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Розглянуто класичні та похідні критерії прийняття рішень в умовах повної невизначеності. Запропоновано трипараметричну математичну модель критерію Гурвіця, яка на відміну від класичної дозволяє аналітично врахувати число станів зовнішнього середовища (розмірність задачі), а також міру її впливу на переваги експерта при прийнятті рішень. Запропонований апарат може бути використано у процедурах групового або індивідуального експертного оцінювання ефективності управлінських рішень

Ключові слова: критерії прийняття рішень, альфа-критерій Гурвіця, невизначеність, експертне оцінювання

Рассмотрены классические и производные критерии принятия решения в условиях полной неопределённости. Предложена трёхпараметрическая математическая модель критерия Гурвица, которая в отличие от классической позволяет аналитически учитывать число состояний внешней среды (размерность задачи), а также степень её влияния на предпочтения эксперта при принятии решений. Предложенный аппарат может быть использован в процедурах группового или индивидуального экспертного оценивания эффективности управленческих решений

Ключевые слова: критерии принятия решения, альфа-критерий Гурвица, неопределённость, экспертное оценивание

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MODELLING THE EXPERT'S PREFERENCES IN DECISIONMAKING UNDER COMPLETE UNCERTAINTY

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1. Introduction

The process of managing a modern organization is characterized by a high degree of uncertainty of the external and internal environment, which entails the need to make informed management decisions based on all sorts of risks. This necessitates the need to facilitate management by using some modern information technology [1] and a decision support system (DSS). Despite the continuous growth of the accompanying organizational information management [2], a significant number of problems in management decisions can be reduced to the classical models of game theory, for

example, to the problem of choosing the optimal pure strategy in conditions of complete uncertainty. This task does not imply a unique solution by virtue of the main limitations, namely the total uncertainty of the external environment. Such circumstances most clearly reveal the problem of choosing a criterion to determine the best strategy. In the case of multiple expert assessments (individual or group) of the effectiveness of managerial decisions, it is important to choose not only a particular criterion but also the tools of its parameter setting for a particular problem to be solved. It determines the relevance of developing mathematical models to compare alternatives that analytically include factors

and performance manifestations, inherent in the process of decision-making.

2. Literature review and problem statement

The task of choosing an optimal pure strategy under a complete uncertainty is described in detail in the classical literature on game theory [3] and decision-making [4]. It involves the use of one of several well-known criteria, namely a function expressing preferences of an expert or a decision-maker, and defines the rules for choosing an acceptable or optimal solution.

In the classical case, the problem can be represented as a matrix whose rows mean appropriate solutions and whose columns are factors that can affect the result obtained by an expert. The intersection of the columns and rows contains gains or efficiencies e_{ij} that correspond to the implementation of the decisions E_i under appropriate conditions F_j (Table 1).

 $\label{eq:table 1} \mbox{Table 1} \\ \mbox{A matrix of solutions' efficiency } |e_{ii}|$

	F ₁	F_2	 F_{m}
E ₁	e ₁₁	e_{12}	 e _{1m}
E ₂	e ₂₁	e_{22}	 e _{2m}
En	e _{n1}	e_{n2}	 e_{nm}

The multitude E_0 of the best options consists of those options E_{i0} that belong to the set E of all the options and whose value of the criterion Z_{i0} is the highest among all the assessments Z_i :

$$E_{0} = \left\{ E_{i0} \mid E_{i0} \in E \land Z_{i0} = \max_{i} Z_{i} \right\}.$$
 (1)

The criteria are divided into classical and derivative [5] (Table 2).

There are various modifications of the above criteria, for example [6, 7], used to solve specific problems, but they do not alter the characteristics inherent in a problem at hand: none of the criteria can be considered a priority in relation to other existing ones. In fact, the uncertainty, inherent in external conditions, is converted by an expert into criteria of selection uncertainty. In this situation, there is an increasing relevance of the parametric adjustment methods to derived criteria as analytically more common.

The Hurwitz criterion for decision-making is also known as the criterion of "optimism-pessimism" [6], or "alpha-test" [8], which is due to the use of the pessimism factor $\alpha \in [0; 1]$ in the analytical form of the criterion:

$$\begin{split} Z_{HW} &= \max_{i} \left(\alpha \cdot \min_{j} \left(e_{ij} \right) + \left(1 - \alpha \right) \cdot \max_{j} \left(e_{ij} \right) \right), \\ \alpha \in [0; 1]. \end{split} \tag{2}$$

This ratio is usually interpreted as a quantitative "measure of pessimism" of an expert

when choosing a strategy. It is determined by the decision-maker's subjective considerations on the basis of statistical studies of decision-making results or the personal experience of acting in similar situations [9]. Choosing this factor correctly can be just as difficult as selecting the right criterion [10]. Therefore, most often, its value is chosen to be 0.5 [11].

