Методами узагальненої матриці розсіювання і часткових областей розв'язано задачу розсіювання хвилі – H10 на структурі, утвореної ланцюжком зв'язаних хрестоподібних резонаторів, які частково заповнюють позамежний хвилевід по висоті. Отримана багатомодова модель, яка верифікована експериментом. Використання нової моделі в методі інтелектуального синтезу і оптимізації конструкцій багатоланкових ХДР-фільтрів дозволяє враховувати довільну кількість хвиль в позамежному хвилеводі, що знижує похибку проектування

Ключові слова: НВЧ-фільтр, хвилеводно-діелектричні резонатори, хрестоподібний хвилевід, узагальнена матриця розсіювання

Методами обобщенной матрицы рассеяния и частичных областей решена задача рассеяния волны – H10 на многозвенной структуре, образованной цепочкой связанных крестообразных резонаторов, частично заполняющих запредельный волновод по высоте. Полученная многомодовая модель верифицирована экспериментом. Использование новой модели в методе интеллектуального синтеза и оптимизации конструкций многозвенных ВДР-фильтров, позволяет учитывать произвольное число волн в запредельном волноводе, что снижает погрешность проектирования

Ключевые слова: СВЧ-фильтр, волноводно-диэлектрические резонаторы, крестообразный волновод, обобщенная матрица рассеяния

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DEVELOPMENT OF A MATHEMATICAL MODEL OF MICROWAVE FILTER BASED ON THE PARTIALLY FILLED CROSS-SHAPED WAVEGUIDE-DIELECTRIC RESONATORS

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1. Introduction

Ultra-high frequency range is traditionally used for designing information channels in space technology, computer networks and mobile providers' networks, also in radars and defense complexes. The need for the super wide-band and super fast transmission of information led to transferring ultramodern telecommunication technologies into the UHF range [1, 2]. The characteristics of radio-relay stations substantially depend on the electrical parameters of multitier band pass filters, installed in their transceivers. Among the known microwave filters, it is expedient to consider the designs on the basis of the partially filled waveguide-dielectric resonators (WDR) with the use of leucosapphire or quartz, due to such high quality indicators as: its own high quality, rare spectrum parasitic oscillations, high level of delivered power [3]. The advantages of these filters also include technological feasibility in the UHF-range.

2. Analysis of scientific literature and the problem statement

The presence of both strict multimode electrodynamic models and efficient methods of optimization is necessary for solving the problem of optimization of the filters designs. Among the latter, preferable are contemporary methods of the artificial intelligence, such as genetic algorithms [4], artificial neural networks [5] and, in particular, expert systems. The key idea of the proposed method of intellectual synthesis and optimization of multitier WDR-filters is the logical analysis of frequency response, which is calculated on the basis of the mathematical model [6]. At the same time, this method, when applied to partially-filled cross-shaped resonators, relies on approximate calculation, which in program implementation takes into account an arbitrary number of waves at the cross point of a regular waveguide and the cut-off waveguide, and limited – in the cross-shaped [7]. Therefore, to reduce an error of the calculations, there appears a necessity to develop more efficient mathematical models, on the basis of the methods of the generalized scattering matrix (GSM) [8, 9], with the use of the method of transverse resonance [10] and numerical-analytical PRM in combination with the method of decomposition [11], allowing for the creation of numerical algorithms, which potentially consider an arbitrary number of modes on all scatterers. Given approaches are advisable to take as the basis of the electrodynamic model of WDR filters with cross-shaped cross-section.

3. The purpose and objectives of the study

The purpose of this study is an error reduction in the known solution of the problem of the H_{10} wave scattering on the chain of partially-filled WDR with cross-shaped cross-section, by developing of a new multimode model on the basis of the GSM method.

To achieve the set goal, the following tasks are to be solved:

– to solve the problem of the scattering by the GSM method, using physically substantiated assumptions;

– to implement numerically the obtained solution and to study the convergence of calculations;

– to conduct experimental verification of the calculated data.

4. Mathematical model of microwave filter with cross-shaped dielectric resonators, partially filled on height

Let us examine a planar junction (Fig. 1) of a cross-shaped layered (port 1) and hollow rectangular (port 2) waveguides. Horizontal dotted line $y=0$ corresponds to ideal electrical wall in the plane of symmetry, z=0 – ideal magnetic wall. The planes of symmetry correspond to the symmetry of the basic H_{10} mode of uniform rectangular waveguide, considered as excitatory. Considering the symmetry, let us confine ourselves to the examination of modes in one fourth of a waveguide. To find the S-matrix of a planar junction, we will use the method of partial regions, assuming the infinite conductivity of waveguide walls and imperfection of a dielectric. Let us designate

$$\vec a^{(i)} = \left\{a_n^{(i)}\right\}_{n=1}^{N_i} \text{ and } \vec b^{(i)} = \left\{b_n^{(i)}\right\}_{n=1}^{N_i}$$

the vectors of the amplitudes of the incident and scattered waves in the port $i=1,2$. The ratio between the number of the considered modes for the best convergence is determined by the following rule: at the master frequency, the basis of the frequency-dependent cross-shaped layered waveguide must be joined by all propagating modes and those damped modes, for which

$$\left|k_z\right|^2 \le \max\chi^2 - k_0^2, \tag{1}$$

where $\max\chi$ is the maximum transverse wave number of modes of the basis of a rectangular waveguide.

