Для випадку плоскої деформації область лінії локалізації пластичної деформації на ділянці зміцнення представлена у вигляді смуги кінцевої довжини. Береги смуги можуть вільно ковзати, але при цьому повинні залишатися в контакті. Побудовано замкнутий аналітичний розв'язок задачі. Визначено орієнтація смуги, поля швидкостей зміни напруг і переміщень, отримана залежність довжини лінії локалізації від величини навантаження

Ключові слова: теорія пластичності, мікромеханіка, полоси зсуву, теорія мікродеформацій, біфуркація, полікристали, критичне навантаження

Для случая плоской деформации область линии локализации пластической деформации на участке упрочнения представлена в виде полосы конечной длины. Берега полосы могут свободно скользить, но при этом должны оставаться в контакте. Построено замкнутое аналитическое решение задачи. Определены ориентация полосы, поля скоростей изменения напряжений и перемещений, получена зависимость длины линии локализации от величины нагрузки

Ключевые слова: теория пластичности, микромеханика, полосы сдвига, теория микродеформаций, бифуркация, поликристалл, критическая нагрузка

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ANALYSIS OF ORIGIN OF SHEAR BANDS IN A REINFORCING ELASTIC-PLASTIC BODY

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1. Introduction

It is known that at a definite stage of loading a reinforcing elastic-plastic body, the process of the homogenous plastic deformation branches off and localization of the plastic deformation occurs. In the formal plan, the problem of localization comes down to determining own values and forms of linearized boundary problem for the difference in fields of the rates of change in stresses and of the rates of deformations and displacements. This problem is usually solved for an infinite body on the assumption that the localization region is the plane of plastic shear.

A localization region at the point of bifurcation is presented in the form of a line of discontinuity of speeds of the finite length. It is a very thin layer of the material, the banks of which can slide freely but in this case they must stay in contact. The problem is reduced to the linearized problem in rates for the abrupt change in the rate of displacements along the line of localization. The theory of microdeformation [1], which leads to the singular surface of fluidity, is used for describing mechanical behavior, which is very important in the problems of localization.

2. Scientific literature analysis and problem setting

In the works [2–4], it is shown that a localized deformation of time-independent materials is connected with the loss of ellipticity of the linearized defining relations of continuous load and leads to bifurcation in the form of the flat band of infinite length. The sensitivity of the bifurcation point to the properties of defining relations is established in these works. The impossibility of using the classical associated law of flow with smooth fluidity surface for realistic description of a slip band in reinforced metals is shown in the studies [5–7]. The need for the introduction of theories of plasticity with the singular surface of fluidity is confirmed in them. Determining relations of this kind emerge with the formulation of the equations of the state of polycrystalline materials based on micromechanical prerequisites [8, 9]. The presence of an angular point on the surface of fluidity allows abrupt changing in the direction of the deformation rate; it is examined in more detail in the papers [10, 1].

In the works [11–13], attention was paid to the existence of a localization line of finite length, which was represented as a crack, the edges of which can slide freely relative to each other but not diverge. Within the framework of this model, it was shown that the crack has special features on its edges and has a tendency of developing along the crack line. In this case, the crack was assigned as an initial disturbance and the dependence of its length on the acting load was not established. This problem has not been solved until now. Let us use the methods proposed in the papers [14, 15] for the formulation of the criterion of brittle fracture. In the work [16–18], the problem of formation of the shear lines was solved with the help of method of finite elements, but the problem of an analytical solution remains open. The influence of the distribution of dislocations and the initial defects on the localization of the shear was explored in the paper [19]. In the work [20] the phenomenon of localization for the case of the combined load was studied.

3. The purpose and tasks of the study

The purpose of the work is building up a closed analytical solution of the problem of localization of the shear deformation in a reinforcing elastic-plastic body.

To achieve the aim, it is necessary to solve the following tasks:

– solution of the problem of localization in the form of a band of discontinuity of rates of the finite length;

– formulation of the criterion of destruction with the use of the Novozhilov condition;

– determination of the critical length of a slip line on the basis of monotonous loading conditions (without partial load reliefs).

