Запропоновано нову реалізацію парадігми предписаного керування на прикладі векторної моделі у вигляді нелінійної нестаціонарної системи диференційних рівнянь першого порядку. Поставлена задача мінімізації квадрату відхилень як задача максимізації одного з критеріїв адекватності – точності. Виведені два додаткові рівняння, які реалізують максимізацію адекватності за двома додатковими її критеріями: глибина та повнота. Продемонстровано, що вирази розв'язків співпадають з виразами часткового випадку – методу скоросного градієнту. Виведено норми похибки

Ключові слова: предписанне управління, максимізація адекватності, додаткові рівняння у задачі оптимального керування

Предлагается новая реализация парадигмы предписанного управления как задача управления на примере векторной модели в виде нелинейной нестационарной системы дифференциальных уравнений первого порядка. Поставлена задача минимизации квадрата отклонений как задача максимизации одного из критериев адекватности – точности. Выведены два дополнительные уравнения, реализующие максимизацию адекватности по двум дополнительным критериям: глубина и полнота. Продемонстрировано, что выражения решений совпадают с выражениями частного случая – метода скоростного градиента. Выведены нормы погрешности

Ключевые слова: предписанное управление, максимизация адекватности, дополнительные уравнения в задаче оптимального управления

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REALIZATION OF THE PARADIGM OF PRESCRIBED CONTROL OF A NONLINEAR OBJECT AS THE PROBLEM ON MAXIMIZATION OF ADEQUACY

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1. Introduction

The difference and commonality of two paradigms of contemporary development of physics and cybernetics, descriptive and prescriptive [1–21], stimulated over recent decades their mutual influence and development. Cybernetic approach [1–15, 21–47] and formation of prescribed behavior [41–45], as the new paradigm of controlled interaction and information connection, is becoming increasingly common. Its realization [2, 8, 10, 11–13, 21, 25, 28, 29, 31, 34, 48], observed in the living organisms, by being artificially copied, is also formed when designing machines [1, 5–8, 15–18, 31–32], in the integrated computer systems [14–17], organized production facilities and public structures [9, 15–18, 19–20]. In the recent decades, it has emerged as the necessity of engineering practice [21–26, 40–46, 49–51]. It must be specially noted that the introduction of the controlling influence, which realizes the prescribed – model functioning of a system from many other possible, distinguishes it as particularly interesting for realization. The widespread implementation of cybernetic approaches in view of its advantages becomes absolutely relevant [40–46, 50]. The most widely used approach that is applied to building up control algorithms in nonlinear systems is the method of speed gradient, which is presented in the paper [43]. It "explores formation of control algorithms in nonlinear systems" for the model of "continuous non–stationary system in the form: $\dot{X} = F(X, U, t)$, with the purpose of control in the form of the smooth objective function $Q(x,t) \geq 0$ ".

However, in spite of the positive results of application, this method based on axiomatic assumptions. For example, about proportionality between the rate of change in the controlling action and the gradient, taken by the controlling action from the speed of change in the objective function. Besides the mentioned, the assumption is frequently used about proportionality between the controlling influence and the rate of change in the objective function. Such assumptions are postulated and require thorough checking and correction. At the same time, for further successful realization of the paradigm of prescribing with other forms of objective functions and availability of constraints in the form of inequalities, solving a whole range of questions is required. They should include: development of generalized methods of analytical synthesis of the expressions of controlling influence; forecasting and evaluation of variable strategies [15–18, 31], forecasting parameters of the system [9, 15–20, 31], filtration and optimization [15, 28]. In this regard, the paradigm of adequacy [32] and its application for analytical evaluation [35–36] and improvement of the model [38] is of interest. It is especially attractive for the formation of analytical scientifically-substantiated methods of control of such non-stationary systems [39]. Thus, a generalized realization of the paradigm of control of a dynamic object, i. e., changing its properties in the control process, as the basis of analytical synthesis of controlling action, is a relevant task, whose solution opens up the prospects of constructing cybernetic systems of control.

2. Literature review and problem statement

At present, there is a number of papers, in which the problem of managing non-stationary systems in small and in the asymptotic sense by cybernetic methods is posed. Its solution, in the form of harmonic excitation control, changes radically both the properties and dynamics of the system, converting chaotic motion to the periodic one [1–6, 43, 44].

However, nonlinearity, as the main reason and special feature of this phenomenon, could not be taken into account in full, due to inefficiency of the used methods of linearization, a piecewise constant approximation and the complexity of the methods of nonlinear analysis. Another reason, which hampers the application of the modern apparatus of nonlinear control, is the qualitative difference of the traditional tasks of control from the cybernetic ones. As shown in the analysis of the papers [43, 50], dedicated to the development of prescribed paradigm, such tasks as: synchronization of chaotic oscillations of systems; creation of modes with assigned properties; change in the phase portraits of model, were even not considered by traditional methods of descriptive paradigm.

