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Розроблено ймовірнісну модель роботи портового терміналу, яка враховує нерівномірність завезення наземним транспортом вантажу та вивезення його суднами. Час вивантаження вантажу на склад вважається малим, а навантаження на судна здійснюється на декількох причалах з заданою інтенсивністю. Знайдено сумісний граничний (при t→∞) ймовірнісний розподіл числа суден на терміналі та кількості вантажу на складі за допомогою апарату марковських процесів зі знесенням. Розраховані показники пропускної спроможності терміналу

Ключові слова: портовий термінал, пропускна спроможність, нерівномірність завезення та вивезення вантажу, система обслуговування

Разработана вероятностная модель работы портового терминала, учитывающая неравномерность завоза груза наземным транспортом и вывоза его судами. Время выгрузки груза на склад считается пренебрежимо малым, а погрузка на суда производится на нескольких причалах с заданной интенсивностью. Найдено предельное (при t→∞) совместное распределение вероятностей числа судов на терминале и количества груза на складе с помощью аппарата марковских процессов специального вида. Рассчитаны показатели пропускной способности терминала

Ключевые слова: портовый терминал, пропускная способность, неравномерность завоза и вывоза груза, система обслуживания

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A METHOD OF DETERMINATION OF PORT TERMINAL CAPACITY UNDER IRREGULAR CARGO DELIVERY AND PICKUP

M. Postan
Doctor of Economic Sciences, Professor, Head of Department*
E-mail: postan@ukr.net

L. Kushnir
PhD, Senior Lecturer*
E-mail: Lyudmila_ku@i.ua

*Department "Management & Marketing in Marine Transport"
Odessa National Maritime University
Mechnikova str., 34, Odesa, Ukraine, 65029

1. Introduction

The port terminals are the most important sections of transportation logistics chains, in which there takes place interaction of traffic streams of related types of transport. On efficiency of such an interaction there depend terms and cost of cargoes delivery, as well as their safety. While working out of projects of organization of new and reconstruction of acting terminals one of the main problems is finding of optimum capacity of fronts of cargoes delivery and pickup, which are determined according to size of delivered cargoes, uninterrupted and regular work of transport. In turn, the stated capacity is determined by numerical values of characteristics of the main technological elements of a terminal, namely: moorages, storehouses, unloaders, access roads and etc. Whereby, the capacity of moorages, storehouses and access roads are to be coordinated with each other.

Typically for formal description of functioning of a terminal there is used combination of methods of the queueing theory (QT) and the storage (inventory) theory. For example, the significance of QT for project calculations lies in the fact that it makes it possible to predict variation of length of queues and time of being in a queue of transport units (TU) in a projected terminal, that wait for beginning of loading or unloading for a remote prospect. Objectively existing variation and irregularity of movement of TU create difficult conditions for exploitation of production capacities of ports, that bring to necessity of organization of their reserves. The inventory theory makes it possible to give correct assessment of a level of expected reserves of cargoes in a storehouse of a terminal and to determine its necessary capacity, to give scientific grounds for a value of operation load on structural elements of moorage constructions and to improve its reliability. Description of a terminal in a form of one or another queueing system is also necessary for giving of scientifically grounded assessment of expected values of economic results and expenses that relate to exploitation of a terminal in the long term.

At the same time, while examination of a problem of formal description of a port terminal in terms of the stated theories there remain many unsolved tasks that relate to finding of key dependence of the main characteristics of technological elements on characteristics of incoming streams of TU. Characteristics of technological elements that were calculated not in the proper way can bring to additional demurrage of TU and ships, as well as to losses that relate to such a demurrage. The mentioned problem is urgent for a theory and practice of projection of port terminals, and solving of theoretical difficulties that occur during it requires non-trivial special researches.

2. Literature review and problem statement

It is known that in the present moment while projecting of port terminals for giving of grounds for their capacity, there are widely used methods of simulation [1, 2] and operations research [3, 4], that make it possible to model a work of a terminal. In the literature that is dedicated to modelling of a work of port terminals, for long time there were used ready models of queueing systems, with not enough considering of a specific character of a work of a port [5–7]. However, classical models of QT were worked out mainly for solving of tasks of telecommunication system projecting, and for most transport systems they are of little use. So, for example, they absolutely do not consider the factor of coordination of oncoming traffics in a terminal, and for description of such traffics there is used only a model of the stationary Poisson stream. At the same time, when there are present regulation of a work of transport (for example, in linear shipping), streams of TU, that come to a terminal, are not the Poisson ones but have some certain level of regularity.