4. Localization in form of a shear band

4. 1. Localization of shear band of infinite length

Let us examine a shear localization in an incompressible elastic-plastic body, which is located under the plane strain conditions. Let the strain occur in the plane $0x_1x_2$, then

$$v_3=0, dv_1/dx_3=dv_2/dx_3=dv_3/dx_3=0$$

the component n_3 of the vector of normal to the line of localization also equals zero, where v_k is the components of the rates vector. Assume the body is under the influence of tensile stresses only

$$\sigma_{11}=\sigma_1, \sigma_{22}=\sigma_2\ (\sigma_{12}=\sigma_{13}=\sigma_{22}=0)$$

and in view of incompressibility of the material

$$\sigma_{33}=(\sigma_1+\sigma_2)/2.$$

Here and throughout, the averaging sign is omitted.

The equations of stability in this case can be represented in the following form:

$$\frac{1}{2}\frac{\partial}{\partial x_1}(\Delta\dot{\tau}_{11}-\Delta\dot{\tau}_{22})+\frac{\partial}{\partial x_2}\Delta\dot{\tau}_{21}=-\frac{1}{2}\frac{\partial}{\partial x_1}(\Delta\dot{\tau}_{11}+\Delta\dot{\tau}_{22}),$$

$$\frac{1}{2}\frac{\partial}{\partial x_2}(\Delta\dot{\tau}_{11}-\Delta\dot{\tau}_{22})-\frac{\partial}{\partial x_1}\Delta\dot{\tau}_{12}=\frac{1}{2}\frac{\partial}{\partial x_2}(\Delta\dot{\tau}_{11}+\Delta\dot{\tau}_{22}), \qquad (1)$$

where $\Delta\dot{\tau}_{ij}=\dot{\tau}_{ij}^{b}-\dot{\tau}_{ij}^{0}$ is the difference of the fields of rates of change in the first tensor of Piola-Kirchhoff.

Condition of incompressibility:

$$\Delta v_{i,i}=0.$$

In this case, the connection of $\Delta\dot{\tau}_{ij}$ with the Jaumann derivative of the Cauchy stress tensor $\Delta\sigma_{ij}^{\nabla}$ follows from:

$$\Delta\dot{\tau}_{11}=\Delta\sigma_{11}^{\nabla}-\sigma_1\frac{\partial v_1}{\partial x_1}, \quad \Delta\dot{\tau}_{22}=\Delta\sigma_{22}^{\nabla}-\sigma_2\frac{\partial v_2}{\partial x_2},$$

$$\Delta\dot{\tau}_{12}=\Delta\sigma_{12}^{\nabla}-\frac{1}{2}(\sigma_1+\sigma_2)\frac{\partial v_1}{\partial x_2}+\frac{1}{2}(\sigma_1-\sigma_2)\frac{\partial v_2}{\partial x_1},$$

$$\Delta\dot{\tau}_{21}=\Delta\sigma_{21}^{\nabla}-\frac{1}{2}(\sigma_1-\sigma_2)\frac{\partial v_1}{\partial x_2}-\frac{1}{2}(\sigma_1+\sigma_2)\frac{\partial v_2}{\partial x_1},$$

where the equation of moments is taken into account:

$$\Delta\dot{\tau}_{12}-\Delta\dot{\tau}_{21}=\sigma_1\frac{\partial v_2}{\partial x_1}-\sigma_2\frac{\partial v_1}{\partial x_2}.$$

We will use the theory of plasticity, which considers microstresses and microdeformations (theory of microdeformations) [1, 21], intended for describing plastic deformation of polycrystalline metals. In theory, the inhomogeneity of plastic deformation at the grain level of polycrystal and the nonuniformity of distribution of defects is represented in the form of the sum of local plastic deformations corresponding to its own surface of fluidity and the system of internal forces.