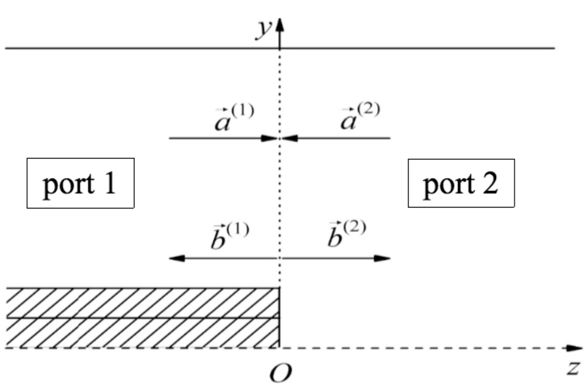

Fig. 1. Planar junction of rectangular and cross-shaped layered waveguides

In Fig. 1 $\max\chi$ is the maximum transverse wave number of modes of the basis of a rectangular waveguide.

The field in the considered waveguide junction is represented in the form

$$\begin{pmatrix}\vec E\\ \vec H\end{pmatrix} = \begin{cases} \sum\limits_{n=1}^{N_1} a_n^{(1)} \begin{pmatrix}\vec E_n^{(1,+)}\\ \vec H_n^{(1,+)}\end{pmatrix} + \sum\limits_{n=1}^{N_1} b_n^{(1)} \begin{pmatrix}\vec E_n^{(1,-)}\\ \vec H_n^{(1,-)}\end{pmatrix}, & z\le 0 \text{ (port 1)},\\ \sum\limits_{n=1}^{N_2} a_n^{(2)} \begin{pmatrix}\vec E_n^{(2,-)}\\ \vec H_n^{(2,-)}\end{pmatrix} + \sum\limits_{n=1}^{N_2} b_n^{(2)} \begin{pmatrix}\vec E_n^{(2,+)}\\ \vec H_n^{(2,+)}\end{pmatrix}, & z\ge 0 \text{ (port 2)}, \end{cases} \tag{2}$$

where

$$b_n^{(i)} = \sum_{p=1}^{N_1} a_p^{(1)} S_{np}^{(i,1)} + \sum_{p=1}^{N_2} a_p^{(2)} S_{np}^{(i,2)},$$
$$i=1,2, \quad n=1,2,\ldots,N_i, \tag{3}$$

$$S_{np}^{(i,j)},\ i,j=1,2,\ n=1,2,\ldots,N_i,\ p=1,2,\ldots,N_j$$

are the elements of S-matrix.

Here additional indices "+" and "–" determine the propagation (or damping) of wave along the axis z or in the opposite direction, accordingly. The indices N_i determine the number of modes of projection basis in the port number i.

Conditions of tangential components of electrical and magnetic fields at the cross point must be fulfilled:

$$\begin{cases} \vec E_t^{(1)} - \chi_{S_2}(x,y)\vec E_t^{(2)} = \vec 0, & z=0, \quad (x,y)\in S_1,\\ \vec H_t^{(1)} - \vec H_t^{(2)} = \vec 0, & z=0, \quad (x,y)\in S_2, \end{cases} \tag{4}$$

where S_1 and S_2 are the considered quarters of cross sections of cross-shaped and rectangular waveguides, respectively.

$$\chi_{S_2}(x,y) = \begin{cases} 1, & (x,y)\in \bar S_2,\\ 0, & (x,y)\notin S_2 \end{cases} \tag{5}$$

is the characteristic function of the set S_2.

The first equation in (4) is multiplied vectorially on the right by the system of functions $\vec h_k^{(1)}$, $k=1,2,\ldots,N_1$. We project the result on the axis z and integrate by the section of the comprehensive cross-shaped waveguide. Let us vectorially on the left multiply the second equation in (4) by the system of functions $\vec e_k^{(2)}$, $k=1,2,\ldots,N_2$. We project the result on the axis $o\ z$ and integrate by the section of the rectangular waveguide. As a result we obtain

$$\begin{cases} \int\limits_{S_1}\left[\vec E_t^{(1)}\times \vec h_k^{(1)}\right]\cdot \hat z ds - \int\limits_{S_2}\left[\vec E_t^{(2)}\times \vec h_k^{(1)}\right]\cdot \hat z ds = 0, & k=1,2,\ldots,N_1,\\ \int\limits_{S_2}\left[\vec e_k^{(2)}\times\left(\vec H_t^{(1)} - \vec H_t^{(2)}\right)\right]\cdot \hat z ds = 0, & k=1,2,\ldots,N_2, \end{cases} \tag{6}$$

where tangential components of the fields are considered only in the joint's plane $z=0$.

