

Досліджено можливості моделювання теплового поля в об'єкті у базисі функцій Гріна, використовуючи перетворення Лапласа. Застосування середовища Matlab дозволяє сформулювати масив значень, вимірних датчиками температури. Візуалізація теплового поля об'єкту здійснюється в результаті адаптації параметрів моделі до реально вимірних значень температури. Відображення інформації про зміну теплового поля у часі забезпечує функції управління процесом термообробки

Ключові слова: теплове поле, рівняння теплопровідності, функція Гріна, перетворення Лапласа, система візуалізації

Исследованы возможности моделирования теплового поля в объекте в базисе функций Грина, используя преобразование Лапласа. Применение среды Matlab позволяет сформировать массив значений, измеренных датчиками температуры. Визуализация теплового поля объекта осуществляется в результате адаптации параметров модели к реально измеренным значениям температуры. Отображение информации об изменении теплового поля во времени обеспечивает функции управления процессом термообработки

Ключевые слова: тепловое поле, уравнение теплопроводности, функция Грина, преобразование Лапласа, система визуализации

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THE STUDY OF THERMAL FIELD OF AN OBJECT REPRESENTED IN THE BASIS OF GREEN FUNCTIONS

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1. Introduction

Electric heating is widely used in various industries: heat furnaces of different scales, autoclaves. The advantages of electric heating are the ability to ensure greater heating rates, high accuracy and uniformity of heating due to easy control of electrical and temperature regimes. The efficiency of electric heat objects (HO) is achieved as a result of the widespread use of automation of technological heat processes.

Regardless of the application of electric heating the technological problem of formation of the corresponding thermal field in a space where the product is handled arises. This problem is related to the monitoring of the corresponding thermal field on the surface of the HO working space. This monitoring determines the need for fixing time variation of the field. To properly manage the process of heating in HO, in addition to the problem of tracking the distribution of the HO temperature field, it is necessary to take into account the behavior of the workpiece temperature [1].

The difference between the nature of the thermal field of the workpiece and the HO leads to different requirements for elements of thermal processes monitoring. Solution of this contradiction is achieved by complicating technical equipment of monitoring and reducing the efficiency of fields changes. The desire to improve the efficiency of field configuration estimation results in the need to build a real-time visualization system of thermal field. At the same time, the resulting field pattern should be adjusted according to actual measured values.

Known research works devoted to the modeling of thermal fields do not provide information on the practical implementation of such models for thermal processes control. To bring the results of theoretical research to implementation at an industrial facility, it is necessary to evaluate the possibility of functioning of the developed algorithms in real time on the basis of the available microprocessor.

2. Literature review and problem statement

In simplified tasks of assessment of the object thermal field state, it is assumed that the temperature of the object corresponds to the average HO temperature. However, the actual measurements of the heated object temperature and space of heating facility at different points indicate that the mentioned temperatures are substantially different. Thus, inaccuracies in the assessment of the real spatial distribution of the temperature in the object lead to the appearance of defects, for example, in the manufacture of ceramic products [2].

In the systems, according to the use directions [3], the data about the distribution of temperatures, which can be used for the process correction are obtained.

In implementing the research models [4], widespread mathematical modeling packages such as Fluent, Ansys, Comsol are used. These solutions are used to investigate the influence of certain process parameters on the final result of the heat treatment. Comparing actual measurements with modeling results [5], that do not involve the management of

the thermal field configuration, cannot satisfy the needs of the operators of industrial facilities in real time.

The software products of the thermal field visualization available in the international market have similar disadvantages [6, 7]. During the studies of the influence of various factors on the formation of an object field [8], the problem of estimating the dynamics of thermal processes is not set, making it difficult to assess the prospects for the use of the results of such research to control the object thermal field.

Assessment of the adequacy of the developed models with a large number of intelligent sensors [9] leads to a significant increase in the cost of the system. Reducing the number of sensors to a minimum, that provides operation of HO in real time, provides information only at the measurement points [10], without calculating the intermediate values or with minimal interpolation. Attempts to obtain a more accurate representation of the thermal field by the compression of the grid of intelligent sensors, again leads to an increase in their number [11].

The effectiveness of the thermal process is traditionally measured by the temperature of the workpiece. However, due to the difficulty of direct measurement of the workpiece temperature, because of the lack of appropriate sensors, this temperature is measured indirectly [12]. Additional difficulties in workpiece temperature measurement are associated with a constant exchange of energy between the fields of HO and the workpiece, so the workpiece temperature is measured indirectly [13], with the implementation of the necessary calculations to ensure the correctness of the measurement results.

