

# GRAPH THEORY METHODS IN ANALYSING COMMUTING NETWORKS OF MUNICIPAL ELECTRIC TRANSPORT

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*Визначено показники розвитку маршрутних мереж міського тролейбуса і трамвая у великих містах України на підставі положень теорій графів і складних мереж. Установлено наявність середнього лінійного кореляційного зв'язку між цими показниками та транспортною рухомістю населення у досліджуваних містах. Виконано змістовний аналіз та подано предметну інтерпретацію отриманих результатів*

*Ключові слова: складна мережа, міський громадський транспорт, маршрутна система, транспортна рухомість населення*

*Определены показатели развития маршрутных сетей городского троллейбуса и трамвая в крупных городах Украины на основе положений теорий графов и сложных сетей. Выявлено наличие средней линейной корреляционной связи между этими показателями и транспортной подвижностью населения в исследуемых городах. Выполнен содержательный анализ и дана предметная интерпретация полученных результатов*

*Ключевые слова: сложная сеть, городской общественный транспорт, маршрутная система, транспортная подвижность населения*

## 1. Introduction

Public passenger transport (PPT) is an integral part of a modern city infrastructure and a means of its residents' commuting. It provides access to performing economic functions and implementing social needs to all segments of the population, including those who can not afford a private car (people with low incomes), lack physical ability to operate a car, or have no driver's licence (underage children, elderly people with health problems, and disabled persons). Thus, municipal public transit is one of the mechanisms to secure citizens' constitutional rights to work, education, medical care, and recreation. Besides, the PPT industry itself is a source of jobs. Research findings show that investments in the PPT infrastructure create by 19 % more jobs than similar investments in the construction of roads and road facilities [1].

The current rapid increase in private car ownership is a worldwide trend, and Ukraine is not an exception. By early 2014, the existing fleet of private cars in Ukraine had reached 6,514,500 units, i. e. almost 1.5 million cars more than in 2000 [2]. During the same period of time, the corresponding rate of car ownership, i. e. the number of registered private cars per 1,000 residents of the country had increased from 104 to 143 units per 1,000 persons (Fig. 1) [3].

The planning and development of cities in the former USSR depended on the standard rate of motorization, which amounted to 60–70 cars per 1,000 residents with a maximum promising value of 180 cars per 1,000 residents [4]. The growing rate of private car ownership in Ukraine (given the fact that concentration of cars in cities is higher than the national average) shows that the maximum level of car ownership will be (if not yet) exceeded in the near future.

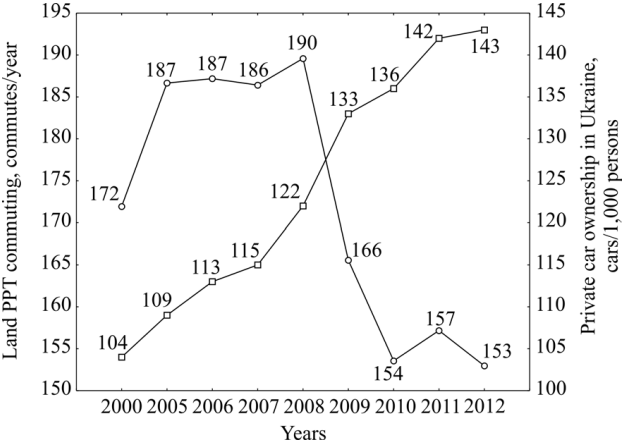


Fig. 1. The dynamics of the rates of private car ownership and PPT commuting in Ukraine during 2000–2012

The increase in the rate of private car ownership leads to a larger fraction of city residents' intercity commutes in personal vehicles and, consequently, reduces the volumes of PPT transits and passenger traffic, which is defined as a number of PPT commutes per capita per year (Fig. 1). According to experts on urban transport [4], the share of commutes in personal vehicles in intercity traffic comprises 54 % when the rate of motorization is 150 cars per 1,000 persons, and it increases to almost 75 % when the rate of motorization is 300 cars per 1,000 people.