Using the representations (2), we obtain

$$
\begin{cases}
\sum\limits_{n=1}^{N_1}\left(\sum\limits_{p=1}^{N_1}a_p^{(1)}S_{np}^{(1,1)}+\sum\limits_{p=1}^{N_2}a_p^{(2)}S_{np}^{(1,2)}\right)\int\limits_{S_1}\left[\vec{E}_n^{(1,-)}\times\vec{h}_k^{(1)}\right]\cdot\hat{z}ds-\\
-\sum\limits_{n=1}^{N_2}\left(\sum\limits_{p=1}^{N_1}a_p^{(1)}S_{np}^{(2,1)}+\sum\limits_{p=1}^{N_2}a_p^{(2)}S_{np}^{(2,2)}\right)\times\\
\times\int\limits_{S_2}\left[\vec{E}_n^{(2,+)}\times\vec{h}_k^{(1)}\right]\cdot\hat{z}ds=\sum\limits_{n=1}^{N_1}a_n^{(1)}\int\limits_{S_1}\left[\vec{E}_n^{(1,+)}\times\vec{h}_k^{(1)}\right]\cdot\hat{z}ds+\\
+\sum\limits_{n=1}^{N_2}a_n^{(2)}\int\limits_{S_2}\left[\vec{E}_n^{(2,-)}\times\vec{h}_k^{(1)}\right]\cdot\hat{z}ds,\quad k=1,2,\ldots,N_1,\\
\sum\limits_{n=1}^{N_1}\left(\sum\limits_{p=1}^{N_1}a_p^{(1)}S_{np}^{(1,1)}+\sum\limits_{p=1}^{N_2}a_p^{(2)}S_{np}^{(1,2)}\right)\int\limits_{S_2}\left[\vec{e}_k^{(2)}\times\vec{H}_n^{(1,-)}\right]\cdot\hat{z}ds-\\
-\sum\limits_{n=1}^{N_2}\left(\sum\limits_{p=1}^{N_1}a_p^{(1)}S_{np}^{(2,1)}+\sum\limits_{p=1}^{N_2}a_p^{(2)}S_{np}^{(2,2)}\right)\times\\
\times\int\limits_{S_2}\left[\vec{e}_k^{(2)}\times\vec{H}_n^{(2,+)}\right]\cdot\hat{z}ds=\sum\limits_{n=1}^{N_1}a_n^{(1)}\int\limits_{S_2}\left[\vec{e}_k^{(2)}\times\vec{H}_n^{(1,+)}\right]\cdot\hat{z}ds+\\
+\sum\limits_{n=1}^{N_2}a_n^{(2)}\int\limits_{S_2}\left[\vec{e}_k^{(2)}\times\vec{H}_n^{(2,-)}\right]\cdot\hat{z}ds,\quad k=1,2,\ldots,N_2.
\end{cases}
\tag{7}
$$

By using the property of the orthogonality of the modes of the waveguides

$$
\vec{E}_{n,t}^{(1,\pm)}=\vec{e}_n^{(1)},\quad \vec{H}_{n,t}^{(1,\pm)}=\pm\vec{h}_n^{(1)}
\tag{8}
$$

$$
\vec{E}_{n,t}^{(2,\pm)}=\sqrt{W_n^{(2)}}\vec{e}_n^{(2)},\quad \vec{H}_{n,t}^{(2,\pm)}=\pm\frac{1}{\sqrt{W_n^{(2)}}}\vec{h}_n^{(2)},
\tag{9}
$$

we obtain

$$
\begin{cases}
\left(\sum\limits_{p=1}^{N_1}a_p^{(1)}S_{kp}^{(1,1)}+\sum\limits_{p=1}^{N_2}a_p^{(2)}S_{kp}^{(1,2)}\right)D_k-\\
-\sum\limits_{n=1}^{N_2}\sqrt{W_n^{(2)}}\left(\sum\limits_{p=1}^{N_1}a_p^{(1)}S_{np}^{(2,1)}+\sum\limits_{p=1}^{N_2}a_p^{(2)}S_{np}^{(2,2)}\right)M_{kn}=\\
=-a_n^{(1)}D_k+\sum\limits_{n=1}^{N_2}\sqrt{W_n^{(2)}}a_n^{(2)}M_{kn},\quad k=1,2,\ldots,N_1,\\
\sqrt{W_k^{(2)}}\sum\limits_{n=1}^{N_1}\left(\sum\limits_{p=1}^{N_1}a_p^{(1)}S_{np}^{(1,1)}+\sum\limits_{p=1}^{N_2}a_p^{(2)}S_{np}^{(1,2)}\right)M_{nk}+\\
+\sum\limits_{p=1}^{N_1}a_p^{(1)}S_{kp}^{(2,1)}+\sum\limits_{p=1}^{N_2}a_p^{(2)}S_{kp}^{(2,2)}=\\
=\sqrt{W_k^{(2)}}\sum\limits_{n=1}^{N_1}a_n^{(1)}M_{nk}+\frac{1}{4}a_k^{(2)},\quad k=1,2,\ldots,N_2.
\end{cases}
\tag{10}
$$

where designation for the integral relation is introduced

$$
\begin{gathered}
M_{kn}=4\int\limits_{S_2}\left[\vec{e}_n^{(2)}\times\vec{h}_k^{(1)}\right]\cdot\hat{z}ds,\\
k=1,2,\ldots,N_1,\quad n=1,2,\ldots,N_2.
\end{gathered}
\tag{11}
$$

Let us write out these integrals for the cases of both polarizations of the waves of a rectangular waveguide.

