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# ANALYSIS OF INDICES OF RELIABILITY OF CASCADE THERMOELECTRIC COOLERS IN VARIOUS CURRENT MODES

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*Розроблена модель взаємозв'язку показників надійності і основних значущих параметрів двокаскадного термоелектричного охолоджувача, що працює у різних струмових режимах при послідовному електричному з'єднанні каскадів. Наведено порівняльний аналіз основних значущих параметрів і показників надійності охолоджувача для різних струмових режимів у широкому діапазоні перепадів температури від  $\Delta T=60$  К до  $\Delta T=90$  К з урахуванням теплового навантаження*

*Ключові слова: термоелектричні пристрої, показники надійності, режим роботи, перепад температури, теплове навантаження*

*Рассмотрена модель взаимосвязи показателей надежности и основных значимых параметров двухкаскадного термоэлектрического охладителя, работающего в различных токовых режимах при последовательном электрическом соединении каскадов. Проведен сравнительный анализ основных значимых параметров и показателей надежности охладителей для различных токовых режимов работы в широком диапазоне перепадов температуры от  $\Delta T=60$  К до  $\Delta T=90$  К с учетом тепловой нагрузки*

*Ключевые слова: термоэлектрические устройства, показатели надежности, режим работы, перепад температуры, тепловая нагрузка*

## 1. Introduction

Thermoelectric coolers relate to the most reliable elements of providing for thermal conditions of radio-electronic equipment due to their special features: absence of mobile components, small overall dimensions and mass, high climatic and mechanical indices, and low inertia. The indices of failure resistance of thermoelectric coolers to the essential degree depend on thermal load, especially under conditions when they exceed the limits of certified regimes, which is characteristic for contemporary equipment, which works under critical conditions. The indices of failure resistance of thermoelectric coolers in this case fall sharply; therefore, the design solutions, directed toward reduction of the failure rate of coolers are extremely relevant. The failure of the thermoelectric system of providing the thermal condition of the thermally loaded element leads not only to the failure of information system, but of the entire system as a whole, the consequences of which may be unpredictable [1].

## 2. Literature review and problem statement

The period of failure-free operation of contemporary thermoelectric coolers under the certified operating conditions exceeds 15 years [2, 3]. Increasingly strict requirements for the thermal loads and dynamic characteristics of coolers lead to a sharp decrease in indices of reliability [4], which is caused, first of all, by the thermal deformations of junction of the material of thermoelements with the base layer of the cooler [5]. Different coefficients of linear temperature expansions lead to the formation of the zones of thermoelements breakaway from the base layer and its cracking [6]. The adhesion of thermoelements connection with the base layer and, accordingly, the indices of reliability of a cooler are influenced by the form and dimensions of thermoelements [7]. Significant attention is paid to the development of thermoelectric materials, since, according to the index of thermoelectric efficiency, thermoelectric coolers are substantially inferior to compressor coolers [8, 9]. As the conducted studies showed, the efficiency of thermo-

electric material is directly connected to the indices of reliability: failure rate and probability of failure-free operation [10]. At the same time, the indices of thermoelectric efficiency of modern industrially manufactured materials of thermoelements have not actually increased for many years, and they are now in a rather narrow range [11].

Increased requirements for the operating conditions of thermoelectric coolers contribute to essential relevance of the problem of increasing the indices of reliability. This leads to the need for further search for the ways of decreasing the failure rate not only due to the use of new materials, but also to the optimization of the current modes of thermoelectric modules. In this case, one should consider that the similar problems for the cascade thermoelectric coolers were not sufficiently explored.

### 3. The aim and tasks of the study

The purpose of this work is the search for ways of increasing the indices of reliability of cascade thermoelectric coolers through the selection of current modes at the varied operating conditions without a change in their design parameters and material of thermoelements.

To achieve this aim, it is necessary to solve the following tasks:

- to develop a reliability-oriented model, which connects the indices of reliability of a cascade thermoelectric cooler with the design and energy indices for different operating modes;
- to carry out comparative analysis of the main design and energy parameters and indices of reliability of a cascade thermoelectric cooler in the range of its temperature functioning.

### 4. Development and analysis of reliability-oriented models for different current modes of a thermoelectric cooler

#### 4.1. Mathematical model of interrelation of indices of reliability and the design and energy indices of the cooler

The application of cascade thermoelectric devices (CTED) in the radio-electronic equipment (REE) is predetermined not only by providing a deeper level of cooling, but also by the increase in energy efficiency [12].

In the process of designing CTED, refrigerating capacity  $Q_0$ , the temperature of the heat-absorbing joint  $T_0$  and the temperature of the heat-releasing joint  $T$ , or ambient temperature  $T_c$ , are usually pre-set. In this case, in the course of designing CTED, different limitations are superimposed on:

- energy characteristics, namely, the required power, the value of operating current, voltage drop;
- mass-and-size characteristics, namely, mass and overall dimensions;
- indices of reliability (failure rate  $\lambda$  and the probability of failure-free operation  $P$ ).

It is necessary to determine design, technical and energy characteristics of CTED, that is, to select such current mode of operation, which would make it possible to consider all the assigned requirements (limitations) comprehensively.

While determining the indices of reliability of TED under operating conditions, namely, of the failure rate  $\lambda$  and the probability of failure-free operation  $P$ , we assume that [13]:

- all thermoelements are connected electrically in series, including cascades;
- failure of any thermoelement leads to the failure of a module and a device as a whole;
- events involving the failure of thermoelements are accepted as independent variables;
- each thermoelement in cascades works in the same current mode under different temperature conditions;
- thermoelectric modules and CTED on their basis relate to the unrestorable articles.

Thus, the indices of reliability of CTED are examined with the following assumptions:

- failures of thermoelements occur suddenly;
- the operating time to the failure is distributed exponentially.

The analysis of dependencies of indices of reliability on the time by the data of tests and operation for different distribution laws demonstrates that for describing the indices of reliability of CTED with the probability of failure-free operation above 0,95, it is justified to apply the law of exponential distribution as the simplest one for quantitative assessment of the indices of reliability [14].