Therefore, attracting car owners to use PPT is a possible way to overcome the negative effects of motorization. This involvement on a voluntary basis is possible only if the ser-

vice quality rate and the PPT commute costs are at least not worse in comparison with commuting in a private car.

The quality of passenger transit services is determined by several factors, among which the most important one is the commuting system development rate. The latter is a geographically and temporally determined complex of all routes and particular PPT types that function within a given transport network [5]. The territorial linkage provides a consistent placement of individual routes, whose sum forms a route network of one or various PPT types, on the city maps. Respectively, parameters and characteristics of a commuting network directly affect the degree of the PPT attractiveness for city residents. Therefore, the urgent task is to assess the nature and extent of the impact of the commuting network development rate on the PPT routes of city residents.

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## 2. Analysis of previous studies and statement of the problem

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Ukrainian and foreign researchers have contributed much attention to the problems of optimal design and improvement of the PPT route networks. The development rate of a route network determines a potential accessibility of residents to PPT and affects the parameters that reflect the level of quality of its services, such as the passenger's commuting time, transit combinations, and travel costs.

Since the emergence of the theory of urban passenger transport, there have been suggested parameters derived analytically from the physical nature of passenger transportation and from certain assumptions about passenger behaviour – *the route network density* and *the route ratio* [5]. The above parameters are useful since their values are quite manageable at the stage of designing a route network, and they directly affect PPT performance characteristics.

In the second half of the 20th century, characteristics of commuting networks such as, in particular, degree of circuitness, complexity index, and degree of connectivity as well as indices of coverage and directness, were commonly based on the classic theory of graphs [6]. Subsequently, these parameters have been analysed from the point of view of their relation to socioeconomic indices of development of areas that are serviced by the studied commuting networks [7].

It is natural that the first objects of study within PPT route networks were urban underground networks, which is due to their topological simplicity, relatively few lines as well as public availability of information on the networks' structure and the passenger traffic. The latter reason has prevented extensive research of traditional route networks of land PPT (bus, trolleybus, and tram) that are much larger and much more complex in big cities. However, in recent years, due to the rapid development of information systems and technologies, the increased computing capacity and the availability of data on the structure of route networks in the Internet, such networks are increasingly becoming objects of research aimed at a comprehensive analysis and a comparison. An important impetus for this development was the relatively new field of knowledge – the theory of complex networks [8] – that laid the basis for a number of additional characteristics that are determined by the route network topology and allow a comprehensive comparative analysis of the structure of various PPT route networks.

Within the framework of the theory of complex networks, there are studies of the topology and development

rate of a number of PPT route networks, including underground networks in 19 cities of the world [9], the route networks of city buses in China [10] and India [11], and land PPT in major cities of Ukraine [12]. However, the research has focused on the comparative analysis of route networks and their classification as well as common characteristics and differences between them but overlooked the task of searching for a connection between characteristics of a route network development and performance indices of passenger traffic.

Study [13] presents a statistical analysis of the dependence between development indices of underground networks in 19 cities of the world and passenger mobility on the subways of the respective cities, whereas another study [14] presents similar research on subways in 50 US cities, with an aim to reveal statistical dependence between the performance of these networks, their load, and specific indicators of a daily mileage of a rolling stock per a passenger transported. However, currently there are no similar studies of land PPT networks; thus, the nature and extent of dependence between network parameters and passenger traffic indices (which reflect attractiveness of PPT to passengers) have been unexplored.

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## 3. The purpose and objectives of the study

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The purpose of the research is to identify the nature and extent of the impact of developing urban land route networks of electric transport in large cities of Ukraine with population of 0.5–1.0 million residents on the population commuting rates in the studied cities as assessment of the transport attractiveness to their residents.

The purpose can be achieved by solving the following objectives:

(1) use data on the configuration of route networks of urban land electric transport in formalizing their representation on the basis of the theory of graphs and the theory of complex networks and calculate their development indices,

(2) use methods of correlation and regression analyses to reveal the nature and extent of the influence of the development of the studied networks on the passenger mobility and give a relevant interpretation of the results.

