UDC 519.23: 004.932.72'1
DOI: 10.15587/1729-4061.2016.75593

AN INVESTIGATION OF THE REDUCTION MODEL POWER INFLUENCE ON THE ACCURACY OF THE OBJECT'S POSITION ASSESSMENT USING RELATIVE METHOD

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Висока точність астрономічних спостережень забезпечується виміром положення об'єктів щодо обраних опорних об'єктів. Аналіз показав наявність синусоїдальної залежності відхилень положень об'єктів при кубічній моделі редукції і її повне усунення при використанні моделі п'ятого ступеня. Введено і проаналізовані показники точності вимірювання положень об'єктів та критерії значимості коефіцієнтів редукційної моделі

Ключові слова: астроредукція, цифровий кадр, небесний об'єкт, модель редукції, оцінка показників точності

Высокая точность астрономических наблюдений обеспечивается измерением положения объектов относительно выбранных опорных объектов. Анализ показал наличие синусоидальной зависимости отклонений положений объектов при кубической модели редукции и её полное устранение при использовании модели пятой степени. Введены и проанализированы показатели точности измерения положений объектов и критерии значимости коэффициентов редукционной модели

Ключевые слова: астроредукция, цифровой кадр, небесный объект, модель редукции, оценка показателей точности

1. Introduction

Nowadays, there are hundreds of ground-based and space-based telescopes operating in the world [1]. Even publicly accessible archives contain petabytes of digital images [2]. Meanwhile, most of the observation results do not contain information about the equatorial coordinates of the measured objects in the digital frame. Therefore, before using such images in most research tasks it is required to determine the spherical coordinates of the measured objects. To solve this problem it is necessary to tentatively perform an astrometric reduction [3].

The classic astrometric reduction was designed for classical astrographs with a comparatively small field of view. These telescopes produce an image of the sky area which with a good approximation may be considered as a central projection of the sphere onto a plane. For such telescopes, the main provisions of Gaussian optics are fair. However, today most of modern survey telescopes are ultra wide-field and super light-sensitive systems with a spherical field of view. The presence of aberrations in such optical systems leads to the violation of the Gaussian optic laws [4]. Nevertheless, the central projection is still the closest to the real astrophoto mathematical model with which it is possible to solve many astronomical problems. But in each case it is necessary to choose the right reduction model and complement it with the necessary members in order to minimize differences between the model and the real image.

Images of objects that are located in the frame center have minor deviations from the central projection laws. Therefore, using of linear reduction model may be enough to assess their coordinates. However, astrometry of all objects in the frame requires a model which, on the one hand, approximates aberrations of the telescope optical system on the entire frame, and eliminates the approximation of noise bursts on the other. That is why one of the most important decisions is choosing the optimal astrometry reduction model for a particular optical system.

2. Literature review and problem statement

The problem of determining object's equatorial coordinates relative to the equatorial coordinates of known reference stars, which position in the image is known, is considered as a classic in many textbooks [3, 5]. The main disadvantage of this classic reduction method is the requirement for the presence of a sufficient number of evenly spaced reference stars in the digital frame. This disadvantage is a serious problem for telescopes with a small field of view. However, publication of high-density star catalogs, that cover entire celestial sphere (such as UCAC4 [6], PPMXL [7], XPM [8, 9]) and contain hundreds of millions of objects, solve this problem.

Papers [10, 11] present a non-classical reduction of digitized images of the sky obtained from telescopes with a wide field of view (more than 1 square degree). In these papers to convert from measured to tangential coordinates a full polynomial function of the sixth power has been used. At the same time, analysis of the importance and correctness of usage of all the reduction model coefficients was not carried out. In papers [12, 13] to obtain precise equatorial coordinates of celestial objects from digitized photographic plates the classic reduction model was used. However, in these papers the assessment of object coordinates obtained using wide-field telescopes, as well as the impact of various aberrations that requires using the reduction model other than the classic was not taken into account. Presented in the paper [14] reduction methods differ from each other both in the number of defined parameters and in the required number of reference stars. Meanwhile, this paper is not considered a particular astronomical reduction when the image contains distortions caused by the characteristics of the optical system.

Methods of assessing of equatorial coordinates of the objects were considered in the paper [15]. But the accuracy indicators of the objects positions measurement were not presented. In the paper [16] according to the research results the dependence between the mean square assessment of equatorial coordinates and the number of identified stars depending on the model of constants plate was shown. However, the results are based on using of USNOB1.0 [17] reference stars catalog, which has lower accuracy compared to the UCAC4 [6] that is used in this paper. This is a significant disadvantage of the paper [16] since the accuracy of astronomical reduction depends on the accuracy of used star catalog. Another disadvantage of the paper is the use only of a cubic reduction model which may be insufficient for wide-field telescopes.

Thus, it is necessary to consider different reduction polynomial models. At the same time, it is appropriate to assess the significance of the reduction model coefficients and determine the influence of the polynomial model power on the accuracy of object's position assessment.