In the course of designing thermoelectric systems of cooling and heat stabilization, the standardized series of thermoelectric modules are widely applied. With the unification of thermoelectric modules, the possibility of their cascading both with the sequential and with the parallel electrical connection of cascades is implied. In the majority of cases, the sequential electrical connection of cascades proves to be the most rational. In addition to the known advantages, this connection makes it possible to use more flexibly the possibility of implementation of one or another mode of cascade operation depending on initial conditions. While designing CTED, it is necessary to consider the possibility of their operation in a wide range of current modes: from  $Q_{0\max}$  to  $\lambda_{\min}$  modes. Therefore, we will further examine the following current modes of CTED operation as the most common [13]:

$Q_{0\max}$  is the mode that provides maximum refrigerating capacity of CTED;

$A_{\max}$  is the mode that provides maximum refrigerating capacity, refrigeration coefficient and voltage  $\left[ \frac{Q_0^2}{I} \right]_{\max}$  ;

$\left[ \sum_i n_i \right]_{\min}$  is the mode providing minimum quantity of thermoelements;

$\left[ \frac{Q_0}{I} \right]_{\max}$  is the mode providing maximum ratio of refrigerating capacity to the current;

$\left[ \frac{Q_0}{I^2} \right]_{\max}$  is the mode, close to mode  $E_{\max}$  ( $B_i = \Theta_i$ ), providing the highest energetic efficiency of CTED;

$\lambda_{\min}$  is the mode providing minimum failure rate and maximum probability of failure-free work [14].

In this work, we examined CTED with the sequential electrical connection of cascades, which are, as a rule, collected on the base of the standardized and identical branches of thermoelements or standard modules on their basis.

Therefore, the following condition must be met:

$$B_i I_{\max_i} = B_{i+1} I_{\max_{i+1}} = \dots = B_N I_{\max_N}, \tag{1}$$

where  $i=1,2,\dots,N$ ;  $B_i = I/I_{\max_i}$  is the relative operating current of the  $i$ -th cascade;  $I_{\max_i} = \frac{e_i T_{i-1}}{R_i}$  is the maximum operating current, A;  $e_i$  is the coefficient of thermo EDS of the branch of thermoelement of the  $i$ -th cascade, B/K;  $T_{i-1}$  is the temperature of heat-absorbing joint of the  $i$ -th cascade, K;  $R_i$  is the electrical resistance of the branch of thermoelement of the  $i$ -th, Ohm.

A temperature difference of CTED can be represented in the form:

$$\Delta T = \sum_{i=1}^N \Delta T_i = \sum_{i=1}^N \Delta T_{\max_i} \Theta_i, \tag{2}$$

where  $\Delta T_{\max_i} = 0.5Z_{i-1} T_{i-1}^2$  is the maximum temperature difference of the  $i$ -th cascade, K;  $Z_i$  is the averaged index of thermoelectrical efficiency of module, 1/K;  $\Theta_i = \frac{\Delta T_i}{\Delta T_{\max_i}}$  is the relative temperature difference of the  $i$ -th cascade, relative units;  $\Delta T_i = T_i - T_{i-1}$  is the operational temperature difference of the  $i$ -th cascade, K.

When constructing CTED, it is also necessary to observe conditions of stationarity of cooling processes, that is, to ensure the thermal mating of cascades, when the amount of heat, released from the previous cascade, must be equal to the refrigerating capacity of the following one.

Given the stationarity condition, the ratio of the number of thermoelements in the adjacent cascades can be written down as

$$\frac{n_{i+1}}{n_i} = \frac{I_{\max_i}^2 R_i}{I_{\max_{i+1}}^2 R_{i+1}} \frac{2B_i \left[ 1 + \frac{\Delta T_{\max_i}}{T_{i-1}} \Theta_i \right] + B_i^2 - \Theta_i}{2B_{i+1} - B_{i+1}^2 - \Theta_{i+1}}, \tag{3}$$

where  $n_i$  is the number of thermoelements in the  $i$ -th cascade, pieces.

Refrigerating capacity of CTED  $Q_0$  is determined «by cold» (further, the first cascade) and can be represented in the form

$$Q_0 = n_1 I_{\max_1}^2 R_1 (2B_1 - B_1^2 - \Theta_1). \tag{4}$$

The power consumption  $W_i$  of the  $i$ -th cascade of CTED can be represented in the form

$$W_i = 2n_i I_{\max_i}^2 R_i B_i \left( B_i + \frac{\Delta T_{\max_i}}{T_{i-1}} \Theta_i \right). \tag{5}$$

Refrigeration coefficient of CTED  $E^N$  can be written down in the form

$$E^N = \frac{Q_0}{\sum_{i=1}^N W_i}. \tag{6}$$

The total failure rate of CTED  $\lambda_\Sigma$  is possible to represent in the form of the total of the failure rates of each cascade:

$$\lambda_\Sigma^N = \sum_{i=1}^N \lambda_i. \tag{7}$$

Taking into account the influence of temperature conditions on the work of each cascade and the thermal load, it is possible to write down the ratio for determining the relative value of the total failure rate of two-stage TED:

$$\frac{\lambda_\Sigma^{N=2}}{\lambda_0} = \frac{n_1 B_1^2 (\Theta_1 + C_1) \left( B_1 + \frac{\Delta T_{\max 1}}{T_0} \right)^2}{\left( 1 + \frac{\Delta T_{\max 1}}{T_0} \Theta_1 \right)^2} \cdot K_{T_1} + \frac{n_2 B_2^2 (\Theta_2 + C_2) \left( B_2 + \frac{\Delta T_{\max 2}}{T_1} \Theta_2 \right)^2}{\left( 1 + \frac{\Delta T_{\max 2}}{T_1} \Theta_2 \right)^2} \cdot K_{T_2}, \tag{8}$$

where  $\lambda_0$  is the nominal failure rate, 1/hour;  $C_1 = \frac{Q_0}{n_1 I_{\max 1}^2 R_1}$  is the relative thermal load of the first cascade;  $C_2 = \frac{Q_0 + W_1}{n_2 I_{\max 2}^2 R_2}$  is the relative thermal load of the second cascade;  $K_{T_1}$ ,  $K_{T_2}$  are the coefficients of significance, considering the influence of decreased temperatures [17].

To assess the probability of failure-free operation  $P$  of CTED, it is possible to write down the ratio:

$$P = \exp \left[ - \sum_{i=1}^N \lambda_i t \right], \tag{9}$$

where  $t$  is the assigned resource, hour.

#### 4. 2. Analysis of the main design and energy parameters and indices of reliability

For convenience of the comparative analysis, in Table 1, we will present calculation data of the main parameters and indices of reliability of CTED at the following initial data:  $Q_0 = 0.5$  W;  $T = 300$  K;  $\Delta T = 60$  K, 70 K, 80 K, 90 K;  $t = 10^4$  h;  $l/S = 10$  and  $\lambda_0 = 3 \cdot 10^{-8}$  1/h.

