Досліджено процес транспортування вантажів у мережевому представленні з метою удосконалення існуючих методів визначення оптимальних характеристик транспортних мереж. Встановлено вплив показників структури мережі, напряму руху і пропускної здатності транспортних комунікацій на визначення фактичної щільності руху на транспортній мережі в статичному стані. Визначено механізм перетворення мережевих моделей процесу вантажних перевезень у матричні моделі, які задаються у вигляді орієнтованих графів і допускають перевезення вантажу через проміжні транспортні вузли

Ключові слова: транспортна мережа, максимальний транспортний потік, найкоротші шляхи, матрична модель

Исследован процесс транспортировки грузов в сетевом представлении с целью усовершенствования существующих методов определения оптимальных характеристик транспортных сетей. Установлено влияние показателей структуры сети, направления движения и пропускной способности транспортных коммуникаций на определение фактической плотности движения на транспортной сети в статическом состоянии. Определен механизм преобразования сетевых моделей процесса грузовых перевозок в матричные модели, которые задаются в виде ориентированных графов и допускают перевозки груза через промежуточные транспортные узлы

Ключевые слова: транспортная сеть, максимальный транспортный поток, кратчайшие пути, матричная модель

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IMPROVEMENT OF THE METHODS FOR DETERMINING OPTIMAL CHARACTERISTICS OF TRANSPORTATION NETWORKS

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1. Introduction

A strategic objective of scientific-technical policy in the field of transportation system of the state is achieving the world level in terms of technical parameters and services quality that are implemented in transport. In this connection, the top priority for the transport sector is to expand scientific research into creation of progressive technologies for the rational organization of cargo transportations, formation and functioning of efficient transportation system, development of fundamentally new management systems using modern information technologies [1, 2].

At present, Ukraine is beneficially different from other countries by the fact that a significant number of its cities are located along traditional transportation and communication routes of the Eurasian continent. The issue of the development of international transport corridors by Ukraine will accelerate not only achieving the strategic goals of integration into the European Community, but also solving such tasks as additional investments into development of the transportation infrastructure of the state, as well as increasing volumes of products for export [1].

Transport in Ukraine is a powerful communication system, which includes all its types (water, road, railway, pipeline, air). The main production funds of transport constitute about 20 % of the production funds of the country [1]. Creating united international transport-logistic system, geographical position of the transportation space of Ukraine, as well as existence of many international transport corridors require the following [1, 2]: separate analysis of transport hubs management; provision of coordination and interaction of all kinds of transport; implementation of modern achievements in scientific and technical progress in the transportation operation.

Designing efficient delivery of cargos with the alignment of all the links of the transportation process necessitated a large number of theoretical and experimental studies on various issues of development of transport systems [1, 2].

Relevance of the research is determined by the need to improve efficiency of the transportation of goods in international traffic through the development and implementation of models, methods and software for the rational organization of international freight traffic.

2. Literature review and problem statement

Many scientific papers in the field of transportation systems, logistics and operations studies address the solution of problems to increase efficiency of cargo transportation in international traffic. The main characteristics of the transport networks include: maximum flow in the transportation network and the shortest distances in the transport network. To solve the problem of optimization of the transportation network, it is necessary to reduce a network representation of the transport problem to the matrix form, for which there is practical mathematical apparatus. An analysis of the literature data that we conducted revealed the following.

The existing methods for solving the problem of maximum flow in the transport network are convenient to use only for a flat network [3]. A new presented algorithm for the maximum flow allows the optimization of solution to the problem, but it does not take into account the peculiarities of transport networks [4]. To solve the problem, it is necessary to extend the method for solving the problems on the optimization of transport networks with and without restrictions of the throughput capacity.

The algorithms of mathematical programming for designing a transport network are developed, which allow finding the optimal ways [5]. But such algorithms do not take into account the large number of intermediate points in the transportation network. The proposed characteristics of transport in the multiplex system enable the optimization, but do not allow the calculation of the shortest distances in the case of a large number of intermediate points [6].

The transportation problem in the matrix and network forms is presented by definition in equivalents [7]. However, sometimes it is more convenient to solve a network problem in matrix form [8]. But we need to improve these methods to solve complex network transportation problems using directed graphs in the Excel environment.