Known algorithms for calculating the temperature of the processed object are complex enough, and can be used on large heating facilities in which there are no sudden changes in temperature. For small thermal facilities based on energy-saving technologies, the use of excessive amounts of equipment and bulky software is economically ineffective. Therefore, to identify the thermal fields parameters used in control of the heating/cooling processes, it is possible to use traditional means of temperature measurement of the furnace with the subsequent processing of the data, based on computer modeling.

From a mathematical point of view, the distribution pattern of the technological object and workpiece thermal fields can be obtained by solving the corresponding differential equations in partial derivatives.

The temperature distribution in three-dimensional Cartesian space can be written in the form of the energy equation:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{1}{k} g(x, y, z, t) = \frac{1}{\alpha} \frac{\partial T}{\partial t}, \quad (1)$$

where T – temperature, x, y, z – spatial coordinates, $\alpha = k/(\rho c)$ – thermal diffusivity, k – thermal conductivity, ρ – density, c – heat capacity, g – internal generation term, t – time.

The thermal field distribution pattern can be obtained from the above equation (1) in several ways:

- bringing the differential equation to a system of algebraic equations using the finite differences method or finite elements method (FEM) [14];
- selecting the approximating polynomials bases [15] with known space nodal values using, for example, the least squares method.

Despite the large resource consumption, which negatively affects the time of calculation of thermal fields, mentioned approaches are widely used in the simulation. However, users rarely analyze the adequacy of the results with actual measurements, because the main interest is the qualitative aspect of the process.

An alternative means for solving differential equations, especially in partial derivatives, is the method of integral transformations. Using this method allows us to reduce the differential equation to algebraic, and also to reduce the dimension of partial differential equations.

Possibilities of classical methods of the heat equation solution, which include the method of integral Laplace transforms, are limited to a number of shortcomings:

- difficulties in solving problems when the initial and boundary conditions are specified in the form of spatial functions;
- in the presence of internal heat sources, it is complicated to solve the multidimensional problems.

For the case when the HO is limited, one should consider the method of finite integral transformations using the Green function method (GF) to find the core of integration.

It should be noted that the computational efficiency of the method of integral transformations depends on the choice of the core (base) of the integral function. A particular advantage of GF is the ability of correct setting of initial and boundary conditions, also GF allows to obtain an analytic representation of the solution of the boundary value problems of mathematical physics [16]. Taking into account the thermal conductivity of the object, which affects the formation of the field, GF represents the interpretation, as a result of the heat source exposure at the boundary or within the space. Since the method of GF is associated with other methods of the heat equation solution, one option is to present the GF as a Fourier series [17]. The subject of research of this approach is to evaluate the accuracy of approximation of the model, depending on the number of harmonics of the Fourier series.

Another method of obtaining of GF is based on building a “truncated” Green’s function, which is sufficient for recording the analytical solution of the problem [18]. This method of finding the “truncated” GF greatly simplifies the process of finding the analytical solution of the problem, and can serve as a basis for the establishment of appropriate algorithms.

Thus, the possibilities of constructing a high-speed model of HO are determined by the selected method of solving the heat equation, which should take into account different boundary conditions of the object and the time-varying intensity of the heat flux of the source. Unlike modeling techniques based on the calculation of a large number of the field points [14], excluding changes in object parameters, methods of integral transforms can reduce the number of calculation points without loss of simulation accuracy.

Selection of the field representation method in the basis of the Green’s function assumes search for its best configuration, as the choice of the GF for specific HO depends on the resolution of the conflict between speed and accuracy of the field model.

Formalization of a methodology for obtaining GF of a real object is problematic, since the changing process conditions change not only the coefficients of the differential equation in partial derivatives, but also the boundary conditions of the solved equation.

Therefore, based on the objectives of the thermal field modeling, a necessary condition for constructing a model by the method of integral transformations is the selection of appropriate GF, which satisfies the requirements of the modeling.

This stage of research, provides for a choice of GF for the proposed HO configuration and assessment of the possibility of using this GF for visualization of the HO field in real time.

3. The purpose and objectives of the research

The purpose of this work is to study the possibilities of mathematical modeling of the thermal field by solving the heat equation using the method of integral transformations in the basis of the Green's functions.

To achieve this goal, the following objectives were set:

- implementation of algorithms for constructing the distribution of thermal field, developed on the basis of selected GF for the analysis of prospects for its use in decision support system according to the data of thermal fields visualization;
- assessment of the performance of the thermal field modeling algorithms on the basis of the investigated GF in terms of their function in real time;
- obtaining the accuracy characteristics of the modeled and measured values of the thermal field.