$$
\begin{aligned}
&M\left(\vec{h}^{(1)},\vec{e}_{h,2p+1,2q}^{(2)}\right)=\\
&=\frac{4(-1)^{p+q}\pi}{N_{h,2p+1,2q}}\Bigg\{-\frac{2p+1}{a}\sum_{m=0}^{M_1}\left(A_{e,m}^{(1)}+A_{h,m}^{(1)}\frac{k_z^2}{|k_z|}\frac{k_{x,m}^{(1)}}{\omega\mu_0\mu}\right)\times\\
&\times M_{mp}^{(c)}(a,a_1)I_3^{(c)}\left(k_{y,m}^{(1)},\frac{2\pi q}{b},0,h_1\right)+\\
&+\frac{2q}{b}\sum_{m=0}^{M_1}\left(A_{e,m}^{(1)}k_{x,m}^{(1)}-A_{h,m}^{(1)}\frac{k_z^2}{|k_z|}\frac{\left(k_{y,m}^{(1)}\right)^2}{\omega\mu_0\mu}\right)M_{mp}^{(s)}(a,a_1)I_3^{(s)}\left(k_{y,m}^{(1)},\frac{2\pi q}{b},0,h_1\right)+\\
&+\frac{2p+1}{4}\Bigg[-\left(A_{e,p}^{(2)}+A_{h,p}^{(2)}\frac{k_z^2}{|k_z|}\frac{k_{x,p}^{(2)}}{\omega\mu_0\mu}\right)I_3^{(c)}\left(k_{y,m}^{(2)},\frac{2\pi q}{b},y_2,h_2\right)+\\
&+\left(B_{e,p}^{(2)}-B_{h,p}^{(2)}\frac{k_z^2}{|k_z|}\frac{k_{x,p}^{(2)}}{\omega\mu_0\mu}\right)I_3^{(c)}\left(k_{y,m}^{(2)},\frac{2\pi q}{b},-y_3,h_2\right)+\\
&+\frac{2p+1}{4}\left(B_{e,p}^{(3)}-B_{h,p}^{(3)}\frac{k_z^2}{|k_z|}\frac{k_{x,p}^{(3)}}{\omega\mu_0}\right)I_3^{(c)}\left(k_{y,m}^{(3)},\frac{2\pi q}{b},-b/2,h_3\right)\Bigg]+\\
&+\frac{qa}{2b}\Bigg[\left(A_{e,p}^{(2)}k_{x,p}^{(2)}-A_{h,p}^{(2)}\frac{k_z^2}{|k_z|}\frac{\left(k_{y,p}^{(2)}\right)^2}{\omega\mu_0\mu}\right)I_3^{(s)}\left(k_{y,m}^{(2)},\frac{2\pi q}{b},y_2,h_2\right)-\\
&-\left(B_{e,p}^{(2)}k_{x,p}^{(2)}+B_{h,p}^{(2)}\frac{k_z^2}{|k_z|}\frac{\left(k_{y,p}^{(2)}\right)^2}{\omega\mu_0\mu}\right)I_3^{(s)}\left(k_{y,m}^{(2)},\frac{2\pi q}{b},-y_3,h_2\right)-\\
&-\left(B_{e,p}^{(3)}k_{x,p}^{(3)}+B_{h,p}^{(3)}\frac{k_z^2}{|k_z|}\frac{\left(k_{y,p}^{(3)}\right)^2}{\omega\mu_0}\right)I_3^{(s)}\left(k_{y,m}^{(3)},\frac{2\pi q}{b},-b/2,h_3\right)\Bigg]\Bigg\},
\end{aligned}
\tag{12}
$$

