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STUDYING THE PECULIARITIES OF BALANCING OF FLEXIBLE DOUBLE-SUPPORT ROTORS BY TWO PASSIVE AUTOMATIC BALANCERS PLACED NEAR SUPPORTS

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Побудована дискретна багатомасова модель гнучкого ротора на двох податливих опорах з двома пасивними автобалансирами, розташованими біля опор. Отримані дві системи диференціальних рівнянь, що, відповідно, описують рух роторної машини в цілому і процес автобалансування. Знайдені основні усталені рухи (на яких автобалансири усувають переміщення ротора в опорах), встановлені умови їх існування

Ключові слова: гнучкий ротор, автобаланcир, автобалансування, основний рух, критична швидкість, резонансна швидкість

Построена дискретная многомассовая модель гибкого ротора на двух податливых опорах с двумя пассивными автобалансирами, расположенными возле опор. Получены две системы дифференциальных уравнений, описывающие, соответственно, движение роторной машины в целом и процесс автобалансировки. Найдены основные установившиеся движения (на которых автобалансиры устраняют прогибы ротора в опорах), установлены условия их существования

Ключевые слова: гибкий ротор, автобалансир, автобалансировка, основное движение, критическая скорость, резонансная скорость

1. Introduction

Rotors of many gas turbine engines, turbine units, centrifugal machines etc. work at high speeds of rotation. As a consequence of this, they behave as flexible [1, 2]. The form and imbalance of a flexible rotor depends on the current speed of rotation, it changes due to temperature or wear of the rotor, etc. That is why such rotors are advisable to balance not only during their manufacture. They can be constantly rebalanced during operation by the passive automatic balancers (AB) [3–15]. In the latter, corrective loads (CL) in the form of pendulums, balls or rollers under certain conditions come by themselves to the position in which they balance a rotor. Such sustained motions are called primary. There are also side sustained motions, with which automatic balancing does not occur while the devices increase vibrations of a rotor.

For balancing a wide range of flexible rotors by passive AB in practice, it is necessary to have a certain method, the efficiency of which is theoretically justified.

2. Analysis of scientific literature and the problem statement

The paper [4] studied a possibility of balancing a flexible rotor on rigid supports by one or two two-ball AB in any correction planes (cross sections), placed at a distance from the supports. A flexible rotor was modeled as a weighty solid homogeneous elastic shaft of a sustained round cross section. In this case they examined stability of the provisions of the balance of balls, when there are no shaft deflections in the correction planes. It was found that in the case of one AB, automatic balancing occurs at the speeds exceeding its unpaired critical velocities and lower than the paired critical speeds of this rotor with an intermediate support at the AB cross section. In the case of two AB, automatic balancing occurs at the speeds exceeding its paired critical velocities and lower than the unpaired critical speeds of this rotor with intermediate supports at the AB cross section. It follows from the results that such a rotor is difficult to drive up to the working rotation speed because AB will increase deflections and vibrations of the rotor in the correction planes at the speeds of rotor rotation, at which the primary motion is not sustained. It should be noted that the applied model of a flexible rotor models the simplest rotors, yet it is difficult for analytical research.

Similar results were obtained by computational methods within the range of different flexible rotors on elastic supports [5, 6]. Thus, in the paper [5] a flexible rotor was modeled as a weightless elastic shaft, on which N two-ball AB are fitted, and in several cross sections of which the point masses form imbalances. In the work [6], in contrast to the work [5], the point masses are replaced by static imbalanced disks.

In the paper [7], a flexible rotor was modeled as a statically imbalanced disk fitted on a weightless, absolutely flexible shaft. For a static balancing of the disk, one AB was fitted on the shaft at some distance from the disk. It was shown analytically that AB cannot completely balance the disk and eliminate deflections of the shaft. It was found that the precision of balancing increases with the AB approaching the disk.

In the work [8], in contrast to the paper [7], the disk is dynamically imbalanced, and two AB are fitted on the shaft from different sides of the disk. It was shown analytically that AB cannot completely balance the disk and eliminate deflections of the shaft. But the precision of balancing increases with the AB approaching the disk.