4. Getting the Green's function basis

The mathematical form of the solution (such as the Green's function G) is defined by the boundary conditions, which are the values of temperatures (or their derivatives) at the boundaries of the calculated area. The combination of the energy equation (1), certain boundary conditions and initial conditions is called boundary value problem.

The boundary conditions can be represented as:

$$k_i \frac{\partial T}{\partial n_i} + h_i T = f_i, \quad i = 1, 2, \dots, 6, \tag{2}$$

where n – the outward unit-normal vector at the boundary, h – heat transfer coefficient.

The initial conditions are presented in the form of:

$$T(x, y, z, t = 0) = F(x, y, z). \tag{3}$$

Here $F(x, y, z)$ – a function of the temperature distribution at time $t=0$.

The boundary conditions (2) apply to each face of the calculated space. They represent one of three types at each surface: type 1 for $k_i=0, h_i=1$, and f_i – a specified temperature; type 2 for $k_i=k, h_i=0, f_i$ – a specified heat flux; and type 3 for $k_i=k, h_i=h, f_i$ – the product of the coefficient of convection and ambient temperature.

Calculation of the thermal field can be divided into multiple sub-problems, calculating the temperature distribution separately from each source, and initial conditions. Using superposition, the solution can be written as:

$$T = T_0 + T_1 + T_2 \dots \tag{4}$$

Temperature can be stated in the integral form with the method of the Green's function. If the Green's function G is known, the temperature that satisfies the equation (1) is given by:

$$\begin{aligned} T(x, y, z, t) = & \iiint_{000}^{abc} G(x, x', y, y', z, z', t, \tau = 0) F(x, y, z) \partial z' \partial y' \partial x' + \\ & + \iiint_{000}^{abc} \frac{\alpha}{k} G(x, x', y, y', z, z', t, \tau) g(x', y', z', \tau) \partial z' \partial y' \partial x' \partial \tau + \\ & + \alpha \iiint_{000}^{abc} \sum_{i=1}^6 \frac{f_i(\tau)}{k_i} G(x, x_i, y, y_i, z, z_i, t, \tau) \partial z' \partial y' \partial x' \partial \tau - \\ & - \alpha \iiint_{000}^{abc} \sum_{i=1}^6 f_i(\tau) \frac{\partial}{\partial n_i} G(x' = x_i, y' = y_i, z' = z_i) \partial z' \partial y' \partial x' \partial \tau. \end{aligned} \tag{5}$$

The first term of the equation (5) only applies to the initial conditions; the second is taken into account if there is a heat source in the considered area; the third – for the boundary conditions for the second and third types; the fourth – only for the boundary conditions of the first type.

Summation of listed integrands represents all non-zero boundary conditions.

Equations (5) can be used to solve sub-problems (4) with the substitution of the appropriate boundary conditions.

The auxiliary problem should be constructed to obtain the desired GF, it is very similar to the boundary value problem for the temperature (1). But the volume energy generation function g is replaced by a product of two Dirac delta functions – $\delta(x) \times \delta(t)$. We assume that the first delta function depends only on the spatial coordinates, and the other – only on time:

$$\frac{\partial^2 G}{\partial x^2} - \frac{1}{\alpha} \frac{\partial G}{\partial t} = -\frac{1}{\alpha} \delta(x - x') \delta(t - \tau),$$

$$G(x, x', t, \tau) = 0 \quad \text{at } t - \tau < 0, \tag{6}$$

where x – the observation location, x' – the heat-source location, τ – time of the thermal pulse.

The Green's function G represents a response to the impact of the pulsed heat source in the space and time coordinates.

The condition of physical realization implies equality of the GF to zero at the initial time.

Using the method of Laplace transform, which consists of three main stages, we obtain the required GF.

In the first stage, we perform the Laplace transform, which allows to move from original function to its transform.

The obtained equation is solved in the operator space. The result of solution of the operator equation allows to obtain the desired GF using the inverse Laplace transform.

For a further solution of the boundary problem with the use of the Green's functions, we will use the numbering system, which is presented in [19]. The specific Green's function and the specific geometry of the calculated space are identified by a "number" of the form XIJYKLZNM in which X, Y, Z represent the coordinate axes, and the letters following each axis name take on the values 1, 2, 3 to represent the type of boundary conditions.

Assume that the calculated space is heated on one surface, the initial heat flux on all other surfaces is equal zero. This case corresponds to the activation of a heater on one of the walls of the HO. These conditions, according to previously accepted designations, are indexed as X12Y22Z22.

