can be realized physically when the order of the transfer function numerator does not exceed the order of the denominator. For most orders, the difference between the denominator and the numerator is 1 or 2. Therefore, some of the coefficients at the high powers of z in the formula (10) may be equal to zero. For example, when the orders' difference between the denominator and the numerator is 2, then $a_k = 0$ and $a_{k-1} = 0$.

The coefficient at z in the highest power in the transfer function (10) denominator should be reduced to one so that the component z^k is part of the polynomial. The image of output signal (8), taking into account the transfer function (10), is represented as follows:

$$Y(z) = \frac{a_k z^{k+1} + a_{k-1} z^k + a_{k-2} z^{k-1} + \ldots + a_1 z^2 + a_0 z}{z^{k+1} + (b_{k-1} - 1) z^k + (b_{k-2} - b_{k-1}) z^{k-1} + \ldots + (b_1 - b_2) z^2 + (b_0 - b_1) z - b_0}. \qquad (11)$$

An infinite power series can be obtained by dividing the numerator of (11) by its denominator:

$$Y(z) = c_0 + c_1 z^{-1} + c_2 z^{-2} + \ldots + c_i z^{-i} + \ldots = \sum_{i=0}^{\infty} c_i z^{-i}, \qquad (12)$$

where c_0, c_1, c_2, ..., c_i are constant coefficients, whereas i is the number of the cycle.

With the determination of Z-transform, it follows that the coefficients of the series (12) are the values of the transitional lattice function $y(iT_0)$ in the moments of quantization, i. e. $c_0 = y(0)$, $c_1 = y(T_0)$, $c_2 = y(2T_0)$, ..., $c_i = y(iT_0)$.

In order to provide a finite duration of the transition function, a series (12) at $i = k$ should reach the value that remains unchanged for the next cycles. This condition can be accomplished when all of the characteristic polynomial coefficients, i. e. the denominator of the transfer function (10), reach point zero.

$$b_{k-1} = b_{k-2} = \ldots = b_1 = b_0 = 0. \qquad (13)$$

When fulfilling the condition (13), the transfer function (8) takes the following form:

$$W(z) = \frac{a_k z^k + a_{k-1} z^{k-1} + a_{k-2} z^{k-2} + \ldots + a_1 z + a_0}{z^k}. \qquad (14)$$

The characteristic polynomial $z^k = 0$ of the transfer function (14) has k roots $z_1 = z_2 = \ldots = z_k = 0$, which are located in the middle of the stability circle with a unit radius (Fig. 5).

The image of the output signal in the system with a transfer function (14) in response to the unit step input action is represented as follows:

$$Y(z) = \frac{a_k z^{k+1} + a_{k-1} z^k + a_{k-2} z^{k-1} + \ldots + a_1 z^2 + a_0 z}{z^{k+1} - z^k}. \qquad (15)$$

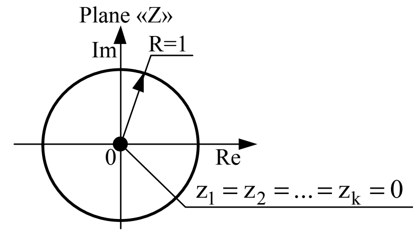

Fig. 5. The stability circle with roots $z_1 = z_2 = \ldots = z_k = 0$

The power series is formed by dividing the numerator of (15) by its denominator. In this series, the coefficients at z^{-i} change only to the cycle number $i = k$; after that, they remain unchanged for the next cycles:

$$Y(z) = c_0 + c_1 z^{-1} + c_2 z^{-2} + \ldots + \\ + c_i z^{-i} + \ldots + = \sum_{i=0}^{k-1} c_i z^{-i} + c_k \sum_{i=k}^{\infty} z^{-i}. \qquad (16)$$

The series (16) gives the values of the lattice transition function at quantization points. Let us define the connection between coefficients $a_k,\ a_{k-1},\ \ldots,\ a_1,\ a_0$ and $c_k,\ c_{k-1},\ \ldots,\ c_1,\ c_0$ by dividing polynomials according to the formula (15) – Fig. 6.