In risky decisions under conditions of uncertainty, the Hurwitz criterion is used by those entities that want to identify the degree of their specific risk preferences as accurately as possible by setting the value of the alpha coefficient [12]. However, such identification requires either a priori formal requirements [13] or a multivariate mathematical model [14], allowing a connection between the factors of the decision-making process and its effectiveness.

Thus, in a situation of selecting the best strategy by the Hurwitz criterion under complete uncertainty of the external environment, there arises the problem of a parametric setting of the criterion. The guidelines for selecting the internal criterion parameter have practical value; ultimately, they are aimed at improving the efficiency of decision-making.

Table 2
Classical and derivative criteria for decision-making under conditions of complete uncertainty in the external environment

complete uncertainty in the external environment							
classical	maximin (Wald's)	$Z_{MM} = \max_{i} \left(\min_{j} \left(e_{ij} \right) \right)$					
	Bayes' and Laplace's	$Z_{BL} = \max_{i} \left(\sum_{j=1}^{m} e_{ij} q_{j} \right)$					
	Savage's	$Z_{s} = \min_{i} \left(\max_{j} \left(\max_{i} \left(e_{ij} \right) - e_{ij} \right) \right)$					
	advanced maximin	$Z_{ME} = \max_{p} \left(\min_{q} \left(\sum_{i=1}^{n} \sum_{j=1}^{m} e_{ij} p_{i} q_{j} \right) \right)$					
	gambler's	$Z_{AG} = \max_{i} \left(\max_{j} \left(e_{ij} \right) \right)$					
deriva- tive	Hurwitz's	$Z_{HW} = \max_{i} \left(\alpha \cdot \min_{j} \left(e_{ij} \right) + \left(1 - \alpha \right) \cdot \max_{j} \left(e_{ij} \right) \right), \ \alpha \in [0, 1]$					
	Hodges- Lehmann's	$Z_{HL} = \max_{i} \left(v \cdot \sum_{j=1}^{n} e_{ij} q_{j} + (1 - v) \cdot \min_{j} \left(e_{ij} \right) \right), \ v \in [0, 1]$					
	Germeier's	$Z_{G} = \max_{i} \left(\min_{j} \left(e_{ij} q_{j} \right) \right)$					
		$I_{_{1}} \coloneqq \left\{ i \middle i \left\{ 1,, m \right\} \& e_{_{i0j0}} - \min_{j} \left(e_{_{ij}} \right) \le \epsilon_{acpt} \right\}$					
	BL(MM)-test	$I_2 := \left\{ i \middle i \left\{ 1,, m \right\} \& \max_{j} \left(e_{ij} \right) - \max_{j} \left(e_{i0j} \right) \ge e_{i0j0} - \min_{j} \left(e_{ij} \right) \right\} \right.$					
		$\boldsymbol{Z}_{\mathrm{BL}(\mathrm{MM})} = \underset{l_i \cap l_2}{max} \Bigg(\sum_{j=1}^{m} \boldsymbol{e}_{ij} \boldsymbol{q}_{j} \Bigg)$					
	of products	$Z_{p} = \max_{i} \left(\prod_{j=1}^{m} e_{ij} q_{j} \right)$					

3. The purpose and objectives of the study

The aim of the study is to ensure the effectiveness of managerial decisions by widening the spectrum of mathematical models of the criteria for their selection.

The following tasks were set to achieve the research objectives:

- to analyze classical and derivative criteria of decision-making under conditions of complete uncertainty,
- to develop a mathematical model of the Hurwitz criterion based on the number of environmental conditions, and
 - to test the suggested model on a methodical example.

4. Devising a mathematical model of the Hurwitz criterion (based on the number of environmental conditions)

The essence of the internal parameter of the Hurwitz criterion is the ability to determine the weight of the maximin and maximax values of the strategy for evaluating its effectiveness.

Since the guidelines for choosing the internal parameter of the considered criterion as set out in the above-mentioned sources are general in nature, let us specify the features that are essential for the formation of a working hypothesis.