The boundary value problem is presented in the form of:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{\alpha} \frac{\partial T}{\partial t}, \quad (7)$$

$$T(x, y, z, t = 0) = T_0,$$

$$T(x = 0, y, z, t) = T_b,$$

$$\begin{aligned} \frac{\partial T}{\partial x}(x = a) &= \frac{\partial T}{\partial x}(y = 0) = \frac{\partial T}{\partial x}(y = b) = \\ &= \frac{\partial T}{\partial x}(z = 0) = \frac{\partial T}{\partial x}(z = c) = 0. \end{aligned}$$

The system of GF equations for the temperature in a space of the size of $a \times b \times c$ will be presented in the form of:

$$\begin{aligned} T(x, y, z, t) &= \iiint_{0 \ 0 \ 0}^{a \ b \ c} G_{X12Y22Z22}(x, x', y, y', z, z', t, \tau = 0) T_0 dz' dy' dx' - \\ &- \alpha \iiint_{0 \ 0 \ 0}^{t \ b \ c} T_b \frac{\partial}{\partial x} G_{X12Y22Z22}(x, x' = 0, y, y', z, z', t, \tau) dz' dy' d\tau, \end{aligned} \quad (8)$$

where T_0 – initial temperature, T_b – temperature at the boundary.

The numbering system allows to create multi-dimensional GF by the product of one-dimensional functions. Thus, the multidimensional GF are represented by:

$$\begin{aligned} G_{X12Y22Z22}(x, x', y, y', z, z', t, \tau) &= \\ &= G_{X12}(x, x', t, \tau) G_{Y22}(y, y', t, \tau) G_{Z22}(z, z', t, \tau), \end{aligned} \quad (9)$$

where G_{X12} , G_{Y22} , G_{Z22} – one-dimensional GF in coordinates x , y , z respectively.

We use the Laplace transform method for finding the GF G_{X12} .

In accordance with (6), for the solution of the auxiliary problem, we write:

$$\frac{\partial^2 G}{\partial x^2} - \frac{1}{\alpha} \frac{\partial G}{\partial u} + \frac{1}{\alpha} \delta(x - x') \delta(u) = 0,$$

$$G(x = 0, x', u) = 0,$$

$$\frac{\partial}{\partial x} G(x = a) = 0,$$

$$G(x, x', u = 0) = 0,$$

$$u = t - \tau. \quad (10)$$

Define the boundary conditions corresponding to the equation (7) (the first type at $x=0$, and the second at $x=a$, where a – the size of the calculated space). For the auxiliary problem, unlike the conditions (7), the boundary conditions for G are homogeneous (equal to zero). These conditions provide the ability to superpose any number

of GF at each other and boundary conditions remain unchanged.

Since the required FG depends on two variables – x and x' , the solution to (10) will be found in the form of:

$$G(x, x', u) = K(x, x', u) + v(x, u), \quad (11)$$

where K is the fundamental heat conduction solution, given [20] by:

$$K(x - x', t - 0) = (4\pi\alpha t)^{-1/2} \exp\left[-\frac{(x - x')^2}{4\alpha t}\right], \quad (12)$$

and $v(x, t)$ satisfies the one-dimensional heat conduction equation:

$$\begin{aligned} \frac{\partial^2 v(x, t)}{\partial x^2} &= \frac{1}{\alpha} \frac{\partial v(x, t)}{\partial t}, \\ t > 0, \quad 0 < x < a, \quad v(x, 0) &= 0. \end{aligned} \quad (13)$$

And v should be such that the boundary conditions (10) are satisfied.

Replace the form (11) G in the auxiliary problem (10) for the boundary value problem for v :

$$\begin{aligned} \frac{\partial^2 v}{\partial x^2} - \frac{1}{\alpha} \frac{\partial v}{\partial u} &= 0, \\ v(x = 0) &= -K(x = 0), \\ \frac{\partial}{\partial x} v(x = a) &= -\frac{\partial}{\partial x} K(x = a), \\ v(x, x', u = 0) &= 0. \end{aligned} \quad (14)$$

We use the Laplace transform to find the image functions \bar{v} in the operator space:

$$\begin{aligned} \frac{\partial^2 \bar{v}}{\partial x^2} - \frac{s}{\alpha} \bar{v} &= 0, \\ \bar{v}(x = 0) &= -\bar{K}(x = 0), \\ \frac{\partial}{\partial x} \bar{v}(x = a) &= -\frac{\partial}{\partial x} \bar{K}(x = a). \end{aligned} \quad (15)$$