Despite the fact that cybernetic methods are characterized by the weakened requirements, demanded of the purpose of control, the requirements for minimum interference, contained in them, are more rigid [15–17, 25–29, 43, 44]. It is the requirement of minimum interference that makes them attractive not only for the synthesis of control in the chaotic systems, but also for the application to a broader class of the tasks of control over oscillating processes [25, 37, 40–46]. The application of cybernetic methods to the control of social-economic [9, 20] or quantum systems in the microcosm [3, 7, 10, 11, 26, 34, 40–48], finding the criteria of manageability [32], makes it possible to build quantum models and to manage molecular systems [34, 40–46], including photobiostimulations [21, 34, 48]. However, the scientific substantiation and generalization of certain attempts of construction of cybernetic methods of control over nonlinear systems is the main unsolved problem. The analysis of articles [40–46, 49, 50, 51] shows that the idea of speed gradient, while efficient at solving a number of problems, in the majority of the cases, when constructing controlling action, is based on the axiomatic hypothesis about proportionality of gradient on the rate of change in the objective function and the rate of change in controlling influence [43, 50]. For this reason, it does not have generality [43, 49–51], and, consequently, the search for the general fundamental regularities that allow establishing the connection between them is of scientific interest and is a relevant task for building up the strategies of control of nonlinear systems. At the same time, the basis for the cybernetic methods of control is cognitive analysis of inner properties and predicted capabilities of both the system itself and of the systems interacting with it, the properties and relative positions of which are considered in accordance with the paradigm of prescribing [11–13, 19]. The comparative approach, proposed in the papers [19, 33], increases their efficiency as the result of replacement of direct measurements with comparison. It formed the comparative theory of cognitive activity and, in particular, of such stages as identification, multifactor assessment, verification of models.

Realization of this theory in the course of cognitive activity makes it possible to verify the formed models and to assess applicability of computational methods, as well as estimate their sensitivity to the peculiarities of the solved problems [19]. Gathering the criteria of comparison is the key element of this theory. The analysis of properties and the selection of the types of these criteria lead to the natural-scientific understanding of adequacy as one of the categories of the theory of cognitive activity. It is widely acknowledged that the criterion of adequacy is comprehensive. It is characterized by such features as: reliability, accuracy, depth, completeness, essentiality, simplicity, applicability. The solution of the problem of quantitative assessment of adequacy, recorded in the form of a uniform expression, which considers several properties simultaneously, is demonstrated in [37]. The application of this concept to the improvement of the processes of design is a promising prerequisite. Thus, for example, in the paper [32] it was applied to the synthesis of different types of regulators, and in the articles [35–38] – to the improvement of the process of constructing a mathematical model. In this connection, let us define the goal of the present work as the substantiation of the method of selection of controlling action, based on cybernetic approach to the control of a dynamic nonlinear object (NO), which realizes the paradigm of the prescribed behavior as the problem on maximization of adequacy in the interval and at the particular point.

3. The purpose and the tasks of the research

The studies, conducted in the present work, set the goal of establishing interrelations between the three criteria that determine the concept of adequacy, and controlling action for the processes of the actual and prescribed functioning of non-stationary, nonlinear system.

To achieve the set goal, on the example of vector model in the form of nonlinear non-stationary system of differential equations of the first order, the following tasks are formulated:

– to form a system of equations that maximizes the adequacy of two processes by one of its criteria, accuracy, as the problem of minimization with the constraints in the form of inequalities;

– to explore the possibility of complementing the system of equations of the problem of minimization with independent equations with the availability of separate target and model;

– to examine the possibility of complementing the system of equations of the problem of minimization with two independent equations with the use of two additional criteria of adequacy: depth and completeness, and to derive equations for the synthesis of controlling action;

– to perform assessment of the norm of error of the vector of strategies and to present it in the form of dependency on the properties of object, error in the function of efficiency, synthesized law of controlling action and the prescribed law of object functioning.

4. Setting and solving the problem on control of efficiency of a nonlinear object

Let us introduce n-component vector of strategies $\overline{X}(t)$, the components of which describe the states of a non-stationary system [43] and they are independent functions of time.

Each of these components and the vector as a whole can be reflected into an n-dimensional space. Let us assume that the prescribed change in the vector of state is set in the interval of time [0, t] also by the n-component vector of strategies $\bar{X}_e(t)$, then we introduce the objective function:

$$\bar{\Psi}(t)=\bar{X}\ (t)-\bar{X}_e(t). \tag{1}$$

Let us also assume that the description of non-stationary, nonlinear system is generalized with the help of a mathematical model:

$$\frac{d}{dt}\bar{\Psi}=f\left(\bar{X}(t),t,[A],[C],[I],\bar{U}\right). \tag{2}$$

In the equation (2), it is designated: $\bar{U}$ is the m-dimensional vector of controlling action; $[A]$ is the kinematic matrix of a housing; which considers its kinematic parameters and spatial positioning (it is introduced by the author in the monograph [32]); $[C]$ and $[I]$ are the matrices of coefficients of resistance and added masses, the consideration of which is necessary for determining separate components of the vector of strategies $\bar{X}(t)$.