The first researches that were dedicated to modelling of interaction of transport streams in a port in assumption of non-limited capacity of fronts of loading and unloading, were made in 70 80s of the last century [8, 9]. In the following two decades the accent of researches were put on building and analysis of stochastic models of work of port terminals with taking into consideration of the finite capacity of loading and unloading fronts [4]. In monograph [4] there is reflected a level of research of this problem up to 2006, where there are given stochastic models of transportation and storage systems with limiting of capacity, that is built with the help of combination of methods of the storage (inventory) theory and QT. However, in it there were analytically researched in general only relatively simple models of interaction of transport streams, where one stream is regular, in other words it is characterized with constant rate of delivery (pickup) of cargoes. At the same time real transport streams have greater or less level of irregularity and that is why they make interest for research of this more theoretically difficult case.

Among the latest researches in the sphere of modelling of interaction of transport streams in transit points there can be singled out a work [10], in which for optimization of a plan of coordination of a stream of ships and rakes there is suggested a model that is based on generalization of a classical model of Wagner-Whitin in the inventory theory. However, such an approach is efficient only for a case of fully controlled transport streams, irregularity of coming of TU is not taken into consideration in it.

In a review article [11] there is given a review of literature in planning of a work of multimodal transport, and there are systematized known results on planning of a work of multimodal and intermodal transport in strategical, tactical and operational levels. However, the given results relate only to determined conditions of transport work, factors of uncertainty, that play a significant role in a work of sea transport, are not reviewed in it.

In the article [12] there is observed a task of optimal management in placement of ships in a group of moorages in a port with use of stochastic dynamics, however, there was not researched an attendant problem of variation of a level of cargo reserves in storehouses.

In [13] there are solved problems of an optimal location of a transport centre (hub) in a transport network with a help of methods of QT with taking into consideration of limited capacity of a hub, that is presented in the form of a queueing system. However, at the same time there was not taken into consideration queues while interaction of transport streams in a hub. In cited works [11 13] a problem of interaction of different types of transport and its influence on capacity of a port (terminal) is not observed, however, coordination of different types of transport is very significant while organization of multimodal transportation. In [14] there was researched the influence of a level of regularity of movement of vessels on a level of cargo reserves in storehouses of a terminal, however, under assumption of regularity of its delivery to a terminal by surface transport, which is a big simplification of real processes of transportation.

In [4] with analytical methods (with a help of a special class of Markov processes - Markov drift processes) there were enough researches of models of interactions (via a storehouse) of transport streams in terminals for a case, when one of streams of means of transport (loaded and empty) is regular, and the other one is described by a model of the Poisson stream. In a case of an interaction of two irregular transport streams because of significant analytical difficulties within Markov models, there was possible to research only the simplest case of interaction of two single TU. For solving of mathematical difficulties that occur and for getting of desired join queue-length distribution of TU and amount of cargo in a storehouse, it is necessary to make one or another simplifying assumptions, for example, about unlimited capacity of one of the fronts of loading/unloading of TU, unlimited capacity of a storehouse and etc.

3. Purpose and tasks of the research

The purpose of the work is building up and analysis of a common enough stochastic model of a work of a port terminal in assumption of irregularity of delivery of cargo to a storehouse by TU of surface transport (with unlimited capacity of a front of unloading) and its pickup by ships. It is a necessary basis for working out of a scientifically grounded method of finding of main dependences of values of capacity of a terminal on set characteristics of TU's streams.

For achievement of the stated purpose, it is necessary to solve the following tasks:

– to give formalized description of a port terminal in terms of the inventory theory and QT with taking into consideration of irregularity of delivery and pickup of a cargo by ships and TU;

– to take necessary simplifying assumptions and give interpretation to a work of a terminal in terms of Markov drift process, but with additional boundary conditions;

– to state a method of solving of a system of integral-differential equations, that was got while building up of stochastic model of a terminal for finding of join distribution of number of ships that are in a terminal, and amount of cargo that is in a storehouse;

– on the basis of a got decision to find formulae for calculation of main values of efficiency of a work of a terminal, to work out a method of calculation of necessary capacity of a storehouse and to find a formula for calculation economically efficient term of recoupment of a project of organization of a terminal.