The general solution for v may be written as

$$\bar{v}(x, x', s) = A \exp\left[\left(s/\alpha\right)^{1/2} x\right] + B \exp\left[-\left(s/\alpha\right)^{1/2} x\right]. \quad (16)$$

Constants A and B may be found by replacing v in the boundary conditions for (16). The result, after some algebra, is

$$\begin{aligned} A &= \frac{\exp\left[-\left(s/\alpha\right)^{1/2} (2a - x')\right] - \exp\left[-\left(s/\alpha\right)^{1/2} (2a + x')\right]}{\sqrt{4\alpha s} \left(1 + \exp\left[-2a\left(s/\alpha\right)^{1/2}\right]\right)}, \\ B &= -\frac{\exp\left[-\left(s/\alpha\right)^{1/2} (2a - x')\right] + \exp\left[-\left(s/\alpha\right)^{1/2} x'\right]}{\sqrt{4\alpha s} \left(1 + \exp\left[-2a\left(s/\alpha\right)^{1/2}\right]\right)}. \end{aligned} \quad (17)$$

We use the obtained coefficients A and B and Laplace transform to find the GF in the operator space:

$$\bar{G}(x, x', s) = \frac{1}{\sqrt{4\alpha s}} \exp\left[-(s/\alpha)^{1/2}|x-x'|\right] + \frac{\exp\left[-(s/\alpha)^{1/2}(2a-x-x')\right] - \exp\left[-(s/\alpha)^{1/2}(2a-x+x')\right]}{\sqrt{4\alpha s}\left(1 + \exp\left[-2a(s/\alpha)^{1/2}\right]\right)} - \frac{\exp\left[-(s/\alpha)^{1/2}(2a+x-x')\right] + \exp\left[-(s/\alpha)^{1/2}(x+x')\right]}{\sqrt{4\alpha s}\left(1 + \exp\left[-2a(s/\alpha)^{1/2}\right]\right)}. \quad (18)$$

The GF original is found from the equation (18) as a result of the inverse Laplace transform. However, it is difficult to perform this operation analytically, so the binomial theorem can be used to get the numerical solution by replacing:

$$\frac{1}{1 + \exp\left[-2a(s/\alpha)^{1/2}\right]} = \sum_{n=0}^{\infty} (-1)^n \exp\left[-2na(s/\alpha)^{1/2}\right]. \quad (19)$$

The compromise ratio precision/calculation time is achieved by limiting the number of members of selected binomial series. Retaining at this stage only the first term from the series (n=0), the required GF in the time domain is given by

$$G_{x12}(x, x', u) \approx \frac{1}{\sqrt{4\pi\alpha u}} \left\{ \begin{aligned} &\exp\left(\frac{-(x-x')^2}{4\alpha u}\right) - \exp\left(\frac{-(x+x')^2}{4\alpha u}\right) + \\ &+ \exp\left(\frac{-(2a-x-x')^2}{4\alpha u}\right) - \exp\left(\frac{-(2a-x+x')^2}{4\alpha u}\right) - \\ &- \exp\left(\frac{-(2a+x-x')^2}{4\alpha u}\right) \end{aligned} \right\}. \quad (20)$$

The approximate expression (20) is accurate for $\alpha u/a^2 < 0.1$ [19]. The GF's for planes Y22, Z22 are obtained similarly by the described method:

$$G_{Y22}(y, y', u) \approx \frac{1}{\sqrt{4\pi\alpha u}} \left\{ \begin{aligned} &\exp\left(\frac{-(y-y')^2}{4\alpha u}\right) + \exp\left(\frac{-(y+y')^2}{4\alpha u}\right) + \\ &+ \exp\left(\frac{-(2b-y-y')^2}{4\alpha u}\right) \end{aligned} \right\}. \quad (21)$$

$$G_{Z22}(z, z', u) \approx \frac{1}{\sqrt{4\pi\alpha u}} \left\{ \begin{aligned} &\exp\left(\frac{-(z-z')^2}{4\alpha u}\right) + \exp\left(\frac{-(z+z')^2}{4\alpha u}\right) + \\ &+ \exp\left(\frac{-(2c-z-z')^2}{4\alpha u}\right) \end{aligned} \right\}. \quad (22)$$

Thus, using the obtained expressions, it is possible to calculate the temperature at any point of the volume with size $a \times b \times c$, while the given space face is heating, in the time period from $t=0$ to $t=t_d$,

$$t_d(x) = \frac{2-x/a}{150} \frac{L^2}{\alpha}, \quad (23)$$

where L is the distance between the edges of the calculated coordinates.

