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# MODEL OF THE CASCADE THERMOELECTRIC COOLING DEVICES IN THE MODE OF THE LARGEST ENERGY EFFICIENCY

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*Розроблено модель каскадного термоелектричного охолоджувача в режимі найбільшої енергетичної ефективності для оцінки економічності функціонування і визначення показників надійності. Аналіз моделі показав, що існує оптимальне співвідношення кількості термоелементів в каскадах, яке відповідає максимуму коефіцієнта охолоджування при заданому перепаді температури. Використання отриманих співвідношень дозволяє вже на етапі проектування прогнозувати показники надійності каскадних термоелектричних охолоджувачів*

*Ключові слова: термоелектричні пристрої, показники надійності, перепад температури, енергетична ефективність*

*Разработана модель каскадного термоэлектрического охладителя в режиме наибольшей энергетической эффективности для оценки экономичности функционирования и определения показателей надежности. Анализ модели показал, что существует оптимальное отношение количества термоэлементов в каскадах, соответствующее максимуму холодильного коэффициента при заданном перепаде температуры. Использование полученных соотношений позволяет уже на этапе проектирования прогнозировать показатели надежности термоэлектрических каскадных охладителей*

*Ключевые слова: термоэлектрические устройства, показатели надежности, перепад температуры, энергетическая эффективность*

## 1. Introduction

Cascade thermoelectric cooling devices (CTED) provide not only the attainment of larger temperature drop $\Delta T$ in comparison with the single-cascade coolers, but higher energy efficiency as well. The standard conditions when designing CTED include the use of the standardized modules and the serial connection of cascades. In this case, it is necessary to determine maximum energy effectiveness at the assigned temperature drop and to select such current regime, which corresponds to the maximum in the energy effectiveness of CTED with the assigned design. In order to achieve the purpose of designing a two-cascade cooler at the assigned temperature range, it is necessary to determine basic significant indicators: relative operating currents of the first $B_1$ and the second $B_2$ cascades, relative temperature drops $\Theta_1$ and $\Theta_2$ in the cascades, a number of thermoelements in the cascades $n_1$ and $n_2$, and then to estimate indicators of reliability of the energy effective CTED.

## 2. Literature review and problem statement

The energy effectiveness of thermoelectric coolers determines not only the possibilities of decreasing the mass and dimensions parameters, but as well an increase in the dynamic and strength indicators for the systems that provide for the thermal modes of thermally loaded elements [1]. A base method for improving the coolers energy effectiveness is an increase in thermoelectric effectiveness of the source materials for thermoelements [2, 3], development of new thermoelectric materials for the promising thin-film modules [4, 5]. A considerable increase in the thermoelectric effectiveness of materials can be achieved by new technologies for creating the thermoelectric materials for coolers and generators [6, 7]. Modern approaches based on the nano-technologies principles [8] allow multi-fold increase in thermoelectric effectiveness through changes in electrical resistance and heat losses by thermoelements, as well as via control of magnetic field [9, 10].

At the same time, any changes in the design of cooler, including the application of new materials that fulfill not only the function of converting electrical energy into cold but also carry mechanical load in the volumetric coolers, lead to changes in the reliability indicators of the device [11, 12].

With an increase in thermoelectric effectiveness, the temperature gradients grow, the adhesive properties of the base layer-thermoelement connection deteriorate, splitting of the contact occurs [13]. A mode of the largest energy effectiveness of cooler in this aspect is indicative since it corresponds to extreme operating conditions. A contradiction between the operation regime of thermoelectric cooler, much needed in practice, and the reliability indicators necessitates further studies as these issues were not properly explored in the literature.

## 3. The aim and tasks of the study

The aim of this work is to develop a model that makes it possible to evaluate the energy efficiency of functioning and predicting the reliability indicators of a two-cascade TED of the chosen design.

To achieve the set aim, it was necessary to solve the following tasks:

– to develop a model of interrelation between the CTED reliability indicators and design and energy indicators under condition of the largest energy effectiveness;

– to analyze the model in order to define conditions for improving the CTED reliability indicators.