4. Description of a general scheme of modelling of a work of a port terminal

We will observe a port terminal that consists of n interchangeable moorages and a storehouse. Similar cargoes are delivered to a terminal with a help of surface type of transport (motor vehicles or rakes) and immediately come to a storehouse. We consider that a stream of such TU is described with a model of a compound renewal process [15]. It means that intervals of time between near moments of arrival of TU are mutually independent random variables with the same distribution function (d. f.) A (t), and their carrying capacity – independent on them and mutually independent random variables with the same d. f. G(x). All cargoes that were delivered to a storehouse are taken out with a help of N, N>n, vessels, each of which works in its separate line. The time of a round voyage is a random variable with d. f. B(t), and d. f. of a net carrying capacity of arbitrary ship is H(x). The rate of loading of a cargo from a storehouse (if it is not empty) to any ship is W.

To make building up and analysis of a mathematical model simpler we will suppose that:

a) capacity of a storehouse is big enough, it means that we will neglect a possibility of additional demurrage of TU, that was caused with complete filling of a storehouse;

b) time of unloading of a cargo from TU to a storehouse is negligible (it is equal to zero); in other words, we consider that a front of unloading of a cargo has an unlimited capacity.

We should mark that the stated assumptions do not except a situation, in which one or several ships can be in demurrage at moorages for additional time because of absence of a cargo in a storehouse in expectance of delivery of a cargo by TU.

We will take the following designations:

$\nu(t)$ – number of ships that are in a terminal (at moorages and in a queue to them) in the moment of time t;

$\xi(t)$ – amount of cargo that are in a storehouse in the moment t.

A structural scheme of the described transportation and storage system is presented in Fig. 1.

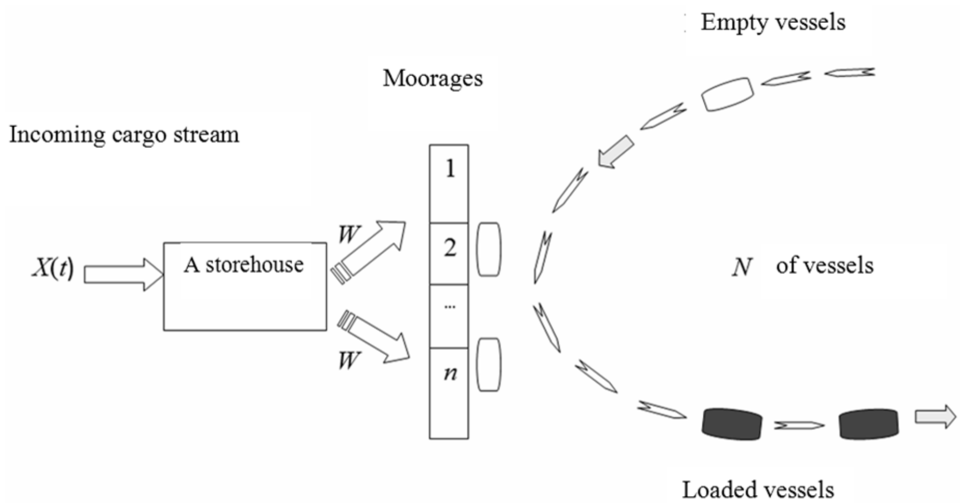

Fig. 1. Structural scheme of interaction of transport streams in a port terminal

5. A system of integral-differential equations in relation to limit probabilistic distribution

From a mathematical point of view our task is to find limit (with $t \to \infty$) distribution of a random vector $(\nu(t), \xi(t))$. From a physical point of view it means that we research a work of a terminal in a steady-state (or in equilibrium) condition.

We will mark:

$$F_k(x) = \lim_{t\to\infty} \mathbf{P}\{\nu(t) = k, \xi(t) \le x\}, \ x \ge 0, k = 0,1,2,...,N,$$

desired limit probabilistic distribution (in assumption of existence of the stated limits). For finding of such a distribution it is necessary to make a number of additional assumptions that make it possible to bring a random process $(\nu(t), \xi(t))$ to Markov process. We put

$$A(t) = 1 - e^{-\lambda_1 t}, t \ge 0;$$

$$B(t) = 1 - e^{-\lambda_2 t}, \ t \ge 0;$$

$$H(x) = 1 - e^{-x/g}, x \ge 0,$$

where λ_1 is intensity of a stream of TU; $1/\lambda_2$ – the average duration of a voyage of one ship; g – the average net carrying capacity of a ship.