In general, the problem of effective control over the international freight transportation process is in the fact that the existing methods do not fully take into account specific features of their fulfillment and, consequently, there is no a unified approach to determining the methods for the determination of optimal characteristics of transportation networks.

3. The aim and tasks of the study

The aim of the study is to improve the methods for determining the optimal characteristics of transportation networks.

To achieve the set aim, the following tasks were solved:

– improvement of the methods for solving the problems on maximum flow in a transport network;

– improvement of the methods for finding the shortest distances in a transport network;

– improvement of the methods for reducing the network representation of the transport problem to the matrix form.

4. Improvement of the methods for solving the problems on maximum flow and the shortest distances in a transportation network

4. 1. Solving the problems on maximum flow

The problem of maximum flow can be formulated as follows: two nodes are connected by a transportation network (TN). Each TN arc is assigned with a number that denotes its throughput capacity in units of transportation vehicles (TV) in a time unit. It is necessary to find the maximum flow that can pass the network from one node, called *source*, to another, called *runoff*. In practice, this problem appears when it is necessary to as quickly as possible for the maximum number of vehicles to pass between any nodes of TN, such as in case of natural disaster, seasonal fluctuations in demand for the transportation of passengers (cargos), etc. A throughput capacity may be full or specially selected for the given transportation only.

The easiest way to solve this problem is under condition that the network is flat, that is, when any two of its vertices can be connected by arc or link, without crossing other links at that [3, 4].

The problem of maximum flow is conveniently solved by the method of trees [5]. The solution can be extended for the problem with multiple sources and runoffs. This will solve the problems on the optimization of transport networks with and without restrictions in throughput capacity.

For this purpose, it is sufficient to build a fake source and connect it by links with the nodes of dispatch. The throughput capacity of these links will be the magnitude of possible dispatch of TV from each node. Similar actions can be performed at the nodes of arrival.

4. 2. Solving the shortest path problem

Since the numbers assigned to the TN links may indicate distance, cost or time, it is equally easy to find the smallest distance, cost or time from one vertex to all others. Solving the transport problem in TN without limitations of throughput capacity, when production is concentrated in one point while consumption – in all others, we find the shortest path [5]. Potentials in this case determine length of the path from the vertex with the potential equal to zero. There were proposed several algorithms for solving the problems on the shortest path. For the solution of the problem, it is not necessary to choose the least total of the potential of the original vertex and evaluate all cost links, and the potentials are assigned to the vertices of network that are considered successively. With such a network, one can create a tree of decisions. A condition of optimality may be violated at certain links that do not belong to the tree. Repeated considerations of these vertices eliminate these violations, the tree is corrected, as well as the potentials of the vertices. Despite the need for repeated considerations of the network, solving this problem is easier using computational technology. This algorithm is the easiest way to compile albums of the shortest paths by criteria of distance (in the first place), cost or time.

While solving the problem of finding the shortest path, in addition to the value of the shortest distance from the selected vertex to all others, we receive the shortest route, namely, a list of nodes that it passes through. In this case, one can use the effect of imposition of flows on the networks. Possessing a matrix of correspondences of freight traffic from each vertex to all others, we build a tree of the shortest paths. Returning from each point of unloading by the shortest route, we summarize flows on the arcs of the network. Passing from a vertex to another vertex, we obtain density of traffic in the network without limitation of throughput capacity. This technique may be used to determine the actual density of traffic in the network in the static state.

When the network has a throughput capacity limitation, imposing flows on the network is a bit complicated. In this case, it is necessary to subtract each elementary flow from the existing throughput capacity of the arc, on which it is imposed. Once a throughput capacity of the arc is filled, it is removed from the network. New trees of the shortest paths are built and the imposition is assigned to another tree, etc., The plan built in this way is not optimal, but if there are no many arcs with the limitations of throughput capacity, then the potentials of the vertices one can correct manually.

4. 3. Improvement of the methods for reducing network representation of the transport problem to the matrix form

The transport problem in the matrix and network forms of representation are equivalent by definition. However, sometimes it is more convenient to solve the network problem in the matrix form. There are two main ways to reduce a network problem to the matrix form [6, 7].

We propose to solve the network transport problems in the Excel environment. A directed graph is called a network, where the following are determined:

– node-source that has only the output arcs (denoted by letter s from "source");

– node-runoff that has only the input arcs (denoted by letter t, from "terminal" – final destination);

– all other nodes – intermediate (transit), interconnected by arcs, which include the input and output arcs.