First, the extreme points of the acceptable range of the internal parameter eliminate the analytical form (1) or produce either a maximin criterion, also known as the Wald test (Abraham Wald), or a maximax criterion (a criterion of a gambler, of extreme optimism). This makes it possible to refer supporters of extreme pessimism and optimism to the users of relevant criteria – that is, to say that the majority of decision-makers using the Hurwitz criterion tend to choose the interior setting in the middle of the allowable range.

Second, the interpretation of the internal parameter as an "expert's attitude to risk" is not quite correct. The classic instrument of the expected utility theory, namely the construction of utility functions [15], is applicable in a situation where the probabilities of outcomes are known, i. e. in the problem of decision-making under risk. Actually, usefulness by the von Neumann-Morgenstern utility theorem is an alternative to the mathematical expectation of a gain as a decision criterion.

Another peculiarity of using the Hurwitz criterion is the absence of an analytical relationship between the value of the internal parameter and the dimension of the solutions' matrix. However, the uncertainty interpreted by the matrix of 2×2 and 20×20 has a different degree. This fact is reflected in the generalization of the Hurwitz criterion [6].

These peculiarities of using the Hurwitz criterion in the task of decision-making in uncertain external conditions allow us to formulate the following working hypotheses.

 H_1 : The preferences of an expert in choosing an internal parameter by the Hurwitz criterion are normally distributed on the interval $\alpha \in [0; 1]$.

H₂: The growth of external environment conditions asymptotically shifts the expert's preferences in a specific choice in the direction of pessimism.

The result of the first hypothesis is the possibility to determine the density of the probability α . Assuming that the interval $\alpha \in [0; 1]$ is equal to six times the standard deviation of the distribution ($\pm 3\sigma$), the analytical form of the density of the probability distribution will be as follows (Fig. 1):

$$p(\alpha) = 3\sqrt{-\cdot e^{-18(\alpha - \cdot)}}.$$
 (3)

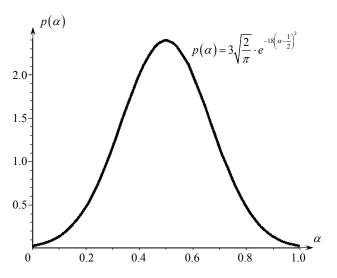


Fig. 1. The density of distributing the probability $\boldsymbol{\alpha}$

According to the hypothesis H_2 , let us find the form of a transformation that asymptotically approximates the top of the distribution (3) to the straight line α =1. The desired transformation can be represented as a piecewise linear function of the following form:

$$\alpha' = \begin{cases} k_1 \alpha, \ 0 \le \alpha \le \frac{1}{2}, \\ 1 - k_2 (1 - \alpha), \ \frac{1}{2} \le \alpha \le 1, \end{cases}$$
 (4)

where k_1 and k_2 are real coefficients of stretching the left pane p (α) and compressing the right pane p(α), respectively.

From the condition of the function continuity (4) at α =0.5, it follows that

$$k_1 = 2 - k_2.$$
 (5)

To ensure the asymptotic properties of the transformation (4), let us use the exponential dependence of the form:

$$k_1 = 2 - e^{-\mu(m-2)}, k_2 = e^{-\mu(m-2)},$$
 (6)

where the argument m>2 is the number of conditions of the external environment in the matrix of solutions (Table 1); μ is the measure of the impact of the problem scope (ambiguity or entropy) on the DM preferences.

Then the mathematical model is as follows:

$$\begin{split} Z_{\mathrm{HW}} &= \underset{i}{\mathrm{max}} \bigg(\alpha' \cdot \underset{j}{\mathrm{min}} \Big(e_{ij} \Big) + \Big(1 - \alpha' \Big) \cdot \underset{j}{\mathrm{max}} \Big(e_{ij} \Big) \bigg), \\ \alpha' &\in \big[0; \, 1 \big], \end{split} \tag{7}$$

$$\alpha' = \begin{cases} \alpha \left(2 - e^{-\mu(m-2)} \right), & 0 \le \alpha \le \frac{1}{2}, \\ 1 - \left(1 - \alpha \right) e^{-\mu(m-2)}, & \frac{1}{2} \le \alpha \le 1, \end{cases}$$
 (8)

$$p(\alpha) = 3\sqrt{\frac{2}{\pi}} \cdot e^{-18\left(\alpha - \frac{1}{2}\right)^2}.$$
 (9)

It defines an explicit expression for the Hurwitz criterion, depending on the parameters: α is the DMP tendency to pessimism, m is the number of conditions of the external environment (the dimension of the problem), μ is the measure of the impact of the problem scope on the decision-maker's preferences (determined empirically).