In a general case, all mentioned magnitudes are the functions of time and, from the point of view of generality, this model is not different from the model of a non-stationary system [43], adopted as the basis of the study, but it additionally takes into account mechanical motions of the housing. For compactness of the record, this dependency on time is not conditionally shown.

Let us also define the magnitude of the efficiency Q as the k-dimensional vector, whose components are calculated by the methodology [36] and which are determined by the set of factors, for example: by technological result, by resource consumption, by probability and time of full realization. However, the formation of this function was discussed in the paper [36] and it is not the subject of the present studies; therefore, let us confine ourselves to the assumption that it exists and that its components are the three times differentiated functions of the vectors of strategies and control, and they also depend on external influences and the properties of actual object, which will be simulated:

$$Q=f_1\left(\bar{Y},\bar{X},[A],[C],[I],\bar{U}\right).$$

It should be noted that the assumption about existence and continuity, and differentiability of this function for each specific case of the object of control is subject to prove, which cannot be realized on the model, selected as the object of the study. It goes without saying that this circumstance and condition defines the limits of applicability of the results of these studies and it is used as the known, well-proven in the natural sciences, method of assumptions. Let us also assume that this function is integrable with square and set by the paradigm of prescribing in the form:

$$Q_*=f_1\left(\bar{Y},\bar{X}_e,[A],[C],[I],\bar{U},t\right),$$

then in the interval [0, t], its norm with the Euclidean metric will be determined:

$$\|Q_*\|=\sqrt{\int_0^t\left[f_1\left(\bar{Y},\bar{X}_e,[A],[C],[I],\bar{U},\tau\right)\right]^2 d\tau}.$$

Now let us examine two processes Q_* – prescribed and Q of the actual functioning (further on, for simplicity and reduction of record, the actual dependency on time formally is not shown). Their adequacy, obviously, will be determined by the magnitude of deviation. It is shown in the paper [37] that the local adequacy, i. e., at the given point in time, is best characterized for the two compared processes by relative error. In this case, the minimization of the square of relative error corresponds to the maximization of adequacy. Since the model is accepted as the object, let us further confine ourselves to examination of relative error.

Thus, we will write down the dimensionless function as a dimensionless deviation of the function of efficiency or relative error:

$$\delta=(Q-Q_*)/\|Q_*\|;\ \|Q_*\|\neq 0. \tag{3}$$

Let us form a quadratic form with the aid of dimensionless vector – function (3):

$$F\left(\bar{Y},\bar{X},[A],[C],[I],\bar{U}\right)=\frac{1}{2}[\delta]^T[P][\delta], \tag{4}$$

where $[P]$ is the positively determined matrix, whose other properties we do not limit as yet. Let us set the problem of minimization of the objective function:

$$\min_{\bar{U}}\left\{F\left(\bar{Y},\bar{X},[A],[C],[I],\bar{U}\right)\right\}=\min_{\bar{U}}\left\{\frac{1}{2}[\delta]^T[P][\delta]\right\},$$

in the presence of constraints in the form of inequalities:

$$g_j=f_{3j}\left(\bar{Y},\bar{X},[A],[C],[I],\bar{U}\right)-b_j\leq 0;\ j=\overline{1,m}. \tag{5}$$

Lagrange's function will be written down as follows:

$$L\left(\bar{X},\bar{\Lambda}\right)=\frac{1}{2}[\delta]^T[P][\delta]+ \\ +\sum_{j=1}^{m}\lambda_j\left(f_{3j}\left(\bar{Y},\bar{X},[A],[C],[I],\bar{U}\right)-b_j\right). \tag{6}$$

Introducing the vector–function

$$\bar{F}_3\left(\bar{Y},\bar{X},[A],[C],[I],\bar{U}\right),$$

the components of which are set in the form

$$f_{3j}\left(\bar{Y},\bar{X},[A],[C],[I],\bar{U}\right),$$

and the matrix-column $[b]$ with the components b_j, we write the Lagrange's function in the form:

$$L\left(\bar{X},\bar{\Lambda}\right)=\frac{1}{2}[\delta]^T[P][\delta]+ \\ +\bar{\Lambda}^T\left[\bar{F}_3\left(\bar{Y},\bar{X},[A],[C],[I],\bar{U}\right)-[b]\right]. \tag{7}$$

Now, in accordance with the method of Lagrange's multipliers, the vector of strategies $\bar{X}$, the vector of Lagrange's multipliers, auxiliary vectors $\bar{V}$ and $\bar{W}$ will be defined as the solution of the system, which minimizes objective function – the square of the relative deviation:

$$\begin{cases}\frac{1}{2}\nabla_X\left([\delta]^T[P][\delta]\right)+\bar{\Lambda}^T\nabla_x\left\{\left[\bar{F}_3\left(\bar{Y},\bar{X},[A],[C],[I],\bar{U}\right)-[b]\right]^T\right\}-\bar{V}=0,\\ \left[\bar{F}_3\left(\bar{Y},\bar{X},[A],[C],[I],\bar{U}\right)-[b]\right]+\bar{W}=0,\\ \bar{X}^T\bar{V}=0,\\ \bar{\Lambda}^T\bar{W}=0.\end{cases} \quad (8)$$