It should be noted that discrete models of a flexible rotor used in the papers [5–8] model any isotropic rotors. On the other hand, these models are subject to analytical analysis. A common drawback of the works [4–8] is also the use of two-ball AB in the models. In practice, all AB are multi-ball.

Described studies reveal that it is not expedient to use passive AB for elimination of deflections of a flexible rotor in non-supporting points. That is why the paper [9] suggested a new way of balancing a flexible double-support rotor by passive AB. According to the method, the supports are made elastic and AB are positioned as close as possible to the supports. The method is based on the fact that a rigid long rotor can be dynamically balanced by two AB at the super-resonance velocities of rotor rotation [10]. It is assumed that at such velocities passive AB will eliminate displacements (and vibrations) of the rotor in supports, not eliminating its deflections at non-supporting points. That is why it is important to examine the efficiency of the method and the peculiarities of such a balancing of a flexible rotor.

As this method is applicable for a wide range of flexible rotors, then its efficiency is expedient to examine analytically. With this purpose, it is necessary to use a model of a flexible rotor and AB subject to analytical research. In the paper [11] a discrete multimass model of a flexible rotor on elastic supports is constructed. It is shown that the model is adequate enough in simulating flexible rotors and can be studied analytically. In the work [12] a possibility of research into dynamics of multiball AB is revealed. Taking into account the experience of these works, in the paper [13] they built a discrete multimass model of a flexible rotor on elastic supports with several AB with many CL. This model describes the dynamics of a rotor machine with AB in general and is not directly suitable for the study of the process of automatic balancing. That is why it is expedient in the research to receive minimal number of differential equations from this model that describes exactly the process of automatic balancing. The respective rules of transformation of differential equations of motion and the examples of their application are presented in the paper [14].

The study of working capacity of AB comes down to determining the primary and side sustained motions, defining the conditions of their existence, and the research into stability. In this case AB is efficient at those angular velocities of rotor rotation, at which the primary motions exist and are stable, and the side motions are not stable or do not exist. The implementation of this approach for a ball AB [3] or AB with two connected CL [15] allow us to suggest that when the primary motions exist and are stable, then the side motions are unstable or do not exist. Therefore, the efficiency of the method will be studied in the research into conditions of existence and stability of the primary motions.

3. The purpose and objectives of the study

The aim of this work is to study the peculiarities of balancing of flexible double-support rotors by two AB, placed near supports of the rotor.

To achieve the set goal, the following problems are to be solved:

– to build a discrete N-mass model of a flexible rotor on two elastic supports with two AB near supports, to receive differential equations of the motion of the system;

– to find the primary motions of a rotor machine;

– to examine conditions of existence of the primary motions;

– to obtain a closed system of differential equations relative to a minimal number of generalized dynamic variables that describe the process of automatic balancing.

4. Methods of research into the peculiarities of balancing of flexible rotors by passive automatic balancers

In theoretical studies, the elements of the theory of rotor machines and the theory of passive automatic balancers are used.

Differential equations that describe the motion of a flexible rotor with two passive AB, placed near supports, are derived from the more general equations found in the paper [13]. Thus, a discrete multimass model of a flexible double-support rotor with two passive AB, placed near supports, with many CL, is used in the studies.

The primary motions and the conditions of their existence are determined by the condition that AB replace displacements in the supports of the rotor.

Differential equations that describe the process of automatic balancing are derived from the differential equations of motion of a rotor machine by the method described in [14]. In this case, a closed system of differential equations is received, compiled relative to the displacements of the rotor in supports and relative to the total imbalances, reduced to two correction planes (planes of supports). On the primary motions, these dynamic variables equal zero.

5. Results of the study of peculiarities of balancing of flexible rotors by passive AB

5. 1. Description of a theoretical-mechanical model of a rotor and AB

We consider the following model of a flexible rotor with two AB (Fig. 1). The shaft of the rotor is assumed by absolutely elastic weightless straight line. N of absolutely flat rigid disks D_j with the mass M_j, $/j=\overline{1,N}/$ is fitted on it. With the unstrained shaft, the centers of disks – points O_j, $/j=\overline{1,N}/$ are on a straight line and the disks' planes are perpendicular to this line.