It is possible to set the following formal relations based on Fig. 6:

$$\left.\begin{array}{l} c_0 = a_k, \\ c_1 = a_{k-1} + a_k, \\ c_2 = a_{k-2} + a_{k-1} + a_k, \\ c_3 = a_{k-3} + a_{k-2} + a_{k-1} + a_k, \\ \ldots \\ c_{k-2} = \sum_{i=2}^{k} a_i, \\ c_{k-1} = \sum_{i=1}^{k} a_i, \\ c_k = \sum_{i=0}^{k} a_i, \\ c_{k+1} = c_{k+2} = c_{k+3} = \ldots = \sum_{i=0}^{k} a_i. \end{array}\right\} \qquad (17)$$

The equation (17) can be written in a generalized form to establish a mathematical interpretation of the transition function of finite duration:

$$\left.\begin{array}{l} c_i = \sum_{j=k-i}^{k} a_j \text{ when } i \le k, \\ c_i = \sum_{i=0}^{k} a_i \text{ when } i > k. \end{array}\right\} \qquad (18)$$

The advantage of the formulas (18) consists in a possibility of an immediate determination of the values of transition functions of finite duration by transfer functions in the form (14). Each of the values c_i is one level of the transition function of finite duration (Fig. 7).

The value c_0 is not equal to zero only when the order of the desired transfer function (14) numerator coincides with the order of the denominator, that is, when $a_k \neq 0$. If the order of the numerator is smaller than the denominator order by value 2, the coefficients c_1 and c_0 are equal to zero ($c_1 = c_0 = 0$).

The duration of the transition functions shown in Fig. 7 is defined as kT_0, where k is the order of the desired transfer function denominator (14). The levels of discretized transition functions are changing at sampling moments and run up to the time quantization value of $t = kT_0$. All levels of extra $t > kT_0$ amount to the value of c_k.

Let us consider an example. Suppose the desired transfer function of the system $W(z)$ is configured for transient processes of finite duration:

$$W(z) = \frac{0{,}05z^4 + 0{,}1z^3 + 0{,}3z^2 + 0{,}5z + 1}{z^4}.$$

In the case under consideration, the greatest degree of the denominator is $k = 4$ because the transition function is completed in 4 cycles, i. e. the time length is equal to $4T_0$. Numerator coefficients have the following values:

$$a_4 = 0{,}05,\ a_3 = 0{,}1,\ a_2 = 0{,}3,\ a_1 = 0{,}5,\ a_0 = 1.$$

Let us calculate the transition function of the system in response to the input unit step action by the formulas (18). The results are listed in Table 1.

Table 1

The values of the transition function of finite duration

Cycle number	0	1	2	3	4	...	i
Value	$c_0 = 0{,}05$	$c_1 = 0{,}15$	$c_2 = 0{,}45$	$c_3 = 0{,}95$	$c_4 = 1{,}95$	...	$c_i = 1{,}95$

The transition function of finite duration (Fig. 8) is based on data from Table 1.

