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STRENGTH AND DURABILITY ANALYSIS OF A FLAT SPRING AT VIBRO-IMPACT LOADINGS

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Досліджується плоска пружина та процес реалізації асиметричної кусково-лінійної пружної характеристики. Враховується контактна жорсткість, що параметрично залежить від переміщення локальної маси. Визначено еквівалентні згинальні та контактні напруження. Проведено аналіз динамічної міцності за значенням коефіцієнта запасу міцності та строку служби плоскої пружини. Отримані результати можна застосовувати під час створення резонансних віброударних систем

Ключові слова: віброудар, віброударна балка, метод скінченних елементів, контактні напруження, довговічність, власні частоти коливань

Исследуется плоская пружина и процесс реализации асимметричной кусочно-линейной упругой характеристики. Учитывается контактная жесткость, которая параметрически зависит от перемещения локальной массы. Определены эквивалентные изгибочные и контактные напряжения. Проведен анализ динамической прочности по значению коэффициента запаса прочности и срока службы плоской пружины. Полученные результаты можно применять при создании резонансных виброударных систем

Ключевые слова: виброудар, виброударная балка, метод конечных элементов, контактные напряжения, долговечность, собственные частоты колебаний

1. Introduction

Vibro-impact systems are endowed with a number of important dynamic properties, especially useful for technological purposes. The scope of application of these systems covers advanced techniques of the surface treatment of materials with complex structure. The capacity to implement high-frequency oscillations with a wide range of multiple harmonics promotes the use of the indicated systems in the fields of nanotechnology and troubled energy intensive industries.

The fabrication of vibro-impact systems requires new and simple design solutions, capable of efficient implementation of the various types of modes. In this case, the main requirements are the strength and durability of the components of the system.

2. Literature review and problem statement

In practice, there is sufficient number of ways of realization of vibro-impact systems. Most of them are pursuing implementation of piecewise linear elastic characteristics by using additional limiters of motion of executive elements. In particular, solution [1] implements non-resonant harmonic and vibro-impact modes. To provide for regulated harmonic and vibro-impact working modes, it is necessary to apply a set of additional elastic limiters and control system of the vibration exciter's drive. This greatly increases labor intensity of the operations and the system's price.

Publication [2], to provide regulated harmonic and vibro-impact working modes, necessitates applying a set of additional elastic limiters, the means of fixation and unfixation of the working mass and control system of the vibration exciter's drive. As a result, the operations get more labor-intensive while the design – more complicated.

There is a known device [3] where, for the provision of regulated vibro-impact working modes, they apply a set of additional elastic elements-limiters, mounted on a rotating disk. For the implementation of regulated harmonic working modes, it is necessary to modify the elastic element that rigidly binds the working and reactive mass. This significantly increases labor intensity of the operations to achieve a relatively basic constructive set-up of the regulated harmonic and vibro-impact modes.

There is a known solution [4] where the implementation of vibro-impact modes is carried out on two flat springs that are set overlap. However, implementation of the regulated harmonic modes and vibro-impact working modes is impossible without using the additional sets of flat springs. It definitely increases labor-intensity of the operations in case of realignment of the system.

It is known [5] that the elastic elements are the most important component of resonant vibration systems and machines. In the case of implementation of vibro-impact modes and asymmetric piecewise linear elastic characteristics, where special importance is acquired by the contact tasks, it is necessary to pay maximum attention to the question of strength and durability [6].

Designing applied methods of project calculation of the vibro-impact systems requires a thorough study of the problems of strength and durability of elastic elements that provide resonant modes. The main problematic issue here is the joint taking into account of existing stresses based on consideration of real dynamic condition of the elastic elements. It should be noted that the use of asymmetric piecewise linear elastic characteristics [7] is an efficient means of realization of vibro-impact systems. Using one elastic beam for realization of vibro-impact modes is presented in [8]. The approach represents reduction of the system with distributed parameters to the discrete one to simplify the procedure of analysis.

