

*Запропоновано метод дослідження чутливості математичних моделей електроенергетичних компенсаційних симетрувальних пристроїв. Досліджено чутливість однієї з існуючих моделей систем симетрування напруг. Це дало змогу визначити точність датчиків для системи симетрування напруг, котра може бути реалізована на основі відповідної математичної моделі*

*Ключові слова: симетрування напруг, симетрувальний пристрій, чутливість математичної моделі, зона нечутливості*

*Предложен метод исследования чувствительности математических моделей электроэнергетических компенсационных симметрирующих устройств. Исследована чувствительность одной из существующих моделей систем симметрирования напряжений. Это помогло определить точность датчиков для системы симметрирования напряжений, которая может быть реализована на основе соответственной математической модели*

*Ключевые слова: симметрирование напряжений, симметрирующее устройство, чувствительность математической модели, зона нечувствительности*

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# EXPLORING SENSITIVITY OF MATHEMATICAL MODEL FOR THE SYSTEM OF VOLTAGE SYMMETRIZATION

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## 1. Introduction

Electricity has characteristics that allow determining the level of its applicability in a variety of industrial processes. The characteristics, with which power receivers are able to carry out their functions, are combined under a general concept of electric power quality.

Voltage asymmetry is an important characteristic of electric power quality. Reliability and performance efficiency of electric power consumers depend on maintaining indicators of voltage asymmetry within permissible limits.

The use of symmetrization devices (SD) occupies important place among the measures and means of reducing voltage asymmetry: symmetrization of condenser installations, symmetrization transformers, static thyristor compensators whose important component is a control system.

Contemporary control systems of SD widely use microprocessor devices whose algorithms of work are based on mathematical models of voltage symmetrization. In this case, important elements of SD control systems are the sensors of electrical magnitudes, on whose error the SD error calculation parameters depend.

For correct selection of sensors of electric magnitudes, it is necessary to solve the inverse problem of sensitivity of mathematical model of the appropriate SD control system.

Thus, the research into sensitivity of mathematical models of SD control systems is a rather relevant task for electroenergy.

## 2. Literature review and problem statement

Sensitivity is one of the main properties of a mathematical model. If a mathematical model has parameters, then sensitivity analysis is a procedure of searching for an answer to the question: "What will happen to the result of modelling if the values of its parameters change?"

Assume the area of insensitivity of the parameter  $u$  of mathematical model is in the interval between  $u^-$  to  $u^+$  while insensitivity zone of the result of modelling  $F$  – in the interval from  $F_0$  to  $F_0 + \partial F$  (Fig. 1) [1, 2]. In this case,  $F_0$  is the optimal value of the result of modelling,  $u_0$  is the optimal value of the parameter.

Solving the inverse problem of sensitivity is in the search for a zone of insensitivity of parameter of mathematical model if the interval of zone of insensitivity of the result of modeling is known. Therefore, to achieve this goal, it is necessary to solve the following system of equations [1, 2]:

$$\begin{cases} F = f(u), \\ F = F_0 + \partial F. \end{cases} \quad (1)$$

As we know from [1, 2], mathematical models of control systems, which have  $n$  input parameters and  $m$  magnitudes at the output, are convenient to represent in the form of a "black box" (Fig. 2).