$$
\begin{aligned}
&M\left(\vec{h}^{(1)},\vec{e}_{e,2p+1,2q}^{(2)}\right)=\\
&=-\frac{4(-1)^{p+q}\pi}{N_{e,2p+1,2q}}\Bigg\{\frac{2q}{b}\sum_{m=0}^{M_1}\left(A_{e,m}^{(1)}+A_{h,m}^{(1)}\frac{k_z^2}{|k_z|}\frac{k_{x,m}^{(1)}}{\omega\mu_0\mu}\right)M_{mp}^{(c)}(a,a_1)I_3^{(c)}\left(k_{y,m}^{(1)},\frac{2\pi q}{b},0,h_1\right)+\\
&+\frac{2p+1}{a}\sum_{m=0}^{M_1}\left(A_{e,m}^{(1)}k_{x,m}^{(1)}-A_{h,m}^{(1)}\frac{k_z^2}{|k_z|}\frac{\left(k_{y,m}^{(1)}\right)^2}{\omega\mu_0\mu}\right)M_{mp}^{(s)}(a,a_1)I_3^{(s)}\left(k_{y,m}^{(1)},\frac{2\pi q}{b},0,h_1\right)+\\
&+\frac{qa}{2b}\Bigg[\left(A_{e,p}^{(2)}+A_{h,p}^{(2)}\frac{k_z^2}{|k_z|}\frac{k_{x,p}^{(2)}}{\omega\mu_0\mu}\right)I_3^{(c)}\left(k_{y,m}^{(2)},\frac{2\pi q}{b},y_2,h_2\right)-\left(B_{e,p}^{(2)}-B_{h,p}^{(2)}\frac{k_z^2}{|k_z|}\frac{k_{x,p}^{(2)}}{\omega\mu_0\mu}\right)\times\\
&\times I_3^{(c)}\left(k_{y,m}^{(2)},\frac{2\pi q}{b},-y_3,h_2\right)-\left(B_{e,p}^{(3)}-B_{h,p}^{(3)}\frac{k_z^2}{|k_z|}\frac{k_{x,p}^{(3)}}{\omega\mu_0}\right)I_3^{(c)}\left(k_{y,m}^{(3)},\frac{2\pi q}{b},-b/2,h_3\right)\Bigg]+\\
&+\frac{2p+1}{4}\Bigg[\left(A_{e,p}^{(2)}k_{x,p}^{(2)}-A_{h,p}^{(2)}\frac{k_z^2}{|k_z|}\frac{\left(k_{y,p}^{(2)}\right)^2}{\omega\mu_0\mu}\right)I_3^{(s)}\left(k_{y,m}^{(2)},\frac{2\pi q}{b},y_2,h_2\right)-\\
&-\left(B_{e,p}^{(2)}k_{x,p}^{(2)}+B_{h,p}^{(2)}\frac{k_z^2}{|k_z|}\frac{\left(k_{y,p}^{(2)}\right)^2}{\omega\mu_0\mu}\right)I_3^{(s)}\left(k_{y,m}^{(2)},\frac{2\pi q}{b},-y_3,h_2\right)-\\
&-\left(B_{e,p}^{(3)}k_{x,p}^{(3)}+B_{h,p}^{(3)}\frac{k_z^2}{|k_z|}\frac{\left(k_{y,p}^{(3)}\right)^2}{\omega\mu_0}\right)I_3^{(s)}\left(k_{y,m}^{(3)},\frac{2\pi q}{b},-b/2,h_3\right)\Bigg]\Bigg\},
\end{aligned}
\tag{13}
$$

where

$$I_3^{(c)}(\alpha,\beta,d,h)=\int_0^h e^{-|\mathrm{Im}\,\alpha|h}\cos(\alpha y)\cos(\beta(y+d))dy=$$

$$=e^{-|\mathrm{Im}\,\alpha|h}\begin{cases} h, \quad \alpha=\beta=0, \\ \dfrac{h}{2}\cos(\alpha d)+\dfrac{\sin(\alpha(2h+d))}{4\alpha}-\dfrac{\sin(\alpha d)}{4\alpha}, \\ \alpha=\beta\neq 0, \\ \dfrac{\sin(\alpha h-\beta(h+d))}{2(\alpha-\beta)}+ \\ +\dfrac{\sin(\alpha h+\beta(h+d))}{2(\alpha+\beta)}+\dfrac{\beta\sin(\beta d)}{\alpha^2-\beta^2}, \\ \alpha\neq\beta. \end{cases} \qquad (14)$$

$$I_3^{(s)}(\alpha,\beta,d,h)=\int_0^h \frac{e^{-|\mathrm{Im}\,\alpha|h}}{\alpha}\sin(\alpha y)\sin(\beta(y+d))dy=$$

$$=e^{-|\mathrm{Im}\,\alpha|h}\begin{cases} 0, \quad \beta=0, \\ -\dfrac{h}{\beta}\cos(\beta(h+d))+\dfrac{1}{\beta^2}\left[\sin(\beta(h+d))-\sin(\beta d)\right], \\ \alpha=0, \quad \beta\neq 0, \\ \dfrac{\beta}{\alpha^2}\left[\dfrac{h}{2}\cos(\alpha d)-\dfrac{\sin(\alpha(2h+d))}{4\alpha}+\dfrac{\sin(\alpha d)}{4\alpha}\right], \\ \alpha=\beta\neq 0, \\ \dfrac{1}{\alpha\beta}\left[\dfrac{\sin(\alpha h-\beta(h+d))}{2(\alpha-\beta)}-\dfrac{\sin(\alpha h+\beta(h+d))}{2(\alpha+\beta)}+\dfrac{\alpha\sin(\beta d)}{\alpha^2-\beta^2}\right], \\ \alpha\neq\beta\neq 0. \end{cases} \qquad (15)$$

Let us write down the system (10) in the matrix form

$$\begin{cases} \left(DS^{(1,1)}-MW_2^{1/2}S^{(2,1)}\right)\vec{a}^{(1)}+ \\ +\left(DS^{(1,2)}-MW_2^{1/2}S^{(2,2)}\right)\vec{a}^{(2)}=-D\vec{a}^{(1)}+MW_2^{1/2}\vec{a}^{(2)}, \\ \left(W_2^{1/2}M^TS^{(1,1)}+S^{(2,1)}\right)\vec{a}^{(1)}+ \\ +\left(W_2^{1/2}M^TS^{(1,2)}+S^{(2,2)}\right)\vec{a}^{(2)}=W_2^{1/2}M^T\vec{a}^{(1)}+\vec{a}^{(2)}. \end{cases} \qquad (16)$$