The whole subject of vibro-impact systems covers the tasks of parametric synthesis, analysis of dynamics and stability of motion. Important are the questions of bifurcation analysis and chaos. Crucial processes that accompany the performance of vibro-impact systems are availability of the contact problems and analysis of the contact forces [9]. They acquire special importance in the systems with clearances [10]. Such problems are strongly-nonlinear, both from the point of view of dynamics and the contact phenomena [11].

Thus, high labor intensity of operations to ensure the regulated resonant harmonic and vibro-impact working modes is a common problem of the known technical solutions. The main idea aimed at eliminating this drawback is to use one elastic element and a scheme capable to realize various elastic characteristics relative to its basic implementation at minimum design changes in the system. The tasks of eliminating the shortcomings, mentioned in publications [1–4], as well as ensuring strength and durability of responsible units of vibro-impact systems are well substantiated.

3. Aims and objectives of the study

The aim of the study is to provide for the strength and durability of a flat spring using regulated resonant harmonic and vibro-impact working modes.

To achieve the set aim, the following tasks were to be solved:

– to determine the maximum bending stresses, taking into account the changes in elastic characteristic of one flat spring;

– to identify the contact stresses and the coefficient of rigidity in the contact zone;

– to consider parametric dependency of the contact rigidity and evaluate its effect on the own oscillation frequencies of a vibro-impact system;

– to determine the strength margin factor and durability of a flat spring under conditions of dynamics of a vibro-impact mode.

4. Materials and methods of the study

The study is aimed at examining dynamic properties of a vibro-impact system, designed on one elastic element – flat spring with locally focused working mass of the inertia m_w. The mass is located in the central cross section of the spring that is rigidly nipped at the edges. This scheme is typical for the most of resonant two mass systems that lead to the considered one mass system.

The schematic of realization of the vibro-impact mode on one flat spring is represented in Fig. 1 by using asymmetric elastic characteristic of the form:

$$c(t)=\begin{cases} c_I, y(t)\geq 0, \\ c_{II}, y(t)<0, \end{cases} \tag{1}$$

where

$$c_I=\frac{192EJ}{L^3},\quad c_{II}=\frac{6EJ\cdot(l_1+4l_2)}{(l_1+l_2)\cdot l_2^{\,3}},$$

where c_I, c_{II} are the coefficients of rigidity for schemes without intermediate and with absolutely rigid intermediate supports of the types I and II, respectively, $c_{II}>c_I$; EJ is the bending rigidity of the spring; $J=bh^3/12$ is the axial inertia moment of the spring; b and h are the width and thickness of the spring.

Application of this scheme makes it possible, by changing location of the intermediate supports, to adjust the form of asymmetric elastic characteristic, in particular, the coefficient c_{II}. Using the intermediate supports that are installed on both sides of a relatively flat spring, it is possible to ensure performance of vibration system in the harmonious mode with the appropriate coefficient c_{II}.

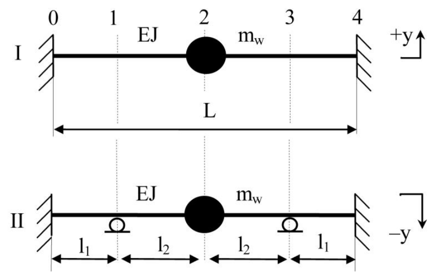


Fig. 1. Schematic of realization of asymmetric piecewise linear elastic characteristic on one flat spring

The ratio of coefficients of rigidity, taking into account $l_2=L/2-l_1$, will have the form:

$$n_c=c_{II}/c_I=\frac{L^2\cdot(L-1{,}5l_1)}{(L-2l_1)^3}. \tag{2}$$

The resulting ratio (2) is used during synthesis of vibro-impact systems and modes.