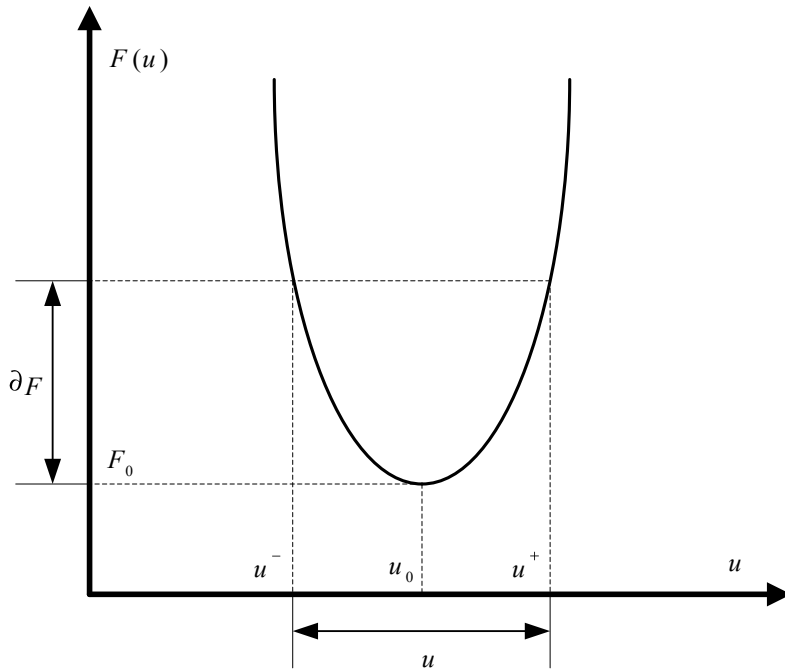


Fig. 1. Zone of insensitivity of parameter of mathematical model and of the result of modelling

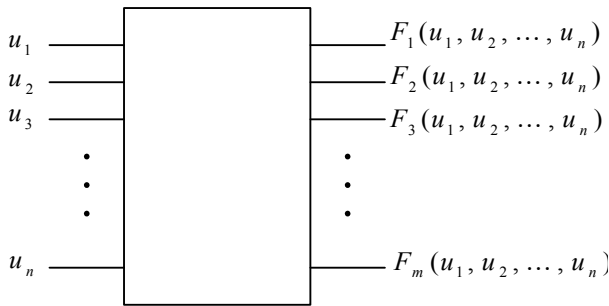


Fig. 2. Representation of mathematical model in the form of a “black box”

Then for  $n$  input parameters and  $m$  output results of mathematical model, the system of equations (1) will be rerecorded as follows:

$$\begin{cases} F_1 = f_1(u_1, u_2, \dots, u_n); \\ F_2 = f_2(u_1, u_2, \dots, u_n); \\ \vdots \\ F_m = f_m(u_1, u_2, \dots, u_n); \\ F_1 = F_{01} + \partial F_1; \\ F_2 = F_{02} + \partial F_2; \\ \vdots \\ F_m = F_{0m} + \partial F_m. \end{cases} \quad (2)$$

Thus, the essence of the problem comes down to solving the system of equations (2).

There are various mathematical models of voltage and current symmetrization systems, the sensitivity of which may be examined. Ref. [3, 4] consider mathematical models of voltage and current symmetrization systems based on the p-q theory. Ref. [5] examines mathematical model of symmetrization

system of the load in four-wire electrical networks. Ref. [6] proposes mathematical model of symmetrization system of the load under nonsinusoidal voltages and currents. Ref. [7] considered modelling of the process of loads symmetrization loads using a static compensator (statcom).

Simulation of the process of voltage and current symmetrization in aircraft electrical systems was performed in [8]. Ref [9] also considered another approach to loads symmetrization. It should be noted that examination of sensitivity of mathematical models of electric power systems is given attention to in domestic papers [1, 2]. However, not a single article cited a study of sensitivity of the models of voltage and current symmetrization systems. This is due to the fact that sensors of electric magnitudes for the voltage and current symmetrization systems are selected in many cases in the following way:

- 1) particular sensors are examined with specific values of errors in measurement of electric magnitudes;
  - 2) as a result of simulation or field experiments, the errors of results of modelling are determined;
  - 3) if permissible values of errors of modelling results are obtained, then the problem of choosing sensors is solved;
  - 4) if the values of errors of modelling results are unacceptable, then it is necessary to return to point 1 and repeat the research for sensors with lower error of measurement of electric magnitudes.
- Solving the inverse problem of sensitivity of mathematical models of voltage and current symmetrization systems makes it possible to immediately obtain the interval of zone of insensitivity of modeling, as a result of which it is possible to estimate accuracy of the sensors. This approach is more convenient because it is not necessary to analyze particular sensors of electric magnitudes in terms of their suitability for practical implementation of the appropriate control systems.

In this work we will examine sensitivity of the model of system of voltage symmetrization, which is proposed in [10]. That is, we will continue the research, part of the results of which is described in [10].

### 3. Aims and objectives of the research

The aim of the research is the solution of inverse problem of sensitivity of the model of voltage symmetrization systems for selecting the sensors of electric magnitudes required for building a SD control system.