5. The results of the field modeling in the basis of the Green's functions

The hardware and software package [1] was used for the implementation of the proposed method, which makes it possible to measure the point values of the temperature inside the heated space of $1 \times 3 \times 2$ meters. The modeled space corresponds to the size of the experimental facility, which is a closed metal container. Thermal insulation from the environment is performed by a heat insulating material (glass-wool) with a thickness of 70 mm, placed between two metal plates 3 mm thick. The 200 W heater is mounted on the side wall, and powered from a regulated power supply of 220 V, 50 Hz. The temperature sensors are installed in the container space. The measured values of the temperature field are transmitted to the personal computer by the RS-232 interface.

The software realized in the form of Matlab m-function, allows to form an array of the mathematical model values and carry out visualization of values calculated in the GF basis.

Visualization of the developed algorithm values was carried out in modeling the thermal field in the GF basis – (20)–(22) for a space with parameters: $a=1$ m, $b=3$ m, $c=2$ m, initial temperature $T_0=25$ °C.

Since it is difficult to present the analytical temperature dependence of the thermal diffusivity coefficient α of the container, this dependence is given in tabular form. The coefficient of thermal diffusivity α is represented as a matrix relationship “temperature – α ”. Heat source parameters for the model are defined as the ratio of “coordinate – temperature” using actual measured values of the boundary temperature.

After switching on the heat source, installed in the coordinate YZ ($X=0$) (the side wall of the container), and proceeding to the stationary heating mode in accordance with the equation (5), the processing of signals from sensors, located on the wall, is carried out. The result is a two-dimensional temperature field in the basis of GF (Fig. 1), which is a boundary condition for a three-dimensional boundary value problem.

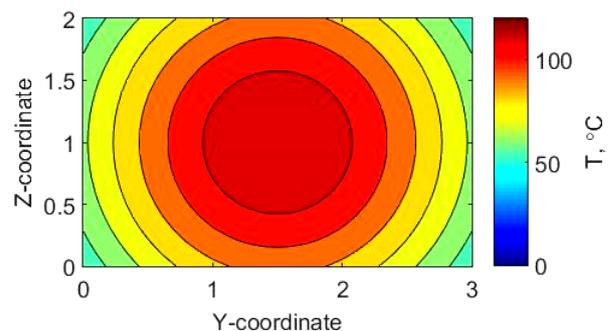


Fig. 1. Temperature field in YZ ($X=0$) coordinates

Changes in the three-dimensional temperature field in time are illustrated in Fig. 2, which shows the temperature distribution at the instants $t=100$ s (Fig. 2, a), $t=10000$ s (Fig. 2, b), $t=50000$ s (Fig. 2, c).

Adequacy of modeling results was estimated by comparing the resulting thermal distribution with the measured temperature values obtained from the temperature sensors T1, T2, T3. The sensors were placed in the XY plane, $Z=1$ in the space of the experimental facility (Fig. 3), which cor-

responds to the coordinates: T1(0.5, 2.7, 1), T2(0.6, 1.5, 1), T3(0.5, 0.5, 1). As the temperature sensors, the thermocouples NAT type K were used, with a measuring range of $-40 \dots +400 \text{ }^\circ\text{C}$, measurement error, according to passport data, is 0.75 %. Conversion and transmission of signals from the sensors to a PC are provided by the remote terminal unit (RTU). RTU is realized on the basis of OWEN PLC 150 UM. This PLC has 4 analog input channels, built-in 16-bit ADC, the response time of one channel is 0.5 seconds, the measurement error is 0.5 %.

Comparing the calculated temperatures $T1c=70 \text{ }^\circ\text{C}$, $T2c=100 \text{ }^\circ\text{C}$, $T3c=80 \text{ }^\circ\text{C}$, with the actual measured values $T1m=73 \text{ }^\circ\text{C}$, $T2m=106 \text{ }^\circ\text{C}$, $T3m=84 \text{ }^\circ\text{C}$, one can conclude that the absolute error of the field displaying does not exceed $6 \text{ }^\circ\text{C}$. If we take for the standard the actual measured values of the temperature, which are validated with the indications of the standard thermometer, it can be assumed that the relative modeling error does not exceed 7 %.