The detailed account of theoretical principles can be found in the works [1, 15, 21]. Determining ratios of the theory of microdeformation at the plane strain on condition of monotonous load (without partial load reliefs) may be presented in the form:

$$\Delta\dot{\tau}_{11}-\Delta\dot{\tau}_{22}=[2G_p-\frac{1}{2}(\sigma_1+\sigma_2)]\left(\frac{\partial\Delta v_1}{\partial x_1}-\frac{\partial\Delta v_2}{\partial x_2}\right),$$

$$\Delta\dot{\tau}_{12}=[G_q+\frac{1}{2}(\sigma_1-\sigma_2)]\frac{\partial\Delta v_2}{\partial x_1}+[G_q-\frac{1}{2}(\sigma_1+\sigma_2)]\frac{\partial\Delta v_1}{\partial x_2},$$

$$\Delta\dot{\tau}_{21}=[G_q-\frac{1}{2}(\sigma_1+\sigma_2)]\frac{\partial\Delta v_2}{\partial x_1}+[G_q-\frac{1}{2}(\sigma_1-\sigma_2)]\frac{\partial\Delta v_1}{\partial x_2}, \quad (2)$$

where G_p, G_q are the rigidity modules of the continuous load and orthogonal additional loading, which are determined by the formulas:

$$\frac{1}{2G_p}=\left[\frac{1}{2G}+\frac{C(\alpha)}{A_1}\right], \frac{1}{2G_q}=\left[\frac{1}{2G}+\frac{B(\alpha)}{A_1}\right],$$

where

$$B(\alpha)=\frac{\pi^2}{30}\left(8-15\cos a+10\cos^3 a-3\cos^5 a\right),$$

$$C(\alpha)=A(\alpha)-\frac{\mu F^2(\alpha)}{1+\mu},$$

$$A(\alpha)=\frac{2\pi^2}{15}\left(2-5\cos^3 a+3\cos^5 a\right),$$

$$F(\alpha)=\frac{\pi^2}{2}\left(1-2\cos^2 a+\cos^4 a\right), \mu=A_3/A_1.$$

It is necessary to use the experimental data for determining the constants of material A_1, A_3, as it is described in the paper [1].

Besides, α is the apex angle at the hypercone top of directions of active microplastic deformation, the axis of which is directed along the guiding deviator. The value of the angle is determined by the formula:

$$\cos\alpha=\frac{\tau_0+\kappa(t)}{r(t)},$$

where τ_0 is the fluidity limit

$$r(t)=\sqrt{r(t):r(t)}, \kappa(t)=\int_0^t\frac{\mu F}{r(t)(1+\mu\Omega)}r:dr.$$

Furthermore, according to the theory of microdeformations, we can define the limits of the use of ratios as:

$$tg\beta \le \frac{\cos\alpha - \kappa F(\alpha)}{\sin\alpha}, \quad (3)$$

where β is the angle of fracture of the loading trajectory at the point of bifurcation.

Let us note that the simple loading occurs with the biaxial tension (compression) of an incompressible body under the plane strain conditions. As a consequence, the theory of microdeformation transfers to the deformation theory of plasticity and the modules of tangential rigidity transfer respectively to the tangent and the secant modules of the diagram of pure shear.

Determining ratios in this case can be represented in the following form (with regard to incompressibility):

$$\Delta\dot{\tau}_{ij} = K_{ijkl} v_{l,k} + \dot{q}\delta_{ij}.$$

Components of the matrix K_{ijkl} take the form:

$$K_{1111} = G_q(\xi - k - \eta), K_{1122} = -G_q\xi, K_{1112} = K_{1121} = 0,$$

$$K_{2211} = -G_q\xi, K_{2222} = G_q(\xi + k - \eta), K_{2212} = K_{2221} = 0,$$

$$K_{1212} = G_q(1+k), K_{1221} = K_{2112} = G_q(1-\eta), K_{2121} = G_q(1-k)$$

and

$$k = \frac{\sigma_1 - \sigma_2}{2G_q}, \eta = \frac{\sigma_1 + \sigma_2}{2G_q}, \xi = \frac{G_p}{G_q}.$$