## 4. Development and analysis of the CTED reliability-oriented model under condition of the largest energy effectiveness

**4. 1. Model of connection between the CTED reliability indicators and the energy and design parameters**

Refrigerating capacity $Q_0$ of the cooler's first cascade can be presented in the form [14]:

$$Q_0 = n_1 I^2_{\max 1} R_1 (2B_1 - B_1^2 - \Theta_1), \quad (1)$$

where $I_{\max}$ is the maximum operating current, A, $I_{\max 1} = \frac{e_1 T_0}{R_1}$; $n_1$ is the number of thermoelements in the first cascade, pieces; $T_0$ is the temperature of the heat-absorbing joint of the first cascade, K; $e_1$ is the coefficient of thermal EMF of the thermoelements branch of the first cascade, V/K; $R_1$ is the electrical resistance of the thermoelement branch of the first cascade, Ohm; $B_1$ is the relative operating current of the first cascade, rel. un., $B_1 = I/I_{\max 1}$; $\theta_1$ is the relative temperature drop of the first cascade, rel. un.,

$$\Theta_1 = \frac{T_1 - T_0}{\Delta T_{\max 1}},$$

where $T_1$ is the intermediate temperature, K; $\Delta T_{\max 1}$ is the maximum temperature drop in the first cascade, K.

For the serial connection of cascades, current in the cascades is identical; therefore, it is possible to write down:

$$I_{\max 1} B_1 = I_{\max 2} B_2, \quad (2)$$

where $B_2$ is the relative operating current of the second cascade, rel. un., $B_2 = \frac{e_2 T_1}{R_2}$; $e_2$ is the coefficient of thermal EMF of the thermoelements branch of the second cascade, V/K; $R_2$ is the electrical resistance of the thermoelement branch of the second cascade, Ohm.

A general temperature drop in a two-cascade CTED can be represented in the form:

$$\Delta T = \Delta T_1 + \Delta T_2 = \Delta T_{\max 1} \theta_1 + \Delta T_{\max 2} \theta_2, \quad (3)$$

where $\Delta T_1$ is the temperature drop in the first cascade, K, $\Delta T_1 = T_1 - T_0$; $\Delta T_2$ is the temperature drop in the second cascade, K, $\Delta T_2 = T - T_1$; $\Theta_2$ is the relative temperature drop in the second cascade, rel. un.,

$$\Theta_2 = \frac{T - T_1}{\Delta T_{\max 2}},$$

where $\Delta T_{\max 2}$ is the maximum temperature drop in the second cascade, K.

Then the condition for the thermal alignment of cascades takes the form

$$\frac{n_1}{n_2} = \frac{I^2_{\max 2} R_2 (2B_2 - B_2^2 - \Theta_2)}{I^2_{\max 1} R_1 \left[ 2B_1 \left( 1 + \frac{\Delta T_{\max 1}}{T_0} \Theta_1 \right) + B_1^2 - \Theta_1 \right]}, \quad (4)$$

where $n_2$ is the number of thermoelements in the second cascade, pieces.

Refrigerating coefficient of the two-cascade CTED can be written down in the form

$$E^{N=2} = \frac{Q_0}{W_1 + W_2}, \quad (5)$$

where $W_1$ is the consumption power of the first cascade, W,

$$W_1 = 2 n_1 I^2_{\max 1} R_1 B_1 \left( B_1 + \frac{\Delta T_{\max 1}}{T_0} \Theta_1 \right); \quad (6)$$

$W_{12}$ is the consumption power of the second cascade, W,

$$W_2 = 2 n_2 I^2_{\max 2} R_2 B_2 \left( B_2 + \frac{\Delta T_{\max 2}}{T_1} \Theta_2 \right). \quad (7)$$

