Використання оцінок вектора стану, отриманого різними вимірювачами одночасно, передбачає їх вагове підсумовування. Однак матриця вагових коефіцієнтів апріорно не завжди відома. Отримано аналітичний вираз для кореляційної матриці помилок прямого визначення незмінної матриці вагових коефіцієнтів при використанні оцінок кореляційних матриць помилок. Показано, що точність оцінювання залежить як від точності незалежних вимірювачів, так і від довжини вибірки, взятої для визначення оцінок параметра

Ключові слова: об'єднання інформації, вимірювання параметрів, незалежні вимірювачі, фільтрація оцінок, матриця вагових коефіцієнтів

Использование оценок вектора состояния, полученного различными измерителями одновременно, предполагает их весовое суммирование. Однако матрица весовых коэффициентов априорно не всегда известна. Получено аналитическое выражение для корреляционной матрицы ошибок прямого определения неизменяющейся матрицы весовых коэффициентов при использовании оценок корреляционных матриц ошибок. Показано, что точность оценивания зависит как от точности независимых измерителей, так и от длины выборки, взятой для определения оценок параметра

Ключевые слова: объединение информации, измерение параметров, независимые измерители, фильтрация оценок, матрица весовых коэффициентов

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ANALYSIS OF ACCURACY IN EVALUATING GRAVIMETRIC COEFFICIENTS IN THE ALGORITHM OF SPATIAL MONITORING UNDER CONDITIONS OF EXCESS

Y. Kulyavets
PhD, Senior Lecturer*
E-mail: yuriy.kulyavec@mail.ru

O. Bogatov
PhD, Senior Lecturer**
E-mail: bogatovoleg@mail.ru

E. Ermakova
PhD, Senior Lecturer
Department of Engineering and Computer Graphics***
E-mail: ermelene@mail.ru

P. Karlash
Senior teacher*
E-mail: peter.karlach@ukr.net

V. Popov
PhD, Senior Lecturer**
E-mail: pvm777@ukr.net

*Department of Life Safety & Environmental Engineering
Kharkiv National University of Construction and Architecture
Sumskay str., 40, Kharkiv, Ukraine, 61002
**Department of Metrology and Life Safety
***Kharkiv National Automobile and Highway University
Petrovskogo str., 25, Kharkiv, Ukraine, 61002

1. Introduction

Hazardous factors of emergencies are determined by using monitoring systems at various levels. Modern technologies allow integrating numerous sources of information into a single information resource available to work precisely on the collected miscellaneous data from various sources. Reliability and high accuracy of the information can facilitate quick response to any man-made or natural disaster, objective assessment of its scope and extent of damage, effective elimination of consequences of accidents and disasters, and significant improvement of the reliability of predicting the onset of a crisis. The results of monitoring are used for early warning and information management so that managerial decisions could be taken in time to make necessary changes in the state and direction of a system, process, or phenomenon.

An evaluation unit is usually a two-step formation. In the first phase, assessment of signal parameters results in an initial measurement – a surveillance vector. However, all measurements should be practically simultaneous so that a parameter change between the moments of measuring could be significantly less than the expected error, could have no impact of the magnitude of this error, and could be easily neglected. The task of the second phase of measurements is to integrate the first phase results and to formulate the evaluation unit, which is a state vector. The total number of primary measurements (the scope of the surveillance vector) is often redundant, which means that it exceeds the minimum sufficient estimates for the parameter evaluation. Then the resulting measurement is a statistical problem of evaluating the state vector through using an excess number of primary values that were obtained simultaneously.

The problem of an optimal use of the state vector values is reduced to a consistent application of a data filtering algorithm, and the resulting estimate is the sum total of the weight estimates from the state vector of the measuring devices, provided that the primary measurements were independently obtained by different meters. In this case, the accuracy of the resulting estimate (i. e. the precision matrix, which is reverse to the correlation matrix of errors, or error correlation matrix) depends not only on the accuracy and nature of the initial measurement but also on the value of the measured parameter. Then the weight coefficient matrix is included in the determination of the resulting estimate; it depends on the accuracy matrices used for its calculation and also on the value of the parameter that is to be determined. Therefore, direct application of the filtering algorithm seems to be difficult.