The ratio of own oscillations frequencies will take the following ratio: $n_\omega=\omega_{0II}/\omega_{0I}=\sqrt{n_c}$. By entering the condition of multiplicity of own bending frequencies of the considered rod systems in the form $n_\omega=2$, we received the expressions for determining the place of location of intermediate supports

$$l_2\cong 0{,}275L,$$

$$l_1\cong 0{,}225L,$$

in this case $2(l_1+l_2)=L$.

For realization of the set own oscillations frequency for the scheme I in the course of project calculation of the bending rigidity EJ of the elastic element, we use the following formula:

$$EJ=\frac{m_w L^3\omega_{0I}^{\;2}}{192}. \tag{3}$$

The model that is analyzed numerically is adopted with the following parameters' values: $\omega=\omega_{0I}=314$ rad/s,

$E=2{,}05\times10^5$ MPa, $b=0{,}080$ m, $m_w=20$ kg, $L=0{,}5$ m. Based on them, we calculated by the established formulas: $EJ=1{,}284$ kN×m^2, $l_1=0{,}113$ m, $l_2=0{,}137$ m, $h=9{,}79\times10^{-3}$ m, $c_I=1{,}972\times10^6$ N/m, $c_{II}=7{,}888\times10^6$ N/m.

Principal during implementation of the elastic characteristic (1) is the availability of the contact problem between the flat spring and intermediate cylindrical support (Fig. 2). The value of the contact force is similar to reaction of the support that can be determined by the known methods of finite elements [12, 13], initial parameters [14].

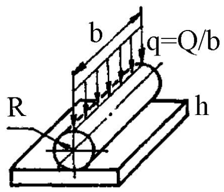


Fig. 2. Schematic of the contact problem

The contact force is parametrically dependent on the instantaneous displacement y(t) of the local mass, therefore, it varies according to the condition:

$$Q\left[y(t)\right]=\begin{cases}0, y(t)>0,\\ \dfrac{c_{II}y(t)\cdot\left(l_1^2+4l_1l_2+3l_2^2\right)}{2l_1\cdot\left(l_1+4l_2\right)}, y(t)\le 0.\end{cases} \quad (4)$$

The value of contact rigidity for the contact materials steel-steel is known from the theory of Hertzian contact stress [14, 15]:

$$c_{contact}\left[y(t)\right]=0{,}81\sqrt[3]{Q\left[y(t)\right]RE^2}, \quad (5)$$

where R is the radius of the intermediate support.

The equivalent contact stresses can be found by the following way:

$$\sigma_{eq}\left[y(t)\right]^{contact}=m_{III(IV)}\times\sigma_m\left[y(t)\right]^{contact}, \quad (6)$$

where $m_{III(IV)}$ are the coefficients, which for the band take the following values according to the hypothesis of strength: $m_{III}=0{,}557$; $m_{IV}=0{,}6$; $\sigma_m^{contact}$ is the maximum contact stress that is determined by the known formula from the theory of Hertzian contact stress

$$\sigma_m\left[y(t)\right]^{contact}=0{,}418\sqrt{\frac{Q\left[y(t)\right]}{bR}}.$$

The value of permissible maximum pressure on the contact area $[\sigma^{contact}]$ is the criterion of contact strength. For the spring steel 50G its values are in the range of $[\sigma^{contact}]=1100...1450$ MPa. The condition of contact strength:

$$\sigma_{eq}^{contact}\le\left[\sigma^{contact}\right]. \quad (7)$$

The values of maximum bending stresses are determined by the known formulas, represented with indexes for the corresponding calculation techniques:

$$\sigma_{max}^{I}\left[y(t)\right]=\frac{M_{max}^{I}\left[y(t)\right]}{W},$$

$$\sigma_{max}^{II}\left[y(t)\right]=\frac{M_{max}^{II}\left[y(t)\right]}{W},$$

where

$$M_{max}^{I}\left[y(t)\right]=\frac{1}{8}c_I y(t)L,$$

$$M_{max}^{II}\left[y(t)\right]=c_{II}y(t)\frac{l_2\cdot\left(l_1+2l_2\right)}{2\cdot\left(l_1+4l_2\right)}.$$

are the maximum bending moments in dangerous cross-sections of the considered rod systems; $W=bh^2/6$ is the moment of resistance of the cross-section.