3. The purpose and research problems

The purpose of the research is to analyze the influence of the reduction model power on the accuracy of the object's position assessment.

To achieve this goal the following tasks were solved:

– assessment of the celestial objects positions using reduction models of the third and fifth power;

– introducing indicators of the measurement accuracy of objects positions in the digital frame;

– investigation of the reduction models power influence on the analyzed accuracy indicators;

– assessment of the coefficients significance of the investigated reduction models using the f – Fisher criteria.

4. Measuring the positions of objects in digital frames

Objects position determination [16] and reference stars selection have been performed on digital frame [18]. The arbitrarily chosen reference star S is known by its coordinates in a digital frame coordinate system (x, y), and the equatorial coordinates (α, δ) [19], obtained from the reference stars catalog.

Distortions of telescope optical system [20–22] violate the laws of central projection. Thus, star coordinates (x, y) in the digital frame coordinate system [23] do not equal to the ideal coordinates (ξ, η) obtained according to formulas of central projection [3].

It is assumed that errors in positions representation of celestial objects on a digital frame were taken into account in the form of the reduction polynomial [24, 25]. The results of the performed reduction provide information about refined coordinates of the frame center (ξ_{0_k}, η_{0_k}) and the vector of the coefficients of selected reduction model constants plates:

$$\theta_x = \{a_{0,} a_1, \ldots, a_p\}, \quad (1)$$

in which:

$$\theta_x = \{a_{0,} a_1, \ldots, a_p\}; \quad (2)$$

$$\theta_y = \{b_{0,} b_1, \ldots, b_p\}, \quad (3)$$

where p is the number of the coefficients used.

This paper assumes that as reduction polynomial the model of the third power has been used [20]:

$$\xi_i = a_0 + a_1 x_i + a_2 y_i + a_3 x_i^2 + a_4 x_i y_i + a_5 y_i^2 + a_6 x_i^3 + a_7 x_i^2 y_i + a_8 x_i y_i^2 + a_9 y_i^3; \quad (4)$$

$$\eta_i = b_0 + b_1 x_i + b_2 y_i + b_3 x_i^2 + b_4 x_i y_i + b_5 y_i^2 + b_6 x_i^3 + b_7 x_i^2 y_i + b_8 x_i y_i^2 + b_9 y_i^3, \quad (5)$$

where (x, y), (ξ_i, η_i) are coordinates of the i object in the celestial coordinate system and in the ideal digital frame coordinate system respectively.

To investigate the influence of the reduction model the polynomial of the fifth power has been used:

$$\xi_i = a_0 + a_1 x_i + a_2 y_i + a_3 x_i^2 + a_4 x_i y_i + a_5 y_i^2 + a_6 x_i^3 + a_7 x_i^2 y_i + a_8 x_i y_i^2 + a_9 y_i^3 + a_{10} x_i^4 + a_{11} x_i^3 y_i + a_{12} x_i^2 y_i^2 + a_{13} x_i y_i^3 + a_{14} y_i^4 + a_{15} x_i^5 + a_{16} x_i^4 y_i + a_{17} x_i^3 y_i^2 + a_{18} x_i^2 y_i^3 + a_{19} x_i y_i^4 + a_{20} y_i^5; \quad (6)$$

$$\eta_i = b_0 + b_1 x_i + b_2 y_i + b_3 x_i^2 + b_4 x_i y_i + b_5 y_i^2 + b_6 x_i^3 + b_7 x_i^2 y_i + b_8 x_i y_i^2 + b_9 y_i^3 + b_{10} x_i^4 + b_{11} x_i^3 y_i + b_{12} x_i^2 y_i^2 + b_{13} x_i y_i^3 + b_{14} y_i^4 + b_{15} x_i^5 + b_{16} x_i^4 y_i + b_{17} x_i^3 y_i^2 + b_{18} x_i^2 y_i^3 + b_{19} x_i y_i^4 + b_{20} y_i^5. \quad (7)$$

Based on the discussed initial data we need to assess the influence of the reduction model on the indicators of accuracy assessment of the objects equatorial coordinates. It is also necessary to assess the significance of coefficients of the investigated reduction models.

5. Building residuals field

Analyzed digital frame with angular dimensions of R_x and R_y along x and y axis respectively is divided into l rectangular fragments. The angular size of l fragment along the x and y axes are defined as:

$$R_{lx} = \frac{R_x}{l}, \quad (8)$$

$$R_{ly} = \frac{R_y}{l}. \quad (9)$$

Belonging of the i-th star to the l-th fragment is determined by the condition of entering its coordinates in the fragment boundaries:

$$\begin{cases} R_{l-1x} \le x_i < R_{lx}, \\ R_{l-1y} \le y_i < R_{ly}. \end{cases} \quad (10)$$

Using the obtained assessments of constants plates coefficients $\hat{\theta}$ (1), values of the refined coordinates of the frame center (ξ_{0_k}, η_{0_k}) and coordinates of the i-th object in the digital frame coordinate system (x_i, y_i) assessment of the object's equatorial coordinates is calculated (α_i, δ_i).