With such assumptions a stream of loaded TU is described with a model of the compound Poisson process X(t), X(0)=0, with non-negative trajectories and zero drift [15], and a process $(\nu(t), \xi(t))$ becomes Markovian. For finding of limit distribution of this process it is necessary first of all to derive a proper system of integral-differential equations. This system of equations is derived with a help of typical probabilistic reasoning that lies in recording of asymptotic relations that connect desired distribution in two infinitely close moments of time, and in further limiting process [4, 15]. For example, according to theorem of total probability in an infinitely small interval $(0, \Delta t)$ of steady-state operating condition of a system we have with x>0

$$F_0(x) = [1-(\lambda_1 + N\lambda_2)\Delta t]F_0(x) + \\ +\lambda_1 \Delta t \int_0^x G(x-y)dF_0(y) + \mu\Delta t(F_1(x) - F_1(0)) + o(\Delta t),$$

$$F_k(x) = [1-(\lambda_1 + \lambda_2(N-k))\Delta t] \times \\ \times F_k(x - W_k \Delta t) + \lambda_1 \Delta t \int_{+0}^x G(x-y)dF_k(y) - \\ -\mu_k \Delta t(F_k(x) - F_k(0)) + \lambda_2(N-k+1)F_{k-1}(x) + \\ o(\Delta t), k = 1,2,\ldots,N-1,$$

$$F_N(x) = [1-(\lambda_1 + \mu_N)\Delta t]F_N(x - W_N \Delta t) + \\ +\lambda_1 \Delta t \int_{+0}^x G(x-y)dF_N(y) + \lambda_2 \Delta t F_{N-1}(x) + o(\Delta t).$$

where

$$W_k = \mu_k g; \;\; \mu_k = \mu \min(k,n); \mu = W/g.$$

After brace expansion in the right parts of these equalities, dividing of both parts by Δt and move to the limit with $\Delta t \to +0,$ we will get the following system of integral-differential equations:

$$0 = -(\lambda_1 + N\lambda_2)F_0(x) + \\ +\lambda_1 \int_0^x G(x-y)dF_0(y) + \mu(F_1(x) - F_1(0)), \; x > 0, \tag{1}$$

$$F_k(x) = [1-(\lambda_1 + \lambda_2(N-k))\Delta t] \times \\ \times F_k(x - W_k \Delta t) + \lambda_1 \Delta t \int_{+0}^x G(x-y)dF_k(y) - \\ -\mu_k \Delta t(F_k(x) - F_k(0)) + \lambda_2(N-k+1)F_{k-1}(x) + \\ +o(\Delta t), \;\; k = 1,2,\ldots,N-1, \tag{2}$$

$$-W_N F_N'(x) = -(\lambda_1 + \mu_N)F_N(x) + \\ +\lambda_1 \int_{+0}^x G(x-y)dF_N(y) + \lambda_2 F_{N-1}(x), x > 0, \tag{3}$$

Boundary conditions are formally got from a system (1)–(3) with limit transfer with $x \to +0$:

$$W_k F_k'(0) = [\lambda_1 + \lambda_2(N-k)]F_k(0) + \\ +\lambda_2(N-k+1)F_{k-1}(0), \\ k = 1,2,\ldots,N, \tag{4}$$

$$F_0(0) = 0. \tag{5}$$

To a system of equations (1)–(5) it is to be added normalization conditions:

$$\sum_{k=0}^{N} F_k(\infty) = 1. \tag{6}$$

The boundary-value problem (1)–(6) can be solved analytically with a help of the Laplace transform method. We will mark through

$$\phi_0(s) = \int_0^\infty e^{-sx} dF_0(x),$$

$$\phi_k(s) = F_k(0) + \int_{+0}^\infty e^{-sx} dF_k(x), \mathrm{Re}\, s \ge 0, \;\; k = 1,2,\ldots,N, \tag{7}$$

the Laplace-Stieltjes transformations of functions

$$F_k(x), \;\; k = 0,1,\ldots,N.$$

We transform a system (1)–(6), with use to the integral terms of a theorem of convolution with taking into consideration of the conditions (4), (5). In the result, we will come to the following system of algebraic equations for finding of functions (7):

$$-[\lambda_1(1-g(s)) + \lambda_2 N]\phi_0(s) + \mu_1\phi_1(s) = \mu_1 F_1(0), \tag{8}$$

$$-[-sW_k + \mu_k + \lambda_1(1-g(s)) + \lambda_2(N-k)]\phi_k(s) + \\ +\lambda_2(N-k+1)\phi_{k-1}(s) + \mu_{k+1}\phi_{k+1}(s) = \\ = (-\mu_k + sW_k)F_k(0) + \mu_{k+1}F_{k+1}(0), k = 1,2,\ldots,N-1, \tag{9}$$