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## 4. General characteristics of the studied cities and their commuting networks

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Ukraine has four cities with a population of 0.5 to 1.0 million residents that according to DBN 360-92\*\* "Town planning. Planning and development of urban and rural settlements" belong to a group of large settlements: Donetsk, Zaporizhia, Kryvyi Rih, and Lviv. PPT in these cities is multimodal and represented in its traditional forms – bus (shuttle bus included), trolleybus, and tram. There are no subway lines in these cities, but Kryvyi Rih has an operating speed tram line with underground sections. The city of Odesa (Ukraine) is also included in the list of the studied cities since its population fluctuates around 1 million residents, and its PPT has similar characteristics to those of the above-mentioned major cities. General characteristics of the studied cities and their route networks are presented in Table 1.

Table 1

General characteristics of the studied cities, their route networks and passenger traffic indices

| Parameter   | Parameter values for the studied cities |            |            |       |       |
|---|---|------------|------------|-------|-------|
|   | Donetsk                                 | Zaporizhia | Kryvyi Rih | Lviv  | Odesa |
| 1. The city area, km <sup>2</sup>                                       | 358                                     | 331        | 410        | 182   | 237   |
| 2. The number of residents (as of 01.03.2014), thsd residents           | 965.2                                   | 765.9      | 654.2      | 757.5 | 1016  |
| 3. The number of PPT routes, particularly:                              | 120                                     | 114        | 120        | 71    | 111   |
| bus and shuttle bus   | 99                                      | 95         | 83         | 51    | 79    |
| trolleybus  | 11                                      | 9          | 22         | 10    | 11    |
| tram  | 10                                      | 10         | 15         | 10    | 21    |
| 4. The number of stops on the route network                             | 706                                     | 458        | 351        | 336   | 553   |
| 5. A yearly volume of passenger transits (as of 2013), thsd passengers: |   |            |            |       |       |
| trolleybus  | 86409                                   | 18295      | 22947      | 22905 | 38092 |
| tram  | 63181                                   | 40861      | 23674      | 35935 | 56002 |
| 6. PPT commuting (as of 2013), commutes/year                            |   |            |            |       |       |
| trolleybus  | 90.96                                   | 23.88      | 35.03      | 30.18 | 37.46 |
| tram  | 63.18                                   | 53.34      | 36.14      | 47.35 | 55.07 |

Unlike subway lines, route networks of urban land PPT are more likely to change over time because of technically and technologically easy procedures of opening and closing of certain routes, introduction and withdrawal of the network stops, and changing the route circuits. The most unstable is the structure of the city bus route circuit, whereas the networks of public electric transport (trolleybuses and trams) are more stable. In addition, land PPT networks lack reliable indices on passenger traffic (it especially concerns bus intercity transits in the cities of Ukraine) because of imperfections in the system of statistical reporting on passenger traffic. Subsequent analysis of the route networks of municipal land electric transport (trolleybus and tram) is based on the volumes of passenger traffic as of 2013 (Table 1) as presented on the website of the “Ukrelektrotrans” corporation [15].

5. Formalising municipal route networks in graphs

It is convenient to present PPT route network as a graph. In mathematics, the graph theory defines the notion of graph G as a combination of two sets  $G = \langle V; E \rangle$ : a nonempty V set, all elements of which are called the graph vertices, and a set of unordered pairs of vertices E of set V, each of which is an edge of the graph [16]. The graph vertices are identified by their serial numbers in set V. Two different vertices of the graph  $v_i, v_j \in V (i \neq j)$  are called adjacent if they are connected by an edge  $(v_i, v_j) \in E$ .

The PPT route network graph can be presented in different ways that are distinguished by approaches to the formation of sets of the graph vertices and edges that are known as spaces [8]. A route network fragment (Fig. 2, a) consisting of seven stops (1–7) and three routes (A, B, and C) is presented as a graph in different spaces, as shown in Fig. 2, b–d.