The maximum shear stresses, operating in the central cross-section of the rod:

$$\tau_{max}^{I}\left[y(t)\right]=\frac{1{,}5c_I y(t)}{bh},\quad \tau_{max}^{II}\left[y(t)\right]=\frac{1{,}5c_{II}y(t)}{bh}.$$

Equivalent stress by Mises:

$$\sigma_{vonMises}=\sqrt{\sigma_{max}^{2}+4\tau_{max}^{2}}.$$

The resultant instantaneous equivalent bending stress is determined by conditions of work of the rod system:

$$\sigma_{vonMises}\left[y(t)\right]=\begin{cases}\sigma_{vonMises}^{I}(t), y(t)\ge 0,\\ \sigma_{vonMises}^{II}(t), y(t)<0.\end{cases} \quad (8)$$

Based on considering the contact rigidity of intermediate supports (5), the scheme of type II can be represented as the rod system with pliable intermediate supports (Fig. 3), the supporting (contact) rigidity of which $c_y[y(t)]$ is determined by the instantaneous displacement of local mass.

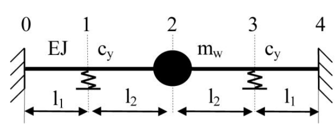


Fig. 3. Calculation scheme of the rod system with elastic intermediate supports

By the finite elements method, we initially obtained a generalized matrix of rigidity of the core system with pliable intermediate supports and the values of own frequency $\Omega[y(t)]$ – based on the solution of the appropriate frequency equation [16]:

$$\Omega\left[y(t)\right]=\frac{8\times\sqrt{\begin{array}{l}3\,EJ\cdot L\cdot m_w\cdot\left(2\cdot c_y\left[y(t)\right]\cdot L\cdot l_1^3-3\cdot c_y\left[y(t)\right]\cdot L^4+3\,EJ\cdot L\right)\times\\ \times\left(\begin{array}{c}2\cdot c_y\left[y(t)\right]\cdot L^3\cdot l_1^3+3\,EJ\cdot L^3-12\cdot c_y\left[y(t)\right]\cdot L^2\cdot l_1^4+\\ +24\cdot c_y\left[y(t)\right]\cdot L\cdot l_1^5-16\cdot c_y\left[y(t)\right]\cdot l_1^6\end{array}\right)\end{array}}}{m_w\cdot\left(\begin{array}{c}2\cdot c_y\left[y(t)\right]\cdot L^4\cdot l_1^3+3EJ\cdot L^4-12\cdot c_y\left[y(t)\right]\cdot L^3\cdot l_1^4+\\ +24\cdot c_y\left[y(t)\right]\cdot L^2\cdot l_1^5-16\cdot c_y\left[y(t)\right]\cdot L\cdot l_1^6\end{array}\right)}. \quad (9)$$

The value of own frequency of the system Ω system is within $\omega_{0I} \le \Omega \le \omega_{0II}$ (Fig. 4) and it asymptotically approaches the value ω_{0II} with the increase of coefficient of rigidity of the support $c_y \to \infty$:

$$\lim_{c_y \to \infty} \Omega = 4 \cdot \sqrt{\frac{6\, EJ \cdot (2L - 3l_1)}{m_w L \cdot (L - 2l_1)^3}} \equiv \omega_{0II}.$$

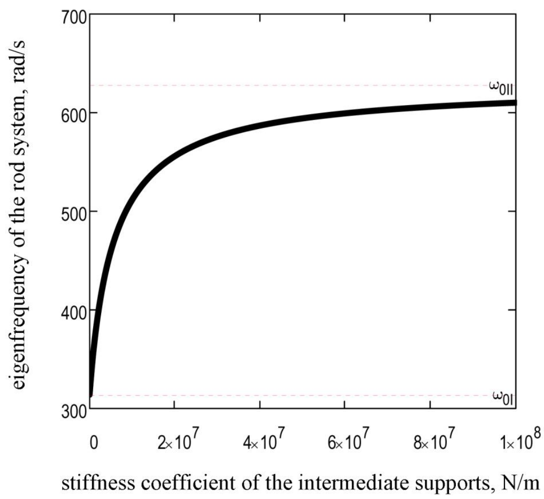


Fig. 4. Influence of coefficient of rigidity of the intermediate supports on the own oscillations frequency of the rod system