To achieve the set aim, the following tasks were to be solved:

- to record functional dependencies between parameters and results of simulation of the model of voltage symmetrization;
- to solve the inverse problem of sensitivity of a mathematical model;
- to assess the accuracy of sensors of electric magnitudes required for building an appropriate SD control system.

**4. Materials and methods the study of sensitivity of mathematical model**

For solving the set problems, in this work we used the following methods:

- theoretical electrical engineering and the theory of linear algebra – to obtain analytic expressions of the laws of symmetrization and mathematical models for the systems of symmetrization devices control;

- capacities of computer simulation – when solving the inverse problem of sensitivity of a mathematical model.

After obtaining mathematical models for the systems of symmetrization devices control, we carried out simulation modelling in the environment of Matlab (Simulink) using the Sim Power Systems library. This helped to confirm the adequacy of the received models. In this case, we used the M-programming language of the Matlab environment to record the obtained models in a functional form.

Next, using the same M-language, we created the program whose algorithm is presented in this work. Using this program, solving of the inverse problem of sensitivity of the mathematical model was carried out. We received values of the intervals of zone of insensitivity of the model's parameters depending on the preset intervals of zone of insensitivity of the modelling results.

**5. Results of solving the inverse problem of sensitivity of mathematical model**

Let us consider a model of control system for the asymmetry of voltages, proposed in [10].

Assume that the phase voltages are to be symmetrized to the magnitudes

$$\overset{\circ}{U}_A = U_\phi \cdot e^{j\arg(\overset{\circ}{U}_A)},$$

$$\overset{\circ}{U}_B = U_\phi \cdot e^{j\left(\arg(\overset{\circ}{U}_A) - \frac{2\pi}{3}\right)},$$

$$\overset{\circ}{U}_C = U_\phi \cdot e^{j\left(\arg(\overset{\circ}{U}_A) + \frac{2\pi}{3}\right)}.$$

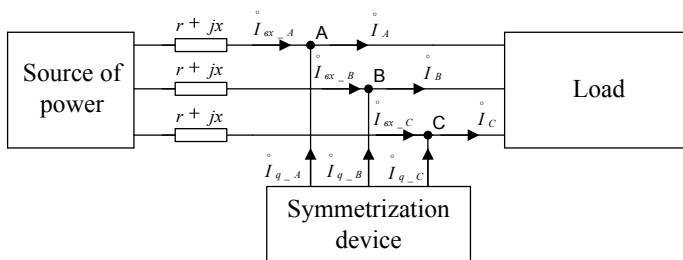


Fig. 3. Voltage symmetrization circuit

The optimum argument of the expected phase voltage  $\arg(\overset{\circ}{U}_A)$  (Fig. 4) must differ little from zero because the argument of phase voltage A of power source is accepted as zero. Incorrect choice of magnitude  $\arg(\overset{\circ}{U}_A)$  may lead to receiving negative real parts of the SD full capacity, or negative active of the SD power. This indicates that SD should perform injection of active power to the network. When us-

ing, for example, condenser SD, such result is unacceptable. Graph (Fig. 4), to simplify the calculations, demonstrates dependencies of the valid parts of phase conductivities of SD on the magnitude  $\arg(\overset{\circ}{U}_A)$ . These valid parts of phase conductivities of SD are directly proportional to the active powers. Therefore,  $\arg(\overset{\circ}{U}_A)$  will be defined as follows:

$$\arg(\overset{\circ}{U}_A) = \begin{cases} \left[ \left| \max(\alpha_{1A}, \alpha_{1B}, \alpha_{1C}) \right| \geq \left| \min(\alpha_{2A}, \alpha_{2B}, \alpha_{2C}) \right|, \right. \\ \left. \min(\alpha_{2A}, \alpha_{2B}, \alpha_{2C}), \right. \\ \left[ \left| \max(\alpha_{1A}, \alpha_{1B}, \alpha_{1C}) \right| < \left| \min(\alpha_{2A}, \alpha_{2B}, \alpha_{2C}) \right|, \right. \\ \left. \max(\alpha_{1A}, \alpha_{1B}, \alpha_{1C}), \right. \end{cases} \quad (3)$$