Residuals between assessments (α_i, δ_i) and catalog values (α_j,) of the equatorial coordinates (right ascension and declination) of the k-th star in the l-th fragment are defined as:

$$\Delta_{\alpha_{ij(k)}} = (\alpha_{i(k)} - \alpha_{j(k)}) \cdot \cos\delta_{i(k)}; \quad (11)$$

$$\Delta_{\delta_{ij(k)}} = \delta_{i(k)} - \delta_{j(k)}, \quad (12)$$

where $\alpha_{i(k)}$, $\delta_{i(k)}$, $\alpha_{j(k)}$, $\delta_{j(k)}$ are right ascension and declination of the l frame measurement and the j catalog item, that constitute the k identified couple.

Assessment of the mean deviation of residuals of object's equatorial coordinates in the l fragment is defined as:

$$E(RA) = \hat{\hat{\Delta}}_{\alpha_{ij(k)}} = \sum_{i=1}^{N_l} \Delta_{\alpha_{ij(k)}} / N_l; \quad (13)$$

$$E(DE) = \hat{\hat{\Delta}}_{\delta_{ij(k)}} = \sum_{i=1}^{N_l} \Delta_{\delta_{ij(k)}} / N_l, \quad (14)$$

where N_l is the number of objects in the l fragment.

Assessment of the standard deviation of residuals assessments of right ascension and declination is calculated as follows:

$$RMS(RA) = \sigma_{\delta_{ij(k)}} = \sqrt{\sum_{i=1}^{N_l} (\Delta_{\alpha_{ij(k)}} - \hat{\hat{\Delta}}_{\alpha_{ij(k)}}) / N_l}; \quad (15)$$

$$RMS(DE) = \sigma_{\delta_{ij(k)}} = \sqrt{\sum_{i=1}^{N_l} (\Delta_{\alpha_{ij(k)}} - \hat{\hat{\Delta}}_{\alpha_{ij(k)}}) / N_l}. \quad (16)$$

Similarly, mathematical expectation ($\hat{\hat{\Delta}}_\alpha$, $\hat{\hat{\Delta}}_\delta$) and standard deviation of residuals (σ_α, σ_δ) are calculated for the entire frame by using all of the N_{obj} reference stars.

The resulting values of the above calculations are presented in tabular form in which each m-th cell of the n-th row corresponds to the l-th fragment (Table 1).

Table 1

Residuals field in table form

Count	39	121	110	102	125	93	123	137	122	152
E(Ra)	–0.05	0.00	0.02	0.02	0.01	0.00	0.00	0.00	0.00	0.00
RMS(Ra)	0.05	0.06	0.06	0.05	0.06	0.06	0.07	0.06	0.06	0.06
E(De)	–0.02	–0.01	0.00	0.02	0.00	0.00	–0.01	0.00	–0.01	–0.01
RMS(De)	0.05	0.06	0.05	0.06	0.06	0.07	0.05	0.06	0.05	0.06
E(X)	–0.02	0.01	0.01	0.02	0.01	0.01	0.01	0.01	0.01	0.00
RMS(X)	0.03	0.04	0.03	0.03	0.03	0.03	0.04	0.03	0.03	0.03
E(Y)	–0.02	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00
RMS(Y)	0.04	0.03	0.03	0.03	0.03	0.04	0.03	0.03	0.03	0.03
Count	131	137	125	154	172	134	170	118	109	119
E(Ra)	–0.01	0.01	0.01	0.00	0.00	–0.01	0.00	–0.02	–0.01	0.01
RMS(Ra)	0.07	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.05
E(De)	–0.02	0.03	0.02	–0.01	–0.01	–0.02	–0.01	0.01	0.01	0.00
RMS(De)	0.06	0.05	0.05	0.05	0.05	0.06	0.05	0.06	0.05	0.05
E(X)	–0.01	0.01	0.01	0.01	0.00	–0.01	0.00	0.00	0.00	0.00
RMS(X)	0.05	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03
E(Y)	–0.01	0.01	0.01	–0.01	–0.01	–0.01	–0.01	0.00	0.01	0.00
RMS(Y)	0.03	0.02	0.02	0.03	0.03	0.03	0.03	0.03	0.03	0.03
Count	148	138	135	146	119	130	131	177	172	129
E(Ra)	0.01	0.02	–0.01	–0.03	–0.03	–0.02	0.01	0.00	–0.01	0.00
RMS(Ra)	0.06	0.06	0.06	0.06	0.05	0.06	0.07	0.06	0.06	0.07
E(De)	0.01	0.00	0.00	–0.02	0.00	–0.01	0.00	0.01	0.01	0.02
RMS(De)	0.04	0.06	0.05	0.06	0.05	0.06	0.05	0.06	0.05	0.05
E(X)	0.01	0.01	0.00	–0.01	–0.01	–0.01	0.01	0.00	0.00	0.00
RMS(X)	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03
E(Y)	0.01	0.00	0.00	–0.01	0.00	0.00	0.00	0.00	0.01	0.01
RMS(Y)	0.02	0.03	0.02	0.03	0.02	0.03	0.03	0.03	0.02	0.03
Count	128	151	193	178	109	182	165	170	171	185
E(Ra)	0.02	–0.01	0.00	–0.02	–0.01	0.01	0.01	0.01	0.01	0.00
RMS(Ra)	0.05	0.05	0.06	0.06	0.05	0.06	0.06	0.07	0.06	0.06
E(De)	0.01	–0.01	–0.01	–0.01	0.00	0.00	0.00	0.00	0.00	0.01
RMS(De)	0.05	0.05	0.05	0.06	0.06	0.05	0.05	0.05	0.05	0.05
E(X)	0.01	0.00	0.00	–0.01	0.00	0.01	0.01	0.01	0.01	0.00
RMS(X)	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03
E(Y)	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
RMS(Y)	0.03	0.03	0.02	0.03	0.03	0.03	0.03	0.02	0.03	0.03