Using ratios (1)–(7), refrigerating coefficient of the two-cascade TED can be written down in the form

$$E^{N=2} = \frac{2aB_1 b - aB_1^2 c + 2a^2 B_1^3 \frac{\Delta T_{\max 1}}{T_0} - a \frac{\Delta T}{\Delta T_{\max 2}}}{2B_1^2 A - 2B_1^3 B + 2B_1 D \frac{\Delta T}{\Delta T_{\max 2}}}, \quad (8)$$

where

$$a = \frac{n_1 I^2_{\max 1} R_1}{n_2 I^2_{\max 2} R_2};$$

$$b = \frac{\Delta T_{\max 1}}{\Delta T_{\max 2}} + \frac{I_{\max 1}}{I_{\max 2}};$$

$$c = \frac{\Delta T_{\max 1}}{\Delta T_{\max 2}} + 2a \left( 1 + 2\frac{\Delta T_{\max 1}}{T_0} \right) + \frac{I^2_{\max 1}}{I^2_{\max 2}};$$

$$A = \left( \frac{\Delta T_{\max 1}}{\Delta T_{\max 2}} + a \right) \left( a + \frac{I^2_{\max 1}}{I^2_{\max 2}} \right) - 2a \frac{\Delta T_{\max 1}}{T_0} \frac{I_{\max 1}}{I_{\max 2}} \frac{\Delta T_{\max 2}}{T_1} \frac{\Delta T}{\Delta T_{\max 2}} + 2 \left( \frac{I_{\max 1}}{I_{\max 2}} \frac{\Delta T_{\max 2}}{T_1} \frac{\Delta T_{\max 1}}{\Delta T_{\max 2}} - a \frac{\Delta T_{\max 1}}{T_0} \right);$$

$$B=\left(a+\frac{I_{max1}^2}{I_{max2}^2}\right)\left(\frac{I_{max1}}{I_{max2}}\frac{\Delta T_{max2}}{T_1}\frac{\Delta T_{max1}}{\Delta T_{max2}}+a\frac{\Delta T_{max1}}{T_0}\right);$$

$$D=a\left(\frac{I_{max1}}{I_{max2}}\frac{\Delta T_{max2}}{T_1}+\frac{\Delta T_{max1}}{T_0}\right).$$

Functional dependence $E^{N=2}=f(B_1)$ has the maximum for different designs of TED ($n_1/n_2$) and temperature drops ΔT=60 K; 70 K; 80 K; 90 K at T=300 K, $n_1=9$, $l_2/s_2=l_1/s_1=10$.

With an increase in temperature drop ΔT, optimum magnitude of relative operating current $B_1$ shifts towards larger values.

From condition $\frac{dE^N}{dB_1}=0$, we will obtain ratio for determining the optimum magnitude of relative operating current $B_1$, which corresponds to the maximum of refrigerating coefficient $E^N$ for the TED of the assigned design ($n_1/n_2$) and to temperature drop ΔT:

$$B_1^4\left(Bc-2aA\frac{\Delta T_{max1}}{T_0}\right)-4B_1^3\left(Bb+aD\frac{\Delta T_{max1}}{T_0}\frac{\Delta T}{\Delta T_{max2}}\right)+$$

$$+B_1^2\left(2Ab+Dc\frac{\Delta T}{\Delta T_{max2}}+3B\frac{\Delta T}{\Delta T_{max2}}\right)-2B_1A\frac{\Delta T}{\Delta T_{max2}}-\left(\frac{\Delta T}{\Delta T_{max2}}\right)^2=0. \quad (9)$$

The represented ratio (9) allows us to determine the magnitude of optimum relative operating current $B_1$, providing for the maximum of refrigerating coefficient $E^N$, at the assigned values of ratio $n_1/n_2$ and temperature drop ΔT.

Then we determine relative temperature drops in cascades $\Theta_1$ and $\Theta_2$, using the method of successive approximations, taking into account the temperature dependence of the parameters (one or two approximations are sufficient):

$$\Theta_1=\frac{B_1^2\left(a+\frac{I_{max1}^2}{I_{max2}^2}\right)-2B_1\left(\frac{I_{max1}}{I_{max2}}-a\right)+\frac{\Delta T}{\Delta T_{max2}}}{\frac{\Delta T_{max1}}{\Delta T_{max2}}+a-2aB_1\frac{\Delta T_{max1}}{T_0}}; \quad (10)$$

$$\Theta_2=\frac{\Delta T}{\Delta T_{max2}}-\frac{\Delta T_{max1}}{\Delta T_{max2}}\Theta_1 \quad (11)$$

and refrigerating capacity $Q_{01}$ of the assigned design ($n_1/n_2$) of TED in regime $E_{max}$ at the assigned ΔT.