To account for the real rigidity of supports c_y, it is necessary to adjust the place of their location with the appropriate solution of the equation (9) by the condition of provision of the corresponding value of the own frequency.

For analysis of dynamic state of a flat spring, taking into account the contact rigidity, one should consider a model of the forced vibrations with regard to instantaneous change in the frequency parameters of the system:

$$y''(t) + 2n \cdot y'(t) + \begin{cases} \omega_{0I}{}^2 \cdot y(t), y(t) \ge 0, \\ \Omega[y(t)]^2 \cdot y(t), y(t) < 0 \end{cases} = f \cdot \sin(\omega t), \quad (10)$$

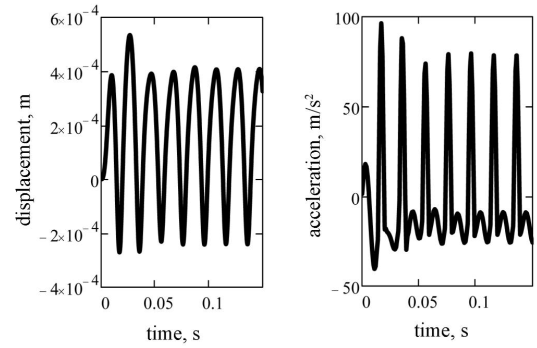
where $n=2b/m_w$, $f=F_0/m_w$.

Thus, formula (10) took into account the contact problem (5) of a vibro-impact system through the change in its own frequency (9).

5. Results of examination of the stressed state

Assuming F_0=600 N and $b=2m_w\omega\xi$ at the coefficient ξ=0.15, by differential equation (10) we numerically obtained time kinematic characteristics – displacements and accelerations (Fig. 5). The presence of a vibro-impact mode is evidenced by asymmetric time characteristic of acceleration of the local mass (Fig. 5, *b*).

Instantaneous dependencies for normal, equivalent and contact stresses are given in Fig. 6. Received normal and equivalent stresses (Fig. 6, *a, b*) have a distinctly expressed asymmetrical character according to the obtained dependency y(t) while the contact stresses – the pulse one (Fig. 6).



a *b*

Fig. 5. Instantaneous kinematic characteristics: *a* — displacement; *b* —acceleration