Analysis of the calculation data, given in Table 1, and their graphic representation makes it possible to draw conclusions of general nature for all current modes of operation of CTED:

- the value of optimum intermediate temperature  $T_1$  (Fig. 1) practically coincides (difference less than 1 %) in the range of temperature differences from  $\Delta T = 60$  K to  $\Delta T = 90$  K and does not depend on the value of thermal load for different operating modes.

With an increase in the general temperature difference  $\Delta T$ :

- the ratio of the number of thermoelements in the adjacent cascades  $\left( \frac{n_2}{n_1} \right)$  increases (Fig. 2);
- the total number of thermoelements  $\left( \sum_i n_i \right)$  increases (Fig. 3);
- the refrigeration coefficient  $E^N$  decreases (Fig. 4);
- the value of operating current  $I$  increases;
- the relative value of failure rate  $\frac{\lambda_\Sigma}{n_1 \lambda_0}$  increases (Fig. 5),

and therefore, so does the value of the total failure rate  $\lambda_\Sigma$  (Fig. 6);

- the probability of failure-free operation  $P$  decreases (Fig. 7).

Table 1

Calculation data of main parameters and indices of reliability of two-stage TED for various temperature differences  $\Delta T$  and current modes of operation

Name of operating mode	$B_1$ , rel. units	$B_2$ , rel. units	$n_1$ , pcs	$n_2$ , pcs	I, A	$E^N$ , rel. units	$\frac{\lambda_{\Sigma}}{n_1 \lambda_0}$	$\lambda_{\Sigma} \cdot 10^8$ , 1/h	P	$\frac{n_2}{n_1}$ , rel. units	$U_{\Sigma}$ , B
$Q_{0max}$	1.0	0.96	4.3	15.5	4.63	0.049	4.1	54.0	0.9946	3.66	2.2
$A_{max}$	0.83	0.79	4.5	13.4	3.8	0.077	1.9	25.3	0.9975	2.97	1.7
$\left(\sum_i n_i\right)_{min}$	0.79	0.75	4.6	13.0	3.6	0.087	1.4	18.4	0.9982	2.82	1.6
$\left(\frac{Q_0}{I}\right)_{max}$	0.65	0.62	5.4	13.3	3.0	0.12	0.55	8.67	0.99913	2.47	1.4
$\left(\frac{Q_0}{I^2}\right)_{max}$	0.42	0.39	10.3	23.7	1.9	0.152	0.076	2.36	0.99976	2.31	1.74
$\lambda_{min}$	0.34	0.32	17.1	39.9	1.57	0.125	0.032	1.64	0.99984	2.33	2.56
$Q_{0max}$	1.0	0.95	6.0	23.5	4.55	0.033	4.6	83.3	0.9917	3.92	3.25
$A_{max}$	0.88	0.83	6.3	21.0	4.0	0.048	2.4	45.5	0.9955	3.37	2.62
$\left(\sum_i n_i\right)_{min}$	0.83	0.79	6.4	20.1	3.8	0.055	1.8	34.3	0.9965	3.16	2.40
$\left(\frac{Q_0}{I}\right)_{max}$	0.74	0.70	7.1	20.7	3.4	0.065	1.1	22.7	0.99977	2.90	2.38
$\left(\frac{Q_0}{I^2}\right)_{max}$	0.55	0.53	10.9	27.4	2.5	0.080	0.250	8.1	0.99919	2.52	2.62
$\lambda_{min}$	0.47	0.44	16.8	42.4	2.1	0.066	0.135	6.8	0.99932	2.53	4.00
$Q_{0max}$	1.0	0.92	10.0	43.3	4.4	0.019	4.35	142	0.9859	4.33	6.0
$A_{max}$	0.92	0.85	10.3	40.3	4.7	0.024	3.3	99.7	0.9900	3.9	5.0
$\left(\sum_i n_i\right)_{min}$	0.89	0.82	10.4	39.4	4.7	0.025	2.6	87.4	0.9913	3.8	4.9
$\left(\frac{Q_0}{I}\right)_{max}$	0.84	0.77	10.9	39.2	4.7	0.029	2.2	66.4	0.9934	3.6	4.6
$\left(\frac{Q_0}{I^2}\right)_{max}$	0.70	0.64	14.3	47.0	3.1	0.035	0.8	32.6	0.9967	3.3	5.0
$\lambda_{min}$	0.61	0.56	20.3	65.2	2.7	0.031	0.47	28.5	0.9972	3.22	5.9
$Q_{0max}$	1.0	0.92	30.8	161.4	4.25	0.0059	5.0	461.3	0.9549	5.25	20.0
$A_{max}$	0.97	0.89	31.0	157.2	4.15	0.0062	4.6	437.7	0.9572	5.07	19.4
$\left(\sum_i n_i\right)_{min}$	0.96	0.88	31.1	154.7	4.10	0.0065	4.2	390.6	0.9617	4.97	19.2
$\left(\frac{Q_0}{I}\right)_{max}$	0.94	0.87	31.8	154.9	4.0	0.0067	3.8	362	0.9644	4.87	18.5
$\left(\frac{Q_0}{I^2}\right)_{max}$	0.89	0.82	34.5	160	3.80	0.0072	2.9	303.6	0.9700	4.64	18.3
$\lambda_{min}$	0.83	0.76	41.8	186.6	3.50	0.007	2.1	265.5	0.9738	4.47	20.3

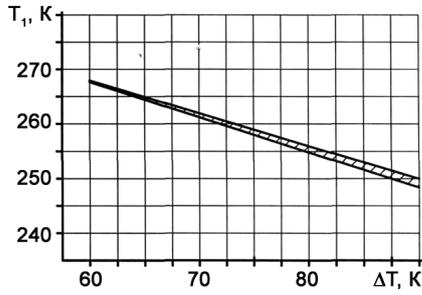


Fig. 1. Dependency of optimum intermediate temperature  $T_1$  on general temperature difference  $\Delta T$  for various current modes of operation of two-stage TED at  $T=300$  K

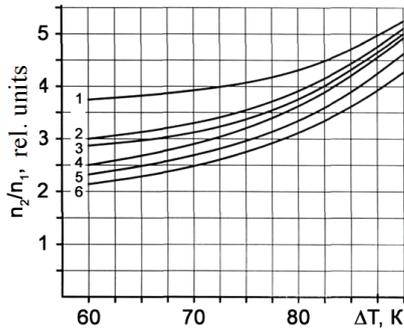


Fig. 2. Dependency of ratio of the number of thermoelements in adjacent cascades  $\frac{n_2}{n_1}$  of two-stage TED on the general temperature difference  $\Delta T$  for various current modes of operation with  $T=300$  K:

$$1 - Q_{0max}; 2 - A_{max}; 3 - (n_1+n_2)_{min}; 4 - \left(\frac{Q_0}{I}\right)_{max}; 5 - \left(\frac{Q_0}{I^2}\right)_{max}; 6 - \lambda_{min}$$