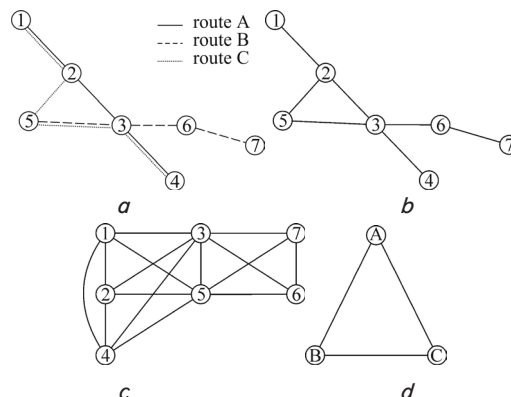


Fig. 2. Ways of presenting a PPT route network as a graph: a – a route network fragment; b – L-space; c – P-space; d – C-space

The space of stops (L-space, Fig. 2, b) is a simple graph whose vertices represent municipal PPT stops. Two vertices are connected by edges if stops that correspond to the vertices are consistent at least on one of PPT routes. Vertices connected by an edge in this space may not be physically consistent on the road network (on which the routes are laid) if public transport of some routes does not use all stops (for example, in case with express traffic mode).

The space of connections (P-space, Fig. 2, c) is a simple graph whose vertices represent public transport stops. Two vertices are connected by an edge if stops that correspond to these vertices at the same time belong to at least one of the routes. In other words, the presence of an edge between vertices in this space means that a passenger can travel between the respective stops without changing the route, i.e. directly.

The space of routes (C-space, Fig. 2, d) is a simple graph whose vertices represent routes of municipal public transport. Two vertices are connected by an edge if routes that correspond to the vertices have at least one common stop.

6. Parameters of the route network development and calculation of their values for the studied cities

The main topological parameters of the route networks that are studied in the theory of complex networks and influence the development of the latter include [17]: the average degree of the network vertex  $\langle k \rangle$ , the average length of the shortest path in the network  $\langle l \rangle$ , the clustering coefficient C, and the assortativity coefficient r.

The average degree of a network vertex  $\langle k \rangle$  is defined as the arithmetic mean of the degrees of all vertices in the network, i. e.

$$\langle k \rangle = \frac{1}{V} \sum_{i=1}^V k_i, \tag{1}$$

where V is the number of vertices in a network and  $k_i$  is the degree of vertex i, i. e. the number of edges which are incidental to the vertex.

The *average length of the shortest path* in a network  $\langle l \rangle$  determines its linear characteristics and is calculated as the arithmetic mean of the shortest paths between all pairs of vertices in the network  $\tilde{l}_{ij}$  that are expressed in the number of edges on each path:

$$\langle l \rangle = \frac{2}{V(V-1)} \sum_{i=1}^V \tilde{l}_{ij} \tag{2}$$

The *clustering coefficient* of some  $i$  vertex in the network determines the probability that two vertices adjacent to it are adjacent to each other; the former is calculated by the following formula:

$$C_i = \frac{2E_i}{k_i(k_i-1)} \tag{3}$$

where  $E_i$  is a sum total of the number of edges that interconnect all vertices adjacent to the given one.

The *clustering coefficient* of the whole network  $C$  is calculated as the arithmetic mean of the clustering coefficients of all its vertices, i. e.

$$C = \frac{1}{V} \sum_{i=1}^V C_i \tag{4}$$

The *assortativity coefficient* of the network  $r$  is calculated as the coefficient of the linear Pearson correlation between pairs of degrees of all adjacent vertices in the network by the following formula:

$$r = \frac{E \sum_{e=1}^E k_{ei} k_{ej} - \left[ \sum_{e=1}^E k_{ei} \right]^2}{E \sum_{e=1}^E k_{ei}^2 - \left[ \sum_{e=1}^E k_{ei} \right]^2} \tag{5}$$

where  $k_{ei}$  and  $k_{ej}$  are degrees of the network vertices that are incidental to the given edge.

Table 2 shows calculated topological indices of municipal electric transport route networks in the studied cities. In Table 2, trolleybus is designated as TR and tram – as T. Indices of the relevant parameters indicate the space for which these values are calculated, i. e. L, P or C (Fig. 2).