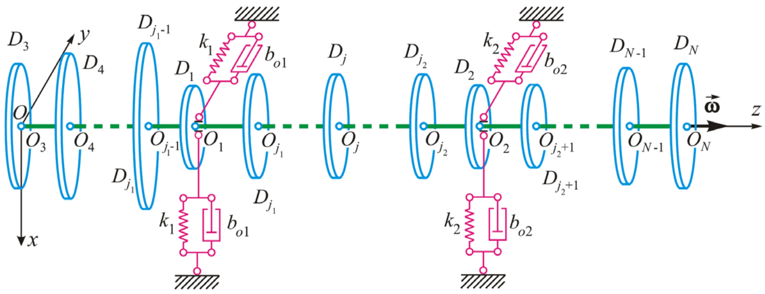

Fig. 1. N-mass model of a flexible double-support rotor

The shaft is held by two elastic-viscous supports at the points O_1 and O_2. The supports are isotropic with linear characteristics.

It is assumed that the shaft of a flexible rotor is a simple, linear, perfectly elastic body that obeys Hooke law, and: shaft deformations are low; a shaft rotates around a fixed axis, which passes through the supports, with constant angular rate ω; there is no shaft torsion; the points O_j, $/j=\overline{1,N}/$ move in transverse planes of unstrained shaft; deformations of the shaft and supports by the forces of weight can be neglected.

The motion of a flexible rotor is defined relative to the right system of fixed rectangular axes x, y, z: z axis is directed along the axis of rotation in the direction of the angular velocity vector ω, axes x and y are directed parallel to the main directions of viscous-elastic supports, the origin of coordinates is the point O_3. Coefficients of rigidity and viscosity of the supports are k_1, k_2 and b_{o1}, b_{o1}, respectively.

The motion of a flexible rotor is completely determined by its rotation around the z axis and the deviation of the disks' centres $O_j(x_j,y_j)$, $/j=\overline{1,N}/$ from the axis of rotation.

In the plane of the j-th disk at the point $G_{0,j}$ (Fig. 2) at the distance $r_{0,j}$ from its longitudinal axis, there is a point mass $m_{0,j}$, which forms static imbalance $\mathbf{s}_{0,j}$, $/j=\overline{1,N}/$. In the initial moment the vector $\mathbf{s}_{0,j}$ forms the angle ϕ_{0_j}, $/j=\overline{1,N}/$. with the axis x. The angle of rotation of the j-th disk equals ωt, and the angle of rotation of the point mass $m_{0,j}$ equals $\phi_{0,j}=\omega t+\phi_{0_j}$, $/j=\overline{1,N}/$.

Disks – D_j, /j=1,2/ contain a pendulum or a ball or a roller AB (Fig. 2). In a pendulum AB number j (Fig. 2, *a*), n_j of pendulums are fitted on the shaft, with the mass m_j and physical length r_j. In a ball or a roller AB number j (Fig. 2, *b*), there are n_j of balls or cylindrical rollers of the mass m_j. They roll without slipping along circular tracks and in this case the distance from the centre of the disk to the center of the ball or the roller equals r_j. As is accepted in the theory of passive AB [4–6, 10, 12–15], we believe that CL move in the planes of the disks and do not impede movement of one another. The action of the force of weight is neglected.

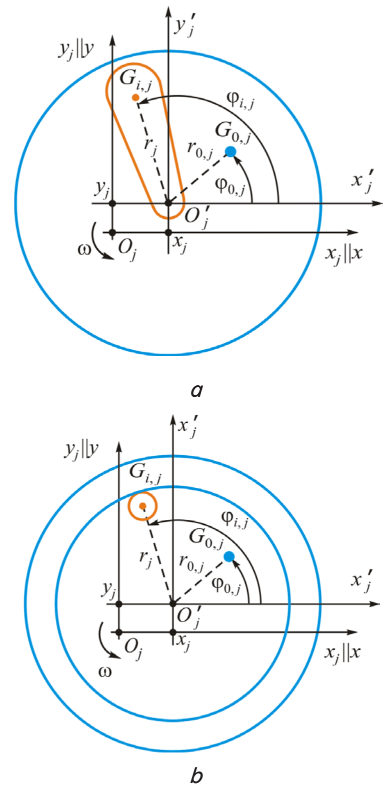

Fig. 2. Kinematics of planar parallel motion of the j-th disk with: *a* — pendulum AB; *b* — ball or roller AB