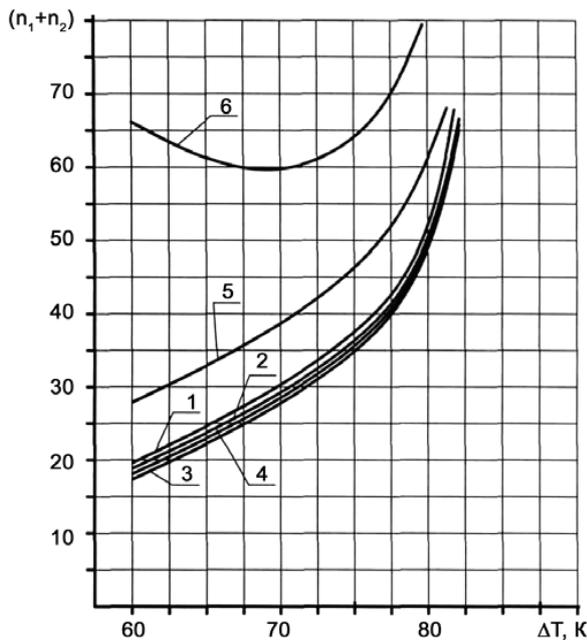


Fig. 3. Dependency of total number of thermoelements  $(n_1+n_2)$  of two-stage TED on the general temperature difference  $\Delta T$  for various current modes of operation with  $T=300$  K and  $Q_0=0.5$  W:

$$1 - Q_{0max}; 2 - A_{max}; 3 - (n_1+n_2)_{min}; 4 - \left(\frac{Q_0}{I}\right)_{max}; 5 - \left(\frac{Q_0}{I^2}\right)_{max}; 6 - \lambda_{min}$$

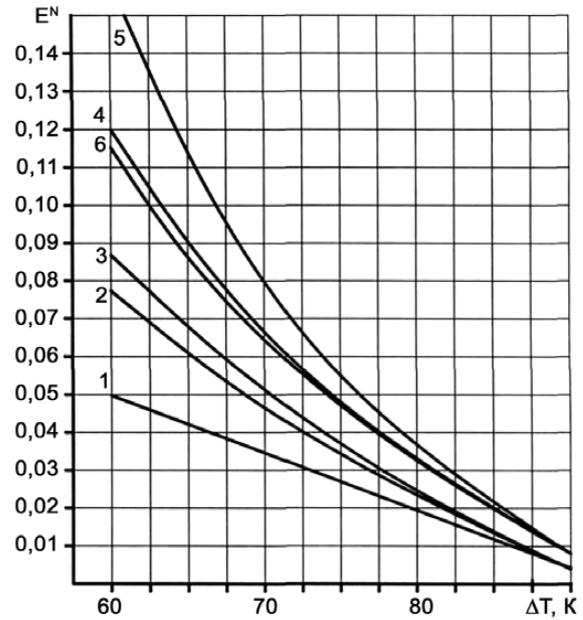


Fig. 4. Dependency of refrigeration coefficient  $E^N$  of two-stage TED on the general difference of temperature  $\Delta T$  for various current modes of operation with  $T=300$  K: 1 -  $Q_{0max}$ ; 2 -  $A_{max}$ ;

$$3 - (n_1+n_2)_{min}; 4 - \left(\frac{Q_0}{I}\right)_{max}; 5 - \left(\frac{Q_0}{I^2}\right)_{max}; 6 - \lambda_{min}$$

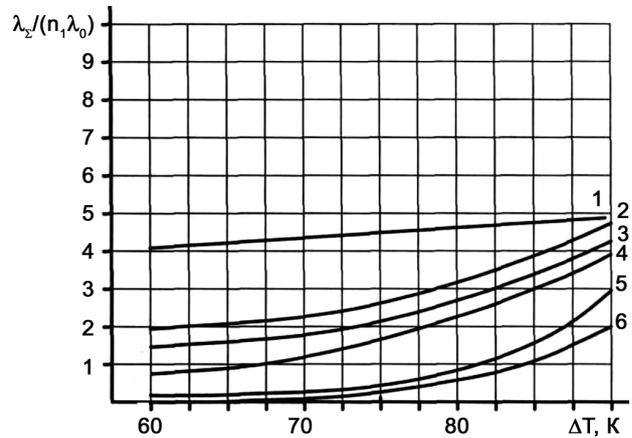


Fig. 5. Dependency of relative value of failure rate  $\frac{\lambda_{\Sigma}}{n_1\lambda_0}$  of two-stage TED on the general difference of temperature  $\Delta T$  for various current modes of operation with  $T=300$  K: 1 -  $Q_{0max}$ ; 2 -  $A_{max}$ ; 3 -  $(n_1+n_2)_{min}$ ; 4 -  $\left(\frac{Q_0}{I}\right)_{max}$ ; 5 -  $\left(\frac{Q_0}{I^2}\right)_{max}$ ; 6 -  $\lambda_{min}$

With the assigned temperature difference  $\Delta T$ :

- the maximum value of ratio of the number of elements of adjacent cascades  $\left(\frac{n_2}{n_1}\right)$  corresponds to mode  $Q_{0max}$ , and the minimum value – to mode  $\lambda_{min}$  (Fig. 2);
- the total number of thermoelements attains the highest values in mode  $\lambda_{min}$ , and the lowest – in the mode  $(n_1+n_2)_{min}$  (Fig. 3);
- the refrigeration coefficient attains maximum values in the mode  $\left(\frac{Q_0}{I^2}\right)_{max}$ , that is, in the mode, close to  $E_{max}$  (Fig. 4), and minimum values – in mode  $Q_{0max}$ ;

– the relative magnitude of failure rate  $\frac{\lambda_{\Sigma}}{n_1 \lambda_0}$  reaches maximum values in the mode  $Q_{0max}$  (Fig. 5), and minimum values – in the mode  $\lambda_{min}$ , and, therefore, the magnitude of the total failure rate  $\lambda_{\Sigma}$  (Fig. 6), decreases from the mode  $Q_{0max}$  to  $\lambda_{min}$ ;