Analysis of the obtained values of the vertex average degree  $\langle k \rangle$ , length of the average shortest path  $\langle l \rangle$  and the clustering coefficient  $C$  shows that the studied route networks in all spaces occupy an intermediate position between the classic Erdős-Rényi (ER) random graphs and regular graphs (D), which can be represented by the following ratios [18]:

$$\langle k_{ER} \rangle = V \cdot C \tag{6}$$

$$\langle k_D \rangle = \text{const} \tag{7}$$

$$\langle l_{ER} \rangle \approx \frac{\ln(V)}{\ln \langle k_{ER} \rangle} \tag{8}$$

$$\langle l \rangle \approx V^{\sqrt{C}} \tag{9}$$

where  $V$  is the number of vertices in a network.

The calculated values suggest an idea of the studied networks and allow drawing some conclusions about their structure and characteristics. For example, in terms of a route network scheme, the most extensive are the networks of electric transport in Lviv and a trolleybus network in Odesa with  $\langle k_L \rangle = 2.53 \dots 2.60$ , whereas the least ramified are route networks of electric transport in Zaporizhia and the tram network in Donetsk, with close to tree-shaped topologies and  $\langle k_L \rangle = 2.06 \dots 2.07$ . Meanwhile, given an equal probability of passenger correspondence between all stops in the network, the average length of a trolleybus travel in Kryvyi Rih ( $\langle l_L \rangle = 24.6$ ) exceeds its counterpart in Odessa almost twice ( $\langle l_L \rangle = 12.1$ ). Under the same assumptions, the tram network in Kryvyi Rih, with  $\langle l_P \rangle = 1.31$ , is a network with the fewest connections, whereas the tram network in Odesa, with  $\langle l_P \rangle = 2.59$ , has most of all connections in comparison with other studied networks.

Table 2

Topological indices of the studied electric transport networks in various spaces

| Parameter             | Parameter values for municipal route networks |       |            |       |            |       |       |       |       |       |
|-----------------------|---|-------|------------|-------|------------|-------|-------|-------|-------|-------|
|                       | Donetsk                                       |       | Zaporizhia |       | Kryvyi Rih |       | Lviv  |       | Odesa |       |
|                       | T   | TP    | T          | TP    | T          | TP    | T     | TP    | T     | TP    |
| $\langle k_L \rangle$ | 2.07  | 2.23  | 2.07       | 2.06  | 2.14       | 2.17  | 2.60  | 2.53  | 2.13  | 2.55  |
| $\langle k_P \rangle$ | 26.7  | 32.6  | 36.7       | 34.4  | 47.3       | 34.1  | 24.8  | 27.7  | 27.8  | 37.8  |
| $\langle k_C \rangle$ | 4.22  | 3.17  | 6.20       | 6.50  | 11.3       | 8.67  | 5.33  | 5.60  | 5.70  | 4.31  |
| $\langle l_L \rangle$ | 22.2  | 19.2  | 15.4       | 15.1  | 14.9       | 24.6  | 6.22  | 14.2  | 23.9  | 12.1  |
| $\langle l_P \rangle$ | 2.22  | 2.70  | 1.63       | 1.67  | 1.31       | 2.48  | 1.61  | 2.01  | 2.59  | 1.95  |
| $\langle l_C \rangle$ | 1.56  | 2.80  | 1.31       | 1.07  | 1.31       | 1.99  | 1.33  | 1.51  | 2.02  | 2.10  |
|                       | 0.057   | 0.109 | 0.000      | 0.017 | 0.067      | 0.045 | 0.180 | 0.200 | 0.045 | 0.093 |
| $C_P$                 | 0.965   | 0.903 | 0.855      | 0.871 | 0.872      | 0.836 | 0.786 | 0.856 | 0.888 | 0.798 |
| $C_C$                 | 0.644   | 0.603 | 0.864      | 0.941 | 0.874      | 0.843 | 0.664 | 0.728 | 0.760 | 0.567 |
| $r_L$                 | 0.220   | 0.004 | 0.045      | 0.141 | 0.467      | 0.153 | 0.264 | -0.10 | 0.167 | 0.176 |
| $r_P$                 | 0.224   | 0.165 | 0.020      | -0.13 | -0.19      | 0.003 | -0.01 | 0.076 | 0.165 | 0.204 |
| $r_C$                 | -0.26   | -0.20 | -0.17      | -0.32 | -0.14      | 0.55  | -0.19 | 0.252 | -0.14 | 0.197 |

The researched networks are characterized by high values of the clustering coefficient in the spaces of routes and connections, the latter having signs of the “tight world”, i. e. there are small values of the average shortest path in comparison with the number of vertices of the network.