Let us introduce the function of current $\psi(x_1, x_2)$, which provides identical fulfillment of incompressibility conditions:

$$\Delta v_1 = \frac{\partial\psi}{\partial x_2}, \Delta v_2 = -\frac{\partial\psi}{\partial x_1}.$$

Then we obtain a differential equation in the partial derivatives of the fourth order relative to the function of current from the equations of stability (1) with regard to determining ratios (2):

$$(1+k)\frac{\partial^4\psi}{\partial x_1^4} + 2(2\xi - 1)\frac{\partial^4\psi}{\partial x_1^2\partial x_2^2} + (1-k)\frac{\partial^4\psi}{\partial x_2^4} = 0.$$

Following the papers [6, 14, 15], the solution to this equation can be presented in the form of the analytic function F:

$$\psi(x_1, x_2) = F(x_1 + \Omega x_2),$$

where F is the arbitrary function, Ω is the complex constant, which satisfies the biquadratic equation:

$$(1+k) + 2(2\xi - 1)\Omega^2 + (1-k)\Omega^4 = 0. \quad (4)$$

In a general case, the equation (4) has four different roots:

$$\Omega_j^2 = \frac{1 - 2\xi + (-1)^j\sqrt{(1-2\xi)^2 - (1-k^2)}}{1-k}.$$

Depending on the nature of roots, the area of ellipticity (E), hyperbolicity (G) and parabolicity (P) is defined.

The boundary between regimes E and P, and G and P is assigned by the line k=1. The boundary between E and G areas is the parabola $2\xi = 1 - \sqrt{1-k^2} > 1$. Two shear bands, the slope of which is assigned by the angle $\pm\theta_0$ between the shear band and the axis $0x_1$, simultaneously appear on the border of the two regions (E/G):

$$\cot^2\vartheta_0 = \frac{1 + 2\text{sign}(k)\sqrt{\xi(1-\xi)}}{1-2\xi}.$$

On the border of E/P mode, only one band, parallel to axis $x_1(x_2)$, appears when

$$k = 1(k = -1): \vartheta_0 = 0, k = 1, \vartheta_0 = \frac{\pi}{2}, k = -1.$$

Since k, η, ξ depend on parameter α, it is possible to plot the dependency $k - \xi$ and to find the point of intersection with the line (E/G), using their parametric representation,

4. 2. Line of localization of finite length

Let us examine, as in the preceding chapter, a uniform, preliminarily plastically deformed, incompressible elastic-plastic body under the plane strain conditions. Meanical behavior of the body is characterized by the defining equations in rates of the theory of microdeformation (2). With a definite load, the line of localization of finite length can be formed in the body, which may be represented as the weakened surface, along which the adjacent layers of material can slide freely but they stay in contact, as it was adopted in the works [11–13]. This slip line is different from the section because it receives normal cohesive forces.

The shear band of finite length in the plane $0\hat{x}_1\hat{x}_2$ will be represented by the line of discontinuity of rates with the length $2l$, located along the axis $0\hat{x}_1$ (Fig. 1). We will consider that the line of discontinuity has slope angle ϑ_0 to axis $0x_1$, i. e., the same as in the case of the infinite band, examined in the preceding chapter.

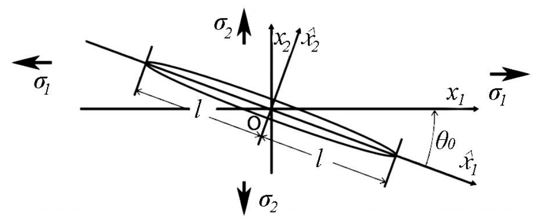

Fig. 1. Shear band of finite length

The transition to the system of coordinates $0\hat{x}_1\hat{x}_2$ is achieved with the use of matrix of rotation Q, determining the turning of axes $0\hat{x}_1\hat{x}_2$ by the angle ϑ_0 in relation to system $0x_1x_2$ (accepted by the positive counterclockwise rotation) according to the formula:

$$\hat{x} = [Q^T]\cdot x, [Q] = \begin{bmatrix} \cos\vartheta_0 & \sin\vartheta_0 \\ -\sin\vartheta_0 & \cos\vartheta_0 \end{bmatrix}.$$