where

$$\alpha_{1,2A} = \arg(\underline{Y}_i) + \arg(\overset{\circ}{U}_{S-A}) \mp \arccos\left(\frac{U_A \cdot (\text{Re}(\underline{Y}_A) + \text{Re}(\underline{Y}_i))}{U_{S-A} \cdot Y_i}\right);$$

$$\alpha_{1,2B} = \arg(\underline{Y}_i) + \arg(\overset{\circ}{U}_{S-B}) \mp \arccos\left(\frac{U_B \cdot (\text{Re}(\underline{Y}_B) + \text{Re}(\underline{Y}_i))}{U_{S-B} \cdot Y_i}\right) + \frac{2\pi}{3};$$

$$\alpha_{1,2C} = \arg(\underline{Y}_i) + \arg(\overset{\circ}{U}_{S-C}) \mp \arccos\left(\frac{U_C \cdot (\text{Re}(\underline{Y}_C) + \text{Re}(\underline{Y}_i))}{U_{S-C} \cdot Y_i}\right) - \frac{2\pi}{3};$$

where  $U_i$  is the current value of the voltage of the  $i$ -th phase, expected after symmetrization ( $i=A, B, C$ );  $\underline{Y}_i$  is the conductivity of the  $i$ -th phase of load ( $i=A, B, C$ );  $\underline{Y}_{q-i}$  is the conductivity of the  $i$ -th phase of SD ( $i=A, B, C$ );  $\overset{\circ}{U}_{S-i}$  is the voltage of the  $i$ -th phase from a power supply source ( $i=A, B, C$ );  $Y_i$  is the conductivity of power line (Fig. 3).

Fig. 4 demonstrates that the real parts of conductivities and, therefore, active SD powers will be the lowest at zero points.

In the next step we find SD phase powers:

$$S_{q-i} = \overset{\circ}{U}_i \overset{\circ}{U}_{S-i} \overset{\circ}{Y}_i - U_i^2 \left( \overset{\circ}{Y}_i + \overset{\circ}{Y}_i \right), \quad i = A, B, C. \quad (4)$$

If necessary, it is possible to find SD phase conductivities by the following ratio:

$$\underline{Y}_{q-i} = \frac{\overset{\circ}{U}_{S-i} - \overset{\circ}{U}_i}{U_i} \cdot \underline{Y}_i - \underline{Y}_i, \quad i = A, B, C \quad (5)$$

and then they are easy to convert to interphase conductivities, as well as to find interphase powers of SD.

The effectiveness of using a mathematical model of voltage symmetrization, proposed in [10], is proven on a particular example for a scheme of two-transformer substation (Fig. 5). In this case, in the course of voltage symmetrization in one node of electricity network, the voltage of reverse sequence is 30 % lower than the voltage obtained using other methods. In the course of voltage symmetrization in two nodes of electricity network, we obtained the reverse sequence voltages, which are about 10 % and 15 % lower than the voltages generated when using other methods.

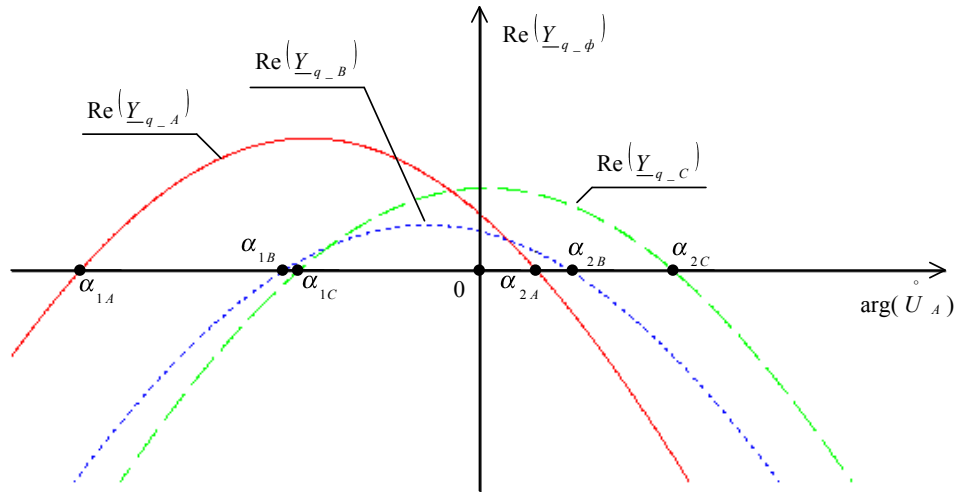


Fig. 4. Graph of dependencies of real parts of the SD conductivities on the argument of the A phase current