By considering separately cases $\vec{a}^{(1)}=\vec{0}$ and $\vec{a}^{(2)}=\vec{0}$, we obtain

$$\begin{cases} DS^{(1,2)}-MW_2^{1/2}S^{(2,2)}=MW_2^{1/2}, \\ W_2^{1/2}M^TS^{(1,2)}+S^{(2,2)}=I, \\ DS^{(1,1)}-MW_2^{1/2}S^{(2,1)}=-D, \\ W_2^{1/2}M^TS^{(1,1)}+S^{(2,1)}=W_2^{1/2}M^T. \end{cases} \qquad (17)$$

From the first equation in (17) we find $S^{(1,2)}$:

$$S^{(1,2)}=D^{-1}MW_2^{1/2}\left(I+S^{(2,2)}\right). \qquad (18)$$

Substituting this expression into the second equation of the system (17), we obtain

$$S^{(2,2)}=\left(I+W_2^{1/2}M^TD^{-1}MW_2^{1/2}\right)^{-1}\times$$
$$\times\left(I-W_2^{1/2}M^TD^{-1}MW_2^{1/2}\right). \qquad (19)$$

From the third equation in the (17) we find $S^{(1,1)}$:

$$S^{(1,1)}=D^{-1}MW_2^{1/2}S^{(2,1)}-I. \qquad (20)$$

Substituting this expression into the fourth equation of the system (17), we obtain

$$S^{(2,1)}=2\left(I+W_2^{1/2}M^TD^{-1}MW_2^{1/2}\right)^{-1}W_2^{1/2}M^T. \qquad (21)$$

Let us define

$X=MW_2^{1/2}$ is the matrix of size $N_1\times N_2$, (22)

then

$$S^{(1,1)}=D^{-1}XS^{(2,1)}-I=$$
$$=2D^{-1}X\left(I+X^TD^{-1}X\right)^{-1}X^T-I, \qquad (23)$$

$$S^{(1,2)}=D^{-1}X\left(I+S^{(2,2)}\right)=$$
$$=D^{-1}X\left(I+\left(I+X^TD^{-1}X\right)^{-1}\left(2I-\left(I+X^TD^{-1}X\right)\right)\right)=$$
$$=2D^{-1}X\left(I+X^TD^{-1}X\right)^{-1}, \qquad (24)$$

$$S^{(2,1)}=2\left(I+X^TD^{-1}X\right)^{-1}X^T, \qquad (25)$$

$$S^{(2,2)}=\left(I+X^TD^{-1}X\right)^{-1}\left(I-X^TD^{-1}X\right). \qquad (26)$$

By defining

$$A=I+X^TD^{-1}X, \qquad (27)$$

we obtain for $S^{(2,2)}$

$$AS^{(2,2)}=I-X^TD^{-1}X \qquad (28)$$

and for the remaining blocks of the scattering matrix

$$AS^{(2,1)}=2X^T, \qquad (29)$$

$$S^{(1,2)}=2D^{-1}XA^{-1}=D^{-1}S^{(2,1)T}, \qquad (30)$$

$$S^{(1,1)}=S^{(1,2)}X^T-I. \qquad (31)$$

For obtaining the formula (30), the symmetry of the matrix A was considered

$$A^T=A. \qquad (32)$$

Thus, for finding S-matrix it suffices to invert the matrix A of the size $N_2\times N_2$. Let us note that the elements of S–matrix do not fulfill the condition of reciprocity ($S^{(2,1)}\neq S^{(1,2)T}$) because the basis of a cross-shaped waveguide was not normalized according to the power ($D\neq 1$).

Since in reality the values of the protrusions of a cross-shaped waveguide are sufficiently low, then as the approximation of the solution of the task, let us examine a symmetrical planar joint of the rectangular waveguide $c\times b$ with a rectangular waveguide $a\times b$ of less width $(a<c)$ and identical height. Fig. 2 displays a half of such a joint. Horizontal dot-and-dash line $x=0$ corresponds to an ideal magnetic plane of symmetry.

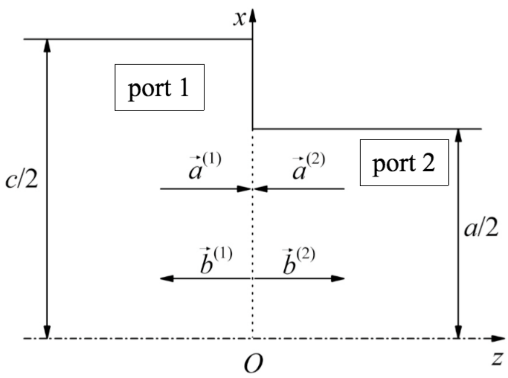

Fig. 2. Symmetrical planar joint of two rectangular waveguides of identical height

For calculating the scattering matrix of this joint, the approach, described above, is used, with the only difference being that the ratio (9) and orthonormality condition is fulfilled for the bases of both ports. Scattering matrices in this case are calculated by the formulas

$$AS^{(2,2)}=I-X^{T}X, \tag{33}$$

$$AS^{(2,1)}=2X^{T}, \tag{34}$$

$$S^{(1,2)}=2XA^{-1}=S^{(2,1)T}, \tag{35}$$

$$S^{(1,1)}=S^{(1,2)}X^{T}-I, \tag{36}$$

where

$$X=W_1^{-1/2}MW_2^{1/2}, \tag{37}$$

$$A=I+X^{T}X, \tag{38}$$

$M=\{M_{kn}\}_{k=1,n=1}^{N_1,N_2}$ is the matrix of moments,

here

$$M_{kn}=4\int_0^{a/2}dx\int_0^{b/2}\left[\vec{e}_n^{(2)}\times\vec{h}_k^{(1)}\right]\cdot\hat{z}dy,$$
$$k=1,2,\ldots,N_1,\quad n=1,2,\ldots,N_2. \tag{39}$$