Directed arcs in the network are marked with arrows, non-directed arc is replaced with two arrows facing each other. Arc with arrow and a certain value of the appropriate parameter specifies universal concept – flow that moves from the initial node of the arc to the final node. The objects of flows in practical problems are the cargos, gas, passengers, vehicles, communication signals, fluids, etc.

Most of the optimizing problems in networks are the problems on flows in the networks (*network flow problems*) [7, 8]. For the network optimization problems, a fundamental principle is the *principle of maintaining the flow* at any node, particularly, the total of flows Fex(x) at the node output is equal to the total of flows at its input Fent(x) + potential p(x) of node (+ proposal/–demand), for example:

– node-source s: Fex(s =0+p(s)=P, where P is the magnitude of total flow along the network; potential p(s)=+P;

– node-runoff t: Fex(t)=P+p(t)=0 because potential p(t)=–P;

– intermediate node x: Fex(x)=Fent(x)±p(x).

A flow in each node of the network is function that satisfies linear equations and inequalities, where each arc (x_i, x_j) of the network is in line with one or more positive numbers. For example, magnitude $d(x_i, x_j)$ in the problem on maximum flow is the throughput capacity of the arc (maximum amount of product that can be delivered with node x_i to node x_j along this arc per unit of time); in the transport problem, this is the distance or the cost of transportation. Hence the magnitude of flow along arc (x_i, x_j) does not exceed throughput capacity of this arc $d(x_i, x_j)$ if it is set.

The purpose of the study is the reduction of network representation of the transport problem to the matrix form that will allow us in future to solve the problems of cargo transportation optimization. Fig. 1 displays TN without limitation for the throughput capacity, Fig. 2 presents TN with limitations for the throughput capacity.

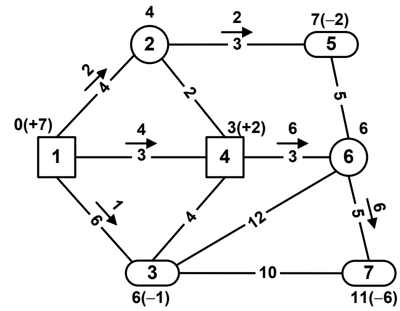

Fig. 1. Example of TN without limitation in the throughput capacity

Fig. 1, 2 display networks with 7 vertices and 11 links. Next to the corresponding vertex in parentheses is the number with a plus sign that indicates the volume of production while the volume of consumption is, respectively, denoted by the number with a minus sign. The cost of cargo transportation is written down in each arc, where the denominator of fraction demonstrates throughput capacity of separate links in the network. Fig. 1, 2 presents distributions of cargo flows and potentials.

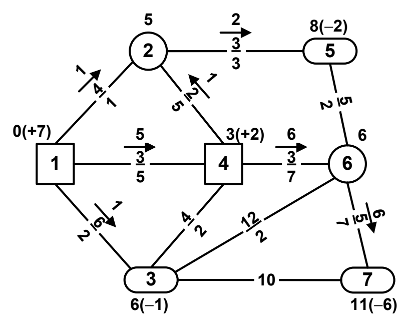

Fig. 2. Example of TN with limitation in the throughput capacity

Production volume is equal to the throughput capacity of the arc, that is

$$a_i = d_{ij}. \quad (1)$$

For the arcs whose throughput capacity is unlimited, in particular for arcs 3–7 and 7–3, it will correspond to the known big number.

The volume of consumption for producing vertices of the network is determined by formula:

$$b_j = \sum_{j \neq i} d_{ij} - a(x). \quad (2)$$

For the vertices that consume cargo – by formula:

$$b_j = \sum_{j \neq i} d_{ij} + b(x). \quad (3)$$

For transit vertices, by formula:

$$b_j = \sum_{j \neq i} d_{ij}. \quad (4)$$

4. 4. Improvement of the methods of searching for the shortest distances in the transportation network

Often, when solving practical problems, there is a need to show the links between certain objects. Directed and non-directed graphs, which are referred to in the scientific literature as networks, are a natural model for the implementation of such links [7, 8].

Let us consider the problem of searching for the best route in terms of the smallest distance. This problem is naturally modeled using networks, that is, we have connected network G, in which positive weight of each edge is equal to its length. Length of the path in such a network is equal to the sum of lengths of the edges that form this path. In the terms of networks, the problem is reduced to finding the shortest path between two set vertices of graph G [7, 8].