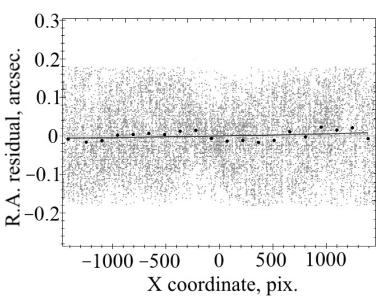
Table 1 was formed using catalog values and the assessments positions of reference stars on the series of frames. Frames were obtained at the observatory ISON-NM (H15). The exposure time was 150 sec. Table 2 shows telescope technical specifications.

Table 2

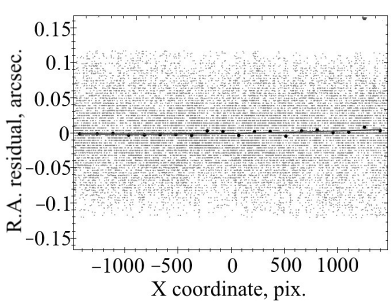
Technical characteristics of Centurion-18 telescope

N	Telescope technical characteristics:	
1	Instrument	0.45–m f/2.8
2	CCD-matrix	FLI ML09000-65
3	Frame size (pixels)	3056×3056
4	Pixel height and width (microns)	12×12
5	Focal length (mm)	1270.0
6	Field of view (arcmin.)	1°39′×1°40′

Using the computational method described in [26] and its implementation in CoLiTec software [27–29], the dependences of assessments of residuals by frame axes were obtained (Fig. 1–6).

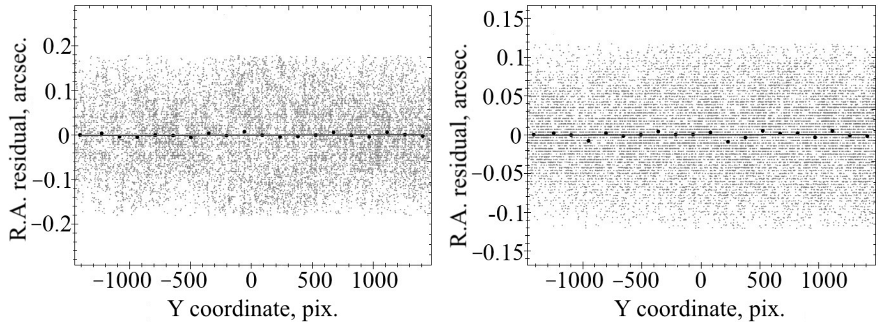


a



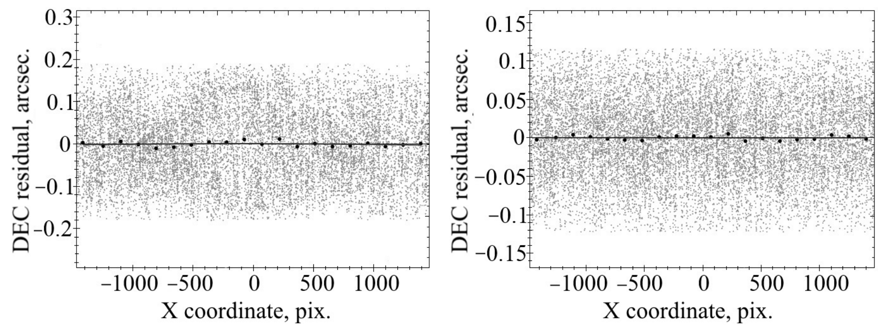
b

Fig. 1. Dependence of the right ascension residuals of the reference stars by frame abscissa: *a* – third-power model, *b* – fifth-power model