$$-[-sW_N + \lambda_1(1-g(s)) + \mu_N]\phi_{N-1}(s) + \\ +\lambda_2\phi_{N-1}(s) = (-\mu_N + sW_N)F_N(0), \;\; \mathrm{Re}\, s \ge 0; \tag{10}$$

$$\sum_{k=0}^{N} \phi_k(0) = 1. \tag{11}$$

Summing all equations of the system (8)–(10), after elementary transformations we will get the following equality:

$$-\lambda_1(1-g(s))\sum_{k=0}^{N}\phi_k(s) = s\sum_{k=1}^{N}[W_k(F_k(0) - \phi_k(s))]. \tag{12}$$

Having divided both parts of the equality (12) by *s* and moved to the limit with $s \to +0$, we will get a relation, which expresses so called conservation law of cargo traffic that is delivered to a storehouse of a terminal and that is taken out from it, that takes place in a steady-state regime of work of a terminal:

$$\lambda_1 g^{(1)} = \sum_{k=1}^{N} W_k(F_k(\infty) - F_k(0)), \tag{13}$$

where

$$g^{(1)} = \int_0^\infty x dG(x) < \infty$$

is an expectation value (the average value) of carrying capacity of one TU. Actually, in the left-hand side of equality (13) there stands intensity of cargo traffic that comes to a storehouse of a terminal, and in the right-hand side – there stands intensity of a stream of a cargo that is uploaded from a storehouse to ships.

Solving of a system (8)–(11) can be found by means of sequential expression of functions $\phi_k(s), k=1,2,...,N,$ through $\phi_0(s)$ and further use of the equality (12). The constants that remain being unknown $F_k(0), k=1,2,...,N,$ are found from conditions of the functions $\phi_k(s), k=0,1,...,N,$ analyticity in the right half-plane $\operatorname{Re} s \geq 0.$ It is possible to show that system's (8)–(10) determinant has exactly N roots in this half-plane.

Having determined functions $F_k(x),\ k=0,1,...,N,$ it is possible to calculate, for example, the following important indices of a work of a terminal:

a) stationary mean amount of cargo that is in a storehouse in an arbitrary moment of time,

$$\mathbf{M}\xi = \int_0^\infty x \sum_{k=0}^N dF_k(x) = -\sum_{k=0}^N \frac{d}{ds}\phi_k(s)\Big|_{s=0};$$

b) stationary mean number of ships that are in a terminal in an arbitrary moment of time,

$$\mathbf{M}\nu = \sum_{k=1}^N kF_k(\infty) = \sum_{k=1}^N k\phi_k(0);$$

c) stationary probability of the fact that a storehouse is empty

$$\sum_{k=1}^N F_k(0);$$

d) capacity (throughput) of a front of loading of a terminal

$$\sum_{k=1}^N W_k F_k(\infty).$$

We should mark that expression

$$\sum_{k=1}^N W_k F_k(\infty) - \lambda_1 g^{(1)}$$

has a meaning of a reserve of capacity of a front of loading of a terminal.

To prevent too great accumulation of cargoes in a terminal with passing of time this reserve, according to (13), is to be positive. From a formal point of view positiveness of a reserve means existing of a steady-state regime of a work of transportation and storage system under observation.

6. Solving of a system (8)–(11) for a special case N=1, n=1

For this special case (one ship, one moorage) system (8)–(10) will take the following form:

$$[\lambda_1(1-g(s))+\lambda_2]\phi_0(s) - \mu\phi_1(s) = -\mu F_1(0),$$

$$-\lambda_2\phi_0(s) + [-sW + \mu + \lambda_1(1-g(s))]\phi_1(s) = \\ = (\mu - sW)F_1(0), \operatorname{Re} s \geq 0. \quad (14)$$

Solving of problem (14) is given with the following formulae:

$$\phi_0(s) = \mu\frac{\phi_1(s) - F_1(0)}{\lambda_1(1-g(s))+\lambda_2}, \quad (15)$$

$$\phi_1(s) = \\ = F_1(0)\frac{[\lambda_1(1-g(s))+\lambda_2](\mu - sW) - \lambda_2\mu}{[\lambda_1(1-g(s))+\lambda_2][\mu - sW + \lambda_1(1-g(s))] - \lambda_2\mu}. \quad (16)$$