The position of CL is defined by the angles $\phi_{i,j}$, $/i=\overline{1,n_j}/$. At the turning of the i-th pendulum around a shaft, it is exposed to the moment of forces of viscous resistance $b_j r_j^2(\omega-\dot{\phi}_{i,j})$, where b_j is the coefficient of the moment of forces of viscous resistance (brought to the shoulder r_j); $(\omega-\dot{\phi}_{i,j})$ is the speed of rotation of the pendulum around the shaft relative to the j-th disk; a dot above the values means a time derivative. During motion of the i-th ball or roller along the track, it is exposed to the action of the force of viscous resistance $b_j r_j(\omega-\dot{\phi}_{i,j})$, where b_j is the coefficient of force of viscous resistance; $r_j(\omega-\dot{\phi}_{i,j})$ is the speed of motion of the centre of the ball or the roller relative to the j-th disk.

The motion of a rotor machine in the moving coordinate system Gξηζ (axis ζ coincides with the axis z, and axes ξ, η rotate around the axis z with angular speed ω synchronously with the rotor) can be described by the following conjugated complex equations [13]:

$$\mathrm{Left}_j=M_{\Sigma j}D_\tau^2\Xi_{zj}-L_j^T K_\Xi+b_{oj}D_\tau\Xi_{zj}+k_j\Xi_{zj}+D_\tau^2 S_{zj}=0,$$

$$\overline{\mathrm{Left}}_j=0,\ \ /j=1,2/,$$

$$\mathrm{Left}=MD_\tau^2 Z+K_\Xi-\omega^2 S_{0,z}=0,\ \overline{\mathrm{Left}}=0, \tag{1}$$

$$\mathrm{Left}_{i,2N+j}=\ddot{s}_{zij}+b_j/(m_j\kappa_j)\dot{s}_{zij}+m_j/(2\kappa_j)\times$$

$$\times(D_\tau^2\Xi_{zj}-e^{2i\tilde{\psi}_{ij}}\bar{D}_\tau^2\bar{\Xi}_{zj})=0,\ \overline{\mathrm{Left}}_{i,2N+j}=0,$$

$$/i=\overline{1,n_j},\ j=1,2/, \tag{2}$$

where

$M_{\Sigma j}=M_j+m_{0j}+n_j m_j$, $/j=1,2/$,

$M_{\Sigma j}=M_j+m_{0j}$, $/j=\overline{3,N}/$ is the mass of the corresponding disk;

$D_\tau \bullet = \dot{\bullet} + i\omega \cdot \bullet$ is the differential operator;

$\Xi_{zj} = (x_j + iy_j)e^{-i\omega t}$, $/j = \overline{1,N}/$ are the generalized coordinates that describe the motion of the centres of disks of a flexible rotor;

$L_1 = (1-\tilde{l}_3, 1-\tilde{l}_4, ..., 1-\tilde{l}_N)^T$, $L_2 = (\tilde{l}_3, \tilde{l}_4, ..., \tilde{l}_N)^T$ are the vectors, in which $\tilde{l}_j = (l_j - l_1)/(l_2 - l_1)$, $/j = \overline{3,N}/$ are the dimensionless parameters that set the positions of non-supporting disks relative to the supporting ones, where l_j, $/j = \overline{1,N}/$ is the coordinate of the centre of masses of the j-th disk on the axis z;

$K_\Xi = K(Z - \Xi_{z1}L_1 - \Xi_{z2}L_2)$ is the auxiliary matrix;

K is the matrix of rigidity, the elements of which are the coefficients of rigidity $(k_{j,p})$, $/j,p = \overline{3,N}/$ (they are determined as the magnitude of static vertical force which is to be applied to the point O_j of the shaft so that as a result of it, a single displacement of the point O_p occurs);

$Z = (\Xi_{z3}, \Xi_{z4}, ..., \Xi_{zN})^T$ is the vector of the rotor's displacement in non-supporting points;