The problems on the shortest paths belong to fundamental problems of combinatorial optimization, because many of them can be reduced to finding the shortest path in a network. There are different types of problems on the shortest path: (1) between two given vertices, (2) between a given vertex and all others, (3) between each pair of vertices in the network, (4) between two given vertices to the paths that pass through one or more of the specified vertices; (5) the first, second, third, etc. shortest path in a network. Of all the described types, the most interesting for solving the network transport problems are the first three. In this case, the first two of them are realized using the Dijkstra's algorithm varieties [4], and the third one by using the Floyd algorithm [5].

Let us assume there is directed graph G=(V, E) whose all arcs have positive marks (arcs costs). It is possible to represent graph *G* in the form of map of route flights from one city to another, where each vertex corresponds to a city, and arc v→w to the shuttle route from city v to city w (Fig. 3). The mark of arc v→w is the flight time from city v to city w. In this case, one can assume that in this case the model matches a non-directed graph because the marks of arcs v→w and w→v may coincide. But the flight time is mostly different in opposite directions between two cities. In addition, assumption about coincidence of the marks of arcs v→w and w→v does not affect essentially the solution of the set problem. In this case, the solution of the problem on finding the shortest path will be minimum time of flights between different cities.

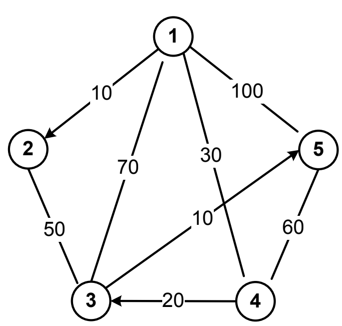

Fig. 3. Directed graph with marked arcs

Method of graphs. Our initial data for this method are the known specified directed graph G(V, E), shown in Fig. 3. In this case, the whole set of its vertices V is divided into two subsets. The first subset includes the cities of departures (m of cities), and the second subset includes the cities of airplanes landing (n of cities).

To resolve this problem, existing algorithms may not be applied because the Dijkstra's algorithm is insufficient (according to it, we find only one line from the matrix of the shortest distances), and the Floyd algorithm is excessive (it generates matrix of the shortest distances between any a/p, that is, m+n to m+n).

It is necessary to find the shortest routes for flights between the airports (a/p) of departures and landings, including landings at intermediate a/p (they can be both a/p of departures and a/p of landings of airplanes). In other words, we must receive the matrix of the shortest distances between the a/p of departures and the a/p of landings (Table 1).

Table 1

Matrix of the shortest distances between departures and landings

Indicators	A/p of landings				
	No.	1	2	...	n
A/p of departures	1	C_{11}	C_{12}	...	C_{1n}
	2	C_{21}	C_{22}	...	C_{2n}
	...	...	...	...	...
	m	C_{m1}	C_{m2}	...	C_{mn}

That is why we consider a fundamentally new algorithm, shown in the listing of program from a pseudo code, which is presented below and in which:

– array D is the resulting matrix of the shortest distances, and at every step element D[i, v] contains length of the current shortest path from vertex i to vertex v;

– array C specifies distances of the flights, where element C[i, j] is equal to the cost of arc i→j. If arc i→j does not exist, then C[i, j] equals ∞ (infinity), that is, larger than any actual cost of arcs;

– element of array P[i, v] contains the number of vertex, preceding vertex v in the shortest path from vertex i;

– set S means the same as in the Dijkstra's algorithm, namely a sequence of vertices of the "special" shortest path:

```
procedure New( var D: array[1 .. m, l .. (m + n)] of real;
 C: array[1 .. (m + n), l .. (m + n)] of real;
 P: array[1 .. m, l .. (m + n)] of integer);
begin
(1) for i := 1 to m do
 begin
 S := {i}; {selecting the next vertex from the subset of a/p of departures}
 for j := 1 to (m + n) do
 begin
 D[i, j] := C[i, j]; { D initialization }
 P[i, j] := i
 end
(2) for j := 1 to (m + n –1) do
 begin
 selecting such vertex w from set V\S that value D[i, w] minimal;
 add w to set S;
 for each vertex v from set V\S do
 begin
 if (D[i, w) + C[w, v] < D[i, v] then
 P[i, j] := w;
 D[i, v] := min(D[i, v], D([i, w] + C[w, v] );
 end
(3) end
(4) end
 end; { New }
```

In the external loop (lines 1–4), we sequentially select all a/p of departures, and in the internal one (lines 2–3) we find the shortest routes from these a/p to all others, and if, along this route, the intermediate vertices are available, they are remembered.