As a result of simple algebraic conversions (from the first equation of the system (8), by multiplying it preliminarily by the transposed vector of strategies, we subtract the second equation, multiplied by the transposed vector of Lagrange's multipliers), and also, taking into account two other equations and conditions of the saddle point:

$$\Delta\bar{X}^T\nabla_X\left([\delta]^T[P][\delta]\right)=0,$$

we will obtain:

$$\sum_{j=1}^{m}\lambda_j b_j=0.$$

From the last equation of the system (8), taking into account the condition of additional non-rigidity, let us find, through simple comparison, the auxiliary vector $\bar{W}$:

$$\bar{W}=[b_1,\ldots,b_i,\ldots,b_n]^T.$$

In addition to this, in accordance with the physical sense, by definition, let us write down expressions for the Lagrange's multipliers:

$$\lambda_j=-\frac{1}{2}\nabla_X\left([\delta]^T[P][\delta]\right)\left[\nabla_X f_{3j}\left(\bar{Y},\bar{X},[A],[C],[I],\bar{U}\right)\right]^{-1},$$

and form additional equation:

$$\sum_{j=1}^{m}\left\{\nabla_X\left([\delta]^T[P][\delta]\right)\left[\nabla_X f_{3j}\left(\bar{Y},\bar{X},[A],[C],[I],\bar{U}\right)\right]^{-1}\right\}b_j=0,$$

which will complement the system (8), but owing to its uniformity it will not allow obtaining a single solution. Taking into account the last equation, let us rewrite the system (8):

$$\begin{cases}\frac{1}{2}\nabla_X\left([\delta]^T[P][\delta]\right)+\nabla_x\sum_{j=1}^{m}\lambda_j f_{3j}\left(\bar{Y},\bar{X},[A],[C],[I],\bar{U}\right)-\bar{V}=0,\\ \nabla_\lambda\sum_{j=1}^{m}\lambda_j\left(f_{3j}\left(\bar{Y},\bar{X},[A],[C],[I],\bar{U}\right)-b_j\right)+[b]=0,\\ \sum_{j=1}^{m}\nabla_x\left([\delta]^T[P][\delta]\right)\left[\nabla_X f_{3j}\left(\bar{Y},\bar{X},[A],[C],[I],\bar{U}\right)\right]^{-1}b_j=0,\\ \bar{X}^T\bar{V}=0,\\ \bar{\Lambda}^T[b]=0.\end{cases}$$

This system is also not complete; therefore, it does not have a single solution. It has n+2m+2 equations at the 3n+ +2m unknowns. For increasing the degree of completeness, we will use the recurrent approximation [32] of objective function, as this was also demonstrated in the papers [38, 39]. Realization of these ideas in the form $n+1$ – recurrent approximation of the objective function:

$$\begin{aligned}F(\bar{X}_{n+1})=&\frac{1}{2}\left\{[\delta]^T[P][\delta]\right\}\Big|_{\bar{X}=\bar{X}_n}+\\ &+\frac{1}{2}\Delta\bar{X}_n^T\left\{\nabla_X\left([\delta]^T[P][\delta]\right)\right\}\Big|_{\bar{X}=\bar{X}_n}+\\ &+\frac{1}{4}\Delta\bar{X}_n^T\left\{\nabla_x^2\left([\delta]^T[P][\delta]\right)\right\}\Big|_{\bar{X}=\bar{X}_n}\Delta\bar{X}_n\end{aligned}$$

leads to the new form of the Lagrange function

$$\begin{aligned}L(\bar{X},\bar{\Lambda})=&\frac{1}{2}\left\{[\delta]^T[P][\delta]\right\}\Big|_{\bar{X}=\bar{X}_n}+\\ &+\frac{1}{2}\Delta\bar{X}_n^T\left\{\nabla_X\left([\delta]^T[P][\delta]\right)\right\}\Big|_{\bar{X}=\bar{X}_n}+\\ &+\frac{1}{4}\Delta\bar{X}_n^T\left\{\nabla_x^2\left([\delta]^T[P][\delta]\right)\right\}\Big|_{\bar{X}=\bar{X}_n}\Delta\bar{X}_n+\\ &+\bar{\Lambda}^T\left[\bar{F}_3\left(\bar{Y},\bar{X},[A],[C],[I],\bar{U}\right)-[b]\right].\end{aligned}$$