Let us give the values of relation integrals between the modes of different polarization

$$M\left(h_{h,2p+1,2q}^{(1)},e_{h,2r+1,2s}^{(2)}\right)=$$
$$=4\int_0^{a/2}dx\int_0^{b/2}\left[e_{h,2r+1,2s}^{(2)}\times h_{h,2p+1,2q}^{(1)}\right]\cdot\hat{z}dy=$$
$$=\delta_{qs}\frac{(-1)^{p+q+r+s}\pi^2 b}{N_{h,2p+1,2q}N_{h,2r+1,2s}}\times$$
$$\times\left[\frac{(2p+1)(2r+1)}{ac}M_{pr}^{(c)}(a,c)+\frac{4qs}{b^2}M_{pr}^{(s)}(a,c)\right], \tag{40}$$

$$M\left(h_{h,2p+1,2q}^{(1)},e_{e,2r+1,2s}^{(2)}\right)=0, \tag{41}$$

$$M\left(h_{e,2p+1,2q}^{(1)},e_{h,2r+1,2s}^{(2)}\right)=$$
$$=4\int_0^{a/2}dx\int_0^{b/2}\left[e_{h,2r+1,2s}^{(2)}\times h_{e,2p+1,2q}^{(1)}\right]\cdot\hat{z}dy=$$
$$=\delta_{qs}\frac{2(-1)^{p+q+r+s}\pi^2}{N_{e,2p+1,2q}N_{h,2r+1,2s}}\times$$
$$\times\left[\frac{q(2r+1)}{a}M_{pr}^{(c)}(a,c)-\frac{(2p+1)s}{c}M_{pr}^{(s)}(a,c)\right], \tag{42}$$

$$M\left(h_{e,2p+1,2q}^{(1)},e_{e,2r+1,2s}^{(2)}\right)=$$
$$=4\int_0^{a/2}dx\int_0^{b/2}\left[e_{e,2r+1,2s}^{(2)}\times h_{e,2p+1,2q}^{(1)}\right]\cdot\hat{z}dy=$$
$$=\delta_{qs}\frac{(-1)^{p+q+r+s}\pi^2 b}{N_{e,2p+1,2q}N_{e,2r+1,2s}}\times$$
$$\times\left[\frac{4qs}{b^2}M_{pr}^{(c)}(a,c)+\frac{(2p+1)(2r+1)}{ac}M_{pr}^{(s)}(a,c)\right]. \tag{43}$$

Because both joined rectangular waveguides have an identical height, the modes with different indices n along y do not interact with each other. Therefore, the efficiency of the calculation of the S-matrix of a planar junction can be increased if we examine separately the tasks of scattering for each value of the index $n=0,2,\ldots$, followed by the assignment of found coefficients of partial S-matrices to the corresponding elements of complete S-matrix.

In contrast to the scattering matrix S, expressing amplitudes of the scattered modes through the amplitudes of the falling modes (3), the matrix of transfer T expresses the amplitudes of modes (falling and scattered) in the port 1 through the amplitudes of modes in the port 2.

$$\begin{cases}\vec{b}^{(1)}=T^{(1,1)}\vec{a}^{(2)}+T^{(1,2)}\vec{a}^{(2)},\\ \vec{a}^{(1)}=T^{(2,1)}\vec{a}^{(2)}+T^{(2,2)}\vec{b}^{(2)}.\end{cases} \tag{44}$$

The blocks of the matrix of transfer can be calculated through the blocks of the scattering matrix according to the formulas

$$T^{(1,1)}=S^{(1,2)}-S^{(1,1)}\left[S^{(2,1)}\right]^{-1}S^{(2,2)}, \tag{45}$$

$$T^{(1,2)}=S^{(1,1)}\left[S^{(2,1)}\right]^{-1}, \tag{46}$$

$$T^{(2,1)}=-\left[S^{(2,1)}\right]^{-1}S^{(2,2)}, \tag{47}$$

$$T^{(2,2)}=\left[S^{(2,1)}\right]^{-1}. \tag{48}$$

5. Results of calculation and experiment

Numerical algorithm was implemented programmatically based on the developed mathematical model, and a study of convergence with the purpose of establishing a necessary number of waves, considered in the cut-off waveguide, was also carried out; the diagrams of the convergence of mathematical model are represented in Fig. 3.

Fig. 3 shows that in order to ensure less than 0,2 % error in the calculation, it suffices to consider 5–7 modes in the cut-off cross-shaped waveguide for the selected parameters of the problem.

Mathematical calculation of the designs of one-tier filters and their comparison to the experimental prototypes was performed for the purpose of the verification of the developed mathematical model; the results of comparison are in Table 1; diagrams of frequency responses of one-tier filters are represented in Fig. 4, 5.