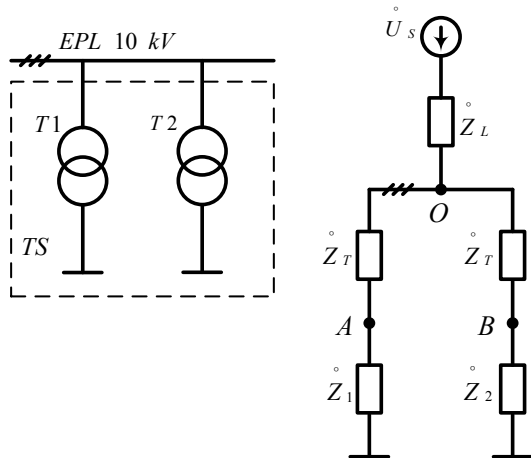


Fig. 5. Simplified one line scheme of connecting two-transformer substation to the 10 kV line (left) and equivalent circuit, recalculated for 0.4 kV (right)

Thus, functional dependencies between parameters of the model and results of the simulation have been synthesized. Since the given dependencies are implicitly defined functions of many variables, then for further solving of the inverse problem of sensitivity, it is necessary to write down these functions in some form. To achieve this goal, it is convenient to use the M programming language in the Matlab environment, which is exactly what was done.

We describe the algorithm for solving the system of equations (1) (Fig. 6).

At the initial stage of execution of the algorithm (Fig. 6), it is necessary to know deviation of the modelling result  $\partial F$ . As a result of execution of this algorithm, we calculate the upper and lower limits of insensitivity zone of parameter. In this case, it is necessary to set such values of these limits, which may be relevant for this case. Thus, for example, array of the upper limits of insensitivity zone of parameter may be set as follows:  $10^{-n}, 10^{-(n-1)}, \dots, 10^{-1}$ ,  $n$  is  $N$ . And the value of  $n$  affects which lowest integral number will start the cycle. The accuracy of determining the limits of insensitivity zone will depend on the base of members of this array. The same applies to the array of lower limits of insensitivity zone of the parameter.

In order to solve the system of equations (2), it is necessary that a solution algorithm of system (1) is repeated for  $n$  input parameters of the model  $n$  and for  $m$  modelling results.

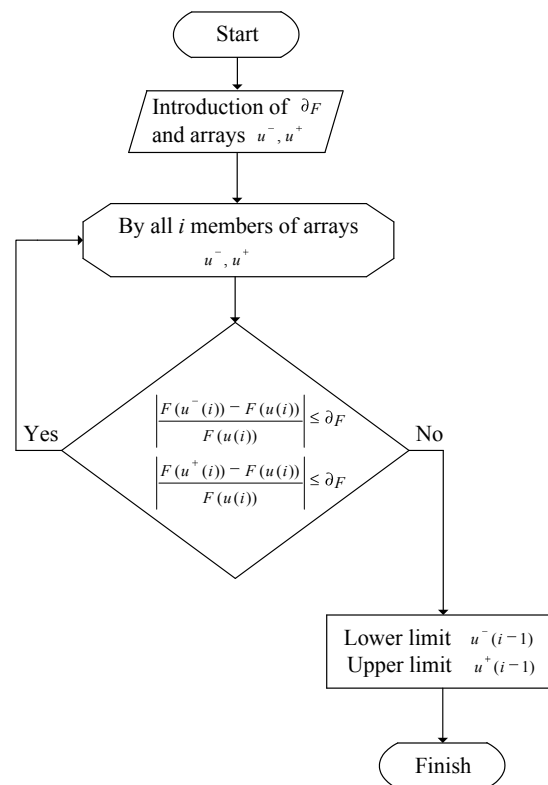


Fig. 6. Algorithm of solution of system (1)