Disclosing directly for the selected objective function the expressions of gradient of the first and the second order:

$$\nabla_X\left([\delta]^T[P][\delta]\right)=\nabla_X\left([\delta]^T[P]\right)[\delta]+[\delta]^T[P]\nabla_X([\delta]);$$

$$\begin{aligned}\nabla_x^2\left([\delta]^T[P][\delta]\right)=&\nabla_x^2\left([\delta]^T[P]\right)[\delta]+\\ &+2\nabla_X\left([\delta]^T[P]\right)\nabla_X([\delta])+[\delta]^T[P]\nabla_x^2([\delta]),\end{aligned}$$

we will find that the auxiliary vector $\bar{V}$ depends only on the form of the objective function:

$$\begin{aligned}\bar{V}=&-\frac{1}{4}\left\{\nabla_x^2\left([\delta]^T[P][\delta]\right)\right\}\Big|_{\bar{X}=\bar{X}_n}\Delta\bar{X}_n=\\ =&-\frac{1}{4}\left\{\begin{aligned}&\nabla_x^2\left([\delta]^T[P]\right)[\delta]+\\ &+2\nabla_X\left([\delta]^T[P]\right)\nabla_X([\delta])+[\delta]^T[P]\nabla_x^2([\delta])\end{aligned}\right\}\Bigg|_{\bar{X}=\bar{X}_n}\Delta\bar{X}_n.\end{aligned}$$

Another possible variant of complementing the system (8) are the cases when the point of the solution is not the saddle point, and then the condition is satisfied:

$$\frac{1}{2}\Delta\bar{X}^T\nabla_X\left([\delta]^T[P][\delta]\right)+\sum_{j=1}^{m}\lambda_j b_j=0$$

or

$$\frac{1}{2}\Delta\bar{X}^T\nabla_X\left([\delta]^T[P][\delta]\right)+\bar{\Lambda}^T[b]=0$$

and the system will be complemented with equation, but with the unknown auxiliary vector:

$$\begin{cases} \frac{1}{2}\nabla_X\left([\delta]^T[P][\delta]\right)+\nabla_x\left\{\sum_{j=1}^{m}\lambda_j f_{3j}\left(\bar{Y},\bar{X},[A],[C],[I],\bar{U}\right)\right\}-\bar{V}=0; \\ \bar{V}=-\frac{1}{4}\left\{\nabla_x^2\left([\delta]^T[P][\delta]\right)\right\}\Big|_{\bar{X}=\bar{X}_n}\Delta\bar{X}_n; \qquad \bar{X}_n=\Delta\bar{X}_n; \\ \nabla_\lambda\left\{\sum_{j=1}^{m}\lambda_j\left(f_{3j}\left(\bar{Y},\bar{X},[A],[C],[I],\bar{U}\right)-b_j\right)\right\}+\bar{W}=0; \\ \sum_{j=1}^{m}\nabla_X\left([\delta]^T[P][\delta]\right)\left[\nabla_X f_{3j}\left(\bar{Y},\bar{X},[A],[C],[I],\bar{U}\right)\right]^{-1}b_j=0; \\ \frac{1}{2}\Delta\bar{X}^T\nabla_X\left([\delta]^T[P][\delta]\right)+\bar{\Lambda}^T[b]=0;\ \bar{X}^T\bar{V}=0;\ \bar{\Lambda}^T\bar{W}=0. \end{cases}$$

However, assuming that as a result of change in the state of the system from a certain initial value to the new, in which the objective function reaches a zero value, i. e., a zero deviation of efficiency from the reference one, we will obtain, taking into account the expression of the objective function (4) from the first equation of the system:

$$\frac{1}{2}\left\{[\delta]^T[P][\delta]\right\}\Big|_{\bar{X}=\bar{X}_n}= =\Delta\bar{X}^T\sum_{j=1}^{m}\lambda_j\nabla_x f_{3j}\left(\bar{Y},\bar{X},[A],[C],[I],\bar{U}\right).$$

The system (8) realizes maximization of adequacy by one of the criteria, which determine adequacy – accuracy [32, 36, 38], since the problem minimizes the relative square of deviation of efficiency, which in turn ensures local correspondence of the model to the object. For the provision of maximization by two other criteria – depth and completeness – let us conduct additional studies. Thus, we examine the properties of the Lagrange's function (7), which satisfies the problem on its minimization in the time interval as the problem on the maximization of adequacy. As shown in the articles [37, 38], the adequacy of any model in the section of determining one of the arguments – time from zero to the current moment t is maximized, if

$$\min_{\bar{X}}\left\{L\left(\bar{X},\bar{\Lambda}\right)^2\right\},$$

the latter is equivalent to the condition:

$$L\left(\bar{X},\bar{\Lambda}\right)\nabla_X\left[L\left(\bar{X},\bar{\Lambda}\right)\right]=0.$$

As a result of heterogeneity and nontriviality of the problem $L\left(\bar{X},\bar{\Lambda}\right)\neq 0,$ and, consequently, $\nabla_X\left[L\left(\bar{X},\bar{\Lambda}\right)\right]=0.$ For a generalized problem, as one of the components of the vector $\bar{X},$ the time is used, i. e., the time is recorded implicitly. In this problem we separate it on purpose, and in this case we note that the controlling influence $\bar{u}\left(\bar{X}(t),t\right)$ is the function of both the vector of strategies and time.