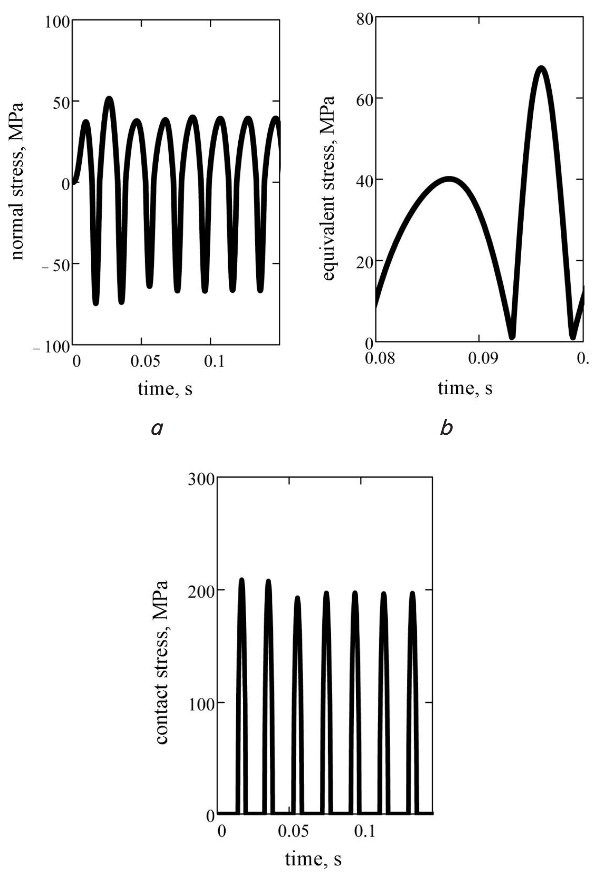


Fig. 6. Instantaneous characteristics of stresses: *a* — normal at bending; *b* — equivalent at bending; *c* — contact at bending

Parametrical dependency of stresses on displacement of the local mass is presented in Fig. 7, *a*. The linear dependency for the normal stresses and nonlinear – for the contact ones, are obvious. The stresses depend on acceleration nonlinearly (Fig. 7, *b*).

Henceforth, the presented elastic element must be calculated by the known techniques for durability and endurance [17, 18], taking into account the effect of asymmetric stress. Characteristics of asymmetric sign-variable negative cycle of the change in bending stresses: $\sigma_{max}{}^{I}$=51,3 MPa, $\sigma_{min}{}^{II}$=–74,6 MPa. Coefficient of asymmetry of the cycle $R_\sigma=\sigma_{min}{}^{II}/\sigma_{max}{}^{I}$=–1,45. The amplitude of stress $\sigma_a=(\sigma_{max}{}^{I}-\sigma_{min}{}^{II})/2=62{,}94$ MPa. Average value: $\sigma_a=(\sigma_{max}{}^{I}+\sigma_{min}{}^{II})/2=-11{,}63$ MPa.

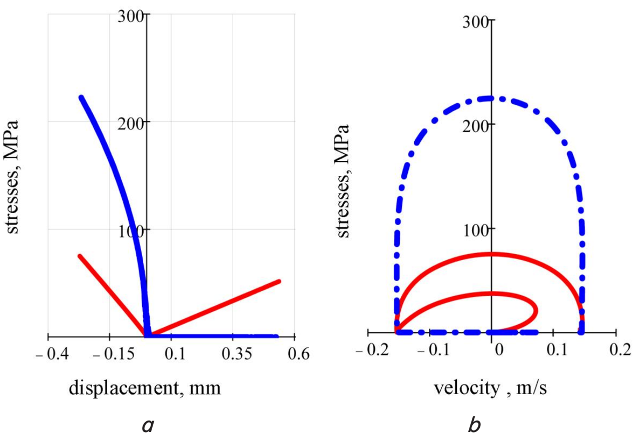


Fig. 7. Parametrical dependency of stress on kinematic parameters of the local mass: *a* — displacement; *b* — acceleration: red color — bending stress, dark blue color — contact stress

Reduced strength margin factor at the joint action of bending and contact stresses will be determined by the modified formula (instead of torsion we considered contact stresses):

$$n=\frac{n_\sigma n_c}{\sqrt{n_\sigma{}^2+n_c{}^2}}, \qquad (11)$$

where n_σ is the strength margin factor at bending,

$$n_\sigma=\frac{\sigma_{-1}}{K_\sigma\sigma_a+\psi_\sigma\sigma_{med}},$$

where K_σ is the effective stress concentration factor, $K_\sigma>1$, $K_\sigma=1+q_\sigma(\alpha_\sigma-1)$; q_σ is the factor of material's sensitivity to the concentration of stresses, tentatively for alloyed steels, $q_\sigma=0{,}6...0{,}8$; α_σ is the theoretical coefficient of concentration of stresses, $\alpha_\sigma=1{,}2...2$; ψ_σ is the factor of the material's sensitivity to asymmetry of the cycle, tentatively for steels with intermittent resistance $\sigma_в=520...750$ MPa – $\psi_\sigma=0{,}05$; for $\sigma_в=700...1000$ MPa – $\psi_\sigma=0{,}1$; n_c is the strength margin factor for contact stresses,

$$n_c=\frac{\left[\sigma^{contact}\right]}{\sigma_{eq}{}^{contact}},$$

where $\sigma_{eq}{}^{contact}=208{,}57$ MPa is the equivalent contact stress, calculated by the fourth hypothesis of strength according to (6).