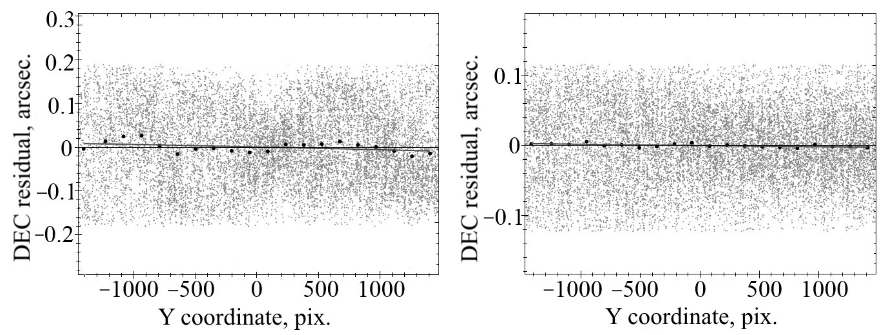
a *b*

Fig. 2. Dependence of the right ascension residuals of the reference stars by frame ordinate: *a* – third-power model, *b* – fifth-power model



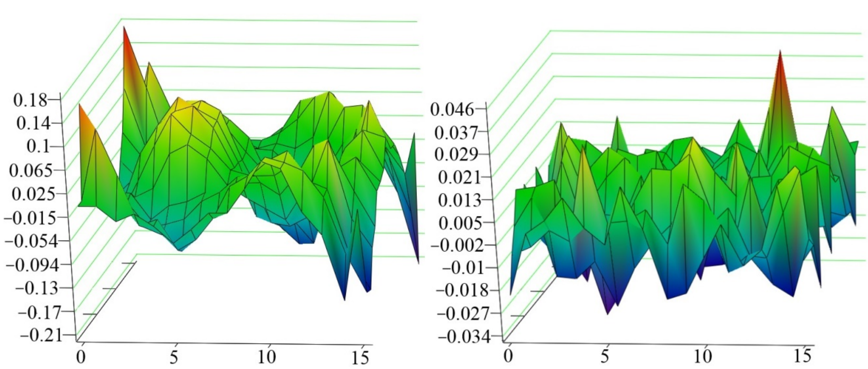
a *b*

Fig. 3. Dependence of the declination residuals of the reference stars by frame abscissa: *a* – third-power model, *b* – fifth-power model



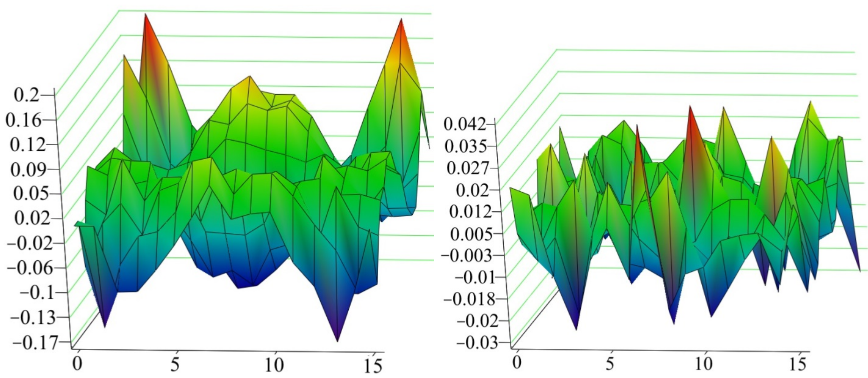
a *b*

Fig. 4. Dependence of the declination residuals of the reference stars by frame ordinate: *a* – third-power model, *b* – fifth-power model



a *b*

Fig. 5. Residuals field of the right ascension: *a* – third-power model, *b* – fifth-power model



a *b*

Fig. 6. Residuals field of the declination: *a* — third-power model *b* — fifth-power model

Fig. 1–6 show presence of a latent sinusoidal component in case of using the model of the third power. In Fig. 5, *a*, 6, *a* its presence is obvious. On the other hand, the use of reduction model of the fifth power (6), (7) practically eliminates it. Table 3 shows the results of analysis of indicators of the measurement accuracy of objects positions for the considered reduction models.

Table 3

Analysis results of influence of reduction model power

Parameter of comparison	Model of the third power	Model of the fifth power
Reference stars		
Number of reference stars	14018	15930
$\hat{\Delta}_{\alpha\delta} < 0.25$ arcsec.	13897	15928
σ_α, arcsec.	0.09	0.05
σ_δ, arcsec.	0,09	0,06
Quantile 0.9 of α residual module, arcsec.	0.139	0.088
Quantile 0.9 of δ residual module, arcsec.	0.141	0.09
Quantile 0.99 of α residual module, arcsec.	0.180	0.12
Quantile 0.99 of δ residual module, arcsec.	0.186	0.12
UCAC4 stars		
Number of UCAC4 stars	39571	39575
$\hat{\Delta}_\alpha$, arcsec.	0.00	0.00
$\hat{\Delta}_\delta$, arcsec.	−0.01	−0.01
σ_α, arcsec.	0.24	0.18
σ_δ, arcsec.	0.26	0.20
Quantile 0.9 of α residual module, arcsec.	0.342	0.256
Quantile 0.9 of δ residual module, arcsec.	0.377	0.285
Quantile 0.99 of α residual module, arcsec.	0.734	0.551
Quantile 0.99 of δ residual module, arcsec.	0.780	0.606

As shown in Table 3 the use of the reduction model of the fifth power immediately gives positive results in terms of accuracy. Similarly, it may be noted that incensement of the reduction model power practically eliminates the dependence of object's position measurement RMS from its location in the frame.