Accuracy of the resulting estimate was primarily analysed as dependent on the accuracy of measuring the weight coefficient matrix (regardless of the methods of obtaining it). It was also specified that the accuracy of the resulting evaluation parameter depended not only on the accuracy of the estimates used but on the accuracy of the evaluation matrix weights: errors in its determination affect the accuracy of the resulting estimate. However, the possible methods of assessing the weight coefficient matrix were not considered, and, consequently, no correlations were disclosed to determine its accuracy. Therefore, it remains necessary and topical to find such possible methods for obtaining a weight coefficient matrix and determining its accuracy.

2. Analysis of previous studies and statement of the problem

Requirements for building structural subsystems to monitor emergencies, including radiation hazards, and a mechanism for mathematical modeling of emergency monitoring systems are both provided in the study [1]. In [2], the method of measuring emergency parameters is specified to include a certain amount of redundancy, which allows critical evaluation of the process of measuring. It is also argued that the program of preliminary processing of the measurement results should involve routine analysis of the measurements' reliability on the basis of trustworthy estimates of the uncertainty limits derived from studying the measurement process performance, which allows controlling the parameters of the measuring process in real time. The structure of the theory of redundant measurements, its essence as well as some fundamental concepts and definitions of this theory are presented in [3]. In [4], it is emphasized that information redundancy implies existence of additional relationships between the measured values and output parameters. In [5], attention is focused on determining the impact of redundant measuring on the accuracy of the estimates. In [6], higher accuracy of trajectory information processing is claimed to occur due to accounting for spatial and temporal data redundancy and cross-correlation of measurement errors. In [7], the possibility of using redundancy is examined for individual assessment methods. In [8], a generalized method is suggested for statistical evaluation of an object if there is excessive primary information obtained by consecutive approximations. Issues related to the analysis of test methods for improving the accuracy of measurement results for electric values are considered in [9]. In [10], there is an example of a method for solving a system of equations overridden by finding a minimum of a quadratic functional to calculate target coordinates by using a difference-and-long-range method alongside redundant data. In [11], it is emphasized that the use of redundant information for the resulting estimate may significantly complicate the problem-solving process, which makes it necessary to find quasi-optimal methods for reliable assessing and efficient building of systems that do not use redundancy of primary measurements [12]. So in [13], there is an analysis of topological data that depend on the network structure and the measurements' alignment on it. The method is reduced to building a balanced hypergraph of measurements and determining the shortest routes. The combining algorithm that is suggested in [14] entails use of automotive wheel rotation speed sensor signals in the absence of consumer navigation equipment of satellite navigation systems. Correction of lost data from satellite navigation systems with the help of an inertial navigation system is considered in [15]. In [16], use of cellular navigation systems is described as appropriate only when there is a loss of signals from satellite navigation systems on the basis of adaptive processing algorithms. In [17], it is noted that a combination of inertial-satellite navigation systems has the following advantages: a simple mathematical model, high reliability, independent work, and a large excess of navigation information. Information for processing [18] is the difference between the initial information about the position and velocity, which is derived from a platformless inertial navigation system and a satellite navigation system; errors are assessed, and the initial information is corrected. In [19], it is suggested to use an approach that consists in combining several sets of measurements of different nature and using mathematical models of motion. An example is the combined processing of data provided by the accelerometer and the video analysis system. It is important to develop methods of combined processing of data from the video analysis system and inertial sensors as it is essential to remodel information about the movement of a person [20]. An algorithmic support of pedestrian inertial navigation systems is developed with regard to choosing a specific algorithm for combining measurement data. If the systems are described by nonlinear equations, the well-known methods of approximation in Bayesian filtering are an extended Kalman filter (EKF), a sigma-point Kalman filter (an unscented Kalman filter (UKF)), a cubature Kalman filter (CKF), and a modified square-root cubature Kalman filter (SRKF) [21]. In [22], a method is describes for combining data obtained in three spectral bands – microwave, infrared, and optical – by using three types of sensors – a radiometer, a thermal imager, and a camera. The results of the three types of sensors are reflected as a digital image of a collection of objects on the basis of spectral filtering of data. In [27], a method is suggested to determine combining filter's setting coefficients in processing flight information. The suggested method is intended to identify the trajectory of the aircraft.