The rates of change in the components of the first tensor of Piola-Kirchhoff, the rates of displacements and the gradient may be expressed in the frame of reference $0\hat{x}_1\hat{x}_2$ as follows:

$$\Delta\hat{\dot{\tau}} = Q^T \Delta\dot{\tau} Q,\ \hat{v} = Q^T \bar{v},\ \hat{\nabla}\hat{v} = Q^T \nabla\bar{v} Q.$$

Let us represent determining ratios in the form:

$$\Delta\hat{\dot{\tau}} = \hat{K}(\hat{\nabla}v)^T + \dot{q}e,$$

where the tensor of conversion of the fourth order is defined as:

$$\hat{K}_{ijhk} = Q_{li}Q_{mj}K_{lmno}Q_{nh}Q_{ok},$$

and the indices take values 1 and 2.

Following the paper [13], let us introduce the function of current $\hat{\psi}(\hat{x}_1, \hat{x}_2)$ in the form:

$$\hat{\psi}(\hat{x}_1,\hat{x}_2) = \frac{\hat{\dot{\tau}}_{21}^{\infty}}{2G_q}\sum_{j=1}^{2}\mathrm{Re}\left[A_j f(\hat{z}_j)\right],$$

where

$$f(\hat{z}_j) = \hat{z}_j^2 - \hat{z}_j\sqrt{\hat{z}_j^2 - l^2} + l^2 \ln\left(\hat{z}_j + \sqrt{\hat{z}_j^2 - l^2}\right),$$

$$\hat{z}_j = \hat{x}_1 + W_j\hat{x}_2,\ W_j = \frac{\sin\vartheta_0 + \Omega_j\cos\vartheta_0}{\cos\vartheta_0 - \Omega_j\sin\vartheta_0}.$$

We will use the boundary conditions on the line of localization for determining the constants A_j They are based on the rate of change in tangent stresses being equal to zero and the continuity of normal components of the first tensor of Piola-Kirchhoff, as well as continuity of rates of normal displacements of the bifurcational solution. For the difference of fields, we obtain respectively:

$$\Delta\hat{\dot{\tau}}_{21}(\hat{x}_1, 0^{\pm}) = -\hat{\dot{\tau}}_{21}^{\infty},\ \forall\left|\hat{x}_1\right| < l,$$

$$\left[\Delta\hat{\dot{\tau}}_{22}(\hat{x}_1, 0)\right] = 0,\ \forall\left|\hat{x}_1\right| < l,$$

$$\left[\Delta\hat{v}_2(\hat{x}_1, 0)\right] = 0,\ \forall\left|\hat{x}_1\right| < l.$$

Using the solution and boundary conditions on the line of discontinuity, we arrive to the system of linear algebraic equations relative to the constants A_j:

$$\begin{bmatrix} c_{11} & c_{21} & c_{12} & c_{22} \\ -c_{21} & c_{11} & -c_{22} & c_{12} \\ c_{31} & c_{41} & c_{32} & c_{42} \\ -c_{41} & c_{31} & -c_{42} & c_{32} \end{bmatrix}\begin{bmatrix} \mathrm{Re}[A_1] \\ \mathrm{Im}[A_1] \\ \mathrm{Re}[A_2] \\ \mathrm{Im}[A_2] \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -1 \\ 0 \end{bmatrix},$$

coefficients of which are determined by the formulas:

$$\begin{cases} 2\mu c_{1j} = \hat{K}_{1112} - \hat{K}_{1222} - \mathrm{Re}[W_j][\hat{K}_{1111} - 2\hat{K}_{1122} - \hat{K}_{1221} + \hat{K}_{2222} + \\ +\mathrm{Re}[W_j](2\hat{K}_{1121} - 2\hat{K}_{2122} + \mathrm{Re}[W_j]\hat{K}_{2121})] + \\ +\mathrm{Im}[W_j]^2(2\hat{K}_{1121} - 2\hat{K}_{2122} + 3\mathrm{Re}[W_j]\hat{K}_{2121})], \\ 2\mu c_{2j} = \mathrm{Im}[W_j][\hat{K}_{1111} - 2\hat{K}_{1122} - \hat{K}_{1221} + \hat{K}_{2222} + \\ +\mathrm{Re}[W_j](4\hat{K}_{1121} - 4\hat{K}_{2122} + 3\mathrm{Re}[\hat{W}_j]K_{2121}) - \mathrm{Im}[W_j]\hat{K}_{2121}], \\ 2\mu c_{3j} = -\hat{K}_{1221} + \mathrm{Re}[W_j][\hat{K}_{1121} - \hat{K}_{2122} + \\ +\mathrm{Re}[W_j]\hat{K}_{2121}] - \mathrm{Im}[W_j]\hat{K}_{2121}], \\ 2\mu c_{4j} = \mathrm{Im}[W_j][-\hat{K}_{1121} + \hat{K}_{2122} - 2\mathrm{Re}[W_j]\hat{K}_{2121}],\ j=1,2. \end{cases}$$

It is possible to show easily that the conversion of the system determinant to zero corresponds to the line of discontinuity of infinite length.

On the line of localization of the band $\hat{x}_2 = 0$, then:

$$\hat{\psi}(\hat{x}_1, 0) = \frac{\hat{\dot{\tau}}_{21}^{\infty}}{2G_q}\left(\mathrm{Re}\left[A_1\right] + \mathrm{Re}\left[A_2\right]\right)\times$$
$$\times\left(\hat{x}_1^2 - \hat{x}_1\sqrt{\hat{x}_1^2 - l^2} + l^2\ln\left(\hat{x}_1 + \sqrt{\hat{x}_1^2 - l^2}\right)\right).$$

Using the obtained solution, it is possible to determine the fields of rates of displacements and deformations, and, with the help of determining ratios (2), to build up the rates of change in the stresses in the vicinity of the apex of the band of localization.

$$\Delta\dot{\tau}_{11} - \Delta\dot{\tau}_{22} = A\frac{l}{\sqrt{x_1^2 - l^2}}\dot{\tau}_{22}^{\infty}, \quad (5)$$

$$\Delta\dot{\tau}_{12} = B\frac{l}{\sqrt{x_1^2 - l^2}}\dot{\tau}_{22}^{\infty},$$

$$\Delta\dot{\tau}_{21} = C\frac{l}{\sqrt{x_1^2 - l^2}}\dot{\tau}_{22}^{\infty},$$

where A, B, C are the constants, which depend on parameters of the material.

As it can be seen from the obtained solution, the root special features appear in the apex of a slip line. A concept of the coefficient of intensity of stresses is used in the mechanics of destruction for determining the critical length of a crack. In our case, the problem is solved in rates, which leads to the need for a new formulation of the criterion of the localization development.

5. Special features of behavior of solution in apexes of slip line

As follows from the presented solution (5), the field of rates of change in the stresses has a root peculiarity, which at first glance makes no physical sense. Let us formulate a criterion of ductile fracture, using the Novozhilov fracture criterion [14, 15] and averaging of infinite stresses in the vicinity of the apex of a crack. Let us consider that in the case with ductile fracture we deal with the occurrence of the localized flow, in which the strength of the body with the section is determined by the average values of stresses in a small vicinity of the apex of the crack. In this case, it is possible to consider the vector of Burgers at the level of a monocrystal or a localized shear within the limits of grain of polycrystalline metal as "the quantum" of slip. As follows from the solution represented above (5), the gradient of the rates of change in stresses in the vicinity of the apex of a slip line is so large that it is not possible to disregard its change even within the limits of one grain. In that case, we can judge the stressed state, using the concept of the average rates within the limits of the grain:

$$\bar{\dot{\sigma}}_{ij} = \frac{1}{b}\int_{l}^{l+b}\dot{\sigma}_{ij}(\hat{x}_1)d\hat{x}_1.$$

This, practically discrete, representation differs from the continual one only in the vicinity of singular points of the field of stresses (Fig. 2).