**7. A correlation-regression analysis of how indices of the transportation network development affects passenger traffic**

The nature and extent of the impact of electric transport networks’ development indices on passenger traffic BPC (commutes on public electric transport per capita per year) are analysed with the help of the linear Pearson correlation coefficient  $r_{xy}$ . Since the latter requires normal distribution of factorial and efficiency features and is very sensitive to random emission as well as correlation between passenger traffic BPC (the result) and each individual parameter of the route network (the factor variable), the research is made in the following sequence:

- (1) excluding abnormal emissions from statistical sampling by the Shovene criterion [19],
- (2) testing emission-verified samples for normal distribution by the Shapiro-Wilk criterion [19],
- (3) calculating the linear Pearson correlation coefficient  $r_{xy}$  for each pair “factor – result” [20].

The main statistical characteristics of factors and results of calculation of the linear correlation coefficient between them and passenger traffic are shown in Table 3.

Table 3

Basic statistics of topological indices of route networks and the calculated Pearson pair correlation coefficients

| Parameter (factorial feature) | Mathematical expectation $\bar{x}$ | Standard deviation $\sigma_x$ | The coefficient of variation $v = \frac{\sigma_x}{\bar{x}}$ | The coefficient of correlation $r_{xy}$ |
|-------------------------------|------------------------------------|-------------------------------|---|---|
| $\langle k_L \rangle$         | 2.255                              | 0.216                         | 0.096   | -0.167                                  |
| $\langle k_P \rangle$         | 32.99                              | 6.733                         | 0.204   | -0.289                                  |
| $\langle k_C \rangle$         | 6.099                              | 2.359                         | 0.387   | -0.573                                  |
| $\langle l_L \rangle$         | 16.78                              | 5.718                         | 0.341   | 0.280                                   |
| $\langle \rangle$             | 2.017                              | 0.470                         | 0.233   | 0.551                                   |
| $\langle l_C \rangle$         | 1.701                              | 0.523                         | 0.307   | 0.171                                   |
| $C_L$                         | 0.081                              | 0.066                         | 0.808   | 0.023                                   |
| $C_P$                         | 0.863                              | 0.051                         | 0.059   | 0.485                                   |
| $C_C$                         | 0.749                              | 0.128                         | 0.171   | -0.557                                  |
| $r_L$                         | 0.154                              | 0.154                         | 1.002   | -0.182                                  |
| $r_P$                         | 0.052                              | 0.141                         | 2.703   | 0.582                                   |
| $r_C$                         | -0.042                             | 0.279                         | 6.681   | -0.387                                  |

An analysis of the calculations shown in Table 3 indicates that parameters  $\langle l_P \rangle$ ,  $r_P$ ,  $\langle k_C \rangle$  and that represent the route network in the spaces of connections and routes have an average correlation dependence ( $r_{xy} = 0.5...0.7$ ). Diagrams of dispersion for these parameters with overlaying calculation lines of regression derived by the method of least squares are shown in Fig. 3, a-d.

Other parameters of electric transport route networks do not have significant statistical effect on passenger traffic.