In accordance with [14], for the two-cascade TED, the magnitude of relative failure rate can be written down in the form

$$\frac{\lambda_\Sigma}{\lambda_0}=\frac{n_1B_1^2(\Theta_1+C_1)\left(B_1+\frac{\Delta T_{max1}}{T_0}\Theta_1\right)^2}{\left(1+\frac{\Delta T_{max1}}{T_0}\Theta_1\right)^2}K_{T_1}+$$

$$+\frac{n_2B_2^2(\Theta_2+C_2)\left(B_2+\frac{\Delta T_{max2}}{T_1}\Theta_2\right)^2}{\left(1+\frac{\Delta T_{max2}}{T_1}\Theta_2\right)^2}K_{T_2}, \quad (12)$$

where $\lambda_0$ is the nominal failure rate, 1/h; $C_1$, $C_2$ is the relative thermal load of the first and second cascades, rel. un.,

$$C_1=\frac{Q_{01}}{n_1I_{max1}^2R_1};\quad C_2=\frac{Q_0+W_1}{n_2I_{max2}^2R_2};$$

$K_{T1}$, $K_{T2}$ are the coefficients of significance taking into account the effect of reduced temperature [14].

**4. 2. Analysis of results of the reliability-oriented simulation of a two-cascade CTED**

Data of the calculations of basic parameters are given in Table 1 for $l_2/s_2=l_1/s_1=10$; T=300 K; ΔT=60 K; 70 K; 80 K; 90 K; $n_1=9$; $n_1/n_2$=1,0; 0,67; 0,5; 0,33; 0,2; 0,1 and the averaged value of thermoelectric modules effectiveness $\overline{z_M}=2,4-2,5\cdot10^{-3}$ 1/K; $\lambda_0=3\cdot10^{-8}$ 1/h; $t=10^4$ h.

With the decrease in ratio $n_1/n_2$ at the assigned value of temperature drop ΔT=60 K:

– relative operating current of the second cascade $B_2$ increases (Fig. 1, pos. 3);

– relative temperature drop of the first cascade $\Theta_1$ decreases (Fig. 1, pos. 4);

– relative temperature drop of the second cascade $\Theta_2$ increases (Fig. 1, pos. 5);

– magnitude of operating current I increases (Fig. 1, pos. 6);

– refrigerating coefficient TED has absolute maximum at $n_1/n_2=0,44$ (Fig. 2, pos. 1);

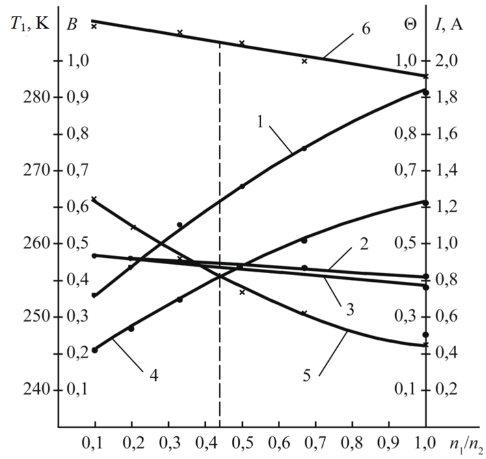


Fig. 1. Dependence of intermediate temperature $T_1$, relative operating currents $B_1$ and $B_2$, relative temperature drops $\Theta_1$ and $\Theta_2$ in the cascades and the magnitude of operating current I of two-cascade TED in regime $E_{max}$ on ratio $n_1/n_2$ at T=300 K; ΔT=60 K; $n_1=9$; $(l/s)_i=10$: 1 – $T_1=f(n_1/n_2)$; 2 – $B_1=f(n_1/n_2)$; 3 – $B_2=f(n_1/n_2)$; 4 – $\Theta_1=f(n_1/n_2)$; 5 – $\Theta_2=f(n_1/n_2)$; 6 – $I=f(n_1/n_2)$