Meanwhile, information redundancy often entails a possibility of data estimation by various relatively simple measuring devices. Then the task of using information redundancy is reduced to finding optimal algorithms of combining these estimates, such as their weighted summation with a weight that is proportional to the accuracy of the estimates. Most studies assume that accuracy of the initial measurement (the surveillance vector) is known and determined, first of all, by the possibilities of the measuring and is independent of the value of the measured parameter. At the same time, when various transformations of the surveillance vector are used to obtain estimates for the state vector, accuracy of the assessment depends on the value of the assessment [5]. In such cases, it is necessary to obtain simultaneously not only the state vector estimates but also the estimates of their accuracy matrix to determine the matrix of weight coefficients, or the weight coefficient matrix.

3. The objective and tasks of the study

The objective of this research is to develop the existing algorithms of combining redundant information, including the analysis of how precise the direct determination of the weight coefficient matrix (WCM) is if it does not change while using the estimates of error correlation matrices (ECM) for a normally distributed state vector of a parameter received simultaneously by different independent meters.

To achieve this objective, it is necessary to fulfil the following tasks:

– to obtain analytical expressions for mathematical expectation and variance of estimation components in an ECM to determine the parameter state vector;

– to specify the mathematical expectation and the ECM of estimating a WCM;

– to analyse the structure of the ECM of the resulting estimates with account for the ECM of estimating the WCM.

4. Analysis of the impact produced by errors in determining the weight coefficient matrix on the accuracy of the resulting estimates

In [28], it is shown that information redundancy reduces the solution of the optimal use problem of the same state vector estimates obtained by different meters simultaneously to a consistent application of the estimates filtering algorithm. The study also demonstrates that, if the data were obtained by different measuring devices, the resulting estimate of the parameter $\hat{\alpha}_p$ is the weight sum of the state vector estimates of one measuring device $\hat{\alpha}_1 = \mathbf{f}\left(\hat{\theta}_1\right)$ and the other measuring device $\hat{\alpha}_2 = \mathbf{f}\left(\hat{\theta}_2\right)$:

$$\begin{aligned}\hat{\alpha}_p &= \hat{\alpha}_1 + \mathbf{C}_p^{-1}\mathbf{C}_2\left(\hat{\alpha}_2 - \hat{\alpha}_1\right) = \\ &= \hat{\alpha}_1 + \mathbf{W}\left(\hat{\alpha}_2 - \hat{\alpha}_1\right) = \left(\mathbf{I} - \mathbf{W}\right)\hat{\alpha}_1 + \mathbf{W}\hat{\alpha}_2,\end{aligned} \tag{1}$$

$$\mathbf{C}_p = \mathbf{C}_1 + \mathbf{C}_2, \tag{2}$$

where $\mathbf{C}_i$ is the accuracy matrix of the estimates i of the measuring device, which is reverse to the ECM of estimating the parameter $\mathbf{C}_i^{-1}$ $(i = p,\ 1,\ 2)$; $\mathbf{I}$ is an identity matrix.

It is known [5, 25–27] that accuracy of parameters' evaluation (i. e. the elements of the accuracy matrix $\mathbf{C}_i$) depends not only on the accuracy and type of the initially measured parameters but also on the values of the parameters that are measured. Then the WCM $\mathbf{W} = \mathbf{C}_p^{-1} \cdot \mathbf{C}_2$ within the expression (3) for determining the resulting estimate also depends on the values of these parameters, which are not always known in advance. Therefore, a direct use of the algorithm (1) seems to be difficult. In [28], analysis was undertaken to determine the effect of accuracy of determining the WCM $\hat{\mathbf{W}}$ (regardless of the methods of obtaining it) on the accuracy of the resulting estimate $\hat{\alpha}_p$. The study also shows that the accuracy $\hat{\alpha}_p$ depends not only on the accuracy of the estimates $\hat{\alpha}_1$ and $\hat{\alpha}_2$ but also on the accuracy of determining the WCM $\hat{\mathbf{W}}$. Thus, errors in determining the WCM affect the accuracy of the resulting value $\hat{\alpha}_p$. However, possible methods of obtaining the WCM were no considered in the study [28], which, consequently, did not consider any correlations that would determine the accuracy of estimating the WCM. It is possible to assume that the WCM can be determined if it is calculated by the estimates of the ECM of measuring $\hat{\mathbf{C}}_1^{-1}$ and $\hat{\mathbf{C}}_2^{-1}$. If the measured parameter does not change while it is being measured, the ECM $\hat{\mathbf{C}}_i^{-1}$ can be determined on the basis of n of independent discrete samples of the state vector estimates for the parameter $\hat{\alpha}_i$ $(i = p,\ 1,\ 2)$:

$$\hat{\mathbf{C}}_i^{-1} = \frac{1}{n-1}\sum_{t=1}^{n}\left(\hat{\alpha}_{it} - M\left[\hat{\alpha}_{it}\right]\right)\cdot\left(\hat{\alpha}_{it} - M\left[\hat{\alpha}_{it}\right]\right)^T. \tag{3}$$

We believe that the WCM $\hat{\mathbf{W}}$ is estimated by the ECM estimates $\hat{\mathbf{C}}_1^{-1}$ and $\hat{\mathbf{C}}_2^{-1}$ for measuring the parameter by independent devices. Then obtaining statistical characteristics of the multivariate law of WCM distribution is a rather complicated statistical problem. In the case of a normal distribution of data [29], the values of their mathematical expectation and variance (ECM) are independent. This fact allows building an independent meter of variance in the parameter state vector. Herewith [30], the density of ECM estimates of the dimension $m \times m$, provided the n samples are distributed independently and equally (normal $N\left(0, \mathbf{C}^{-1}\right)$), looks as follows:

$$dF = \frac{\left|\hat{\mathbf{C}}^{-1}\right|^{\frac{1}{2}(n-m-1)} \exp\left(-\frac{1}{2}\operatorname{sp}\hat{\mathbf{C}}^{-1}\Sigma^{-1}\right)}{2^{\frac{1}{2}nm}\pi^{\frac{m(m-1)}{4}}\left|\Sigma\right|^{\frac{1}{2}n}\prod_{i=1}^{m}\Gamma\left[\frac{1}{2}(n+1-t)\right]},$$

where $\hat{\mathbf{C}}^{-1}$ is a positively determined matrix that is assessed according to (3), and $n > m$; Σ is a determinant of the matrix elements $\hat{\mathbf{C}}^{-1}$.

The moments of the elements $\hat{\mathbf{C}}^{-1}$ can be obtained both from the characterizing function and the initial normal distribution [30].

The mathematical expectation ij of the element $\hat{C}_{ij}^{-1}$ in the matrix $\hat{\mathbf{C}}^{-1}$ is equal to:

$$\begin{aligned}M\left[\hat{C}_{ij}^{-1}\right] &= M\left[\frac{1}{n-1}\sum_{t=1}^{n}\left(\hat{\alpha}_{it} - M\left[\hat{\alpha}_i\right]\right)\left(\hat{\alpha}_{jt} - M\left[\hat{\alpha}_j\right]\right)\right] = \\ &= M\left[\frac{1}{n-1}\sum_{t=1}^{n}\sigma_{ijt}\right] = \frac{n}{n-1}\sigma_{ij},\end{aligned}$$

where σ_{ij} is a ij element of the matrix Σ, and the co-variation between $\hat{C}_{ij}^{-1}$ and $\hat{C}_{kl}^{-1}$ looks as follows [30]:

$$\begin{aligned}&M\left[\left(\hat{C}_{ij}^{-1} - \frac{n}{n-1}\sigma_{ij}\right)\left(\hat{C}_{kl}^{-1} - \frac{n}{n-1}\sigma_{kl}\right)\right] = \\ &= \frac{n}{(n-1)^2}\left(\sigma_{ik}\sigma_{jl} + \sigma_{il}\sigma_{jk}\right).\end{aligned} \tag{4}$$