With $s \to +0$ from formulae (15), (16), with use of the L'Hôpital's rule we will get the following relations

$$F_0(\infty) = \frac{\mu}{\lambda_2}(F_1(\infty) - F_1(0)),$$

$$F_1(\infty) = F_1(0)\frac{\mu\lambda_1 g^{(1)} - \lambda_2 W}{\lambda_1 g^{(1)}(\lambda_2+\mu) - \lambda_2 W}. \quad (17)$$

From (11), (17) we find

$$F_1(0) = 1 - \frac{\lambda_1 g^{(1)}(\lambda_2+\mu)}{\lambda_2 W},$$

$$F_0(\infty) = \frac{\lambda_1 g^{(1)}\mu}{\lambda_2 W},$$

$$F_1(\infty) = 1 - \frac{\lambda_1 g^{(1)}\mu}{\lambda_2 W}. \quad (18)$$

Formulae (15)–(18) give solving of tasks.

As the expressions (18) are probabilities, all of them are to be not negative and less than a unit from which there comes the necessity to fulfil condition

$$\lambda_1 g^{(1)} < \frac{\lambda_2 W}{\lambda_2 + \mu}.$$

Note that relation (13) in this case will take the form:

$$\lambda_1 g^{(1)} = (1 - F_1(0))\frac{\lambda_2 W}{\lambda_2 + \mu}.$$

The Laplace-Stieltjes transformation of d.f. of cargo amount, that is in a storehouse, is

$$\phi(s) = \phi_0(s) + \phi_1(s),$$

That is why two initial moments of this distribution can be found with the formulae

$$\mathbf{M}\xi = -\phi'(0), \mathbf{M}\xi^2 = \phi''(0).$$

With a help of formulae (15), (16) after a number of not complicated calculations we find

$$\mathbf{M}\xi = \frac{1}{2}\{\lambda_1(2Wg^{(1)} + \mu g^{(2)})F_1(0) + \\ +\lambda_1 F_1(\infty)[(\mu - \lambda_2)g^{(2)} + \\ +2g^{(1)}(\lambda_1 g^{(1)} - W)]\}[\lambda_2 W - \lambda_1 g^{(1)}(\mu - \lambda_2)]^{-1},$$

$$\mathbf{M}\xi^2 = \frac{1}{\lambda_2}[(\lambda_2+\mu)\phi_1''(0) - 2\lambda_1 g^{(1)}\phi_0'(0) + \lambda_1 g^{(2)}F_0(\infty)], \quad (19)$$

where

$$g^{(i)} = \int_0^\infty x^i dG(x) < \infty,\ i = 1, 2, 3;$$

$$\phi_0'(0) = \frac{1}{\lambda_2}[\mu\phi_1'(0) - \lambda_1 g^{(1)} F_0(\infty)];$$

$$\phi_1'(0) = \frac{\lambda_1}{2}\{F_1(0)(2Wg^{(1)} + \mu g^{(2)}) + F_1(\infty)[(\mu - \lambda_2)g^{(2)} + \\ +2g^{(1)}(\lambda_1 g^{(1)} - W)]\}[\lambda_2 W - \lambda_1 g^{(1)}(\lambda_2 + \mu)]^{-1};$$

$$\phi_1''(0) = \frac{\lambda_1}{3}\{(3g^{(2)}W + \mu g^{(3)})(F_1(\infty) - F_1(0)) + \\ +F_1(\infty)(\lambda_2 g^{(3)} - 6\lambda_1 g^{(1)} g^{(2)}) - 3\phi_1'(0)[(\lambda_2 + \mu)g^{(2)} + \\ +2g^{(1)}(W - \lambda_1 g^{(1)})]\}[\lambda_2 W - \lambda_1 g^{(1)}(\lambda_2 + \mu)]^{-1}.$$

7. Discussion of results of the researches of stochastic model of a terminal

We will state some practical tasks, for solving of which there can be used the results that were got below. We will present two such tasks.

a) Determination of the necessary capacity of a storehouse.