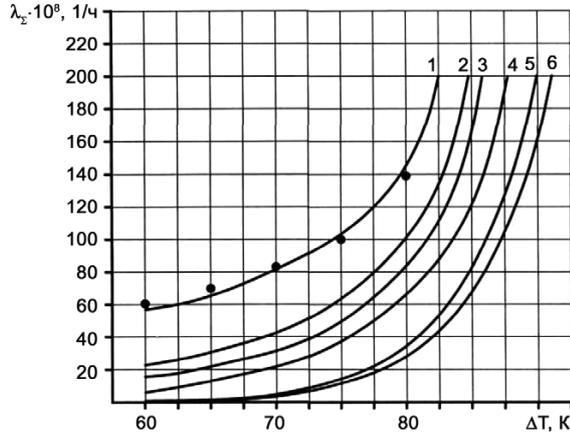


Fig. 6. Dependency of failure rate  $\lambda_{\Sigma}$  of two-stage TED on the general temperature difference  $\Delta T$  for various current modes of operation with  $T=300$  K and  $Q_0=0.5$  Bt: 1 –  $Q_{0max}$ ; 2 –  $A_{max}$ ; 3 –  $(n_1+n_2)_{min}$ ; 4 –  $\left(\frac{Q_0}{I}\right)_{max}$ ; 5 –  $\left(\frac{Q_0}{I^2}\right)_{max}$ ; 6 –  $\lambda_{min}$  (points – experimental data)

– the probability of failure-free operation  $P$  reaches maximum values in the mode  $\lambda_{min}$  and minimum values – in the mode  $\lambda_{min}$ , that is, decreases by the magnitude from the mode  $\lambda_{min}$  to  $Q_{0max}$  (Fig. 7);

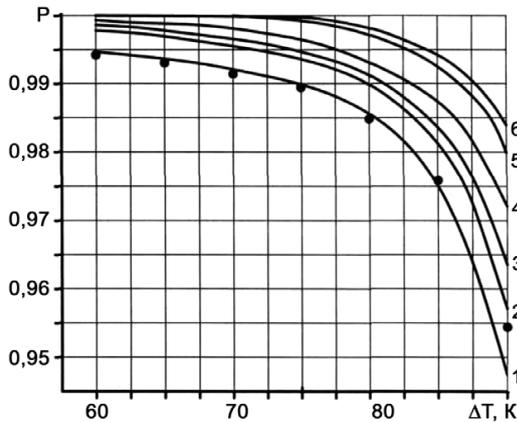


Fig. 7. Dependency of probability of failure-free operation  $P$  of two-stage TED on the general temperature difference  $\Delta T$  for various current modes with  $T=300$  K and  $Q_0=0.5$  W: 1 –  $Q_{0max}$ ; 2 –  $A_{max}$ ; 3 –  $(n_1+n_2)_{min}$ ; 4 –  $\left(\frac{Q_0}{I}\right)_{max}$ ; 5 –  $\left(\frac{Q_0}{I^2}\right)_{max}$ ; 6 –  $\lambda_{min}$  (points – experimental data)

– the magnitude of operating current decreases from the mode  $Q_{0max}$  to  $\lambda_{min}$ .

Now we will examine in more detail two extreme current modes of operation of CTED:

a) mode  $\left(\sum_i n_i\right)_{min}$ , providing minimum number of thermo-elements;

b) mode  $\lambda_{min}$ , providing minimum value of failure rate in comparison with other modes.

$$\text{Mode } \left(\sum_i n_i\right)_{min}$$

Fig. 8 displays dependency  $(n_1+n_2)=f(B_1)$  for various  $\Delta T$  and operating modes.

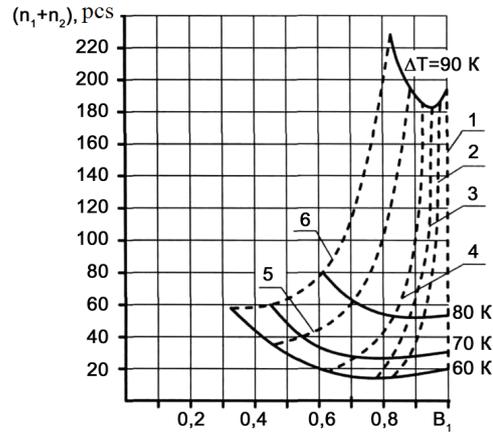


Fig. 8. Dependency of total number of thermoelements  $(n_1+n_2)$  of two-stage TED on relative operating current  $B_1$  for various temperature difference  $\Delta T=60$  K; 70 K; 90 K in various current modes of operation at  $T=300$  K and  $Q_0=0.5$  Bt: 1 –  $Q_{0max}$ ; 2 –  $A_{max}$ ; 3 –  $(n_1+n_2)_{min}$ ; 4 –  $\left(\frac{Q_0}{I}\right)_{max}$ ; 5 –  $\left(\frac{Q_0}{I^2}\right)_{max}$ ; 6 –  $\lambda_{min}$

The function  $(n_1+n_2)=f(B_1)$  has minimum for various  $\Delta T$ .

Fig. 9 presents dependency of the ratio  $\frac{(n_1+n_2)^B}{(n_1+n_2)_{min}}$  on the general temperature difference  $\Delta T$  for various operating modes. With the assigned temperature difference  $\Delta T$ , the relative changes in the total number of thermoelements in percentage in comparison with modes  $Q_{0max}$  and  $\left(\frac{Q_0}{I^2}\right)_{max}$  are given in Table 2.

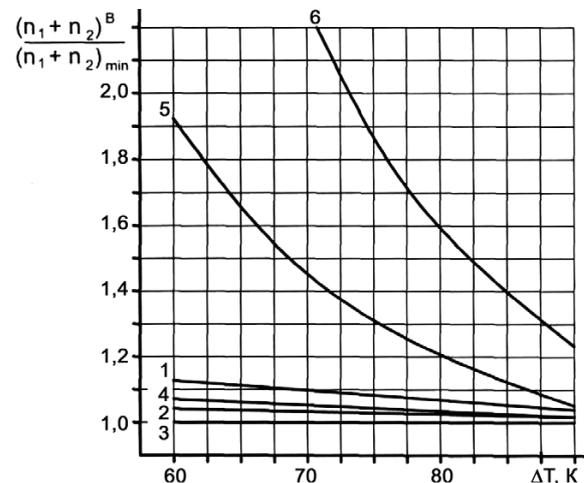


Fig. 9. Dependency of ratio of total number of thermoelements  $(n_1+n_2)$  for any current mode of operation  $B$  and number of thermoelements in mode  $(n_1+n_2)_{min}$  on the general temperature difference  $\Delta T$  at  $T=300$  K and  $Q_0=0.5$  Bt: 1 –  $Q_{0max}$ ; 2 –  $A_{max}$ ; 3 –  $(n_1+n_2)_{min}$ ; 4 –  $\left(\frac{Q_0}{I}\right)_{max}$ ; 5 –  $\left(\frac{Q_0}{I^2}\right)_{max}$ ; 6 –  $\lambda_{min}$