For a mathematical model of symmetrization system, which is represented by ratios (3)–(5), the number of input parameters  $n=12$  (6 modules and 6 arguments of voltages and currents), and the number of results of simulation  $m=6$  (3 modules and 3 arguments of SD powers). For the electro technical calculations, length of the interval of insensitivity zone by the results of simulation (by modules and arguments of SD powers) is chosen as 2.5 %. Arrays of the upper and lower limits of insensitivity zone for parameters are chosen symmetric relative to the origin of coordinates and equal to  $10^{-9}, 10^{-8}, \dots, 10^{-1}$  and  $-10^{-9}, -10^{-8}, \dots, -10^{-1}$ , respectively. We also required the values of parameters of electric mode, based on which the simulation will be performed. In this work, their following values were taken:

- 1) module of the expected interphase voltage 10 kV;
- 2) power line impedance  $0,5+j\cdot 0,4$  Ohms;

3) phase voltages of the load prior to symmetrization  $5765,4 \cdot e^{j0}$  V;  $4941,3 \cdot e^{-j100^\circ}$  V;  $5289,5 \cdot e^{j150^\circ}$  V for phases A, B and C, respectively;

4) phase voltages of power source  $5773,5 \cdot e^{j0}$  V;  $4949,7 \cdot e^{-j100^\circ}$  V;  $5303,3 \cdot e^{j150^\circ}$  V for phases A, B and C, respectively;

5) currents in the load prior to symmetrization;  $18,4 \cdot e^{-j25^\circ}$  A;  $22,0 \cdot e^{-j107^\circ}$  A;  $30,4 \cdot e^{j110^\circ}$  A for phases A, B and C, respectively.

By executing the program, compiled on the basis of the obtained algorithm, for 72 times, we receive deviations in parameters not less than 0.5 %, that is, the intervals of insensitivity zone of the parameters of modeling were received not narrower than  $[-0,5 \%, 0,5 \%]$ .

For some parameters we obtained deviations that significantly exceed 0.5 %. This suggests that the result of simulation does not or weakly depends on the parameter, which is considered.

## 6. Discussion of results of solving the inverse problem of sensitivity of mathematical model for the system of voltage symmetrization

For the existing sensors of electric magnitudes and network analyzers, errors in measurement of the real magnitudes of voltages and currents do not exceed 0.2 %, for the shift angles between phase voltages and coefficients of powers – do not exceed 0.5 %. Therefore, we conclude that for practical realization of the system of voltage symmetrization based on the examined model, the existing sensors of electrical magnitudes are fully applicable.

The algorithm presented in Fig. 6, certainly, has such a drawback as iterativity. However, when using modern computing machines and with a proper choice of elements of arrays of parameters of insensitivity zones, it is possible to get a solution of any necessary accuracy.

By using the developed algorithm, it is possible to examine other mathematical models of the voltage and current symmetrization systems. However, these models should be also presented in the form of a “black box”, and functional dependency between the parameters and the results of modelling must be known.

## 7. Conclusions

1. Using the M-programming language of the Matlab environment, we recorded functional dependency between parameters and results of simulating a mathematical model for the system of voltage symmetrization. As a result, we received a possibility to analyze the resulting implicitly defined functional dependency, as well as to use it in the course of developing other programs in Matlab.

2. We developed algorithm for solving the inverse problem of sensitivity of mathematical model of the system of voltage symmetrization. As a result, the obtained intervals of insensitivity zone of parameters of the considered mathematical model of the system of voltage symmetrization are not narrower than  $[-0,5 \%, 0,5 \%]$ . The designed algorithm may also be used in the study of other mathematical models of voltages and currents symmetrization systems, set in the form of a “black box”.

3. The accuracy of sensors of electrical magnitudes was assessed by the found values of intervals of insensitivity zone of parameters of mathematical model of the system of voltage symmetrization. It is revealed that this accuracy must not exceed 0.5 %. Contemporary sensors of electric magnitudes and network analyzers, the errors of measurement of which for the real magnitudes of voltages and currents do not exceed 0.2 %, for the shift angles between phase voltages and coefficients of powers do not exceed 0.5 %, are fully applicable for practical implementation of the system of voltage symmetrization based on the examined model.

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