$$\begin{array}{l|l}
\underline{\begin{array}{l} a_k z^{k+1} + a_{k-1} z^k + a_{k-2} z^{k-1} + \ldots + a_1 z^2 + a_0 z \\ a_k z^{k+1} - a_k z^k \end{array}} & z^{k+1} - z^k \\
\underline{\begin{array}{l} (a_{k-1} + a_k) z^k + a_{k-2} z^{k-1} + \ldots + a_1 z^2 + a_0 z \\ (a_{k-1} + a_k) z^k - (a_{k-1} + a_k) z^{k-1} \end{array}} & a_k + (a_{k-1} + a_k) z^{-1} + (a_{k-2} + a_{k-1} + a_k) z^{-2} + \\
\underline{\begin{array}{l} (a_{k-2} + a_{k-1} + a_k) z^{k-1} + a_{k-3} z^{k-2} \ldots + a_1 z^2 + a_0 z \\ (a_{k-2} + a_{k-1} + a_k) z^{k-1} - (a_{k-2} + a_{k-1} + a_k) z^{k-2} \end{array}} & + (a_{k-3} + a_{k-2} + a_{k-1} + a_k) z^{-3} + \ldots + \\
\underline{\begin{array}{l} (a_{k-3} + a_{k-2} + a_{k-1} + a_k) z^{k-2} + \ldots + a_1 z^2 + a_0 z \\ (a_{k-3} + a_{k-2} + a_{k-1} + a_k) z^{k-2} - (a_{k-3} + a_{k-2} + a_{k-1} + a_k) z^{k-3} \end{array}} & + \sum_{i=1}^{k} a_i z^{-k+1} + \sum_{i=0}^{k} a_i z^{-k} + \sum_{i=0}^{k} a_i z^{-k-1} + \ldots \\
\quad \vdots & \\
\underline{\begin{array}{l} \sum_{i=1}^{k} a_i z^2 + a_0 z \\ \sum_{i=1}^{k} a_i z^2 - \sum_{i=1}^{k} a_i z \end{array}} & \\
\underline{\begin{array}{l} \sum_{i=0}^{k} a_i z \\ \sum_{i=0}^{k} a_i z - \sum_{i=0}^{k} a_i \end{array}} & \\
\underline{\begin{array}{l} \sum_{i=0}^{k} a_i \\ \sum_{i=0}^{k} a_i - \sum_{i=0}^{k} a_i z^{-1} \end{array}} & \\
\sum_{i=0}^{k} a_i z^{-1} \quad \vdots &
\end{array}$$

Fig. 6. The division of polynomials according to the formula (15)

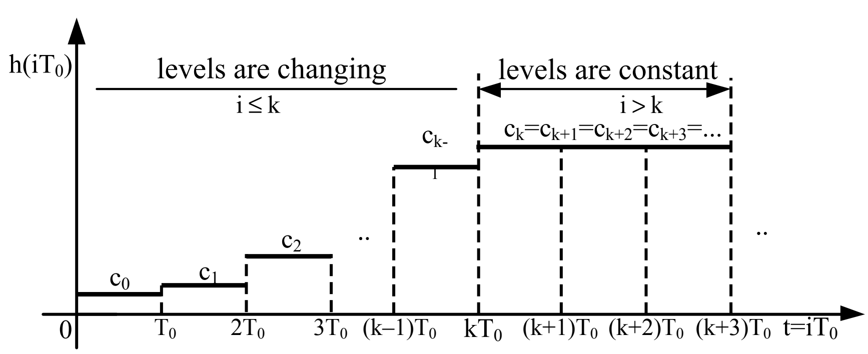

Fig. 7. The transition function of finite duration (built on values c_i)

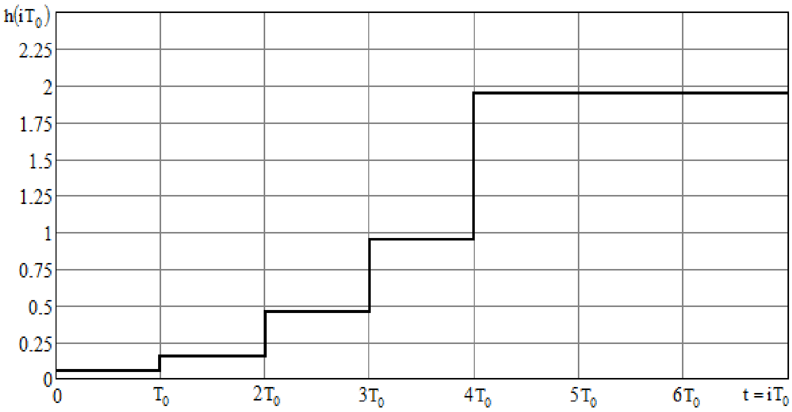

Fig. 8. The transition function of finite duration (data from Table 1)

Thus, the theory of transition functions of finite duration gives a quite simple and obvious mathematical apparatus to set the desired quantized transition functions that are based only on the step sizes at quantization moments and a quantization period.