As an example, let us take, similarly to [43], the system, linear by inputs (2), in which:

$$f\left(\bar{X}(t),t,[A],[C],[I],\bar{U}\right)= =G\left(\bar{X}(t),t\right)+B\left(\bar{X}(t),t\right)\bar{u}\left(\bar{X}(t),t\right),$$

where $G\left(\bar{X}(t),t\right)$ is the n-component vector, $B\left(\bar{X}(t),t\right)$ is the matrix – function of dimensionality of nxm.

In connection with the presented above, the Lagrange's function is a complex function, so for its analysis let us find the substantial derivative:

$$\frac{dL\left(\bar{X},\bar{\Lambda}\right)}{dt}=\frac{\partial L\left(\bar{X},\bar{\Lambda}\right)}{\partial t}+[\delta]^T[P]\nabla_x\left\{[\delta]^T\right\}\frac{\partial}{\partial t}\left(\bar{X}^T\right)+ +[\delta]^T[P]\nabla_u\left\{[\delta]^T\right\}\frac{\partial}{\partial t}\left(\bar{u}^T\right).$$

Further, taking into account that the time t is an independent variable, on which the vector of strategies $\bar{X}(t)$ depends, and, consequently, deviation from the prescribed behavior of the function of objective of the system $[\delta]$, let us convert the substantial derivative to the form:

$$\frac{dL\left(\bar{X},\bar{\Lambda}\right)}{dt}= =[\delta]^T[P]\left\{B\left(\bar{X}(t),t\right)\nabla_x\left\{[\delta]^T\right\}\bar{u}\left(\bar{X}(t),t\right)+\nabla_u\left\{[\delta]^T\right\}\frac{\partial}{\partial t}\left(\bar{u}^T\right)\right\}+ +\frac{\partial L\left(\bar{X},\bar{\Lambda}\right)}{\partial t}+[\delta]^T[P]\nabla_x\left\{[\delta]^T\right\}\left(G\left(\bar{X}(t),t\right)+\frac{d}{dt}\bar{X}_e\right). \qquad (9)$$

The controlling action $\bar{u}\left(\bar{X}(t),t\right)$ is the magnitude, dependent on the time and on the vector of strategies, but it, in particular, influences the Lagrange's function; therefore, let us calculate, as the indicator of sensitivity of the system to the changes in controlling influence, its gradient by controlling influence on the rate of change in the Lagrange's function. The latter, after a series of conversions, will take the form:

$$\nabla_u\left[\frac{dL\left(\bar{X},\bar{\Lambda}\right)}{dt}\right]= =\nabla_u\left\{[\delta]^T[P]\nabla_x\left\{[\delta]^T\right\}\frac{\partial}{\partial t}\left(\bar{X}^T\right)+[\delta]^T[P]\nabla_u\left\{[\delta]^T\right\}\frac{\partial}{\partial t}\left(\bar{u}^T\right)\right\}= =[\delta]^T[P]\nabla_x\left\{[\delta]^T\right\}\nabla_u\left\{G\left(\bar{X}(t),t\right)+B\left(\bar{X}(t),t\right)\bar{u}\left(\bar{X}(t),t\right)+\frac{d}{dt}\bar{X}_e\right\}+ +\nabla_u\left\{[\delta]^T[P]\nabla_u\left\{[\delta]^T\right\}\frac{\partial}{\partial t}\left(\bar{u}^T\right)\right\}= =[\delta]^T[P]\nabla_x\left\{[\delta]^T\right\}B\left(\bar{X}(t),t\right)+\nabla_u\left\{[\delta]^T[P]\nabla_u\left\{[\delta]^T\right\}\frac{\partial}{\partial t}\left(\bar{u}^T\right)\right\}. \qquad (10)$$

Now let us examine the equation (9), which, relative to controlling action for the fixed point, will be reduced to a nonhomogeneous ordinary differential equation with variable coefficients:

$$[\delta]^T[P]\left\{\nabla_u\left\{[\delta]^T\right\}\left(\dot{\bar{u}}^T\right)+B\left(\bar{X}(t),t\right)\nabla_x\left\{[\delta]^T\right\}\bar{u}\left(\bar{X}(t),t\right)\right\}= =-\frac{\partial L\left(\bar{X},\bar{\Lambda}\right)}{\partial t}-[\delta]^T[P]\nabla_x\left\{[\delta]^T\right\}\left(G\left(\bar{X}(t),t\right)+\frac{d}{dt}\bar{X}_e\right). \qquad (11)$$

Thus we obtained two additional equations (10), (11), complementing the system (8), which realize the paradigm of prescribed control, due to the application of the paradigm of adequacy. Let us designate them the equations of adequate synthesis of controlling action.