To ensure performance efficiency and durability of the flat spring in a vibro-impact mode, the condition must be satisfied: $n\geq[n]$, where the permissible value of the strength margin factor according to [18] for the symmetric cycle of stresses $[n]=2{,}4...3$.

Assuming the limit of endurance for the material of the flat spring, steel 50 G, according to [14] as $\sigma_{-1}=300$ MPa, we receive for $K_\sigma=1{,}3$ the values of strength margin factors by the types of stresses $n_\sigma=3{,}69$ and $n_c=5{,}27$. The adjusted strength margin factor $n=3{,}02$ will be considered satisfactory for the operation of the spring under these conditions.

Knowing the base number of cycles $N_0=10^6$, the durability at the current stress will be determined:

$$N_h=N_0\cdot\left(\sigma_{-1}/\left|\sigma_{min}{}^{II}\right|\right)^m=$$
$$=10^6\cdot(300/74{,}6)^9=2{,}757\cdot10^{11}\ \text{cycles},$$

where $m=6...10$ and $N_0=(1...4)\times10^6$ cycles for small models of the spring with stress concentrators.

The estimated service time of the spring under the given working conditions at oscillations frequency $f=50$ Hz will be practically limitless and will amount to $h=N_h/f=174{,}7$ years.

The implementation of vibro-impact mode of a vibration system should not be accompanied by resonant phenomena in the elements of design, including the spring. Therefore, for efficient work of the spring, it is necessary to recalculate its own oscillations frequencies as the element with distributed elastic characteristics. With regard to the scheme of nipping the spring at the edges, the range of its bending p_k own frequencies is determined by the design parameters [12]:

$$p_k=\lambda_k{}^2\sqrt{EJ/\left(\rho bhL^4\right)},\ (k=1,2),$$

where $\lambda_1=4{,}73$, $\lambda_2=7{,}853$.

Thus, the first two own frequencies of transverse (bending) oscillations of the flat spring

$$\frac{p_1}{2\pi}=206{,}44\ \text{Hz},\ \frac{p_2}{2\pi}=569{,}03\ \text{Hz},$$

which is satisfactory in terms of discrepancy between the frequency of own oscillations and the forced, multiples to it according to the following condition:

$$p_k\neq l\omega,\ (l=1,2,3\ldots).$$

If the preceding condition is not met, or the resulting strength margin factor is less than the permissible one, then it is necessary to increase the spring length L.

6. Discussion of results of the analysis of dynamic stressed state of a flat spring

The contact stresses that are of pulse character are crucial to ensure the strength and durability of the flat spring, which implements the corresponding asymmetric piecewise linear elastic characteristic. The larger stresses at bending are those that act in the direction of implementation of elastic characteristic with the coefficient of rigidity c_{II} (Fig. 7).

Thus, the method of considering contact rigidity in the course of implementation of asymmetric elastic characteristic is determined by the parametric dependency and nonlinear characteristic in relation to the contact force (Fig. 8). The maximum value of the contact force due to implementation of a vibro-impact mode is $Q_{max}=1{,}878$ kN.

Considering the nonlinear character of the influence of force parameters, in the contact problem we considered the behavior of strength margin factors and, accordingly, the period of spring's operation time, on the magnitude of amplitude of the perturbation effort F_0 (Fig. 9). Dependencies in Fig. 9, *a* point to the higher sensitivity of bending stresses to the amplitude of external perturbation. The selection of amplitude value of the force of disturbance is carried out based on the required maximum value of acceleration of the local mass. The estimated lifetime of the spring is also nonlinearly dependent on the amplitude value of the force (Fig. 9, *b*).