$S_{zj} = m_j r_j e^{-i\omega t} \sum_{i=0}^{n_j} e^{i\phi_{i,j}}$, $/j=1,2/$ are the generalized coordinates (they describe total imbalances, which are formed by static imbalance and CL in the corresponding correction plane);

$Left = (Left_3, Left_4, ..., Left_N)^T$ is the vector of the left parts of equations that describe the motion of non-supporting disks;

$M = diag(M_{\Sigma 3}, M_{\Sigma 4}, ..., M_{\Sigma N})$ is the diagonal matrix;

$S_{0,z} = (S_{0,z3}, S_{0,z4}, ..., S_{0,zN})^T$ is the vector compiled from $S_{0,zj} = m_j r_j e^{i(\phi_{0,j} - \omega t)}$, $/j = \overline{3,N}/$ static imbalances of non-supporting disks;

$s_{zij} = m_j r_j e^{i(\phi_{i,j} - \omega t)}$, $/i = \overline{1,n_j}, j = 1,2/$ is the imbalance formed by the i-th CL in the j-th correction plane;

$$\kappa_j = \begin{cases} 1, & \text{for pendulums;} \\ 7/5, & \text{for balls;} \\ 3/2, & \text{for cylindrical rollers,} \end{cases}$$

/j=1,2/ is the coefficient that characterizes kinetic energy of CL rotation motion; $\tilde{\psi}_{i,j}$, $/i = \overline{1,n_j}, j = 1,2/$ are the angles that set the position of CL in AB in a certain primary motion (from a family of primary motions if such motions create a family); i is the imaginary unit; a line over value means complex conjugation.

5. 2. Primary motions of a flexible rotor

A shaft of a flexible rotor is balanced on the primary motions and:

a) CL caught up with the rotor, that is why

$$D_\tau^2 S_{zj} = -\omega^2 S_{zj}, \ /j = 1,2/; \quad (3)$$

b) displacements of the centres of masses of supporting discs are absent, and deflections in the planes of non-supporting disks are sustained

$$\Xi_{z1} = \Xi_{z2} = 0, \ \Xi_{zj} = const_j, \quad /j = \overline{3,N}/.$$

That is, at the points of intersection of a non-deformed shaft with the AB planes, the hinge supports are ostensibly formed that hold the shaft.

Thus, on the primary motions, the system (1) takes the form:

$$Left_j = -L_j^T KZ - \omega^2 S_{zj} = 0, \ \overline{Left}_j = 0, \ /j=1,2/,$$

$$Left = (K - \omega^2 M)Z - \omega^2 S_{0,z} = 0, \ \overline{Left} = 0. \quad (4)$$

From the last equation of the system at $\det(K - \omega^2 M) \neq 0$ we find the vector Z:

$$Z_0 = \omega^2 (K - \omega^2 M)^{-1} S_{0,z}, \quad (5)$$

the coordinates of which determine deflections of the shaft on the primary motions in the planes of non-supporting disks. These deflections are caused by imbalances in non-supporting points and depend on the angular velocity of rotation of the rotor.

Substituting (5) in the first two equations of the system (4), we obtain:

$$Left_j = -\omega^2 [S_{zj} + L_j^T K (K - \omega^2 M)^{-1} S_{0,z}] = 0,$$

$$\overline{Left}_j = 0, \ /j=1,2/. \quad (6)$$

In square brackets we recorded total imbalances reduced to two correction planes

$$S_{\Sigma j} = S_{zj} + L_j^T K Z_0 / \omega^2 = S_{zj} + L_j^T K (K - \omega^2 M)^{-1} S_{0,z}, /j=1,2/. \quad (7)$$

Therefore, the total imbalance in the j-th /j=1,2/ correction plane:

– on the primary motions equals zero;

– is created by imbalances from AB and the corresponding disk in the plane j and by imbalances of non-supporting disks D_j, $/j = \overline{3,N}/$ reduced to this plane;

– depends on the rotor deflections in the non-supporting points and the angular velocity of rotation of the rotor (the effect of flexibility of a rotor).