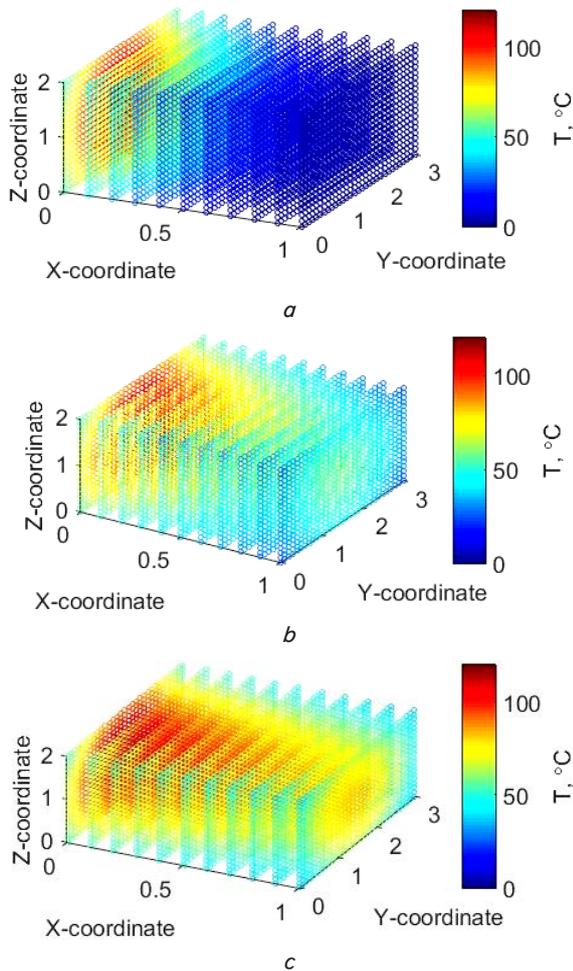


Fig. 2. Temperature field:
 a – $t=100 \text{ s}$, b – $t=10000 \text{ s}$, c – $t=50000 \text{ s}$

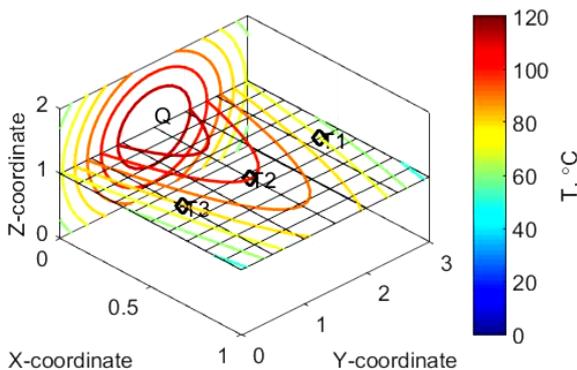


Fig. 3. Estimates of the accuracy of modeling parameters (T1, T2, T3 – sensor installation point; Q – heater)

6. Discussion of the results of the thermal field research, represented in the basis of the Green's functions

Based on the objectives of the research, that involve the assessment of the possibilities of thermal processes modeling in real time, at this stage of the work the goal of development of the original software that implements the model of the field was not declared. At the same time, attempts to use existing software products, that solve the thermal field modeling problem, indicate that modeling system performance, which is estimated by the field calculation time, does not allow a thermal process to be controlled quickly.

From the analysis of the fields modeling algorithms, it follows that most of the really functioning models are based on FEM. The increase in modeling system performance is limited to the modeling accuracy associated with a number of field points, that are modeled by the system.

A search of the alternative to FEM approaches leads to the evaluation of the field modeling capabilities by the integral transformations method. Taking into account the results, obtained by measuring the temperature point values in HO [1], and the results of thermal fields modeling, the possibility of realization of the hardware and software complex in the basis of the Green's functions was investigated. The choice of this basis has shown the possibility of HO field visualization in real time, which is the basic condition for the application of the proposed method for controlling real thermal process of the described experimental facility.

When using this method to calculate the temperature at a given point in the object space, frame calculation takes time about 0.05–0.1 sec. Accordingly, the calculation of the three-dimensional temperature field distribution in one thread required up to 10 minutes.

Investigating the possibility of reducing the frame of field obtaining time, we can compare the results of a two-dimensional field modeling for different types of GF. Fig. 4, a shows the flat field, calculated in the GF basis, obtained by separation of variables (Fourier) method, the number of series members is 10; Fig. 4, b – separation of variables method, the number of series members is 15; Fig. 4, c – Laplace transform method, the number of series members is 2.

Fig. 4 shows that increasing the number of series members to describe GF allows to increase the accuracy of field representation (with a simultaneous increase in computing time), however, significant advantages can be obtained only by changing the method of GF obtaining.

Testing the methods and criteria for the GF selection is submitted as a separate research task.