7. Discussion of the research results of quantized transition functions of finite duration

Standard polynomials allow only rough setting of the required dynamic properties of automatic control systems. It is necessary to perform a mathematical modelling for determining the coefficients of some polynomials. The advantages of the desired dynamics assignment for automatic control systems by using quantized transition functions are the following: visibility of the analytical synthesis process and ease of subsequent technical implementation of discrete regulators programmatically. The use of the desired transition functions of finite duration provides the following benefits:

– the time range of levels variation for desired transition functions is known in advance, so there is no need for any trust zones or calculations of the estimated time of the transition function duration with accuracy;

– the quantization moments divide the time range into equal segments each of which can be set the required value of the desired transition function;

– the relationship between the coefficients of the polynomial transfer function of a system that is configured for transient processes of finite duration and the level values that correspond to the quantized transition function are determined simply and clearly by equations (18);

– all direct and inverse transformations that are associated with the passage from the transfer function of the system to the desired transition function are rather easy to represent in the program code; they will be used in further research to perform synthesis of a discrete time equalizer [19].

8. Conclusion

1. Analysis has enabled us to find a possible use of desired quantized transition functions for regulators synthesis: realization of the idea of splitting the desired transition function into parts should be solved naturally in its application not to analogue systems with continuous signals but to discrete systems that have, at one or more points, a sequence of impulses or a digital code.

2. An analytical method was suggested for regulators synthesis in open-loop automated control systems. This method allows representing the desired quantized transition functions as the number of time-shifted Heaviside functions, but its use is limited by features of the Heaviside transformation.

3. The theory of transient processes of finite duration can provide one of the possible approaches to forming quantized transition functions that has no faults of the Heaviside transformation. The developed mathematical apparatus can perform an analytical determination of operator images for desired quantized transition functions of finite duration, relying only on the values of signal levels at quantization points and a quantization period value.

References

1. Bianchi, G. Electronic filter simulation and design [Text] / G. Bianchi, R. Sorrentino. – McGraw-Hill Professional, 2007. – 606 p.
2. Kuzovkov, N. T. Modalnoe upravlenie i nablyudayuschie ustroystva [Text] / N. T. Kuzovkov. – Moscow: Mashinostroenie, 1976. – 184 p.
3. Burchnall, J. L. The Bessel polynomials [Text] / J. L. Burchnall // Canadian Journal of Mathematics. – 1951. – Vol. 3. – P. 62–68. doi: 10.4153/cjm-1951-009-3
4. Graham, D. The synthesis of optimum transient response: criteria and standard forms [Text] / D. Graham, R. C. Lathrop // AIEE Transactions. – 1953. – Vol. 72, Issue II. – P. 1365–1391.
5. Tolochko, O. I. Analiz ta sintez elektromehanichnih sistem zi sposterigachami stanu: navchalniy posibnik dlya studentiv vischih navchalnih zakladiv [Text] / O. I. Tolochko. – Donetsk: Nord-Pres, 2004. – 298 p.
6. Horla, D. Pole-placement controller with full adaptation for plant with unknown structure in application to servo control [Text] / D. Horla // 2011 International Conference on Communications, Computing and Control Applications (CCCA), 2011. – P. 1–4. doi: 10.1109/ccca.2011.6031468
7. Wu, F. Dynamic dimensional synthesis of a precision 6-DOF parallel manipulator [Text] / F. Wu, L. Wang, W. Rong, L. Sun // 2012 IEEE International Conference on Mechatronics and Automation, 2012. – P. 831–836. doi: 10.1109/icma.2012.6283250
8. Maruschak, Ya. Yu. Sintez astatichnoyi pozitsiynoyi SPR metodom uzagalnenogo harakteristichnogo polinoma [Text] / Ya. Yu. Maruschak, A. P. Kushnir // Resp. mizhvidomchiy nauk.-tehn. zb. Elektromashinobuduvannya ta elektroobladnannya. – 2000. – Issue 55. – P. 3–10.