It follows from the equalities (7) that the effect of flexibility of a rotor is most evident at the speeds, which are solutions of the equation

$$\det(K - \omega^2 M) = 0. \quad (8)$$

The matrix $K - \omega^2 M$ is symmetric. Therefore, the equation (8) always has N–2 real positive solutions. These N–2 velocities are the analogues of critical speeds of a flexible rotor on two hinge supports.

From (7) we find the conditions of existence of the primary motions:

$$\left| S_{0,zj} + L_j^T K (K - \omega^2 M)^{-1} S_{0,z} \right| \leq S_{jmax}, \ /j=1,2/, \quad (9)$$

where $S_{0,zj}$ $m_j r_j e^{i(\phi_{0,j} - \omega t)}$, /j=1,2/ are the static imbalances of supporting disks; S_{1max}, S_{2max} are the balancing capacities of AB.

By the condition (9), total imbalances reduced to two correction planes must be smaller than the balancing capacities of the rcorresponding AB.

The equation (5) and the condition (8) show that in the vicinity of critical speeds of a flexible rotor on two hinge supports, the primary motions do not exist due to big deflections of a flexible shaft that theoretically grow to infinity.

5. 3. Peculiarities of the study of stability of the primary motions

If any primary motion occurs, then the deflections in the supports of a rotor and the total imbalances reduced to two correction planes equal zero:

$$\Xi_{z1}=\Xi_{z2}=0,\ S_{\Sigma 1}=0,\ S_{\Sigma 2}=0. \tag{10}$$

So it is natural to examine the stability of the primary motions (in particular, their families) by the dynamic variables $\Xi_{z1}, \Xi_{z2}, S_{\Sigma 1}, S_{\Sigma 2}$.

To obtain the equations that describe the process of balancing of a flexible rotor, let us proceed in the system of equations (1), (2) to the variables $S_{\Sigma 1}, S_{\Sigma 2}$. Taking into account (7), the system of equations (1), (2) takes the form

$$\begin{gathered}\mathrm{Left}_j = M_{\Sigma j}D_\tau^2\Xi_{zj} - L_j^T K(Z-\Xi_{z1}L_1-\Xi_{z2}L_2)+\\ +b_{oj}D_\tau\Xi_{zj}+k_j\Xi_{zj}+D_\tau^2 S_{\Sigma j}+\omega^2 L_j^T K(K-\omega^2 M)^{-1}S_{0,z}=0,\\ \overline{\mathrm{Left}_j}=0,\quad /j=\overline{1,2}/,\\ \mathrm{Left}=MD_\tau^2 Z+K(Z-\Xi_{z1}L_1-\Xi_{z2}L_2)-\omega^2 S_{0,z}=0,\ \overline{\mathrm{Left}}=0, \end{gathered} \tag{11}$$

$$\begin{gathered}\mathrm{Left}_{2N+j}=\ddot{S}_{\Sigma j}+b_j/(m_j\kappa_j)\cdot\dot{S}_{\Sigma j}+\\ +n_j m_j/(2\kappa_j)\cdot(D_\tau^2\Xi_{zj}-p_j e^{i\vartheta_j}\bar{D}_\tau^2\Xi_{zj})=0,\\ \overline{\mathrm{Left}_{2N+j}}=0,\ /j=\overline{1,2}/,\end{gathered} \tag{12}$$

where

$$\vartheta_j=\arccos(p_{1j}/p_j)/2,$$

$$p_{1j}=\left(\sum\nolimits_{i=1}^{n_j}\cos 2\tilde{\psi}_{i,j}\right)/n_j,\ p_{2j}=\left(\sum\nolimits_{i=1}^{n_j}\sin 2\tilde{\psi}_{i,j}\right)/n_j,$$

$$p_j=\sqrt{p_{1j}^2+p_{2j}^2},\ /j=1,2/.$$

It is assumed that after reaching cruising speed of rotation by a flexible rotor, at first shaft deflections occur rather quickly and then the deflected shaft behaves like a rigid one. In this case, the third equation in (11) takes the form

$$\mathrm{Left}=(\omega^2 M-K)Z+K(\Xi_{z1}L_1+\Xi_{z2}L_2)+\omega^2 S_{0,z}=0,\ \overline{\mathrm{Left}}=0.$$