Table 2

Calculation data of relative magnitude of the total number of thermoelements for the assigned mode B to the mode  $(n_1+n_2)_{min}$  of two-stage TED for various temperature differences  $\Delta T$

No.	Mode name	$\frac{(n_1+n_2)^B - (n_1+n_2)_{min}}{(n_1+n_2)^B} \cdot 100\%$			
		$\Delta T=60\text{ K}$	$\Delta T=70\text{ K}$	$\Delta T=80\text{ K}$	$\Delta T=90\text{ K}$
1	$Q_{0max}$	13.0	12.0	7.0	3.0
2	$\left(\frac{Q_0}{I^2}\right)_{max}$	90.0	45.0	22.0	5.0

Fig. 9 vividly demonstrates that with an increase in the temperature difference, the magnitude of the total number of thermoelements decreases from 13 % to 3 % in comparison with mode  $Q_{0max}$  and from 90 % to 5 % with mode  $\left(\frac{Q_0}{I^2}\right)_{max}$ . A considerable decrease in the total number of thermoelements is observed at temperature fall  $\Delta T=60\text{ K}$  to  $70\text{ K}$ . At maximum temperature differences, the insignificant decrease in the total number of thermoelements is observed.

Fig. 10 presents dependency of ratio of the refrigeration coefficients  $E^B/E^{B=\Theta}$  in any current mode to mode  $\left(\frac{Q_0}{I^2}\right)_{max}$  ( $B=\Theta$ ) on the general temperature difference  $\Delta T$  at  $T=300\text{ K}$  and  $Q_0=0.5\text{ W}$ .

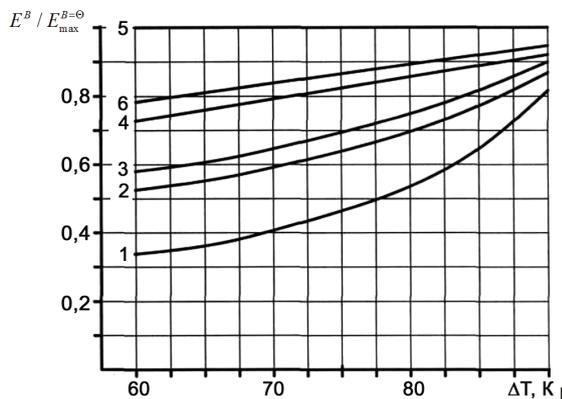


Fig. 10. Dependency of ratio of refrigeration coefficients  $E^B/E^{B=\Theta}$  in any current mode of operation B to the maximum value of refrigeration coefficient in mode  $\left(\frac{Q_0}{I^2}\right)_{max}$  ( $B=\Theta$ ) on the general temperature difference  $\Delta T$  of two-stage TED at  $T=300\text{ K}$  and  $Q_0=0.5\text{ W}$ : 1 –  $Q_{0max}$ ; 2 –  $A_{max}$ ; 3 –  $(n_1+n_2)_{min}$ ; 4 –  $\left(\frac{Q_0}{I^2}\right)_{max}$ ; 5 –  $\left(\frac{Q_0}{I^2}\right)_{max}$ ; 6 –  $\lambda_{min}$

With the assigned temperature difference  $\Delta T$ , the magnitude of a relative change in the refrigeration coefficient, expressed in percents, in comparison with mode  $\left(\frac{Q_0}{I^2}\right)_{max}$ , is given in Table 3.

With an increase in the temperature difference  $\Delta T$ , the magnitude of a relative change in the refrigeration coefficient  $E^B/E^{B=\Theta}$  decreases from 40 % to 10 %. A considerable decrease in the refrigeration coefficient occurs at  $\Delta T=60\text{ K}$  and  $\Delta T=70\text{ K}$ . At large temperature differences, the insignificant decrease in refrigeration coefficient is observed.

Table 3

Calculation data of relative change in refrigeration coefficient of mode  $\lambda_{min}$  to mode  $\left(\frac{Q_0}{I^2}\right)_{max}$  of two-stage TED for various temperature difference  $\Delta T$

Name of compared mode	$E^B/E^{B=\Theta}$			
	$\Delta T=60\text{ K}$	$\Delta T=70\text{ K}$	$\Delta T=80\text{ K}$	$\Delta T=90\text{ K}$
$\lambda_{min}$	40.0	35.0	25.0	10.0

Fig. 11 displays dependency of the ratio  $\left(\frac{n_2}{n_1}\right)^B / \left(\frac{n_2}{n_1}\right)_{min}$  on the general temperature difference  $\Delta T$  at  $T=300\text{ K}$ , where  $\left(\frac{n_2}{n_1}\right)^B$  is the ratio of a number of thermoelements in adjacent cascades for any operating mode;  $\left(\frac{n_2}{n_1}\right)_{min}$  is the minimum of the ratio of a number of thermoelements in adjacent cascades.

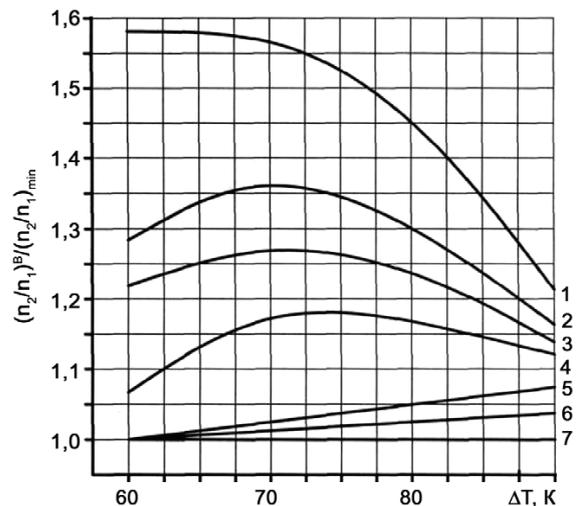


Fig. 11. Dependency of ratio  $\left(\frac{n_2}{n_1}\right)^B / \left(\frac{n_2}{n_1}\right)_{min}$  on the general temperature difference  $\Delta T$  of two-stage TED for various current modes of operation at  $T=300\text{ K}$ : 1 –  $Q_{0max}$ ; 2 –  $A_{max}$ ; 3 –  $(n_1+n_2)_{min}$ ; 4 –  $\left(\frac{Q_0}{I^2}\right)_{max}$ ; 5 –  $\left(\frac{Q_0}{I^2}\right)_{max}$ ; 6 –  $\lambda_{min}$ ; 7 –  $\left(\frac{n_2}{n_1}\right)_{min}$

Let us examine functional dependency  $\left(\frac{n_2}{n_1}\right) = f(B_1)$ , which has the minimum (Fig. 12) at the assigned value of temperature difference.