– refrigerating coefficient of the first cascade $\varepsilon_1$ increases (Fig. 2, pos. 2), of the second cascade $\varepsilon_2$ – decreases (Fig. 2, pos. 3);

– the point of intersection of graphs of dependence of refrigerating coefficients of cascades $\varepsilon_1$ and $\varepsilon_2$ corresponds $n_1/n_2=0,44$ (Fig. 2);

– refrigerating capacity $Q_{01}$ (Fig. 2, pos. 4) and its relative magnitude $C_1$ (Fig. 2, pos. 5) increase;

– total magnitude of failure rate $\lambda_\Sigma$ increases (Fig. 3, pos. 3);

– failure rates of the first $\lambda_1$ and the second $\lambda_2$ cascades also increase (Fig. 3, pos. 1, 2);

– total probability of failure-free operation P decreases (Fig. 4, pos. 3);

– probability of failure-free operation of the first ($P_1$) and the second ($P_2$) cascade decreases (Fig. 4, pos. 1, 2).

Table 1

Results of the calculations of basic parameters for temperature range ΔT=60 K, 70 K, 80 K, 90 K

| $n_1/n_2$ | $B_1$ | $B_2$ | I, A | $T_1$, K | $\Theta_1$ | $\Theta_2$ | $\varepsilon_1$ | $\varepsilon_2$ | E | $Q_{01}$, W | $C_1$ | $W_1$, W | $W_2$, W | $W_\Sigma$, W | $U_\Sigma$, W | $\lambda\cdot10^{-8}$, 1/h | P |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ΔT=60 K | | | | | | | | | | | | | | | | | |
| 1,0 | 0,41 | 0,38 | 1,91 | 279,8 | 0,61 | 0,22 | 0,1 | 1,13 | 0,051 | 0,09 | 0,05 | 0,89 | 0,86 | 1,75 | 0,92 | 1,18 | 0,999882 |
| 0,67 | 0,43 | 0,405 | 2,0 | 272,8 | 0,51 | 0,31 | 0,35 | 0,83 | 0,133 | 0,32 | 0,17 | 0,92 | 1,49 | 2,4 | 1,21 | 1,79 | 0,99982 |
| 0,5 | 0,44 | 0,42 | 2,10 | 268,3 | 0,44 | 0,37 | 0,50 | 0,64 | 0,151 | 0,46 | 0,24 | 0,93 | 2,15 | 3,1 | 1,5 | 2,4 | 0,99976 |
| 0,45 | 0,447 | 0,43 | 2,13 | 267,2 | 0,427 | 0,40 | 0,54 | 0,59 | 0,148 | 0,50 | 0,27 | 0,94 | 2,44 | 3,4 | 1,6 | 2,9 | 0,99971 |
| 0,33 | 0,45 | 0,44 | 2,14 | 262,5 | 0,35 | 0,46 | 0,70 | 0,45 | 0,146 | 0,64 | 0,34 | 0,92 | 3,45 | 4,37 | 2,0 | 3,7 | 0,99963 |
| 0,2 | 0,45 | 0,45 | 2,13 | 257,0 | 0,27 | 0,55 | 0,92 | 0,27 | 0,114 | 0,79 | 0,42 | 0,86 | 6,05 | 6,9 | 3,2 | 6,5 | 0,999352 |