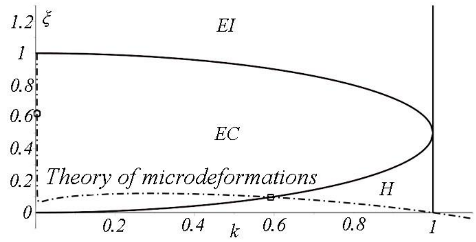

Fig. 2. Classification of regimes

We will use the condition of complete loading (3), applied to the averaged stresses $\bar{\dot{\sigma}}_{ij}$. to plot the dependency of the length of the line of localization on the load.

The point of intersection of the curve $\xi \sim k$, built up according to the theory of microdeformations, with the curve, determining the boundary (E/G) on the diagram of the classification of the regimes of solution, makes it possible to determine the value α_{crit} and $\theta_{crit} = \vartheta_0$.

6. Comparison of results for finite and infinite band of localization

In Fig. 2, the results of calculation at $\gamma_s = 0.002$, $a_1 = 0.0063$, $\mu = -\frac{3}{2\pi^2}$, $\alpha_{crit} = 1.084$, $\theta_{crit} = 0.618$. are presented. Assuming that the initial band of localization of plastic deformation is formed on condition that l/b=1, we can obtain parameters ξ, k, with which the line of discontinuity is formed. Using the formulas enumerated above, we receive that $\xi = 0.62$, $k = 0.0035$ at $\alpha_{crit} = 1.084$, $\vartheta_{crit} = 0.618$, $a_{initial} = 0.1687$. Thus, localization of the band in the initial state can have dimension that is possible to compare with the grain size, which is proved by numerous studies of plastic deformation of polycrystal.

In Fig. 3, the initial point of formation of the localization line is shown by a circle, and the point, at which the band of localization of the infinite length is formed, is shown by a square. As can be seen from the obtained results, localization in the form of the slip line of finite length precedes the localization at the point.

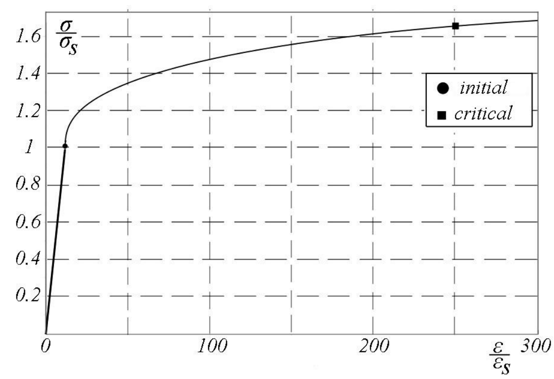

Fig. 3. Diagram of material

7. Conclusions

1. Within the framework of the theory of microdeformations, for the case of biaxial tension under the plane strain conditions of incompressible body, the fields of rates of displacements and changes in stresses in the vicinity of the apex of the assigned line of discontinuity of displacements were built in a closed analytical form.

2. It was established that the solution has the root peculiarity at the apex of the band. For the formulation of a fracture criterion (advance of the line of discontinuity), the average values of the fields of rates of change in stresses in the vicinity of the apex of the line of discontinuity were introduced, by analogy with the concept of the averaged stresses under the Novozhilov fracture condition.

3. The dependence of the length of localization line on subcritical stresses was obtained from the condition of the limitation of the angle of the fracture of the trajectory of load (the angle between the directions of the deviator of subcritical stress and the rate of change in stresses), which follows from the theory of microdeformation. It was established that the localization line in the initial state can have dimension, compared with the characteristic size of the material (in our case, with the size of a grain).

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