If $i = k$ and $j = l$, the latter expression determines the variance $\hat{C}_{ij}^{-1}$:

$$M\left[\left(\hat{C}_{ij}^{-1} - \frac{n}{n-1}\sigma_{ij}\right)^2\right] = \frac{n}{(n-1)^2}\left(\sigma_{ij}^2 + \sigma_{ii}\sigma_{jj}\right). \quad (5)$$

In [31] provides an expression for the distribution density in the correlation of variance estimates of two independent random processes as well as for the variance of this correlation. The correlation of the variance estimates in their form is a one-dimensional analogue of WCM assessment. However, the existing dependence of the WCM on the ECM of measuring does not allow for an assumption that the components within the formula are independent.

Thus, it is rather difficult to obtain an analytical expression of the ECM for evaluating the WCM. However, if there is an assumption of the errors being small and if there is a priori information available on some margin of the true value of the WCM, then nonlinear approximate dependencies in the margin can be approximated by linear dependencies. The main advantage of the linearization method is that it allows obtaining explicitly optimal estimates (and, in this case, the most plausible ones) of the WCM and the ECM.

As we have noted before, the WCM is connected with the ECM of evaluation through a dependency of the following type:

$$\mathbf{W} = \mathbf{C}_p^{-1}\mathbf{C}_2 = \left(\mathbf{C}_1 + \mathbf{C}_2\right)^{-1}\mathbf{C}_2 = \\ = \mathbf{C}_p^{-1}\left(\mathbf{C}_p - \mathbf{C}_1\right) = \mathbf{I} - \mathbf{C}_p^{-1}\mathbf{C}_1. \quad (6)$$

When ECM values are used to assess the WCM, the correlation is the following:

$$\hat{\mathbf{W}} = \hat{\mathbf{C}}_p^{-1}\hat{\mathbf{C}}_2 = \left(\hat{\mathbf{C}}_1 + \hat{\mathbf{C}}_2\right)^{-1}\hat{\mathbf{C}}_2 .$$

While evaluating the WCM, it is necessary to take into account the nature of the interrelation between its elements and the nature of its change over time. We believe that during the time of measuring the measured parameter does not change. Thus, discrete surveillance is used to assess the WCM elements that do not change over time. Random changes of the WCM can arise from parameter measurement errors.

The model of changes in the square matrix $\hat{\mathbf{W}}$ of the dimension $m \times m$ with arbitrary real elements can be "stretched" into a column vector $\hat{\mathbf{W}}^{\#}$ of the dimension $m^2 \times 1$, where the symbol # denotes the operation of "stretching" the matrix into the column vector [5].

For example, if $m = 2$, then:

$$\hat{\mathbf{W}}^{\#} = \left\| \hat{W}_{11} \quad \hat{W}_{12} \quad \hat{W}_{21} \quad \hat{W}_{22} \right\|^{T},$$

where the symbol T denotes the act of transposition.

Then the latter expression can be rewritten as follows:

$$\hat{\mathbf{W}}^{\#} = \left(\hat{\mathbf{C}}_p^{-1}\hat{\mathbf{C}}_2\right)^{\#} = \left[\left(\hat{\mathbf{C}}_1 + \hat{\mathbf{C}}_2\right)^{-1}\hat{\mathbf{C}}_2\right]^{\#} = \\ = \mathbf{w}\left(\left[\hat{\mathbf{C}}_1^{-1}, \hat{\mathbf{C}}_2^{-1}\right]^{\#}\right) = \mathbf{w}\left(\mathbf{W}_{\mathbf{C}}^{\#}\right).$$