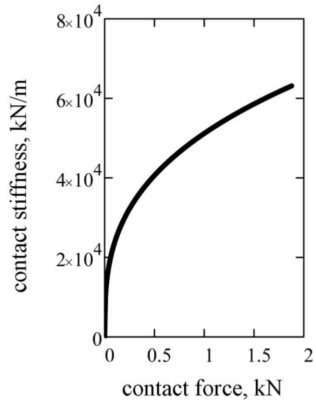
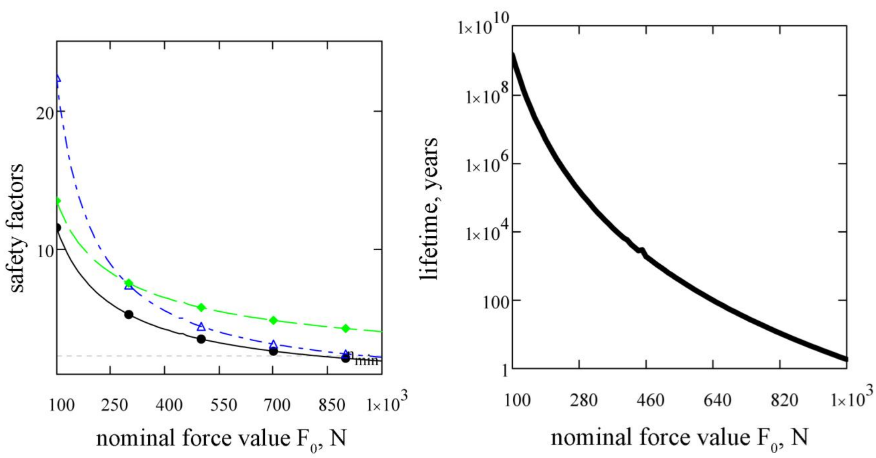


Fig. 8. Parametric characteristic of contact rigidity and strength in the intermediate supports during displacement of the local mass



a *b*

Fig. 9. Effect of nominal value of the effort: *a* — on the strength margin factors, ● — n, ♦ — n_c, Δ — n_σ; *b* — on the spring's lifetime

The existing method of dynamic analysis of stressed state and durability allows comprehensive estimation of characteristics of strength with kinematic and dynamic parameters of vibro-impact or harmonic modes, implemented on one flat spring.

7. Conclusions

In the study we examined the scheme of change in the elastic characteristic of pinned at the edges flat spring by two intermediate cylindrical supports. Asymmetric piecewise linear elastic characteristic is implemented when installing the supports on one side of the flat spring. If the supports are not used, then the stable elastic characteristic with lower coefficient of rigidity is implemented. When installing the supports on both sides of the flat spring, then the stable characteristic with the larger coefficient of rigidity is provided. The last two variants are used for realization of harmonic working modes.

To ensure the strength of the flat spring:

1. We determined the bending stresses that take into account the asymmetry of change in the elastic characteristic and instantenous displacement of the local mass.

2. The contact stresses and the coefficient of rigidity in the contact zone were calculated based on the Hertz theory in the form of function of instantaneous displacement of the local mass.

3. The parametric dependency was taken into account of the contact rigidity on the displacement in the formula for own frequency of bending oscillations of the spring with intermediate supports. It enabled, in the course of dynamic analysis, to consider the influence of contact deformations on the change in the value of own frequency of oscillations in the form of a nonlinear function.

4. We examined dynamics of the stressed state of a flat spring with regard to parametric dependency of the contact rigidity on displacement of the local mass. Through the joint account of the contact and bending stresses, we modified the formula for determining the strength margin factors and lifetime of a flat spring with different values of nominal perturbation effort. The nonlinear effect was established of the effort on the strength margin factors by the types of stresses.

In general, the obtained results confirm availability of a vibro-impact mode and workability of a flat spring in accordance with the proposed scheme within the appropriate parameters of perturbation.

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