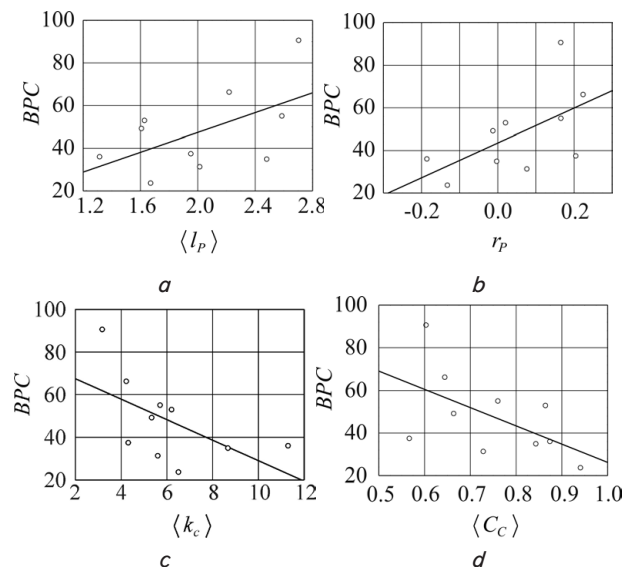


Fig. 3. Diagrams of dispersion and an empirical regression line of the dependence of the number of commutes per capita per year (BPC) on the topological parameters of electric transport route networks: a – the average shortest path in the space of connections  $\langle l_P \rangle$ ; b – the coefficient of assortativity in the space of connections  $\langle l_P \rangle$ ; c – the average degree of vertices in the space of routes  $\langle k_C \rangle$ ; d – the clustering coefficient of the network in the space of routes  $C_C$

The results of the pair linear regression analysis of the parameters shown in Fig. 3 are shown in Table 4.

Table 4

Results of the regression analysis of the dependence of passenger traffic on municipal electric transport upon the parameters of a route network development

| Factor                | Regression equation                            | The coefficient of determination $R^2$ | The standard error S |
|-----------------------|--|--|----------------------|
| $\langle l_P \rangle$ | $BPC = 1.08 + 26.13 \cdot \langle l_P \rangle$ | 0.30                                   | 17.51                |
| $r_P$                 | $BPC = 43.54 + 81.88 \cdot r_P$                | 0.34                                   | 17.06                |
| $\langle k_C \rangle$ | $BPC = 77.11 - 4.81 \cdot \langle k_C \rangle$ | 0.33                                   | 17.19                |
| $C_C$                 | $BPC = 112.1 - 85.89 \cdot C_C$                | 0.31                                   | 17.43                |

According to the data on the calculated coefficient of determination  $R^2$ , 30–35 % of the changing passenger traffic on municipal electric transport in the studied cities is statistically predetermined by the varying selected factors.

**8. Discussing the research findings**

The researched route networks show that the degree of their development is not decisive in terms of attractiveness to passengers. Lower fares, in comparison with bus transport fares, and free of charge travel for a considerable proportion of the urban population of Ukraine have led to a situation

where municipal electric transport is mostly used by persons with relatively low incomes. However, as the above calculations show, some parameters of route networks still have a significant correlation with passenger traffic in these networks, so they require substantive interpretation.

The average length of the route network shortest path in the space of connections  $\langle l_p \rangle$  reduced by a unit reflects the average number of connections passengers make when travelling between two randomly selected stops. Thus, higher values  $\langle l_p \rangle$  correspond to the route network that has a potentially higher shuttle connectivity assuming a constant value for passenger correspondence between all the stops. The possibility of travelling almost without connections attracts potential passengers, but on the other hand, a more extensive connectivity combination results in more route commutes. Respectively, the latter factor prevails, thus increasing the value by 1 % leads to almost the same increase in the proportion of annual route travels.

The factor of the network assortativity  $r_p$  ( $0 \leq r_p \leq 1$ ) is represented by the property of its vertices to be adjacent. This is especially true for high-degree vertices (the so-called "hubs"). Networks with positive assortativity coefficient values are called assortative, while those with negative values are disassortative. In the networks under consideration, high-degree vertices are the stops from which passengers can get directly to a large number of other stops. Obviously, these stops are common to many routes and, therefore, represent major interchange nodes. Negative values  $r_p$  correspond to route networks whose interchange nodes are located at a considerable distance from each other; positive – on the contrary – reveal the concentrations of interchange nodes in the city. Thus, route networks which are assortative in the space of connections are characterized by relatively larger passenger traffic; when the assortativity coefficient is raised by 0.01 %, the BPC value increases by 1.8 %.