By assuming that the function $\mathbf{w}\left(\mathbf{W}_{\mathbf{C}}^{\#}\right)$ on any fairly narrow section is approximately linear, it can be presented as a Taylor series around the point $\mathbf{w}\left(\mathbf{W}_{\mathbf{C}0}^{\#}\right)$, and the elements of the order should be below the first element:

$$\hat{\mathbf{W}}^{\#} = \mathbf{w}\left(\mathbf{W}_{\mathbf{C}0}^{\#}\right) + \mathbf{P}\left(\hat{\mathbf{W}}_{\mathbf{C}}^{\#} - \mathbf{W}_{\mathbf{C}0}^{\#}\right), \quad (7)$$

where $\mathbf{P} = \left\|\mathbf{P}_1 \quad \mathbf{P}_2\right\|$ is a block matrix of recalculating the ECM of the $\hat{\mathbf{C}}_1^{-1}$ and $\hat{\mathbf{C}}_2^{-1}$ measurements in the WCM $\mathbf{W}$ of the dimension $m^2 \times 2m^2$, and μ $(\mu = 1,\ 2)$ is a block of this matrix:

$$\mathbf{P}_\mu = \frac{\partial \mathbf{w}\left(\mathbf{W}_{\mathbf{C}}^{\#}\right)}{\partial\left(\mathbf{C}_\mu^{-1}\right)^{\#}}, \quad (8)$$

which is a two-dimensional matrix of the dimension $m^2 \times m^2$. This understanding of the matrix $\mathbf{P}$ is determined by the introduced model (7).

Then the mathematical expectation and the ECM of determining the WCM of the dimension $m^2 \times m^2$ can be expressed as follows:

$$\hat{\mathbf{W}}^{\#} = \mathbf{w}\left(\mathbf{W}_{\mathbf{C}0}^{\#}\right); \quad (9)$$

$$\mathbf{C}_{\mathbf{W}}^{-1} = \mathbf{P}\mathbf{C}_{\mathbf{C}}^{-1}\mathbf{P}^{T} = \mathbf{P}_1\mathbf{C}_{\mathbf{C}1}^{-1}\mathbf{P}_1^{T} + \mathbf{P}_2\mathbf{C}_{\mathbf{C}2}^{-1}\mathbf{P}_2^{T}, \quad (10)$$

where

$$\mathbf{C}_{\mathbf{C}}^{-1} = \begin{Vmatrix} \mathbf{C}_{\mathbf{C}1}^{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{C}_{\mathbf{C}2}^{-1} \end{Vmatrix}$$

is a block-diagonal (if the assessment meters are independent) ECM of measuring the WCM of parameter evaluation by different measuring devices, with the dimension of $2m^2 \times 2m^2$. The blocks of this matrix are determined by the following correlation:

$$\mathbf{C}_{\mathbf{C}\mu}^{-1} = M\left[\left(\hat{\mathbf{C}}_\mu^{-1} - \mathbf{C}_\mu^{-1}\right)^{\#}\left(\hat{\mathbf{C}}_\mu^{-1} - \mathbf{C}_\mu^{-1}\right)^{\#T}\right], \ (\mu = 1,\ 2),$$

and the elements of the matrix blocks are determined by the expressions (4) and (5).

Thus, the presence of the matrix $\mathbf{C}_{\mathbf{C}}^{-1}$ in the expression (10) shows that the accuracy of evaluating the WCM depends, according to (4) and (5), on the quantity of samples taken to find the ECM of a parameter assessment. Besides, the quantity of such samples should be no less than the dimension of the estimated ECM.

For further analysis of the WCM accuracy estimation, it is necessary to obtain an expression for the matrix $\mathbf{P}$, i. e. to determine the derivative (8).

In general, the matrix

$$\mathbf{P}_\mu = \frac{\partial \mathbf{w}\left(\mathbf{W}_{\mathbf{C}}\right)}{\partial\left(\mathbf{C}_\mu^{-1}\right)} \ (\mu = 1,\ 2)$$

is a result of finding a matrix-to-matrix derivative and a quadrodimensional matrix of the dimension $m \times m \times m \times m$ [31]. It must be taken into account when determining the derivative (8). By assuming that during the measurement the parameter does not change and by using the theorem about a derivative in matrix derivation and a derivative in a reverse matrix [31], we can write the following:

$$\frac{\partial(\mathbf{W_C})}{\partial \mathbf{C}_\mu^{-1}} = \mathbf{C}_p^{-1}\mathbf{C}_1 \frac{\partial \mathbf{C}_1^{-1}}{\partial \mathbf{C}_\mu^{-1}} \mathbf{C}_1 \mathbf{W} +$$