8. Research of significance of reduction model coefficients

To investigate the significance of the coefficients of constants plates we used the catalog and the assessed values of coordinates of reference stars in one frame.

The research was carried out using a direct reduction model of the third power (4), (5). To get LS-estimation of coefficients of constants plates the set of N reference stars has been used. Each i-th reference star is known by its equatorial coordinates from the catalog (α_i, δ_i) and its estimated rectangular coordinates (x_i, y_i). Using the formulas of spherical geometry for each i-th star from its catalog equatorial coordinates (α_i, δ_i) the coordinates were obtained in the ideal coordinate system (ξ_i, η_i).

According to [24] the residual sum of squares $R_{0\alpha}$, $R_{0\delta}$ for α and δ was calculated respectively:

$$R_{0\alpha} = \sum_{i=1}^{N} (\alpha_i - \alpha_i^{'})^2; \qquad (17)$$

$$R_{0\delta} = \sum_{i=1}^{N} (\delta_i - \delta_i^{'})^2, \qquad (18)$$

where α_i, δ_i is the catalog equatorial coordinates of the i-th reference star; $\alpha_i^{'}$, $\delta_i^{'}$ is the equatorial coordinates of the i-th reference star obtained from its rectangular coordinates (x_i, y_i) using the cubic model (4), (5) by a known vector of constants plates θ (1) and converted into the equatorial coordinates by formulas of spherical geometry [3].

The value of the Fisher f-criteria for $(R_{0\alpha}, R_{1\alpha})$ and $(R_{0\delta}, R_{1\delta})$ is defined by [24]:

$$F_{1\alpha} = \frac{R_{1\alpha}^2 - R_{0\alpha}^2}{k} \Big/ \frac{R_{0\alpha}^2}{N-r}; \qquad (19)$$

$$F_{1\delta} = \frac{R_{1\delta}^2 - R_{0\delta}^2}{k} \Big/ \frac{R_{0\delta}^2}{N-r}, \qquad (20)$$

where k is the number of investigated factors of regression model; r – rank of the matrix of partial derivatives F_x $(\mathrm{rang}\, F_x = r \le \min(m, N_{mea})$, m=2, since two parameters are evaluated – (x, y)); $R_{1\alpha}$, $R_{1\delta}$ is the residual dispersion when portions of the reduction model are excluded.

To calculate the value of the Fisher f-criterion of linear coefficients significance of constants plates as $R_{1\alpha}$, $R_{1\delta}$ were used a residual sum of squares $R_{1\alpha_{linear}}$ and $R_{1\delta_{linear}}$, which values were defined by the model with excluded linear component:

$$\xi_i = a_3 x_i^2 + a_4 x_i y_i + a_5 y_i^2 + a_6 x_i^3 + a_7 x_i^2 y_i + a_8 x_i y_i^2 + a_9 y_i^3; \qquad (21)$$

$$\eta_i = b_3 x_i^2 + b_4 x_i y_i + b_5 y_i^2 + b_6 x_i^3 + b_7 x_i^2 y_i + b_8 x_i y_i^2 + b_9 y_i^3. \qquad (22)$$

Similarly, for the investigation of significance of the reduction coefficients of the residual model of the second power the sum of squares $R_{1\alpha_{quadratic}}$, $R_{1\delta_{quadratic}}$, was calculated. The calculation of $\alpha_i^{'}$, $\delta_i^{'}$ was performed according to the expression:

$$\xi_i = a_0 + a_1x_i + a_2y_i \; + a_6x_i^3 + a_7x_i^2y_i + a_8x_iy_i^2 + a_9y_i^3; \quad (23)$$

$$\eta_i = b_0 + b_1x_i + b_2y_i \; + b_6x_i^3 + b_7x_i^2y_i + b_8x_iy_i^2 + b_9y_i^3. \quad (24)$$

The calculation of residual dispersions $R_{1\alpha cubic}$, $R_{1\delta cubic}$ were carried out in the absence of cubic components in the reduction model:

$$\xi_i = a_0 + a_1x_i + a_2y_i + a_3x_i^2 + a_4x_iy_i + a_5y_i^2; \quad (25)$$

$$\eta_i = b_0 + b_1x_i + b_2y_i + b_3x_i^2 + b_4x_iy_i + b_5y_i^2. \quad (26)$$

The calculation results are presented in Table 4.