We find vector Z from it:

$$Z=-(\omega^2 M-K)^{-1}K(\Xi_{z1}L_1+\Xi_{z2}L_2)-\omega^2(\omega^2 M-K)^{-1}S_{0,z}. \tag{13}$$

Taking (13) into consideration, the first two equations of (11) take the form:

$$\begin{gathered}\mathrm{Left}_j=M_{\Sigma j}D_\tau^2\Xi_{zj}+q_{j1}\Xi_{z1}+q_{j2}\Xi_{z2}+\\ +b_{oj}D_\tau\Xi_{zj}+k_j\Xi_{zj}+D_\tau^2 S_{\Sigma j}=0,\ \overline{\mathrm{Left}_j}=0,\ /j=1,2/,\end{gathered} \tag{14}$$

where

$$q_{ij}=L_i^T K[(\omega^2 M-K)^{-1}K+E]L_j,\ /i,j=1,2/;$$

E is the identity matrix N of the 2nd order

The stability of the primary motions of N-mass model of a flexible rotor can be examined by the equations (12), (14).

5. 4. Example – a case of a three-mass model of a rotor

For a three-mass model of a flexible double-support rotor with two AB (Fig. 3), the following results are received:

– motion of the model is described by the equations (2), which will remain unchanged, and by the equations (1), which will take the form:

$$\begin{gathered}\mathrm{Left}_1=M_{\Sigma 1}D_\tau^2\Xi_{z1}-(1-\tilde{l}_3)k_{33}[\Xi_{z3}-\Xi_{z1}(1-\tilde{l}_3)-\\ -\Xi_{z2}\tilde{l}_3]+b_{o1}D_\tau\Xi_{z1}+k_1\Xi_{z1}+D_\tau^2 S_{z1}=0,\\ \mathrm{Left}_2=M_{\Sigma 2}D_\tau^2\Xi_{z2}-\tilde{l}_3 k_{33}[\Xi_{z3}-\Xi_{z1}(1-\tilde{l}_3)-\Xi_{z2}\tilde{l}_3]+\\ +b_{o2}D_\tau\Xi_{z2}+k_2\Xi_{z2}+D_\tau^2 S_{z2}=0,\\ \mathrm{Left}_3=M_{\Sigma 3}D_\tau^2\Xi_{z3}+k_{33}[\Xi_{z3}-\Xi_{z1}(1-\tilde{l}_3)-\Xi_{z2}\tilde{l}_3]-\omega^2 S_{0,z3}=0,\\ \overline{\mathrm{Left}_j}=0,\quad /j=\overline{1,3}/;\end{gathered} \tag{15}$$

– total imbalances reduced to two correction planes:

$$\begin{gathered}S_{\Sigma 1}=S_{z1}-(1-\tilde{l}_3)k_{33}/(\omega^2 M_{\Sigma 3}-k_{33})\cdot S_{0,z3},\ S_{\Sigma 2}=\\ =S_{z2}-\tilde{l}_3 k_{33}/(\omega^2 M_{\Sigma 3}-k_{33})\cdot S_{0,z3};\end{gathered}$$

– the speed at which the effect of flexibility of the rotor manifests itself most vividly

$$\omega_{кр}=\sqrt{k_{33}/M_{\Sigma 3}};$$

– the equations (14) take the form

$$\begin{gathered}\mathrm{Left}_1=M_{\Sigma 1}D_\tau^2\Xi_{z1}+(1-l_3)^2\omega^2 M_{\Sigma 3}k_{33}/(\omega^2 M_{\Sigma 3}-k_{33})\cdot\Xi_{z1}+\\ +(1-\tilde{l}_3)\tilde{l}_3\omega^2 M_{\Sigma 3}k_{33}/(\omega^2 M_{\Sigma 3}-k_{33})\cdot\Xi_{z2}+\\ +b_{o1}D_\tau\Xi_{z1}+k_1\Xi_{z1}+D_\tau S_{\Sigma 1}=0,\ \overline{\mathrm{Left}}=0,\\ \mathrm{Left}_2=M_{\Sigma 2}D_\tau^2\Xi_{z2}+(1-\tilde{l}_3)\tilde{l}_3\omega^2 M_{\Sigma 3}k_{33}/(\omega^2 M_{\Sigma 3}-k_{33})\cdot\Xi_{z1}+\\ +\tilde{l}_3^2\omega^2 M_{\Sigma 3}k_{33}/(\omega^2 M_{\Sigma 3}-k_{33})\cdot\Xi_{z2}+b_{o2}D_\tau\Xi_{z2}+k_2\Xi_{z2}+D_\tau^2 S_{\Sigma 2}=0,\\ \overline{\mathrm{Left}_2}=0\end{gathered} \tag{16}$$

and the stability of the primary motions can be examined by the equations (12), (16).