From the condition  $\frac{d\left(\frac{n_2}{n_1}\right)}{dB_1} = 0$  we will obtain expression for determining the optimum relative operating current  $B_{opt}$ , which corresponds to the minimum value of the function  $\frac{n_2}{n_1} = f(B_1)$ .

$$aB_1^2 - bB_1 + c = 0, \tag{20}$$

where

$$a = \frac{I_{\max 1}}{I_{\max 2}} \left[ 1 + \frac{I_{\max 1}}{I_{\max 2}} \left( 1 + \frac{\Delta T_{\max 1}}{T_0} \Theta_1 \right) \right];$$

$$b = \frac{\Delta T}{\Delta T_{\max 2}} + \Theta_1 \left( \frac{I_{\max 1}^2}{I_{\max 2}^2} - \frac{\Delta T_{\max 1}}{\Delta T_{\max 2}} \right);$$

$$c = \frac{I_{\max 1}}{I_{\max 2}} \Theta_1 - \left( \frac{\Delta T}{\Delta T_{\max 2}} - \frac{\Delta T_{\max 1}}{\Delta T_{\max 2}} \Theta_1 \right) \left( 1 + \frac{\Delta T_{\max 1}}{T_0} \Theta_1 \right).$$

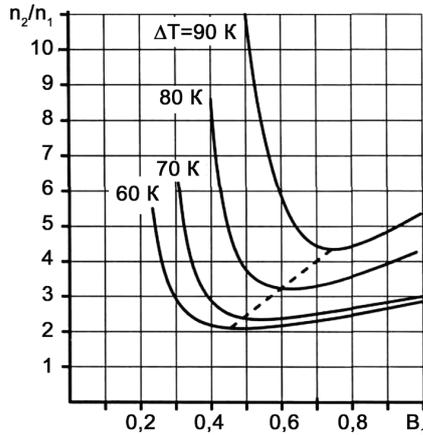


Fig. 12. Dependency of ratio  $n_2/n_1$  of two-stage TED on the relative operating current  $B_1$  for various temperature differences  $\Delta T=60$  K; 70 K; 90 K in various current modes of operation at  $T=300$  K:

1 –  $Q_{0\max}$ ; 2 –  $A_{\max}$ ; 3 –  $(n_1+n_2)_{\min}$ ; 4 –  $\left(\frac{Q_0}{I}\right)_{\max}$ ;  
 5 –  $\left(\frac{Q_0}{I^2}\right)_{\max}$ ; 6 –  $\lambda_{\min}$

The results of calculating the magnitude  $\left(\frac{n_2}{n_1}\right)^B / \left(\frac{n_2}{n_1}\right)_{\min}$  in the mode  $\left(\sum_i n_i\right)_{\min}$  for various temperature differences  $\Delta T$  in comparison with mode  $Q_{0\max}$  and  $\left(\frac{Q_0}{I^2}\right)_{\max}$  are given in Table 4.

Table 4

Calculation data of the magnitude of relative number of thermoelements in adjacent cascades of tw-stage TED for various temperature differences  $\Delta T$

No.	Name of compared mode	$\left(\frac{n_2}{n_1}\right)^B / \left(\frac{n_2}{n_1}\right)_{\min}, \%$			
		$\Delta T=60$ K	$\Delta T=70$ K	$\Delta T=80$ K	$\Delta T=90$ K
1	$\left(\sum_i n_i\right)_{\min}$	22.0	27.0	23.0	14.0
2	$Q_{0\max}$	58.0	57.0	45.0	21.0
3	$\left(\frac{Q_0}{I^2}\right)_{\max}$	1.0	2.0	5.0	7.0

It follows from Table 4 that in mode  $\left(\sum_i n_i\right)_{\min}$  the magnitude of ratio of a number of thermoelements in the adjacent cascades is 1,3÷1,1 times lower than in the mode  $Q_{0\max}$  and 1,3÷1,1 times larger than in the mode  $\left(\frac{Q_0}{I^2}\right)_{\max}$  depending on the magnitude of general temperature difference in the range of temperatures from  $\Delta T=60$  K to  $\Delta T=90$  K.

The relative value of the failure rate  $\frac{\lambda_{\Sigma}}{n_1 \lambda_0}$  (Fig. 13) is (67÷14) % lower in comparison with the mode  $Q_{0\max}$  and (80÷27) % higher in comparison with mode  $\left(\frac{Q_0}{I^2}\right)_{\max}$  in the range of temperature differences from  $\Delta T=60$  K to  $\Delta T=90$  K.

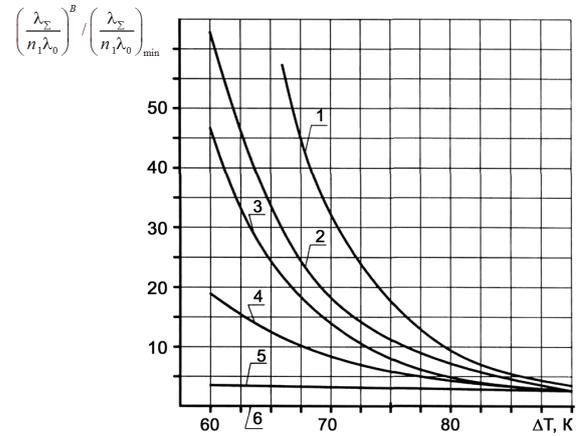


Fig. 13. Dependency of ratio of relative values of failure rate in mode B  $\left(\frac{\lambda_{\Sigma}}{n_1 \lambda_0}\right)^B$  and in mode  $\lambda_{\min} \left(\frac{\lambda_{\Sigma}}{n_1 \lambda_0}\right)_{\min}$  of two-stage TED on the general temperature difference  $\Delta T$  for various current modes of operation at  $T=300$  K and  $Q_0=0.5$  W: 1 –  $Q_{0\max}$ ; 2 –  $A_{\max}$ ; 3 –  $(n_1+n_2)_{\min}$ ; 4 –  $\left(\frac{Q_0}{I}\right)_{\max}$ ; 5 –  $\left(\frac{Q_0}{I^2}\right)_{\max}$ ; 6 –  $\lambda_{\min}$

Calculation data are presented in Table 5.