Table 4

The results of calculation of the Fisher f-criteria without third-power polynomial coefficients

№	Excluded model coefficients	$\sqrt{R_\alpha / N}$	$\sqrt{R_\delta / N}$	F_α	F_δ
1	$a_0 + a_1x_i + a_2y_i$; $b_0 + b_1x_i + b_2y_i$;	1444	1377	$3.02\cdot10^{19}$	$6.73\cdot10^{19}$
2	$a_3x_i^2 + a_4x_iy_i + a_5y_i^2$; $b_3x_i^2 + b_4x_iy_i + b_5y_i^2$;	0.1	0.08	817.08	696
3	$a_6x_i^3 + a_7x_i^2y_i + a_8x_iy_i^2 + a_9y_i^3$; $b_6x_i^3 + b_7x_i^2y_i + b_8x_iy_i^2 + b_9y_i^3$.	0.3	0.38	$7.6\cdot10^4$	$2.8\cdot10^5$

Note: $\sqrt{R_\alpha / N} = 0.06$; $\sqrt{R_\delta / N} = 0.04$

Analogous investigations were carried out with a fifth-power polynomial (Table 5). In this case, the residual sum of squares $R_{1\alpha_{forth}}$, $R_{1\delta forth}$, was obtained using the values $\alpha_i^{'}$, $\delta_i^{'}$ that had been calculated without the fourth-power polynomial components:

$$\begin{aligned}\xi_i &= a_0 + a_1x_i + a_2y_i + a_3x_i^2 + a_4x_iy_i + \\ &+ a_5y_i^2 + a_6x_i^3 + a_7x_i^2y_i + a_8x_iy_i^2 + a_9y_i^3 + \\ &+ a_{15}x_i^5 + a_{16}x_i^4y_i + a_{17}x_i^3y_i^2 + a_{18}x_i^2y_i^3 + \\ &+ a_{19}x_iy_i^4 + a_{20}y_i^5; \end{aligned} \quad (27)$$

$$\begin{aligned}\eta_i &= b_0 + b_1x_i + b_2y_i + b_3x_i^2 + b_4x_iy_i + \\ &+ b_5y_i^2 + b_6x_i^3 + b_7x_i^2y_i + b_8x_iy_i^2 + b_9y_i^3 + \\ &+ b_{15}x_i^5 + b_{16}x_i^4y_i + b_{17}x_i^3y_i^2 + b_{18}x_i^2y_i^3 + \\ &+ b_{19}x_iy_i^4 + b_{20}y_i^5. \end{aligned} \quad (28)$$

The values of $R_{1\alpha fifth}$ and $R_{1\delta fifth}$ were defined by the model with excluded fifth-power components:

$$\begin{aligned}\xi_i &= a_0 + a_1x_i + a_2y_i + a_3x_i^2 + a_4x_iy_i + \\ &+ a_5y_i^2 + a_6x_i^3 + a_7x_i^2y_i + a_8x_iy_i^2 + a_9y_i^3 + \\ &+ a_{10}x_i^4 + a_{11}x_i^3y_i + a_{12}x_i^2y_i^2 + a_{13}x_iy_i^3 + a_{14}y_i^4; \end{aligned} \quad (29)$$

$$\begin{aligned}\eta_i &= b_0 + b_1x_i + b_2y_i + b_3x_i^2 + b_4x_iy_i + \\ &+ b_5y_i^2 + b_6x_i^3 + b_7x_i^2y_i + b_8x_iy_i^2 + b_9y_i^3 + \\ &+ b_{10}x_i^4 + b_{11}x_i^3y_i + b_{12}x_i^2y_i^2 + b_{13}x_iy_i^3 + b_{14}y_i^4. \end{aligned} \quad (30)$$

The results of calculation of the Fisher criteria for exclusion of each coefficient of reduction models are presented in Table 6.

Table 5

The results of calculation of the Fisher f-criteria when parts of the fifth-power reduction model are excluded

№	Excluded model coefficients	$\sqrt{R_\alpha / N}$	$\sqrt{R_\delta / N}$	F_α	F_δ
1	$a_0 + a_1x_i + a_2y_i$; $b_0 + b_1x_i + b_2y_i$;	$2\cdot10^6$	$1.9\cdot10^6$	$8.2\cdot10^{19}$	$1.05\cdot10^{20}$
2	$a_3x_i^2 + a_4x_iy_i + a_5y_i^2$; $b_3x_i^2 + b_4x_iy_i + b_5y_i^2$;	0.09	0.02	$1.6\cdot10^5$	$2.4\cdot10^4$
3	$a_6x_i^3 + a_7x_i^2y_i + a_8x_iy_i^2 + a_9y_i^3$; $b_6x_i^3 + b_7x_i^2y_i + b_8x_iy_i^2 + b_9y_i^3$;	0.27	0.09	$6.4\cdot10^5$	$1.8\cdot10^5$
4	$a_{10}x_i^4 + a_{11}x_i^3y_i + a_{12}x_i^2y_i^2 + a_{13}x_iy_i^3 + a_{14}y_i^4$; $b_{10}x_i^4 + b_{11}x_i^3y_i + b_{12}x_i^2y_i^2 + b_{13}x_iy_i^3 + b_{14}y_i^4$;	0.03	0.01	$1.3\cdot10^4$	$2.3\cdot10^3$
5	$a_{15}x_i^5 + a_{16}x_i^4y_i + a_{17}x_i^3y_i^2 + a_{18}x_i^2y_i^3 + a_{19}x_iy_i^4 + a_{20}y_i^5$; $b_{15}x_i^5 + b_{16}x_i^4y_i + b_{17}x_i^3y_i^2 + b_{18}x_i^2y_i^3 + b_{19}x_iy_i^4 + b_{20}y_i^5$.	0.06	0.006	$3.6\cdot10^4$	426