5. Analysis of the obtained equations and estimation of error

As shown in the papers [32, 39, 47], the system (8) has a set of decisions, since the number of unknowns in it exceeds the number of equations. The latter sets a constraint on the quantity of inequalities (5), the quantity of which substantially decreases. Until now, the resolution of this contradiction has been accomplished by the application of approximate approximations [32] and introduction of new representations of the condition of the saddle point for non-quadratic forms [39]. These solutions do not depend on the principles of control both in the local – small and in the asymptotic sense, and essentially provide certain approximate solutions. They are reflecting only static properties and, respectively, require the decrease in time intervals to account for the influence of nonlinearity and non–stationarity. The proposed equations (10), (11) – the equations of adequate synthesis of controlling action consider both dynamic properties of the object and the properties of objective function.

They are obtained based on the fundamental provision – maximum accuracy is reached at the minimum magnitude of the square of error. The error it self for the complex function at the particular point is determined by the substantial derivative, gradient of the vector-function, by time intervals, by deviations of the vector of states and controlling action over this time interval.

Besides the mentioned, it should be noted that the obtained equations (10), (11) complement the system with two additional equations, which is especially important in the case of setting and solving optimization problems with the inequalities constraints [32, 38, 47]. For comparing the results, received in the articles [43], let us write down for the introduced designations in accordance with the algorithm of the method of speed gradient:

$$\nabla_u\left[\frac{dL(\bar{X},\bar{\Lambda})}{dt}\right]=[\delta]^T[P]\nabla_x\left\{[\delta]^T\right\}B(\bar{X}(t),t), \tag{12}$$

$$\frac{d\bar{u}}{dt}=-MB(\bar{X}(t),t)^T\nabla_x\delta(\bar{X}(t),t)^T[P][\delta].$$

Let us note that the first of them coincides with the equation derived in the article (10) only under condition of equality to zero of its second term:

$$\nabla_u\left\{[\delta]^T[P]\nabla_u\left\{[\delta]^T\right\}\frac{\partial}{\partial t}(\bar{u}^T)\right\}=0$$

or

$$\nabla_u\left\{[\delta]^T[P]\right\}\nabla_u\left\{[\delta]^T\right\}\frac{d}{dt}(\bar{u}^T)=$$
$$=-[\delta]^T[P]\left\{\nabla_u^2\left\{[\delta]^T\right\}\frac{\partial}{\partial t}(\bar{u}^T)+\nabla_u\left\{[\delta]^T\right\}\nabla_u\left\{\frac{\partial}{\partial t}(\bar{u}^T)\right\}\right\}$$

or

$$\frac{d}{dt}(\bar{u}^T)=$$
$$=-[\delta]^T[P]\nabla_u^2\left\{[\delta]^T\right\}\left\|\nabla u\left\{[\delta]^T[P]\right\}\nabla u\left\{[\delta]^T\right\}\right\|^{-1}[D]. \tag{13}$$

That, in turn, leads to the second equation (12) only on the assumption that the rate of change in controlling action does not depend on the magnitude of controlling action, while the matrix M is synthesized, as reducing (13) to the expression, identical with the second equation of the system (12).

For conducting quantitative analysis of the results of the synthesis of controlling action according to the method of speed gradient [43] and results of realization of the proposed paradigm (10), (11), let us examine NO [32]. The results of numerical experiments are represented in Fig. 1, 2. As the data of curves testify, with the increase in the magnitude of the time interval, local deviations from the prescribed behavior of NO considerably increase. The decrease in the time interval improves conformity, but only in this section, significantly increasing the mismatch outside its limits in this case.

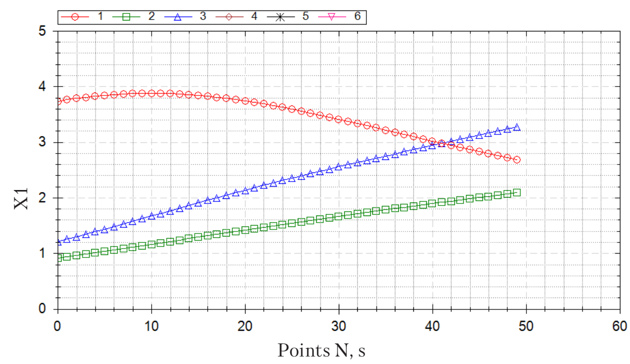

Fig. 1. Dependency of the vector of strategies on time: 1 is the prescribed vector of strategies; 2 is the vector of strategies, determined in accordance with the proposed paradigm (10), (11); 3 is the vector of strategies, determined in accordance with the method of speed gradient [43]

The given graphs are a special case and are not considered as the means of demonstration of advantages of one or another approach. They rather demonstrate the complexity of constructing such conclusions than are the way of graphic reflection, even more so on the plane.