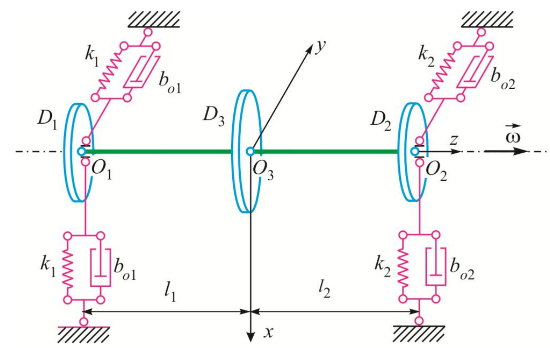

Fig. 3. Three-mass model of a flexible double-support rotor with two AB, placed in the planes of supports

6. Discussion of the results of study of the peculiarities of balancing flexible rotors by passive AB

The built discrete N-mass model of a flexible rotor on two elastic supports with two AB near the supports and the obtained differential equations of its motion make it possible to set the following peculiarities of balancing of the examined flexible rotor:

– on the primary motions, AB eliminate deflections of the rotor and vibrations in elastic viscous supports, but do not eliminate shaft deflections in non-supporting points;

– on the primary motions, elastic viscous supports are conditionally converted to hinge supports;

– shaft deflections and primary motions change with the change in angular velocity of rotation of the rotor;

– the primary motions exist at a certain distance of the speed of rotor rotation from the critical velocities of flexible rotor rotation with hinge supports instead of elastic viscous supports;

– N-mass model allows simulating N–2 critical speeds of flexible rotor rotation;

– at the speeds of rotation of a rotor shaft close to any of these velocities, the conditions of existence of the primary motions are disrupted because shaft deflections theoretically grow to infinity and the balancing capacity of AB is not sufficient for compensating for the imbalances of the rotor;

– in practice, these deflections are limited and, therefore, proper selection of the balancing capacity of AB (and balancing of a rotor before the beginning of operation) can ensure the existence of primary motions at all speeds of rotation of the rotor.

Therefore, the examined method of balancing of flexible rotors is applicable in a much wider range of speeds of rotation of the rotor than the existing methods. In fact one can achieve the working capacity of the method at super-resonance speeds of rotation of the rotor. But to limit the shaft deflections, it is expedient to eliminate working rotation frequency of a flexible rotor from the critical speeds of such a rotor with two hinge supports instead of elastic supports.

It should be noted that the method can be used only for the rotors, in which there is a place for adjusting AB near the supports. This limits the scope of application of the method.

For further substantiation of the studied way of balancing of flexible rotors, we plan, in the range of the constructed model of a rotor machine, to explore:

– stability of the primary motions;

– peculiarities of transition processes that occur at automatic balancing.

7. Conclusions

1. Constructed discrete N-mass model of a flexible rotor on two elastic supports with two AB near supports is subject to analytical analysis and effective at the research into peculiarities of automatic balancing.

2. On the primary motions of a rotor machine:

– due to automatic balancers, two hinge supports are ostensibly formed in a flexible rotor instead of elastic viscous supports;

– automatic balancers eliminate deflections in the supports (vibrations of supports), but do not eliminate shaft deflections in non-supporting points;

– a shaft deflection and the primary motions change with the change in angular velocity of rotation of the rotor.

3. The primary motions exist at a certain distance of the speed of rotation of the rotor from the critical speeds of flexible rotor rotation with hinge supports instead of elastic viscous supports.

4. Stability (of families) of primary motions can be examined by a part of variables, including displacements of the rotor in the supports and total imbalances of a flexible rotor and AB reduced to two correction planes.

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