$$+\mathbf{C}_p^{-1}\mathbf{C}_2 \frac{\partial \mathbf{C}_2^{-1}}{\partial \mathbf{C}_\mu^{-1}} \mathbf{C}_2 \mathbf{W} - \mathbf{C}_p^{-1}\mathbf{C}_2 \frac{\partial \mathbf{C}_2^{-1}}{\partial \mathbf{C}_\mu^{-1}} \mathbf{C}_2 =$$

$$= (\mathbf{I}-\mathbf{W}) \frac{\partial \mathbf{C}_1^{-1}}{\partial \mathbf{C}_\mu^{-1}} \mathbf{C}_1 \mathbf{W} - \mathbf{W} \frac{\partial \mathbf{C}_2^{-1}}{\partial \mathbf{C}_\mu^{-1}} \mathbf{C}_2 (\mathbf{I}-\mathbf{W}),$$

$$(\mu = 1,\ 2). \qquad (11)$$

The matrix that looks as

$$\frac{\partial \mathbf{C}_\mu^{-1}}{\partial \mathbf{C}_\mu^{-1}} \quad (\mu = 1,\ 2),$$

which refers to the latter expression, is a quadrodimensional identity matrix of the dimension $m \times m \times m \times m$ [31], as the derivative C_{ij}^{-1} in the element C_{kl}^{-1} is nonzero only if $i = k$ and $j = l$, i. e.

$$\frac{\partial C_{ij}^{-1}}{\partial C_{kl}^{-1}} = \delta_{ik}\delta_{jl}.$$

Therefore, for the introduced model (7), the matrix derivative [31] is arranged as follows:

$$\mathbf{P}_\mu = \frac{\partial \mathbf{w}(\mathbf{W_C^\#})}{\partial(\mathbf{C}_\mu^{-1})^\#} = \left\| \frac{\partial W_{ij}}{\partial C_{\mu kl}^{-1}} \right\| =$$

$$= \left\| \frac{\partial \mathbf{W}^\#}{\partial C_{\mu 11}^{-1}} \quad \frac{\partial \mathbf{W}^\#}{\partial C_{\mu 12}^{-1}} \quad \ldots \quad \frac{\partial \mathbf{W}^\#}{\partial C_{\mu mm}^{-1}} \right\|. \qquad (12)$$

Then

$$\frac{\partial \mathbf{w}(\mathbf{W}\ )}{\partial C_{1ij}^{-1}} = \frac{\partial}{\partial C_{1ij}^{-1}} =$$

$$= \left[(\mathbf{I}-\mathbf{W}) \frac{\partial}{\partial\ _{1ij}} \mathbf{C}_1 \mathbf{W} \right] = \left[(\mathbf{I}-\mathbf{W}) \mathbf{E}_{ij} \mathbf{C}_1 \mathbf{W} \right] \qquad (13)$$

$$\frac{\partial \mathbf{w}(\mathbf{W_C^\#})}{\partial C_{2ij}^{-1}} = \frac{\partial \mathbf{W}^\#}{\partial C_{2ij}^{-1}} =$$

$$= \left[-\mathbf{W} \frac{\partial \mathbf{C}_2^{-1}}{\partial C_{2ij}^{-1}} \mathbf{C}_2 (\mathbf{I}-\mathbf{W}) \right]^\# = \left[-\mathbf{W}\mathbf{E}_{ij}\mathbf{C}_2(\mathbf{I}-\mathbf{W}) \right]^\#, \qquad (14)$$

where $\mathbf{E}_{ij} = \|\delta_{ij}\|$ is the matrix is which the element ij is equal to 1 and all others are equal to zero.

Consequently, expressions have been obtained for the blocks of the matrix $\mathbf{P}$ in recalculating the ECM of $\mathbf{C}_1^{-1}$ and $\mathbf{C}_2^{-1}$ is measuring the WCM $\mathbf{W}$. The completion of (10) with (13) and (14) allows determining the ECM of estimating the WCM. It is obvious from (13) and (14) that the accuracy of estimating the WCM $\mathbf{W}$ depends on the accuracy of values produced by the measuring devices according to the accuracy matrix $\mathbf{C}_\mu$ $(\mu = 1,\ 2)$. Meanwhile, the accuracy of estimating the parameter (i. e. the accuracy matrix expressed by $\mathbf{C}_1^{-1}$ and $\mathbf{C}_2^{-1}$) depends on the accuracy and nature of the primarily measured parameters as well as on the value of the parameter itself. The accuracy of evaluating the WCM $\mathbf{W}$, in its turn, also depends on the aforementioned parameters.