The value of the average degree of the network vertex represented in the space of routes  $\langle k_c \rangle$  is an average number of routes with which this route has at least one common stop. If the parameter value increases, there is an increase in the route overlays, i.e. their full or partial duplication. Such networks are characterized by a high value of the route coefficient [5]. If the latter increases while the capacity of transport vehicles remains unvaried (which is typical of the studied networks), the network traffic interval and, re-

spectively, the average waiting time at stops become longer. Thus, an increasing value  $\langle k_c \rangle$  makes the route network less attractive to potential passengers, and a 1 % increase of this index leads to a lower passenger traffic on average by 0.61 %.

A similar characteristic of the route network is reflected by its clustering coefficient  $C_c$  ( $0 \leq C_c \leq 1$ ) in the space of routes. It suggests a probability that any random pair of routes, each of which has at least one common stop with the given route, also have at least one common stop. Such situations usually occur when a stop is common for three or more routes. Respectively, higher values of clustering coefficients correspond to relatively larger traffic intervals on the network and reduce the transport traffic on average by 1.37 % with an increase in value  $C_c$  by 1 %.

Thus, the studied characteristics allow making a comparative analysis of different route networks in terms of passenger traffic and forecasting the volumes of transport traffic in networks with varied topological structures.

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## 9. Conclusions

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1. Graph theory methods were used to formalize representation of route networks of urban electric transport in large cities of Ukraine in the spaces of stops, connections and routes and calculate the indices of their development, such as the average degrees of vertices, the values of average shortest paths, clustering coefficients as well as the coefficients of assortativity and centrality of mediation vertices – on the basis of the theory of complex networks. It is found that the studied networks occupy an intermediate position between the Erdős-Rényi (ER) random graphs and regular graphs.

2. The method of correlation and regression analysis was used to research the nature and the extent of impact of the studied route network characteristics on the number of commutes per capita per year in the respective network. It is found that the average degree of correlation with passenger traffic characterizes the indices of the shortest path, the coefficient of assortativity of the route network represented in the space of connections, the average degree of the vertex and the clustering coefficient of the network represented in the space of routes. The suggested substantive interpretation presents a correlation observed between factors and passenger traffic on municipal electric transport in the studied cities.

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*Представлені результати аналізу моделей нелінійної кінетики екосистемних процесів при впливі забруднювачів і поширенні органічних домішок у довкіллі. Обґрунтовано необхідність врахування при аналізі антропогенного впливу на біосферу процесів автокаталізу і самоорганізації живих систем. Розроблено модель синергетичних закономірностей еволюції видів у контексті традиційного континуальної уявлення при обліку біфуркаційних механізмів синергетичної теорії пізнання*

*Ключові слова: нелінійна кінетика, екосистемні процеси, моделі, техногенні чинники, синергетичні закономірності*

*Представлены результаты анализа моделей нелинейной кинетики экосистемных процессов при воздействии загрязнителей и распространении органических примесей в окружающей среде. Обоснована необходимость учета при анализе антропогенного влияния на биосферу процессов автокатализа и самоорганизации живых систем. Разработана модель синергетических закономерностей эволюции видов в контексте традиционного континуального представления при учете бифуркационных механизмов синергетической теории познания*

*Ключевые слова: нелинейная кинетика, экосистемные процессы, модели, техногенные факторы, синергетические закономерности*

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# ФОРМАЛІЗАЦІЯ НЕЛІНІЙНИХ ЗАКОНОМІРНОСТЕЙ РОЗВИТКУ ЕКОСИСТЕМНИХ ПРОЦЕСІВ ПРИ ВПЛИВІ АНТРОПОГЕНЕЗУ

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## 1. Вступ

Екосистема с ее структурными единицами (подсистемами) рассматривается как открытая термодинамическая система, в которой постоянно происходит

обмен энергией и веществом с внешней средой. При подаче энергии в экосистему часть ее рассеивается, а другая часть превращается в тепло. Рассмотренная система соответствует требованиям, предъявляемым к системам способным к самоорганизации [1, 2]: