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Запропонована методика побудови системи багатомоментних рівнянь дворівневого опису виробничої потокової лінії. Отримані незамкнені балансові рівняння. Розглянуто відомі моделі, в яких використані різні способи замикання системи рівнянь. Показані обмеження, що дозволяють отримати перехід до одномоментної PDE-моделі опису конвеєрної лінії і двухмоментної PDE-моделі з використанням рівняння Бюргерса

Ключові слова: PDE-модель, конвеєр, кінетичне рівняння, виробничий процес, багатомоментні рівняння, дворівневе опис, предмет праці, технологічні ресурси, фазовий простір, модель конвеєр

Предложена методика построения системы многомоментных балансовых уравнений для двухуровневого описания производственной поточной линии. Полученные незамкнуты балансовые уравнения. Рассмотрены известные модели, в которых использованы разные способы замыкания системы уравнений. Показаны ограничения и уравнения связей, позволяющие осуществить переход к одномоментной PDE-модели описания конвейерной линии и двухмоментной PDE-модели поточной линии с использованием уравнения Бюргерса

Ключевые слова: PDE-модель, поточная линия, кинетическое уравнение, производственный процесс, многомоментные уравнения, двухуровневое описание, предмет труда, технологические ресурсы, фазовое пространство, модель конвейер

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STATISTICAL VALIDITY AND DERIVATION OF BALANCE EQUATIONS FOR THE TWO-LEVEL MODEL OF A PRODUCTION LINE

O. Pihnastyi
Doctor of Technical Sciences, Associate Professor
Department of Computer Monitoring and Logistics
National Technical University "Kharkiv Polytechnic Institute"
Bagaliya str., 21, Kharkiv, Ukraine, 61002
E-mail: pom7@bk.ru

1. Introduction

The models containing partial differential equations (PDE models) for designing production lines have been developed in recent decades [1–7]. PDE models provide an opportunity to consider the impact of internal production factors and technological constraints of a production system. An essential advantage of this class of models is that they allow describing the motion of objects of labor from one operation to another, enable a closed form solution and do not require significant computational resources. The emergence of new types of models is due to trends of modern industrial production, the main of which is the trend to a continuous reduction of the product life cycle. This trend leads to the fact that, on the one hand, production lines operate for a considerable time in transient unsteady conditions, [3–7], on the other hand, the time for searching for the control mode of technological sites of the production line, therefore, is reduced [9]. This reduction has resulted in the need to develop fundamentally new types of models of production lines [3], as well as control programs and algorithms. Publications on the use of PDE models of production lines have appeared in 2003 [3, 5]. However, the issue of justification of a method of constructing closed equations that define the model requires further development and now determines the relevance of the chosen direction of research and its practical significance for modern flow production.

2. Literature review and problem statement

The papers [1, 3–8] deal with a new class of models of production systems with the flow production method, widely used at present for developing effective production management systems. The review of the publications on the most common PDE models of production lines is made. The factors that have given rise to a class of PDE models are shown. The history of their development is presented. The description of the PDE model, containing the Graves' equation; nonlinear Lighthill-Whitham PDE model; quasi-static PDE model using the nonlinear Karmarkar's equation of state; two-moment PDE model with the Burgers' equation;

diffusion PDE model is given. Special attention is paid to the closed multi-moment PDE model for transient unsteady conditions. Historically, construction of a new type of models of production lines involves the application of two approaches – phenomenological approach [6–8] and statistical approach [1, 3, 4]. The phenomenological approach has provided an opportunity to construct a number of models of production lines by complementing the transfer equation with the equation of state in the form of a clearing function. This has allowed to write down the equations of the PDE model of production lines for the most basic cases of operation. The validity of application was determined by comparative analysis of the results obtained using a discrete-event simulation model (DES model) and the PDE model under study. It is clearly shown that the PDE models constructed using the phenomenological approach are limited. The limitation is due to the following fact. All the main patterns of behavior of a production system are determined experimentally (phenomenologically). The description of the production phenomena abandons the extra details of the manufacturing process. Abandonment of strict description of patterns of behavior of the individual elements constituting a production system allows constructing a production system model with a small number of macroscopic quantities. Most often such macroscopic quantities are the rate of movement of objects of labor from one technological operation to another and the size of the operational reserves between them. The phenomenological model provides satisfactory accuracy when the production process is quasi-stationary. However, it is not suitable to describe transient production processes. The lifetime of these processes is constantly growing and has reached a half of the life cycle for many leading companies at present. The quasi-stationary conditions of production allowed determining phenomenological patterns between the key production parameters, while these patterns almost cannot be determined for transient processes owing to constant changes of external and internal production factors over time. Attempts to create pilot laboratories (simulating the production process on certain sites) at a manufacturing enterprise for predicting changes in phenomenological patterns between the key production parameters were unsuccessful. Researchers are forced to look for new approaches to constructing production system models with the required description accuracy.

The statistical approach [3], based on accounting the laws of interaction of objects of labor with manufacturing equipment and with each other during processing has been proposed for the construction of production system models. The papers [1, 3] consider the evaluation of the calculation accuracy of flow line parameters. Attention is given to the models of statistical dynamics of flow production control systems. Their relation to the class of PDE models is demonstrated. The focus is on the fact that the current methods of statistical dynamics of control systems provide a powerful apparatus that can be used to construct PDE models of control and stabilization systems of the production line parameters.

The review of publications presented in [1, 3, 4, 6, 9, 10] demonstrates that further development and use of PDE models of production lines requires the solution of the following issues:

1) derivation of non-stationary equations of state based on a detailed processing technology of the object of labor considering the arrangement of equipment;

2) construction of multi-moment closed balance equations for steady-state and transient non-stationary operation conditions of the production line;

3) construction of two-level control models of the production line parameters for steady-state and transient conditions considering the parameters and arrangement of equipment, and movement priorities of objects of labor.

3. Goals and objectives

The goal of the research is to develop a method of constructing multi-moment closed balance equations for steady-state and transient non-stationary operation conditions of the production line.

The following objective was set to achieve the goal: to construct and substantiate the system of equations of flow parameters for the production system model in the one-, two-, three-moment description of the production process followed by a generalization of the results for the models in a multi-moment description:

4. The kinetic equation of the production process

The production process state is determined by the states of the total number N of objects of labor [1, 4, 5]. Upon transition of the object of labor from one state to another, there is a transformation of resources (raw materials, human labor) into a finished product as a result of the targeted impact of equipment.

The state of the j-th object of labor in the phase space can be described by state parameters

$$\vec{S}_j=\left(S_{j,1},..,S_{j,\alpha},..,S_{jA}\right),\ \ \vec{\mu}_j=\left(\mu_{j,1},..,\mu_{j,\alpha},..,\mu_{jA}\right),$$

where $S_{j,\alpha}$ (UAH) is the cost of the transferred a-th technological resource or its part on the j-th object of labor, $\mu_{j,\alpha}$ (UAH/hour) is the transfer rate of the cost of the a-th resource on the j-th subject of labor, $0<j\le N,\ \ 0<\alpha\le A$ [4]. The state of the production process parameters at some point in time will be determined if the state parameters of objects of labor

$$\left(\vec{S}_1,\vec{\mu}_1....\vec{S}_N,\vec{\mu}_N\right)$$

and the objective function

$$J\left(t,\vec{S}_j,\vec{\mu}_j\right),$$

are determined, and at any other time point found from the equations of state of objects of labor [3, 4]. As the number of objects of labor N is much greater than unity, we use appropriately normalized distribution function $\chi(t,S,\mu)$ of the number N of objects of labor in the phase space (t,S,μ), that satisfies the kinetic equation of the production process instead of solving the system of N second-order equations [4]:

$$\frac{\partial\chi}{\partial t}+\frac{\partial\chi}{\partial S}\cdot\mu+\frac{\partial\chi}{\partial\mu}\cdot f=$$
$$=\lambda_{\text{Plant}}\cdot\left\{\int_0^\infty\left[\phi(t,S,\tilde{\mu},\mu)\cdot\tilde{\mu}\cdot\chi(t,S,\tilde{\mu})\right]\cdot d\tilde{\mu}-\mu\cdot\chi\right\},$$

$$\frac{d\mu}{dt}=f(t,S)\,, \quad (1)$$

where the product $\chi(t,S,\mu)\cdot d\Omega$ is the number of objects of labor in the cell $d\Omega$ of the phase space with coordinates $S_j\in[S,S+dS[$, $\mu_j\in[\mu,\mu+d\mu[$ (S_d – cost of products). The integration over the volume Ω of the phase space (S,μ) gives the total number N of objects of labor in progress [1, 8–10]:

$$\int_0^{S_d}\int_0^{\infty}\chi(t,S,\mu)d\mu dS=N,$$

$$\Omega=\int_0^{S_d}\int_0^{\infty}d\mu dS. \quad (2)$$

The function f(t, S) determines the law of changes in the state of the object of labor for the regulatory manufacturing process. It is based on the data on the use of technological resources when performing the production operation. The stochastic process of the impact of equipment on the object of labor is described by the distribution density $\phi(t,S,\tilde{\mu},\mu)$ of the random variable μ, where $\tilde{\mu}$ and μ are the transfer rate of resources on the object of labor before and after the impact [1, 3]:

$$\int_0^{\infty}\phi(t,S,\tilde{\mu},\mu)d\mu=1. \quad (3)$$

A detailed derivation of the kinetic equation of the production process (1) has been given in [1, 4].

Production process macroparameters. Let us introduce the numerical characteristics that reflect the essential features of the distribution of objects of labor in progress over states

$$\int_0^{\infty}\mu^k\cdot\chi(t,S,\mu)d\mu=[\chi]_k\,, \quad (4)$$

which we define as the k-th order moments for the distribution function $\chi(t,S,\mu)$. The variation of the distribution function $\chi(t,S,\mu)$ of objects of labor over states is due to the stochastic nature of interaction of objects of labor with equipment and each other [7]. In most interesting cases from a practical point of view, the distribution density $\phi(t,S,\tilde{\mu},\mu)$ does not depend on the state of objects of labor until testing the impact of the manufacturing equipment. Then, integration in the right part of (1) leads to simplification of the integral-differential equation:

$$\frac{\partial\chi}{\partial t}+\frac{\partial\chi}{\partial S}\cdot\mu+\frac{\partial\chi}{\partial\mu}\cdot f=\lambda_{Plant}\cdot\left\{\phi(t,S,\mu)\cdot[\chi]_1-\mu\cdot\chi\right\},$$

$$\frac{d\mu}{dt}=f(t,S). \quad (5)$$

The kinetic equation (5) is used to derive balance equations of the PDE model of the production process. Its solution makes it possible to obtain the law of distribution of the objects of labor over states. The law of distribution exhaustively describes the distribution of objects of labor over states, allows determining numerical characteristics (4). Among them, two numerical characteristics – distribution density of objects of labor in progress over manufacturing positions $[\chi]_0(t,S)$ and the rate of processing of objects of labor $[\chi]_1(t,S)$ in operations along the flow route are of particular importance [3–5, 7]. Often it is required to solve the problem, leaving aside the laws of distribution, operating with the numerical characteristics only $[\chi]_0(t,S)$, $[\chi]_1(t,S)$. The flow parameters $[\chi]_0(t,S)$, $[\chi]_1(t,S)$, $[\chi]_2(t,S)$ and the related method of moments play an important role in the construction of the general theory of production line control systems. If you managed to identify the characteristics of the state parameters of objects of labor, the flow parameters that describe the production process state are determined by the moments of the distribution function of objects of labor over states $\chi=\chi(t,S,\mu)$. In this case, the parameters introduced shall match the production process parameters used [2]. The k-th order balance equation with respect to the moments of the distribution function $\chi=\chi(t,S,\mu)$ of objects of labor over states means the balance equation aggregated over the entire variation range of the value μ

$$\int_0^{\infty}\mu^k\frac{\partial\chi}{\partial t}d\mu+\int_0^{\infty}\mu^{k+1}\frac{\partial\chi}{\partial S}d\mu+\int_0^{\infty}\mu^k\frac{\partial\chi}{\partial\mu}fd\mu=$$

$$=\lambda_{Plant}\int_0^{\infty}\mu^k\left\{\phi(t,S,\tilde{\mu},\mu)[\chi]_1-\mu\chi\right\}d\mu, \quad (6)$$

$$\chi(t,S,\infty)=\chi(t,S,\infty)=0. \quad (7)$$

The conditions (7) indicate that the number of objects of labor in the state with an infinitely small and infinitely large processing time is equal to zero. The initial moments $[\chi]_k$ are connected by the balance relations (6). The connection of the micro- and macro-level of description of the production process under a given distribution law of objects of labor over states is performed through the kinetic equation in the form of a self-consistent problem [1]. To determine the distribution function of objects of labor over states, it is necessary to know the behavior of its first moments that define the form of the function f (t, S). On the other hand, to determine the values of the first moments, you need to get the form of the distribution function $\chi(t,S,\mu)$ by solving the kinetic equation (5). The law of distribution of objects of labor over states is determined by the manufacturing process features by solving the kinetic equation (5), namely, the production technology of the object of labor. Certain forms of the engineering-production function f (t, S) and the transfer function of technological resources on the object of labor $\phi(t,S,\tilde{\mu},\mu)$. correspond to each manufacturing process.

5. The balance equation for the zero moment. The law of conservation of the number of objects of labor in the production process

Let us integrate the terms of the balance equation (6) when k=0:

$$\int_0^{\infty}\frac{\partial\chi}{\partial t}d\mu+\int_0^{\infty}\mu\frac{\partial\chi}{\partial S}d\mu+\int_0^{\infty}\frac{\partial\chi}{\partial\mu}f(t,S)d\mu=$$

$$=\lambda_{Plant}\int_0^{\infty}\left\{\int_0^{\infty}\phi(t,S,\tilde{\mu},\mu)\tilde{\mu}\cdot\chi(t,S,\tilde{\mu})d\tilde{\mu}-\mu\chi(t,S,\mu)\right\}d\mu. \quad (8)$$

Using the notations for the initial moments (4), we obtain

$$\int_0^\infty \frac{\partial\chi(t,S,\mu)}{\partial t}d\mu = \frac{\partial}{\partial t}\int_0^\infty \chi(t,S,\mu)d\mu = \frac{\partial[\chi]_0}{\partial t}, \qquad (9)$$

$$\int_0^\infty \frac{\partial\chi(t,S,\mu)}{\partial S}\cdot\mu d\mu = \frac{\partial}{\partial S}\int_0^\infty \mu\cdot\chi(t,S,\mu)d\mu = \frac{\partial[\chi]_1}{\partial S}, \qquad (10)$$

$$\int_0^\infty f(t,S)\cdot\frac{\partial\chi(t,S,\mu)}{\partial\mu}d\mu = f(t,S)\cdot\int_0^\infty d\chi(t,S,\mu) = 0, \qquad (11)$$

$$\int_0^\infty\left\{\int_0^\infty[\phi(t,S,\tilde{\mu},\mu)\tilde{\mu}\chi(t,S,\tilde{\mu})]d\tilde{\mu} - \mu\chi(t,S,\mu)\right\}d\mu =$$

$$= \int_0^\infty \tilde{\mu}\chi(t,S,\tilde{\mu})d\tilde{\mu} - \int_0^\infty \mu\chi(t,S,\mu)d\mu = 0. \qquad (12)$$

By substituting (9)–(12) into (8), we obtain the equation of the production line model in the one-moment description [4–6, 8].

$$\frac{\partial[\chi]_0}{\partial t} + \frac{\partial[\chi]_1}{\partial S} = 0, \qquad (13)$$

representing the law of conservation of the number of objects of labor in the production process. The equation (13), used to simulate the behavior of the state of the production line parameters in the one-moment description is not closed. There are a number of models using different approaches to close the equation (13). Let us consider one of them in detail. The approximate PDE model, containing the equation (13) and the closing Graves equation of state $[\chi]_1 = \alpha\cdot[\chi]_0\cdot c$, where a=const is the technological constant, c=const (m/h) is the assembly line speed is used for description of assembly lines:

$$\frac{\partial[\chi]_0(t,S)}{\partial t} + \frac{\partial[\chi]_1(t,S)}{\partial S} = 0,$$

$$[\chi]_1 = \alpha\cdot[\chi]_0\cdot c. \qquad (14)$$

By substituting the second equation into the first, we obtain

$$\frac{\partial[\chi]_0}{\partial t} + g\frac{\partial[\chi]_0(t,S)}{\partial S} = 0,$$

$$a = \alpha\cdot c = 0. \qquad (15)$$

Let us supplement (15) with initial and boundary conditions

$$[\chi]_0(0,S) = \theta(S),$$

$$[\chi]_1(t,0) = \varphi(t), \qquad (16)$$

the form of which determines the initial distribution of objects of labor along the production line and the inflow rate of objects of labor, defined by the order book at the beginning of the production line, where

$$[\chi]_0(0,0) = \theta(0) = [\chi]_1(0,0)/a = \varphi(0)/a.$$

Let us write down the characteristic system of equations and the corresponding first integral of movement for (15):

$$\frac{dt}{1} = \frac{dS}{a}, \quad S - gt = \text{const}. \qquad (17)$$

In view of (17), the solution of the equation (15) has the form

$$[\chi]_0(t,S) = W(R), \quad R = S - at. \qquad (18)$$

By substituting the solution (18) into the equation (15), we obtain the identity

$$\frac{dW(R)}{dR}\frac{\partial R}{\partial t} + a\frac{dW(R)}{dR}\frac{\partial R}{\partial S} =$$

$$= \frac{dW(R)}{dR}\left(\frac{\partial R}{\partial t} - a\frac{\partial R}{\partial S}\right) = \frac{dW(R)}{dR}(a - a\cdot 1) \equiv 0. \qquad (19)$$

Using the initial condition (16) and the boundary condition (16) for the equation (15), we write down the solution

$$[\chi]_0(t,S) = \theta(R)H(R) + \varphi(R/a)H(-R),$$

$$H(R) = \begin{cases} 0, & \text{if } R < 0; \\ 0.5, & \text{if } R = 0; \\ 1, & \text{if } R > 0, \end{cases} \qquad (20)$$

where H(R) is the Heaviside function. In general, the model shall be supplemented with the equations of constraints that impose limitations on the equipment performance and the use of technological resources.

6. The balance equation for the first moment. The law of conservation of the rate of movement of objects of labor on the flow route

Let us integrate the terms of the balance equation (6) when k=1:

$$\int_0^\infty \mu\frac{\partial\chi}{\partial t}d\mu + \int_0^\infty \mu^2\frac{\partial\chi}{\partial S}d\mu + \int_0^\infty \mu\frac{\partial\chi}{\partial\mu}f(t,S)d\mu =$$

$$= \lambda_{Plant}\int_0^\infty\left\{\int_0^\infty \phi(t,S,\tilde{\mu},\mu)\tilde{\mu}\chi(t,S,\tilde{\mu})d\tilde{\mu} - \mu\chi\right\}\mu d\mu. \qquad (21)$$

Using the notations for the initial moments (4), we obtain by analogy with (9)–(12)

$$\int_0^\infty \mu\frac{\partial\chi(t,S,\mu)}{\partial t}d\mu = \frac{\partial[\chi]_1}{\partial t},$$

$$\int_0^\infty \frac{\partial\chi(t,S,\mu)}{\partial S}\mu^2 d\mu = \frac{\partial[\chi]_2}{\partial S}, \qquad (22)$$

$$\int_0^\infty f(t,S)\frac{\partial\chi(t,S,\mu)}{\partial\mu}\mu d\mu =$$

$$= f(t,S)\int_0^\infty\left(\frac{\partial[\mu\chi(t,S,\mu)]}{\partial\mu} - \chi(t,S,\mu)\right)d\mu = -f(t,S)[\chi]_0, \qquad (23)$$

$$\int_0^\infty \left\{ \int_0^\infty \left[\phi(t,S,\tilde{\mu},\mu)\tilde{\mu}\cdot\chi(t,S,\tilde{\mu}) \right] d\tilde{\mu} - \mu\chi(t,S,\mu) \right\} \mu d\mu =$$

$$= \left(\frac{[\chi]_{1\psi}}{[\chi]_0}[\chi]_1 - [\chi]_2 \right). \quad (24)$$

By substituting (22)–(24) into (21), we write down the first-order balance equation with respect to the initial moments (6)

$$\frac{\partial[\chi]_1}{\partial t} + \frac{\partial[\chi]_2}{\partial S} - f(t,S)[\chi]_0 =$$

$$= \lambda_{\text{Plant}} \left(\frac{[\chi]_{1\psi}}{[\chi]_0}[\chi]_1 - [\chi]_2 \right). \quad (25)$$

By supplementing the equation (25) by the equation (13), we obtain a system of equations of the two-moment description of the production line:

$$\frac{\partial[\chi]_0}{\partial t} + \frac{\partial[\chi]_1}{\partial S} = 0, \quad \frac{\partial[\chi]_1}{\partial t} + \frac{\partial[\chi]_2}{\partial S} - f(t,S)[\chi]_0 =$$

$$= \lambda_{\text{Plant}} \left(\frac{[\chi]_{1\psi}}{[\chi]_0}[\chi]_1 - [\chi]_2 \right). \quad (26)$$

The system of equations (26) is not closed. By supplementing it with the equations

$$\frac{[\chi]_{1\psi}}{[\chi]_0}[\chi]_1 - [\chi]_2 = 0, \; [\chi]_{1\psi} = [\chi]_1, \; f(t,S) = 0, \quad (27)$$

we obtain a system of equations to describe the production line, known as two-moment PDE model using the Burgers' equation:

$$\frac{\partial[\chi]_0}{\partial t} + \frac{\partial[\chi]_1}{\partial S} = 0, \; \frac{\partial[\chi]_1}{\partial t} + [\chi]_1 \frac{\partial[\chi]_1}{\partial S} = 0. \quad (28)$$

The system of equations (28) is closed. We will not dwell on the study of this model in the present work.

7. The balance equation for the second moment. The general system of balance equations for the flow parameters

Let us integrate the terms of the balance equation (6) when k=2:

$$\frac{\partial[\chi]_0}{\partial t} + \frac{\partial[\chi]_1}{\partial S} = 0,$$

$$\frac{\partial[\chi]_1}{\partial t} + \frac{\partial[\chi]_2}{\partial S} - f(t,S)[\chi]_0 = \lambda_{\text{Plant}} \left(\frac{[\chi]_{1\psi}}{[\chi]_0}[\chi]_1 - [\chi]_2 \right). (29)$$

Using the notations for the initial moments (4), we obtain by analogy with (9)–(12)

$$\int_0^\infty \mu^2 \frac{\partial\chi(t,S,\mu)}{\partial t} d\mu = \frac{\partial[\chi]_2}{\partial t}, \; \int_0^\infty \frac{\partial\chi(t,S,\mu)}{\partial S}\mu^3 d\mu = \frac{\partial[\chi]_3}{\partial S}, \; (30)$$

$$\int_0^\infty f(t,S) \frac{\partial\chi(t,S,\mu)}{\partial\mu}\mu^2 d\mu =$$

$$= f(t,S)\int_0^\infty \left(\frac{\partial[\mu\cdot\chi(t,S,\mu)]}{\partial\mu} - 2\mu\chi(t,S,\mu) \right) d\mu =$$