Capacity of a storehouse is the most important characteristic, which determines capacity of a terminal and serves as a damper, which smoothes irregularity of arrival of TU of related types and which reduces a risk of demurrage of ships in waiting of delivery of a cargo or its taking out from a storehouse, which is completely filled. It is clear that this capacity depends of a level of regularity of TU/ships arrivals and, as well as dynamics of variation of a level of cargo reserves that is stored in a storehouse. We will mark the capacity of a storehouse with E. Then it can be determined from conditions of small probability of its exceeding with a random value ξ of capacity E, it means that from condition

$$\mathbf{P}\{\xi \le E\} \ge 1 - \varepsilon,$$

where ε – the given small probability. With use of one of modifications of the Chebyshev's inequality [16], we get

$$\mathbf{P}\{\xi \le E\} \ge \frac{(\mathbf{M}\xi - E)^2}{(\mathbf{M}\xi - E)^2 + \mathbf{D}\xi},$$

where $\mathbf{D}\xi = \mathbf{M}\xi^2 - (\mathbf{M}\xi)^2$. Having equated the right part of the last inequality with $1-\varepsilon$, we will get the following formula for calculation of the necessary capacity of a storehouse:

$$E = \mathbf{M}\xi + \sqrt{\frac{1-\varepsilon}{\varepsilon}\mathbf{D}\xi}.$$

b) Determination of a term of recoupment of the project of construction (reconstruction) of a terminal.

In project calculations characteristics W, n, E are assumed to be controlled. Realization of technological elements of a terminal with these characteristics requires certain investments, therefore, there occurs a task to assess a term of recoupment. For this it is required to assess expected expenses that relate to organization of a terminal and its exploitation, as well as economic results of a work of an operator of a terminal, it means a financial result of his business activity. With such a purpose we will use additional designations:

$\Pi(t)$ – level of profit of a port operator from exploitation of a terminal in the moment of time t;

$D(t)$ – total income that was got by an operator in time interval $(0, t]$ for carrying out of loading-unloading operations and for storage of cargoes, where

$$D(t) = a_1 X(t) + a_2 \sum_{m=1}^{\omega(t)} \gamma_m + d_{xp}\int_0^t \xi(\tau)d\tau, \tag{20}$$

where a_1 is a tariff rate for unloading of 1 t of cargo from TM to a storehouse; a_2 is a tariff rate for loading of 1 t of cargo from a storehouse to a vessel; $\omega(t)$ is a number of vessels with a cargo, that left a terminal in time interval (0, t]; γ_m is a net carrying capacity of the m^{th} vessel with a cargo, that left a terminal in time interval (0, t]; d_{xp} is an income for day storage of 1 t of a cargo in a storehouse; $R(t)$ is the total expenses of an operator in interval $(0,t]$, that relate to exploitation of a terminal, with

$$R(t) = rt + r_{xp}\int_0^t \xi(\tau)d\tau, \tag{21}$$

where r is regular daily exploitation expanses for a terminal; r_{xp} is daily expenses for storage of 1 t of a cargo in a storehouse.

With taking into consideration of the stated designations and relations (20), (21) it is possible to write

$$\Pi(t) = \Pi(0) + (D(t) - R(t))(1-f) = \\ = \Pi(0) + [a_1 X(t) + a_2 \sum_{m=1}^{\omega(t)} \gamma_m + \\ + p_{xp}\int_0^t \xi(\tau)d\tau - rt](1-f), t \ge 0, \tag{22}$$

where $p_{xp} = d_{xp} - r_{xp}$; f is a tax rate for profit; $\Pi(0)$ is the initial value of profit.

If to observe a work of a terminal in the stationary (steady-state) regime , then relation (22) can be presented in the following way (we moved the initial moment of time to $-\infty$ and rejected the initial value of profit):

$$\Pi(t) = [a_1 X(t) + a_2 \sum_{m=1}^{\omega(t)} \gamma_m + p_{xp}\int_{-\infty}^t \xi(\tau)d\tau - rt](1-f), \\ -\infty < t < \infty. \tag{23}$$

The mean value of random variable (22) is equal to

$$\mathbf{M}\Pi(t) = [a_1 \lambda_1 g^{(1)} t + a_2 \Lambda g t + \\ + p_{xp}\int_{-\infty}^t \mathbf{M}\xi(\tau)d\tau - rt](1-f), \tag{24}$$

where Λ is the intensity of a stationary stream of ships with cargoes, that leave a terminal, with

$$\Lambda = \sum_{k=1}^N \mu_k (F_k(\infty) - F_k(0)).$$

While getting of the formula (24) there was used the Wald identity [16], as well as the following known characteristic of the compound Poisson process [15]:

$$\mathbf{M}X(t)=\lambda_1 g^{(1)}t.$$

On differentiating equality (23), we will get

$$(\mathbf{M}\Pi(t))'=[a_1\lambda_1 g^{(1)}+a_2\Lambda g+p_{xp}\mathbf{M}\xi-r](1-f).$$