An analysis of the commonly known network algorithms for constructing the shortest paths between the vertices of directed graph reveals that the proposed new method for constructing the shortest paths between specified sets of vertices in the network has the following advantages:

– it fully solves the set problem that could not fundamentally be solved using the Dijkstra's algorithm, due to the lack of obtained results;

– it solves the problem of finding the shortest paths between the given infinities of vertices in the network more effectively, that is, easier and faster, compared to, though adequate but redundant, results, that we receive, using the Floyd algorithm.

The new algorithm for constructing the shortest paths between specified sets of vertices in the network was implemented in the form of software package, which was verified at a large number of examples, thus proving its reliability and universality in the network TVs of large dimensions.

The matrix method. First, we compile adjacency matrix S of the known graph G=(V, E) shown in Fig. 3. The lines of matrix S correspond to vertices V_i $(i=\overline{1,5})$, columns – vertices V_j $(j=\overline{1,5})$. Element S_{ij}, which is located at the intersection of the i-th line and the j-th column, is assigned equal to the value that is set on the corresponding arc E_{ij} between vertices V_i and V_j and 0 – in the absence of direct link between them (Table 2).

Table 2

Matrix S

No.	1	2	3	4	5
1	0	10	0	30	100
2	0	0	50	0	0
3	70	0	0	0	10
4	0	0	20	0	0
5	0	0	0	60	0

Next we determine matrix S^2=S+S by the following rule of adding elements of matrices S:

$$S^2_{ij} = \min\left\{\sum_{k=1}^{n}\left(S_{ik}+S_{kj}\right)\right\},$$

provided

$$((S_{ik}\times S_{kj})\neq 0)(i=\overline{1,n};\, j=\overline{1,n}). \quad (5)$$

Upon completion of the formation of all matrices S^m, we define matrix D – resulting matrix of the shortest paths between vertices V_i and V_j of graph G whose elements are calculated by the following formula:

$$D_{ij} = \min\left\{S^1_{ij}\cdots S^m_{ij}\right\}, \text{at } S^1_{ij}\cdots S^m_{ij}\neq 0. \quad (6)$$

Described new method for finding the shortest paths on directed weighted graph by its functional capabilities is fully comparable to the Floyd method. It should also be noted that the new method described, similar to the Dijkstra's algorithm with its various modifications and the Floyd algorithm, may also be used when processing the network models of representation of cargo transportation in TN of various structure [8, 9].

A new method for constructing the shortest paths between different sets of vertices on a graph, which we examined, is also implemented as a software package.

5. Results of research into improvement of the methods for finding the shortest distances in a transportation network

5. 1. Improvement of the method for maximum flow

Improvement of the method for maximum flow is conveniently resolved by the method of trees [10, 11]. Let us explore this method on the example of TN with a node-source and a node-runoff (Fig. 4).

It is necessary to find maximum flow from point 1 to point 6.

Let the links of the network experience permissible two-way motion and their throughput capacity in both directions of motion is the same. The entire network is divided arbitrarily into two trees. One is point 1 (source) and the other one is point 6 (runoff). In Fig. 6, *a,* one tree consists of four edges 1–2, 1–3, 1–4 and 3–5; the second one is from one vertex 6.

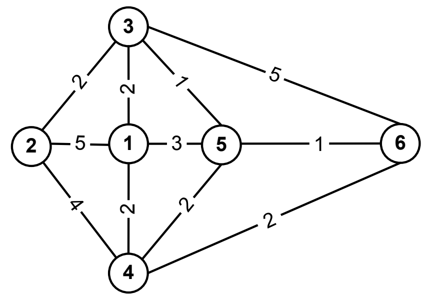

Fig. 4. Transport network with node-source (1) and node-runoff (6)

First, let the flow between vertices 1 and 6 equals zero. Then the trees are connected by arc shown in dotted line 5–6 (Fig. 5). In this regard, from vertex 1 to vertex 6, flow Q_1 may pass, equal to the minimum throughput capacity of one of the arcs. In Fig. 5, there are 2 links with minimal throughput capacity – 3–5 and 5–6. Let the flow equal to 1 pass along route 1–3–5–6. Next, one of the links (we select, for example, 5–6) is eliminated from the network, and we marking this action with a cross in Fig. 6.

The network is again split into two trees. The first one includes vertices 1, 2, 3, 4, 5, and the second one – vertex 6. Let us connect them by link 4–6 (Fig. 6), along which additional flow Q_2 may pass. Its size, due to the minimal throughput capacity of links of route 1–4–6, is equal to 2. Let this flow pass and then exclude in subsequent transformations link 4–6 from the network.

By continuing the same transformations over TN links, we receive at the last step 7 in Fig. 7 the maximum flow in the network, equal to 8. The crossed out links determine minimum section in the network that separates source (vertex 1) and runoff (vertex 6) and whose throughput capacity equals the maximum flow.

The solution may be applied to the problem with multiple sources and runoffs. For this purpose, it is sufficient to build a fake source and connect it by links with nodes of dispatch. A throughput capacity of these links will be the magnitude of possible dispatch of a vehicle from each node. Similar actions can be performed with the nodes of arrival.

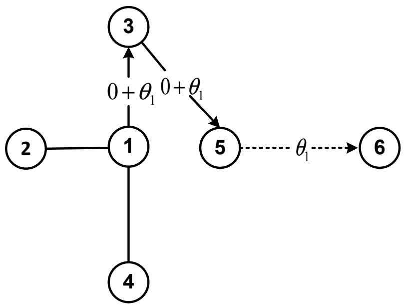

Fig. 5. Step 1 of finding maximum flow in TN by the tree method

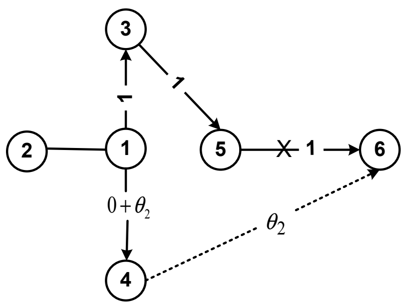

Fig. 6. Step 2 of finding maximum flow in TN by the tree method

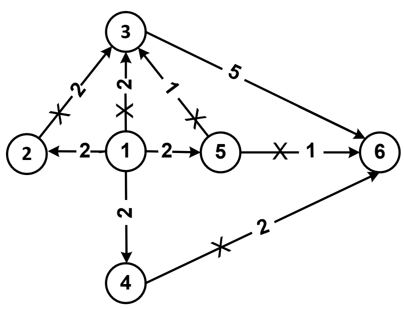

Fig. 7. Step 7 of finding maximum flow in TN by the tree method

5. 2. Improvement of the methods for reducing a network representation of the transport problem to the matrix form

The first way is the improvement of the method Ordena [10, 11], shown in Table 3. Every vertex of the network shown in Fig. 1 is assigned with a line and a column. Thus, in our case, the table consists of seven lines and seven columns. It should always be square. In the cells of the main diagonal in Table 3, the cost of transportation is equal to 0, because the output and, at the same time, input arcs to the same vertex cannot exist.

For the vertices, interconnected by a link, in the cells of the table at the crossing of the corresponding lines and columns is the cost of transportation by this link. Other cells are blocked by the numbers that are larger than the costs of transportation (in Table 3, it is 99).

Table 3

Reducing a network transport problem to the matrix form by method Ordena

No.	1	2	3	4	5	6	7	∑
1	0 $\boxed{9}$	4 $\boxed{2}$	6 $\boxed{1}$	3 $\boxed{4}$	99	99	99	16
2	4	0 $\boxed{7}$	99	2	3 $\boxed{2}$	99	99	9
3	6	99	0 $\boxed{9}$	4	99	12	10	9
4	3	2	4	0 $\boxed{5}$	99	3 $\boxed{6}$	99	11
5	99	3	99	99	0 $\boxed{9}$	5	99	9
6	99	99	12	3	5	0 $\boxed{3}$	5 $\boxed{6}$	9
7	99	99	10	99	99	5	0 $\boxed{9}$	9
∑	9	9	10	9	11	9	15	72

For convenience of the calculation, the value of production (consumption) volume at each vertex is added with any positive number. In Table 3, it is number 9. Thus, the volume of production in vertex 1 will equal 7+9=16; in transit vertex 2 – 9, similar to the volume of consumption; the volume of consumption in vertex 3 will equal 1+9=10, etc. Then the transport problem is solved by any known tabular method, for example, the method of potentials. In Table 3, values of the optimal plan of cargo transportation are in italics, in Fig. 1 – arrows.

The second way is improving the Wagner method [11]. It is more convenient for the networks with throughput capacity limitations. Such a network is depicted in Fig. 2, where an optimal plan of transportation is also presented. Table 4 demonstrates reducing this network to the matrix form.

Table 4

Reducing a network problem to the matrix form by the Wagner method

No.	1	2	3	4	5	6	7	∑
1–2	0	4 $\boxed{1}$	99	99	99	99	99	1
2–1	4	0 $\boxed{1}$	99	99	99	99	99	1
1–3	0 $\boxed{1}$	99	6 $\boxed{1}$	99	99	99	99	2
...	...	...	...	...	...	...	...	...
6–5	99	99	99	99	5	0 $\boxed{2}$	99	2
6–7	99	99	99	99	99	0 $\boxed{1}$	5 $\boxed{6}$	7
7–6	99	99	99	99	99	5	0 $\boxed{7}$	7
∑	1	9	107	17	7	18	113	72

Arcs here are in lines, the vertices are in columns. In the upper-left corner of the table cell is the cost of transportation along the arc. The cells that contain no digits are supposed to be blocked by the numbers that are larger than the costs of transportation (in Table 4, it is 99).

Production volume is equal to the arc's throughput capacity (1). For the arcs whose throughput capacity is unlimited, in particular, arcs 3–7 and 7–3, it corresponds (in our example) to a number of 100.

Consumption volumes for the production vertices of the network are determined by formula (2), for the vertices that consume the goods – by formula (3), and for the transit vertices – by formula (4).

Thus, for vertex 1, the volume of consumption is equal to 1+5+2–7=1, for vertex 7 – 7+100+6=113, and for vertex 2 – 1+5+3=9.

Table 4 also shows the final result of solving the problem – the optimal plan for the transportation of cargo, which is represented in the form of italicized values that correspond to the flows in Fig. 2.

5. 3. Improvement of the methods for finding the shortest distances in the transportation network

The method of graphs. Tables 5–7 present matrices C, D and P, respectively, obtained by using a new algorithm for directed graph, shown in Fig. 3, which mean the following:

– array C assigns distances of flights;

– array D is the resulting matrix of the shortest distances;

– element of array P[i, v] contains the number of the vertex, preceding vertex v along the shortest path from vertex i.

Table 5

Matrix C

Indicators	A/p of departures and landings					
	No.	1	2	3	4	5
A/p of departures and landings	1	∞	10	∞	30	100
	2	∞	∞	50	∞	∞
	3	70	∞	∞	∞	10
	4	∞	∞	20	∞	∞
	5	∞	∞	∞	60	∞

Table 6

Matrix D

Indicators	A/p of departures and landings					
	No.	1	2	3	4	5
A/p of departures	1	∞	10	50	30	60
	2	120	∞	50	120	60

Table 7

Matrix P

Indicators	A/p of departures and landings					
	No.	1	2	3	4	5
A/p of departures	1	∞	10	∞	30	100
	2	∞	∞	50	∞	∞

Using data from matrix P, it is possible to build the routes of flights from each a/p of departures (1 and 2) to each of the a/p of landings (3, 4 and 5):

$1 \xrightarrow{30} 4 \xrightarrow{20} 3 = \mathbf{50}$

$1 \xrightarrow{30} 4 = \mathbf{30}$

$1 \xrightarrow{30} 4 \xrightarrow{20} 3 \xrightarrow{10} 5 = \mathbf{60}$

$2 \xrightarrow{50} 3 = \mathbf{50}$

$2 \xrightarrow{50} 3 \xrightarrow{10} 5 \xrightarrow{60} 4 = \mathbf{120}$

$2 \xrightarrow{50} 3 \xrightarrow{10} 5 = \mathbf{60}$

The matrix method. Using formula (5), we determine matrices S^2=S+S, S^3=S+S^2 and so on, until the last resulting matrix does not contain any zero (Tables 8, 9).