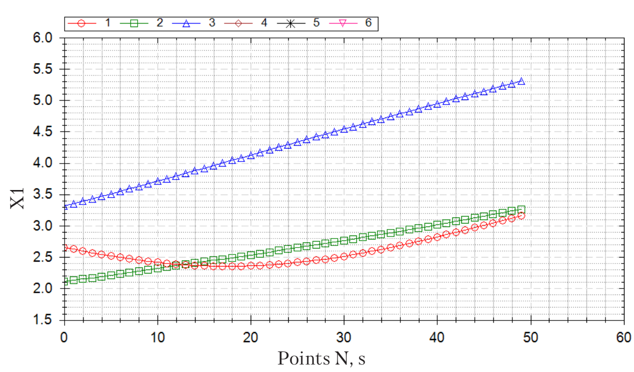

Fig. 2. Dependency of the vector of strategies on time (time countdown, starting from the point of the end of the interval Fig. 1): 1 is the prescribed vector of strategies; 2 is the vector of strategies, determined in accordance with the proposed paradigm (10), (11); 3 is the vector of strategies, determined in accordance with the method of speed gradient [43]

For the demonstration of more general and more efficient comparisons, we will use the norm:

$$\|R\|=\sqrt{\int_0^t\left[R(\tau)\right]^2 d\tau}.$$

Let us assume that the system with the aid of controlling action is driven to the point on the curve of the minimum of deviation from the prescribed process. In this state, the left side of the equation (10) will be reduced to zero. After decomposing the right side (10) in the area of this realization of the process, we will obtain:

$$-[\delta]^T[P]\nabla_x\left\{[\delta]^T\right\}\left[\Delta\bar{X}(t)\right]^T\nabla_x\left(B\left(\bar{X}(t),t\right)\right)=$$
$$=[\delta]^T[P]\nabla_x\left\{[\delta]^T\right\}B\left(\bar{X}(t),t\right)+$$
$$+\nabla_u\left\{[\delta]^T[P]\nabla_u\left\{[\delta]^T\right\}\frac{\partial}{\partial t}\left(\bar{u}^T\right)\right\}$$

After applying the norm to both parts of the equation, let us write down the estimation of error of the vector of strategies

$$\left\|\left[\Delta\bar{X}(t)\right]^T\right\|$$

as the function of norm $\left\|B\left(\bar{X}(t),t\right)\right\|$ and the rate in change of controlling action:

$$\left|[\delta]^T[P]\nabla_x\left\{[\delta]^T\right\}\right|_{min}\left\|\left[\Delta\bar{X}(t)\right]^T\right\|\left|\nabla_X\left(B\left(\bar{X}(t),t\right)\right)\right|_{min}\le$$
$$\le\left|[\delta]^T[P]\nabla_x\left\{[\delta]^T\right\}\right|_{max}\left\|B\left(\bar{X}(t),t\right)\right\|+$$
$$+\left\|\nabla_u\left\{[\delta]^T[P]\nabla_u\left\{[\delta]^T\right\}\frac{\partial}{\partial t}\left(\bar{u}^T\right)\right\}\right\|.$$

The synthesis of controlling action as the solution of the equations of the system (8) and equation (11) decreases the error rate

$$\left\|\left[\Delta\bar{X}(t)\right]^T\right\|.$$

The estimation of error for the method of speed gradient, taking into account the equations (12), will be written down in the form:

$$\left|[\delta]^T[P]\nabla_x\left\{[\delta]^T\right\}\right|_{min}\left\|\left[\Delta\bar{X}(t)\right]^T\right\|\left|\nabla_X\left(B\left(\bar{X}(t),t\right)\right)\right|_{min}\le$$
$$\le\left|[\delta]^T[P]\nabla_x\left\{[\delta]^T\right\}\right|_{max}\left\|B\left(\bar{X}(t),t\right)\right\|+$$
$$+\left\|\nabla_u\left\{[\delta]^T[P]\nabla_u\left\{[\delta]^T\right\}M\nabla_u\left[\frac{dL\left(\bar{X},\bar{\Lambda}\right)}{dt}\right]\right\}\right\|.$$

Comparison of two estimations shows that for the method of speed gradient, the mean square error

$$\left\|\left[\Delta\bar{X}(t)\right]^T\right\|$$

is larger. This is caused by the use of axiomatic assumptions about proportionality of the rate in controlling action to speed gradient and its independence from the magnitude of controlling action.

Thus, based on the presented above, it is possible to assert that realization of the paradigm of prescribed control as the problem of maximization of adequacy, even by the three of the enumerated criteria, gives, in a particular case, the algorithm of speed gradient. In other words, the proposed solution to the problem of prescribed control is a generalizing expression of the algorithm of speed gradient, for whose determining the axiomatic assumptions about the rate of change in controlling action are not made. However, in contrast to the latter, the control is accomplished with a variable speed of change in the controlling influence. In this case, maximization is ensured, or minimization of the arbitrary, additionally selected, objective function, even for the model of the object of control that is not included in the equations.

6. Conclusions

1. Maximization of adequacy makes it possible to realize the paradigm of prescribed control as the solution of optimization problem with the inequalities constraints, it complements the system of equations and allows to synthesize controlling action.

2. Realization of two criteria of adequacy – depth and completeness – forms two equations for the synthesis of controlling action in accordance with the paradigm of prescribing.

3. We determined estimation of the norm of error in the vector of strategies depending on the properties of object, error in the function of efficiency, synthesized law of controlling action and the prescribed law of the object functioning.

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