Therefore, with taking into consideration of discount rate δ hence we find the average value of profit for recoupment period T:

$$\int_0^T e^{-\delta(T-t)}(\mathbf{M}\Pi(t))'dt=$$
$$=\frac{1-e^{-\delta T}}{\delta}[a_1\lambda_1 g^{(1)}+a_2\Lambda g+p_{xp}\mathbf{M}\xi-r](1-f). \qquad (25)$$

A project of organization (reconstruction) of a terminal is compensated in general for period T, if the right part of expression (25) is equal to the cost of a project I_0.Therefore, the expected term of recoupment makes

$$T=-\frac{1}{\delta}\ln\left\{1-\frac{\delta I_0}{[a_1\lambda_1 g^{(1)}+a_2\Lambda g+p_{xp}\mathbf{M}\xi-r](1-f)}\right\}. \qquad (26)$$

We should note that the last formula has the meaning only with meeting of conditions

$$I_0\delta<[a_1\lambda_1 g^{(1)}+a_2\Lambda g+p_{xp}\mathbf{M}\xi-r](1-f).$$

We will give a numerical example, that shows application of formula (26), restricting ourselves with case N=1, n=1. We will take the following value of the initial characteristics, that are included to the right part of formula (26):

$\lambda_1=0{,}5,\ 1/\text{day},\ \lambda_2=0.25$ ships/day,

$g^{(1)}$=5 thousand t, $g^{(2)}=(g^{(1)})^2$=25 (thousand t)^2,

g=25 thousand t, W=11 thousand t/day,

$a_1=a_2$=10 monetary unit/t, p_{xp}=0.05 monetary unit/t day,

r=10 thousand of monetary unit/day, I_0=25 million of monetary unit,

$\delta=$ 0.15 year^{-1}, f=0.25.

According to these initial data, the daily cargo turnover of a terminal is equal to $\lambda_1 g^{(1)}=2.5$ thousand t and, moreover,

$$F_1(0)\approx 0{,}373,\ F_1(\infty)=0{,}6,$$

$$\Lambda=\frac{11}{25}(0{,}6-0{,}373)\approx 0{,}1.$$

For the stated initial data the average amount of cargo, that is stored in a storehouse, that was calculated under formula (19), will make

$$M\xi\approx 5{,}45 \text{ thousand t.}$$

Formula (26) gives the following value for expected term of recoupment:

$$T=-\frac{1}{0{,}15}\times$$
$$\times\ln\{1-\frac{0{,}15\cdot 25000}{[(10\cdot 2{,}5+10\cdot 0{,}1\cdot 25+0{,}05\cdot 5{,}45-10)(1-0{,}25)]365}\}\approx$$
$$\approx 2{,}77 \text{ years.}$$

We should mark that the found solution of a system of equations (8)–(11) for a case of unlimited capacity of a front of unloading serves as the basis for analysis of a system with finite capacity; it can be observed as a null approximation for the stated probabilistic distribution in case of a great enough but finite rate of unloading of cargoes from TU to a storehouse.

The method, that was worked out above, makes it possible to analyze a number of other problems, that have applied meaning, for example, to research the influence of a level of irregularity of arrival of ships and TU on a level of cargo reserves in a storehouse (and on its capacity); for such a purpose it is possible to use a method of the Erlang fictitious phases, with taking of the fact that d.f. A(t) is the Erlang distribution [14].

8. Conclusion

1. A work of a port terminal is described in the terms of the theory of inventory and QT with taking into consideration of irregularity of arrival of TU with a cargo and empty ships in assumption of unlimited capacity of a front of unloading. Such a description does not match with an earlier known scheme of similar Markov processes with drift [4] in a result of necessity of taking into consideration of additional boundary conditions, that were caused with possible interruption of loading to ships because of absence of cargoes in a storehouse.

2. With a help of the used type of the Markov process with drift there is got a system of integral-differential equations and boundary conditions for finding of limit join distribution of number of ships, that are in a terminal in an arbitrary moment of time, and amount of cargo that are stored in a storehouse.

3. There is stated a method of solving of the mentioned system of integral-differential equations in a closed form for a case of arbitrary number of moorages and ships under service, that is based on the Laplace transformation.

4. On the basis of the got solution there are got formulae from calculation of main indices of efficiency of a work of a terminal, there is worked out a method of calculation of necessary capacity of a storehouse and there is got a formula for calculation of economically viable term of recoupment of a project of construction of a terminal.

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