The graphical interpretation of the model (7)–(9) is shown in Fig. 2–4.

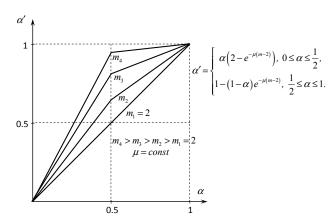


Fig. 2. The graphical interpretation of the transformation (4) in the form of sections of a piecewise linear function

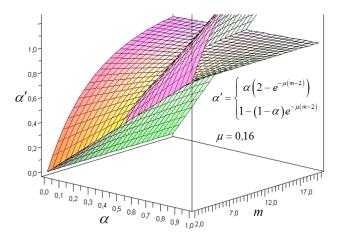


Fig. 3. The graphical interpretation of the transformation (4) in the space of the parameters α and m

Thus, we have suggested a three-parameter mathematical model of the Hurwitz criterion, which, in contrast to the classical model, makes analytical use of the number of conditions of the external environment (the dimension/scope of the problem) and considers the extent of its influence on the decision-maker's preferences in decision-making. Its essence can be summarized as follows: experts, when evaluating the attractiveness of strategies by the Hurwitz criterion at an increase in the number of conditions of the external environment, tend to shift their preferences towards pessimism, thus increasing the share of maximin in the final judgment.

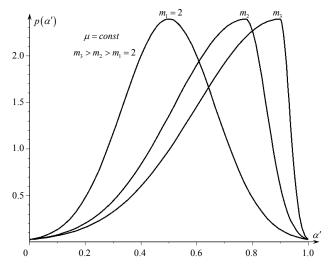


Fig. 4. The deformation density function, depending on the scope of the problem

5. The discussion and approval of the research results

As an example, let us consider a situation where an expert, who is inclined in decision-making neither to optimism nor to pessimism, i.e. is focused on a traditional use of the classical Hurwitz criterion with a parameter α =0.5, is faced with the need of a simultaneous evaluation of policies with different quantitative sets of conditions of the external environment. This situation occurs at the stage of formalizing the problem of decision-making with varying degrees of detail on environmental conditions (or a different degree of awareness about them). Information about the strategies is presented in the form of several classical matrices of solutions (Table 1) of various dimensions (Fig. 5).

Matrix of solutions 1 Matrix of solutions 2

	\mathbf{F}_1	F_2		\mathbf{F}_1	F_2	F_3	F_4
E_1	9	1	E_1	9	1	3	5
E_2	6	3	E_2	6	3	5	4

Fig. 5. The initial data of the problem

When using the classical Hurwitz criterion at α =0.5, equation (1) for the two matrices will produce the following:

$$Z_{HW1} = \frac{1}{2} \cdot 1 + \frac{1}{2} \cdot 9 = 5, \quad Z_{HW2} = \frac{1}{2} \cdot 6 + \frac{1}{2} \cdot 3 = 4.5.$$

Now, on the basis of the suggested model (7), let us take into account an increased dimension of the matrix when the Hurwitz criterion is set parametrically.

Suppose an expert assumes that the dimension m=10 means a 90 percent prevalence of maximin in the criterion, i. e. at m>10 the expert tends to play safe and reduce the Hurwitz criterion to the Wald criterion.

Then from any part of (7), it follows that

$$\mu = -0.1 \ln 0.2 \approx 0.16$$
.

Then for the second matrix

$$\alpha' = \frac{1}{2} \Big(2 - e^{-0.16(4-2)} \Big) = - \bigg(1 - \frac{1}{2} \bigg) e^{-0.16(4-2)} \approx 0.637.$$

Therefore, the values of the Hurwitz criterion for the strategies are:

$$Z_{HW_1} = 0.637 \cdot 1 + 0.363 \cdot 9 = 3.904,$$

$$Z_{HW2} = 0.637 \cdot 3 + 0.363 \cdot 6 = 4.089.$$

The example shows that, with an increasing dimension of the solutions' matrix, the best strategies by the Hurwitz criterion may stop being such because the analytical form of the criterion asymptotically approaches the maximin (Wald's).

Thus, the suggested three-parameter mathematical model of the Hurwitz criterion, in contrast to the classical analytic model, allows taking into account the number of conditions of the external environment (the dimension/scope of the problem) and the degree of its influence on the expert's preferences when making decisions.

It is easy to verify that a correction of the initial hypotheses H_1 and H_2 , for example, a change in the distribution law of the internal parameter, leads to a corresponding parametric correction of the suggested model without any essential change of its nature. Therefore, the advantages of the suggested model include the possibility to maintain the form of the parameters' interrelation, regardless of the correction of the initial hypotheses.