9. Maruschak, Ya. Yu. Metod uzagalnenogo harakteristichnogo polinoma dlya sintezu sistem avtomatichnogo regulyuvannya [Text] / Ya. Yu. Maruschak // Pratsi Mizhnar. konf. z avtomatichnogo keruvannya. Avtomatika 2000. – 2000. – Vol. 4. – P. 32–37.
10. Sadovoy, O. V. Sintez sistem avtomatichnogo keruvannya zi zminnim harakteristichnim polinomom [Text] / O. V. Sadovoy, O. I. Sheremet // Visnik Kremenchutskogo derzhavnogo politehnichnogo universitetu imeni Mihayla Ostrogradskogo. – 2009. – Vol. 4/2009 (57), Issue 1. – P. 32–35.
11. Madani-Esfahani, S. M. Variable Structure Control of Dynamical Systems with Bounded Controllers [Text] / S. M. Madani-Esfahani, S. H. Zak // American Control Conference, 1987. – P. 90–95.
12. Ikbal, M. M. Variable structure control of a magnetic levitation system [Text] / M. M. Ikbal, M. Abdelfatah // Proceedings of the 2001 American Control Conference, 2001. – P. 3725–3730. doi: 10.1109/acc.2001.946215
13. Bartolini, G. Chattering avoidance by second-order sliding mode control [Text] / G. Bartolini, A. Ferrara, E. Usai // IEEE Transactions on Automatic Control. – 1998. – Vol. 43, Issue 2. – P. 241–246. doi: 10.1109/9.661074
14. Sadovoy, O. V. Analitichniy sintez regulyatoriv za kvantovanoyu formoyu bazhanoyi perehidnoyi funktsiyi [Text] / O. V. Sadovoy, O. I. Sheremet // Zbirnik naukovih prats Dniprodzerzhinskogo derzhavnogo tehnichnogo universitetu: (tehnichni nauki). – 2010. – Vol. 1 (14). – P. 258–264.
15. Stroustrup, B. The C++ programming language. Fourth edition [Text] / B. Stroustrup. – Addison-Wesley, 2013. – 1347 p.
16. Computer Engineering Curricula 2016 [Electronic resource]. – Available at: https://www.computer.org/cms/Computer.org/professional-education/curricula/ComputerEngineeringCurricula2016.pdf (Last accessed: 19.03.2016). – Title from the screen.
17. Isermann, R. Identification of dynamic systems [Text] / R. Isermann, M. M nchhof. – Springer-Verlag Berlin Heidelberg, 2011. – 705 p. doi: 10.1007/978-3-540-78879-9
18. Sheremet, O. I. Vikoristannya rozkladannya Hevisayda dlya sintezu regulyatoriv sistem avtomatichnogo keruvannya [Text] / O. I. Sheremet // Visnik Donbaskoyi derzhavnoyi mashinobudivnoyi akademiyi: Zbirnik naukovih prats. – 2009. – Vol. 1 (4E). – P. 189–193.
19. Sheremet, O. I. Ponyattya diskretnogo chasovogo ekvalayzera [Text] / O. I. Sheremet, O. V. Sadovoy, Yu. V. SohIna // Zbirnik naukovih prats Donbaskogo derzhavnogo tehnichnogo universitetu. – 2014. – Vol. 1. – P. 147–151.