5. Discussion of the research results

The problem of optimizing the use of the same state vector estimates with an excessive number of primary estimates obtained simultaneously is reduced to a consistent application of an algorithm of filtering the estimates, whereas the resulting estimate in the case of independent primary measurements is the amount of weight estimates of the meters' state vector. If estimates are obtained through the state vector of various transformations of the surveillance vector, the accuracy of the assessment depends on the value of the assessment. In such cases, it is necessary to obtain simultaneously not only the state vector estimates but also the estimates of their accuracy matrices in order to determine the WCM. The impact of the WCM assessment accuracy (regardless of the methods of obtaining it) on the accuracy of the resulting estimate was previously analysed in the study [28] that showed that the accuracy of the resulting estimate depended on the accuracy of both the meters' estimates and the WCM assessment. One of the possible methods for WCM assessment consists in calculating it according to ECM estimates of the measured data. If the measured parameter does not change during the assessment process, the ECM of the estimates can be determined by independent discrete samples of the state vector estimates. This condition is justified because a change in the parameter may be ignored if this change between the measuring points is much less than the expected error and if it does not affect the magnitude of the error. In most cases, measuring is made with errors caused by various interferences. Therefore, density of the estimates' distribution usually appears to be normal or close to normal. If the data distribution is normal [29], the assessments of their mathematical expectation and variance (ECM) are independent. This fact allows building an independent variance meter of the parameter state vector. However, the existing dependence of the WCM on ECM estimates does not allow an assumption that the components of its formula are independent. If it is assumed that the errors are small and that there is some a priori information about some marginal true value of the WCM, then nonlinear dependencies in this margin can be approximated by linear dependencies. Such a priori information is justified if it is possible to know the scope of the parameter values or to determine the parameter value by one measuring device. The main advantage of the linearization method is that it allows obtaining explicit optimal estimates (in this case, the most plausible ones) of both the WCM and its ECM. The obtained expressions allow taking into account the impact of accuracy in a direct determination of the WCM of the resulting estimate accuracy.

The study was undertaken to develop the algorithms of using excess information, which were set forth in [24, 28]. Further research can be aimed at the following: finding other methods of assessing the WCM, analysing the impact of their accuracy on the accuracy of the resulting estimates, and implementing the findings in specific assessment filtration systems under conditions of data excess. The findings can be used in constructing measurement systems in various industries, as evidenced by the analysis of previous studies.

6. Conclusion

The research has resulted in obtaining analytical expressions for mathematical expectation and variance of components in assessing the error correlation matrix used to determine the parameter state vector. A method of linearization made it possible to determine the mathematical expectation and the error correlation matrix in estimating the weight coefficient matrix.

The study considered the structure of the error correlation matrix of the resulting estimate with account for the error correlation matrix in assessing the weight coefficient matrix. It has been proven that accuracy of a direct evaluation of the weight coefficient matrix depends on both the accuracy of the meters and the number of samples taken to determine the error correlation matrix for evaluating the parameter by different meters. Besides, it has been found that the quantity of the samples should be no less than the dimension of the estimated error correlation matrix. Accounting for the meters' data accuracy entails that the accuracy of estimating the weight coefficient matrix will in turn additionally depend on the parameters that affect the accuracy of such estimates. These include the accuracy and types of the initial measurements as well as the location of the object.

The obtained expressions make it possible not only to evaluate the accuracy of the direct assessment of the weight coefficient matrix but, as shown in the previous studies, to determine the effects of errors on the accuracy of the parameter evaluation. At a given accuracy of measuring the parameter by different meters and the acceptable errors of the resulting estimate, it is possible to expect a particular weight of errors in determining the weight coefficient matrix.

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