These features of the model suggest that the maximum benefit from its use can occur in a multiple group or individual expert assessment, when the calibration of subjective preferences of an expert is particularly important.

A disadvantage is the fact that some voluntarism in determining the internal parameters is present. However, in one form or another, it is inherent in all of the criteria of decision-making under conditions of complete uncertainty.

Further research may be directed to the generalization of the model, taking into account the shape of the distribution, i. e. a "mitigation" of the hypothesis H_1 .

6. Conclusions

- 1. The analysis of the criteria of decision-making under conditions of complete uncertainty shows that there are objective difficulties of parametric settings. Setting a criterion on the basis of an expert's subjective assessment of the extent of his/her own pessimism prevents from ensuring the reliability of managerial decisions under a multiple expert assessment, which makes it important to develop multifactor models of criteria.
- 2. We have suggested a three-parameter mathematical model of the Hurwitz criterion, which, in contrast to the classical model, makes analytical use of the number of conditions of the external environment (the dimension/scope of the problem) and considers the extent of its influence on the decision-maker's preferences in making decisions. The developed model helps choose the adjusting parameters of a criterion in accordance with objective quantitative measures of uncertainty, which increases its value for real working conditions of an expert.
- 3. The model's approbation on a methodological example illustrates certain differences between the suggested model and the classical type. The reliability of the model is confirmed by its complete extreme-case transformation into the known criteria, and its practical value is manifested in its simple algorithmic implementation.

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Запропонована методика побудови системи багатомоментних рівнянь дворівневого опису виробничої потокової лінії. Отримані незамкнені балансові рівняння. Розглянуто відомі моделі, в яких використані різні способи замикання системи рівнянь. Показані обмеження, що дозволяють отримати перехід до одномоментної РDЕ-моделі опису конвеєрної лінії і двухмоментної РDЕ-моделі з використанням рівняння Бюргерса

Ключові слова: PDE-модель, конвеєр, кінетичне рівняння, виробничий процес, багатомоментні рівняння, дворівневе опис, предмет праці, технологічні ресурси, фазовий простір, модель конвеєр

Предложена методика построения системы многомоментных балансовых уравнений для двухуровневого описания производственной поточной линии. Полученные незамкнуты балансовые уравнения. Рассмотрены известные модели, в которых использованы разные способы замыкания системы уравнений. Показаны ограничения и уравнения связей, позволяющие осуществить переход к одномоментной PDE-модели описания конвейерной линии и двухмоментной PDE-модели поточной линии с использованием уравнения Бюргерса

Ключевые слова: PDE-модель, поточная линия, кинетическое уравнение, производственный процесс, многомоментные уравнения, двухуровневое описание, предмет труда, технологические ресурсы, фазовое пространство, модель конвейер

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STATISTICAL VALIDITY AND DERIVATION OF BALANCE EQUATIONS FOR THE TWO-LEVEL MODEL OF A PRODUCTION LINE

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1. Introduction

The models containing partial differential equations (PDE models) for designing production lines have been developed in recent decades [1-7]. PDE models provide an opportunity to consider the impact of internal production factors and technological constraints of a production system. An essential advantage of this class of models is that they allow describing the motion of objects of labor from one operation to another, enable a closed form solution and do not require significant computational resources. The emergence of new types of models is due to trends of modern industrial production, the main of which is the trend to a continuous reduction of the product life cycle. This trend leads to the fact that, on the one hand, production lines operate for a considerable time in transient unsteady conditions, [3-7], on the other hand, the time for searching for the control mode of technological sites of the production line, therefore, is reduced [9]. This reduction has resulted in the need to develop fundamentally new types of models of production lines [3], as well as control programs and algorithms. Publications on the use of PDE models of production lines have appeared in 2003 [3, 5]. However, the issue of justification of a method of constructing closed equations that define the model requires further development and now determines the relevance of the chosen direction of research and its practical significance for modern flow production.

2. Literature review and problem statement

The papers [1, 3–8] deal with a new class of models of production systems with the flow production method, widely used at present for developing effective production management systems. The review of the publications on the most common PDE models of production lines is made. The factors that have given rise to a class of PDE models are shown. The history of their development is presented. The description of the PDE model, containing the Graves' equation; nonlinear Lighthill-Whitham PDE model; quasi-static PDE model using the nonlinear Karmarkar's equation of state; two-moment PDE model with the Burgers' equation;