Table 1

Study of experimental prototypes

Parameters of one-tier filters	First	Second
Calculated frequency, GHz	20,77	20,73
Experimental frequency, GHz	20,76	20,71
Width of the regular waveguide, mm	11,0	11,0
Width of the cut-off cross-shaped waveguide (“protrusion” – 0.3 mm), mm	4,5	4,5
Height of the waveguides, mm	5,5	5,5
Length of the cut-off waveguide, mm	15,0	15,0
Width of resonator, mm	4,5	4,5
Height of resonator, mm	3,2	2,9
Length of resonator, mm	2,4	3,0
Dielectric permittivity of the resonator	11,5	11,5

Conducted experimental study showed that the divergence between the calculated and measured frequencies did not exceed an error of measurements, which is a confirmation of the adequacy of the calculations, fulfilled on the base of the developed model.

The obtained mathematical model of microwave filter with cross-shaped WDR makes it possible to consider an arbitrary number of waves in the cut-off waveguide, and so to increase the accuracy of calculations. This model is implemented in the original method of intellectual synthesis and optimization of designs of multitier microwave filters [12]. The results of the study are planned to apply in the development of a new model of microwave filter with resonators of different classes.

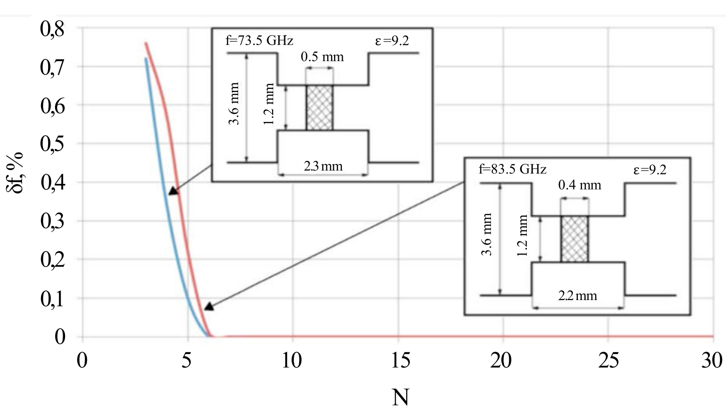

Fig. 3. Diagrams of convergence of the microwave filter for the frequencies 73,5 and 83,5 GHz, where δf is the frequency change in per cents, N is the number of calculated waves in the cut-off waveguide

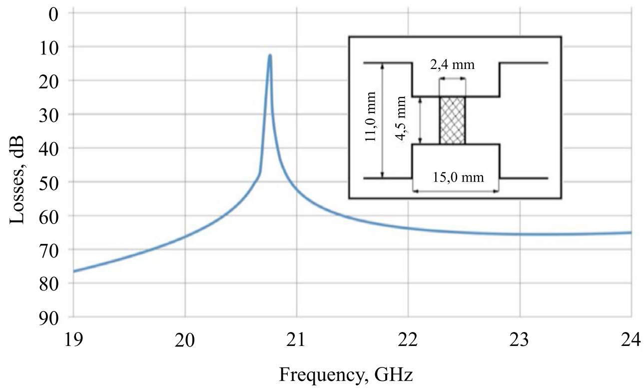

Fig. 4. Calculated frequency response of the first prototype

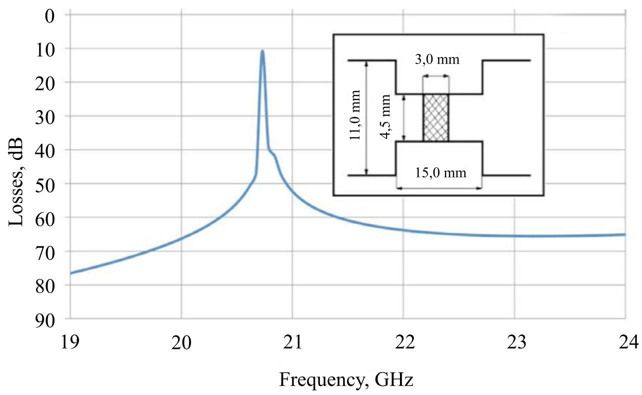

Fig. 5. Calculated frequency response of the second prototype

6. Conclusions

1. Mathematical model of microwave filter with the cross-shaped cross section of a waveguide, partially filled with dielectric by height, was developed. This model is the solution to the problem of H_{10} wave scattering along the chain of partially filled WDR. The solution to the problem was carried out by applying the GSM and partial regions methods. It was shown that the applying of a matrix apparatus ultimately led to the system of equations, taking into consideration both the propagating and damped waves. Therefore, with the programmatic implementation of the model, there appears a possibility to consider an arbitrary number of waves in the cut-off waveguide, which increases the accuracy of calculations when designing filters of this type.

2. Numerical algorithm was programmatically implemented and the convergence of calculations was studied, as a result of which we established that in order to ensure the error in the calculation less than 0.2 %, it was sufficient to consider not more than 5–7 modes in the cut-off cross-shaped waveguide.

3. The experimental study of the calculated designs of one-tier filters was performed. This research showed that the divergence between the calculated and measured frequencies did not exceed one percent that is comparable to an error of measurement and is the confirmation of the adequacy of the calculations, fulfilled on the basis of the developed model.

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