Table 8

Matrix S^2

No.	1	2	3	4	5
1	0	0	50	160	0
2	120	0	0	0	60
3	0	80	0	70	170
4	90	0	0	0	30
5	0	0	80	0	0

Table 9

Matrix S^{10}

No.	1	2	3	4	5
1	430	310	520	300	370
2	620	470	320	460	560
3	340	580	430	570	280
4	590	440	290	430	530
5	490	340	580	330	430

Elements of matrix S_{ij}^m determine length of the shortest path between vertices V_i and V_j that contains m links (arcs).

In the process of forming matrices S^m, we obtain matrix P whose elements are the quantities of arcs that make up the shortest paths between vertices V_i and V_j of graph G (Table 10).

Table 10

Matrix P

No.	1	2	3	4	5
1	3	1	2	1	3
2	2	3	1	3	2
3	1	2	3	2	1
4	2	3	1	3	2
5	3	4	2	1	3

Upon completion of the formation of all matrices S^m, we define matrix D (Table 11) – the resulting matrix of the shortest paths between vertices V_i and V_j of graph G, whose elements are calculated by formula (6).

Table 11

Matrix D

No.	1	2	3	4	5
1	120	10	50	30	60
2	120	130	50	120	60
3	70	80	90	70	10
4	90	100	20	90	30
5	150	160	80	60	90

In the end, by analyzing the contents of Tables S...S^m, P and D, we build routes for the shortest paths between all vertices V_i and V_j of graph G.

6. Discussion of results of the research into the impact of indicators of transportation network on the solution of the problems on maximum flow and the shortest paths in transportation network

Improvement of the method for maximum flow is conveniently resolved by the method of trees. The solution can be applied to the problem with multiple sources and runoffs. This will solve problems for the optimization of transportation networks with and without limitations of their throughput capacity.

For this purpose, it is sufficient to build a fake source and connect it by links with the nodes of dispatch. A throughput capacity of these links will be the magnitude of possible dispatch from each node. Similarly, these actions can be performed with the nodes of arrival.

The improvement of the method for the shortest paths is resolved by using the modified Dijkstra's algorithm. Solving the problem on finding the shortest path, in addition to the value of the shortest distance from a given vertex to all others, we obtain the shortest route, in particular, a list of vertices that it passes. It might be used for imposing flows on the networks. By having matrix of correspondences of freight traffic from each vertex to all others, we build a tree of the shortest paths and then, returning from each point of unloading by the shortest route, we summarize flows at the arcs of the network. Going from one vertex to another vertex, we receive density of traffic in the network without limitation in the throughput capacity. This technique might be used to determine actual density of traffic in the network in the static state.

When a network has throughput capacity limitations, imposing a flow on the network is a bit complicated. In this case, it is necessary to subtract each elementary flow from the existing throughput capacity of the arc, on which it is imposed. Once the capacity of the arc is filled, it is removed from the network, new trees of the shortest paths are built and the imposition is applied to another tree, and so on. The plan built in this way is not optimal, but, if there are no many arcs with limitations in throughput capacity, then, after a machine imposes flows on the network, it is possible to determine bandwidth capabilities and potentials of the vertices and adjust the flows manually.

The improvement of the methods for reducing a network representation of the transport problem to the matrix form is carried out by the more effective modified Dijkstra's method that has algorithmic and software provision of its implementation.

Studies we conducted were performed within the framework of implementation of applied work by requests from motor transport enterprises of the Association of International Automobile Carriers of Ukraine. The results might be used to optimize the routes of transportation of cargoes and the optimization of carriers' loading. Further studies may be extended in the direction of optimization of multimodal transportation of goods by different types of transport.

7. Conclusions

1. It is proposed to improve the method for maximum flow in the transportation network through the use of the method of trees. The solution can be applied to a problem with multiple sources and runoffs. This will solve the problems on the optimization of transportation networks with and without limitations in throughput capacity.

2. We proposed an improved method for building the shortest paths in a transport network between different sets of vertices on the graph, namely, sets of providers and consumers. The method is implemented in the form of software package that might be used for the transport problems of large dimensionality.

3. We defined a conversion mechanism for the network models of the process of cargo transportation in the matrix model, which are set in the form of directed graphs and which allow the transportation of cargo through intermediate transportation nodes.

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