Table 5

Calculation data of relative value of failure rate of two-stage TED for various temperature differences  $\Delta T$

No.	Name of compared mode	$\left(\frac{\lambda_{\Sigma}}{n_1 \lambda_0}\right)^B / \left(\frac{\lambda_{\Sigma}}{n_1 \lambda_0}\right)_{\min}$			
		$\Delta T=60$ K	$\Delta T=70$ K	$\Delta T=80$ K	$\Delta T=90$ K
1	$Q_{0\max}$	67.0	57.0	41.0	14.0
2	$\left(\frac{Q_0}{I^2}\right)_{\max}$	80.0	80.0	67.0	27.0

Data of Table 5 vividly display the gains of the indices of TED reliability with the use of different current modes.

### 5. Discussion of analysis results

According to the results of theoretical studies of the model of interrelation of indices of reliability and main significant parameters of two-stage TED, we carried out comparative analysis of the main parameters for different current modes of operation in the range of temperatures from  $\Delta T=60$  K to  $\Delta T=90$  K at the assigned thermal load.

In mode  $\left(\sum_i n_i\right)_{\min}$  :  
 – the course of dependency of failure rate  $\lambda_{\Sigma}$  is identical  $\frac{\lambda_{\Sigma}}{n_1 \lambda_0}$  (Fig. 14);

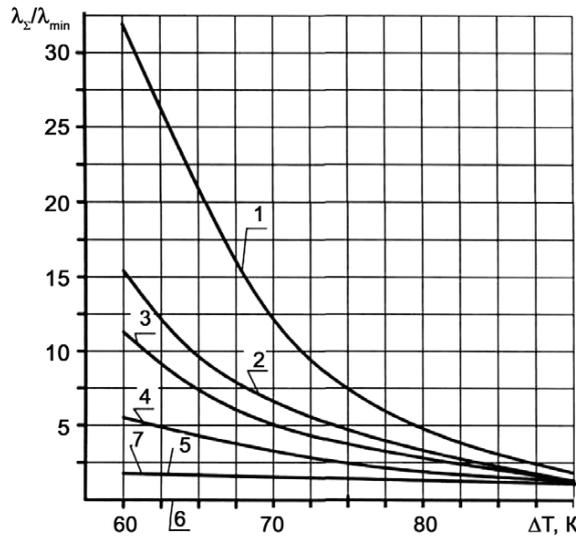


Fig. 14. Dependency of ratio of failure rate  $\lambda_{\Sigma}$  of two-stage TED to the failure rate  $\lambda_{min}$  on the general temperature difference  $\Delta T$  for different current modes of operation at  $T=300$  K and  $Q_0=0.5$  W:

- 1 –  $Q_{0max}$ ; 2 –  $A_{max}$ ; 3 –  $(n_1+n_2)_{min}$ ; 4 –  $\left(\frac{Q_0}{I}\right)_{max}$ ; 5 –  $\left(\frac{Q_0}{I^2}\right)_{max}$ ;
- 6 –  $\lambda_{min}$ ; 7 –  $\left(\frac{n_2}{n_1}\right)_{min}$

– the probability of failure-free operation  $P$  is higher than in the mode  $Q_{0max}$  and it is lower than in the mode  $\left(\frac{Q_0}{I^2}\right)_{max}$ ;

– the magnitude of operating current decreases by  $(22 \div 4)\%$  in comparison with the mode  $Q_{0max}$  depending on temperature difference from  $\Delta T=60$  K to  $\Delta T=90$  K.

Mode  $\lambda_{min}$ .

At the assigned temperature difference  $\Delta T$  in comparison with mode  $Q_{0max}$ :

- the number of thermoelements  $\sum_i n_i$  increases by  $(65 \div 16)\%$ ;
- the refrigeration coefficient  $E^N$  increases by  $(61 \div 16)\%$ ;
- the ratio of the number of elements  $\frac{n_2}{n_1}$  decreases by  $(36 \div 15)\%$ ;
- the value of operating current  $I$  decreases by  $(66 \div 18)\%$ ;
- the relative value of failure rate  $\frac{\lambda_{\Sigma}}{n_i \lambda_0}$  decreases by  $(67 \div 42)\%$ , failure rate  $\lambda_{\Sigma}$  decreases by  $(66 \div 18)\%$  and attains minimum values;

– the probability of failure-free work  $P$  increases and attains maximum values in the interval of temperature difference from  $\Delta T=60$  K to  $\Delta T=90$  K.

With the assigned temperature difference  $\Delta T$  in comparison with the mode  $Q_{0max}$ , close to mode  $E_{max}$ :

- the number of thermoelements  $\sum_i n_i$  increases by  $(40 \div 1)\%$ ;
- the refrigeration coefficient  $E^N$  decreases by  $(18 \div 3)\%$ ;
- the ratio of the number of thermoelements in adjacent cascades  $\frac{n_2}{n_1}$ ; remains stable;
- the value of operating current  $I$  decreases by  $(17 \div 8)\%$ ;
- the relative value of failure rate  $\frac{\lambda_{\Sigma}}{n_i \lambda_0}$  decreases by  $(58 \div 27)\%$ , the value of failure rate  $\lambda_{\Sigma}$  decreases by  $(30 \div 12)\%$  and attains minimum values;
- the probability of failure-free work  $P$  increases and attains maximum values in the interval of temperature difference from  $\Delta T=60$  K to  $\Delta T=90$  K.

The given results demonstrate the possibility of significant decrease in the failure rate at the substantiated selection of current regimes and decreasing the mass-and-size parameters of TED. Obtained results of the work may be used for designing two-stage TED of improved reliability.

This work continues the cycle of papers on the increase in the indices of reliability of one-stage and two-stage thermoelectric coolers and implies further development in the direction of increase in the cascade sequence of TED and consideration of the influence of external associated factors.

## 6. Conclusions

1. Authors proposed the model of interrelation of main indices of reliability with the design and energy indices of a thermoelectric cooler, which considers the influence of thermal load and current modes at various temperature differences, which provides the possibility for the optimization of the design parameters by the criterion of minimum of failures.

2. The reliability-oriented analysis, which made it possible to propose two characteristic current modes of operation, was carried out: the mode  $\left(\sum_i n_i\right)_{min}$ , providing a minimum quantity of thermoelements, that is, minimum mass and overall dimensions of a thermoelectric cooler; and the mode  $\lambda_{min}$ , providing the minimum failure rate and the maximum probability of failure-free operation. The proposed modes may be applied for designing thermoelectric devices of minimum mass and increased reliability.

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