Table 6

The results of calculation of the Fisher f-criteria when coefficients of the fifth-power reduction model are excluded

№	Excluded model coefficients	$\sqrt{R_\alpha / N}$	$\sqrt{R_\delta / N}$	F_α	F_δ
1	a_0	0.002	0.002	$2.9\cdot10^{-8}$	$-9\cdot10^{-9}$
2	a_1x_i	58.8	$1.9\cdot10^6$	$1.9\cdot10^{13}$	$3.1\cdot10^{20}$
3	a_2y_i	$2\cdot10^6$	$6.4\cdot10^2$	$2.4\cdot10^{20}$	$3.5\cdot10^{13}$
4	$a_3x_i^2$	0.003	0.002	362.7	159.2
5	$a_4x_iy_i$	0.002	0.014	23.4	$1.6\cdot10^4$
6	$a_5y_i^2$	0.08	0.015	$4.2\cdot10^5$	$2\cdot10^4$
7	$a_6x_i^3$	0.08	0.05	$3.8\cdot10^5$	$2.3\cdot10^5$
8	$a_7x_i^2y_i$	0.02	0.002	$2.8\cdot10^4$	77.40
9	$a_8x_iy_i^2$	0.03	0.017	$4.8\cdot10^4$	$2.3\cdot10^4$
10	$a_9y_i^3$	0.02	0.002	$3.9\cdot10^4$	7.1
11	$a_{10}x_i^4$	0.002	0.002	1.4	50.05
12	$a_{11}x_i^3y_i$	0.002	0.003	0.7	547
13	$a_{12}x_i^2y_i^2$	0.005	0.002	$1.4\cdot10^3$	102
14	$a_{13}x_iy_i^3$	0.002	0.003	11.7	925
15	$a_{14}y_i^4$	0.02	0.006	$3.2\cdot10^4$	$3\cdot10^3$
16	$a_{15}x_i^5$	0.02	0.003	$2.4\cdot10^4$	513
17	$a_{16}x_i^4y$	0.002	0.002	1.9	89
18	$a_{17}x_i^3y_i^2$	0.01	0.002	$6.8\cdot10^3$	297
19	$a_{18}x_i^2y_i^3$	0.002	0.002	12.5	174
20	$a_{19}x_iy_i^4$	0.007	0.002	$2.2\cdot10^3$	0.35
21	$a_{20}y_i^5$	0.007	0.002	$2.8\cdot10^3$	0.8

Based on Tables 4–6 we can conclude that the most significant coefficients of investigated reduction models are linear components, specifically a_1x_i and a_2y_i.

Concurrently, the significance of the fifth-power reduction model coefficients is comparable with the coefficients of the cubic model. This means that the fifth-power model coefficients have an important influence on the approximation of the aberrations of the optical system, which justifies the application of the investigated reduction model.

9. Conclusions

1. The estimation of celestial objects positions using cubic and fifth-power reduction models was carried out. The reduction models were built using the selected set of reference stars. In this paper as the reference stars catalog the UCAC4 was used.

2. For carrying out the comparative analysis of the investigated reduction models the accuracy indicators of objects positions measuring have been introduced. These include estimates of the mean and standard deviation of assessment of objects equatorial coordinates in each frame fragment; dependence of right ascension and declination residuals by frame's ordinate and abscissa; number of reference stars from the catalog and its spread in the digital frame.

3. Using entered accuracy indicators the investigation of the third and fifth-power reduction model was carried out. The investigation established the presence of a sinusoidal component in the dependence of the deviations of parameters of celestial objects when the cubic reduction model is used. At the same time, for the fifth-power reduction model the dependence was almost completely absent. Analysis of reduction models also showed improvement of the investigated accuracy indicators of assessments of celestial objects positions for the fifth-power model.

4. We assessed the significance of the investigated reduction model coefficients using the Fisher f-criteria. Assessment was carried out for parts of reduction models, as well as individually for each coefficient. The assessment showed that the significance of the fifth-power model coefficients is quite comparable to the coefficients of the cubic model which proves the validity of its application for assessing the positions of celestial objects in digital frames.

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