At this stage, we tested the possibility of using a different number of members of an infinite series (19), that represent the GF. Despite the fact that it is possible to use two to four members of the series to calculate the field point values up to the third sign, to find the values at the boundary of the space

that number rapidly increases (from 15 to 200). This feature leads to a dramatic increase in the calculation time at the heat object boundaries.

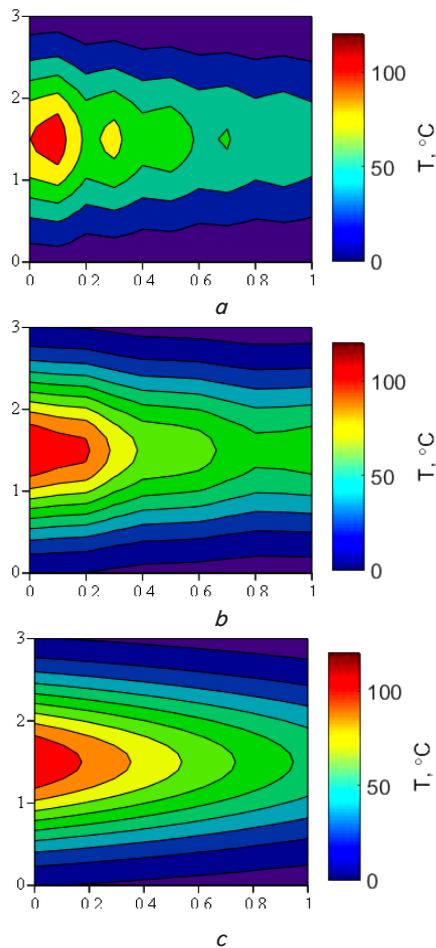


Fig. 4. Flat field calculated in the different GF basis:
a, *b* – GF obtained by the Fourier method, *c* – GF obtained by the Laplace method

To improve the convergence of the results at the boundaries of the field, in fact, not the boundary of the thermal object was calculated, but the closest layer (step of calculation 0.1 m). Boundary values were given in accordance with the data obtained from the experimental facility.

An additional reserve of productivity increasing is based on the properties of the proposed calculation algorithm, in which the temperature of each point in space is calculated independently. This feature allows to speed up the modeling process by separating the procedure into several independent solutions threads using high-performance parallel computing (HPC) or multi-core processors. This approach is expected to be implemented in subsequent phases of real-time thermal field visualization research.

A consideration of the appropriateness of suppositions and assumptions made in the model plays the decisive role in the analysis of modeling errors.

In general, comprehensive description of thermal processes is represented by three components: thermal conductivity, convection and thermal radiation. In our case – the study of the experimental facility thermal field, the assumption that most of the energy is transferred by conduction was made. This is due to the fact that there is no forced air flow inside the facility (convective component). In addition, the small size and small temperature of the controlled heating source have minimal radiation impact on the temperature distribution within the space of the tested HO.

The difficulty to assess the influence of thermal radiation and the lack of accuracy in the evaluation of the degree of blackness of bodies involved in the heat exchange; also affects the accuracy of the simulation. To improve the accuracy of the model it is necessary to update its parameters, envisaging assessment of the impact of the process components from the results of the full-scale experiment with the use of additional sensors, for example, heat flux sensors. Installation of temperature sensors on the experimental facility heat radiator panel allows to test the chosen basis of Green's functions in the construction of the thermal field configuration in conditions of changing the heat boundary conditions, which change by varying the heat treatment conditions. Tracking heat treatment conditions in the chemical industry and metallurgy is required for control of thermal process based on the generated field visualization in order to achieve a given product quality.

7. Conclusions

1. Hardware and software components of the system to construct the distribution of thermal fields were developed. A feature of the hardware component is the principle of openness, which makes it possible to use different types of sensors, as well as to change the number of control points without rebuilding the structure of the system. The advantage of the software component is an independent calculation of all points of the test model space that can be used in the process of parallel computing simulation. In contrast to the widely used finite element modeling, the independent calculation of separate points in space allows to adjust the degree of influence of the thermal process model components on the final result of modeling. The possibility of real-time process visualization was experimentally confirmed, which is proved by actual displayed images of the thermal field of the experimental facility.

2. The obtained data about the speed of the system, that are estimated by the field one frame calculating time, indicate the need for its further improvement by parallelizing the modeling process using, for example, high-performance computing on multicore processors.

3. The evaluation of the accuracy characteristics of the model leads to the need for additional studies to assess the degree of influence of components of the heating process for adjusting the model coefficients tables.

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