$$= -2f(t,S)[\chi]_1, \quad (31)$$

$$\int_0^\infty \left\{ \int_0^\infty \left[\phi(t,S,\tilde{\mu},\mu)\tilde{\mu}\chi(t,S,\tilde{\mu}) \right] d\tilde{\mu} - \mu\chi(t,S,\mu) \right\} \mu^2 d\mu =$$

$$= \left(\left(\frac{[\chi]_{1\psi}}{[\chi]_0} \right)^2 \left(1 + \frac{\sigma_\psi^{\ 2}}{\langle\mu_\psi\rangle^2} \right)[\chi]_1 - [\chi]_3 \right), \quad (32)$$

By substituting (30)–(32) into (29), we write down the balance equation

$$\frac{\partial[\chi]_2}{\partial t} + \frac{\partial[\chi]_3}{\partial S} - 2f[\chi]_1 =$$

$$= \lambda_{\text{Plant}} \left(\left(\frac{[\chi]_{1\psi}}{[\chi]_0} \right)^2 \left(1 + \frac{\sigma_\psi^{\ 2}}{\langle\mu_\psi\rangle^2} \right)[\chi]_1 - [\chi]_3 \right). \quad (33)$$

By combining (13), (25), (33), we write down the general system of equations for the flow parameters of the production process:

$$\frac{\partial[\chi]_0}{\partial t} + \frac{\partial[\chi]_1}{\partial S} = 0, \quad \frac{\partial[\chi]_1}{\partial t} + \frac{\partial[\chi]_2}{\partial S} - f(t,S)[\chi]_0 =$$

$$= \lambda_{\text{Plant}} \left(\frac{[\chi]_{1\psi}}{[\chi]_0}[\chi]_1 - [\chi]_2 \right),$$

$$\frac{\partial[\chi]_k}{\partial t} + \frac{\partial[\chi]_{k+1}}{\partial S} + kf[\chi]_{k-1} =$$

$$= \int_0^\infty \mu^k \lambda_{\text{Plant}} \left\{ \int_0^\infty \left[\phi(t,S,\tilde{\mu},\mu)\tilde{\mu}\chi(t,S,\tilde{\mu}) \right] d\tilde{\mu} - \mu\chi \right\}.$$

Note that the production line models in the three-moment description are not given in the literature.

8. Discussion of the results of constructing a system of equations for the multi-moment model of the production-line

The research justified the use of the kinetic equation in the development of a method for constructing a system of equations for the multi-moment model of the production line, which is based on the statistical approach to the description of the production system. The basic observed macroscopic flow quantities, determining the state of the production line are formalized. The use of the integral-differential kinetic equation of the production process, considering the processing of objects of labor as they move on the flow route has allowed obtaining the balance equations for the two-level description of the production line. This opportunity to construct a closed system of equations is based on the methods of closure of the self-linking chain of balance equations by the small-parameter methods or by setting the equations of state for higher-order moments. It should be noted that the equation of state in the form of a clearing function for the balance equation closure with respect to the zero moment is widely used in foreign literature. In this regard, the limitations related to this method of constructing a closed system of equations are examined in detail. Another important factor is that the description of production lines does not involve the use of the initial moments $[\chi]_k$ (4) higher than the second due to both the complexity of construction of high-order balance equations using the phenomenological approach and definition of conditions for their closure. Due to this fact, the proposed construction method is of practical interest.

9. Conclusions

The statistical approach based on accounting the laws of interaction of objects of labor with manufacturing equipment and with each other during processing is used when constructing a system of equations for the flow parameters of the production line.

The method of constructing multi-moment balance equations, allowing, unlike the known methods based on the phenomenological approach, to write down the system of equations containing the required amount of flow parameters is given. The results are a continuation of the research carried out in [1, 3, 4] and are of scientific and practical interest to the design of control systems for modern production lines operating in transient conditions.

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