| 0,1 | 0,455 | 0,46 | 2,2 | 253,4 | 0,21 | 0,62 | 1,05 | 0,14 | 0,069 | 0,91 | 0,49 | 0,86 | 12,4 | 13,2 | 6,0 | 13,3 | 0,99867 |
| ΔT=70 K | | | | | | | | | | | | | | | | | |
| 0,67 | 0,53 | 0,49 | 2,43 | 270,3 | 0,69 | 0,34 | 0,12 | 0,68 | 0,044 | 0,15 | 0,086 | 1,28 | 2,114 | 3,4 | 1,4 | 4,3 | 0,99957 |
| 0,50 | 0,54 | 0,50 | 2,45 | 264,6 | 0,60 | 0,41 | 0,26 | 0,53 | 0,0767 | 0,33 | 0,19 | 1,26 | 3,0 | 4,26 | 1,7 | 5,5 | 0,99945 |
| 0,37 | 0,55 | 0,53 | 2,51 | 260,0 | 0,55 | 0,50 | 0,40 | 0,40 | 0,083 | 0,45 | 0,25 | 1,26 | 4,3 | 5,6 | 2,2 | 7,8 | 0,99922 |
| 0,33 | 0,547 | 0,53 | 2,52 | 258,7 | 0,50 | 0,52 | 0,40 | 0,36 | 0,0823 | 0,50 | 0,29 | 1,26 | 4,85 | 6,11 | 2,42 | 9,0 | 0,99910 |
| 0,20 | 0,57 | 0,55 | 2,64 | 253,6 | 0,42 | 0,61 | 0,51 | 0,23 | 0,0673 | 0,66 | 0,39 | 1,28 | 8,5 | 9,8 | 3,7 | 15,8 | 0,99842 |
| 0,10 | 0,573 | 0,56 | 2,66 | 248,2 | 0,33 | 0,71 | 0,66 | 0,12 | 0,0433 | 0,82 | 0,49 | 1,25 | 17,7 | 18,9 | 7,1 | 31,9 | 0,99682 |
| ΔT=80 K | | | | | | | | | | | | | | | | | |
| 0,50 | 0,65 | 0,60 | 2,9 | 263,4 | 0,85 | 0,45 | 0,03 | 0,44 | 0,0089 | 0,051 | 0,033 | 1,71 | 4,0 | 5,71 | 1,97 | 12,3 | 0,99877 |
| 0,33 | 0,67 | 0,63 | 3,0 | 257,1 | 0,73 | 0,56 | 0,14 | 0,31 | 0,030 | 0,25 | 0,16 | 1,75 | 6,55 | 8,3 | 2,75 | 19,7 | 0,9980 |
| 0,20 | 0,68 | 0,65 | 3,1 | 250,2 | 0,60 | 0,67 | 0,26 | 0,19 | 0,034 | 0,45 | 0,29 | 1,72 | 11,5 | 13,2 | 4,3 | 32,5 | 0,99676 |
| 0,10 | 0,70 | 0,69 | 3,2 | 245,2 | 0,51 | 0,78 | 0,35 | 0,10 | 0,023 | 0,61 | 0,40 | 1,75 | 24,9 | 26,7 | 8,4 | 71,9 | 0,99283 |
| ΔT=90 K | | | | | | | | | | | | | | | | | |
| 0,20 | 0,83 | 0,77 | 3,6 | 249,4 | 0,89 | 0,71 | 0,05 | 0,16 | 0,0067 | 0,12 | 0,085 | 2,3 | 15,1 | 17,4 | 4,9 | 64,9 | 0,99353 |
| 0,10 | 0,84 | 0,81 | 3,7 | 243,6 | 0,77 | 0,83 | 0,12 | 0,08 | 0,0081 | 0,28 | 0,21 | 2,3 | 32,4 | 34,7 | 9,4 | 140 | 0,9861 |

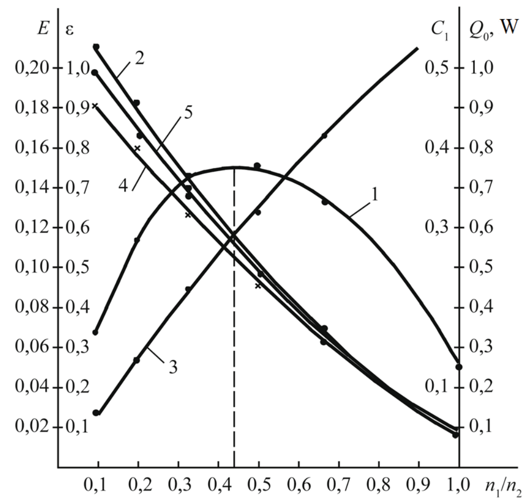


Fig. 2. Dependence of general refrigerating coefficient E and by the cascades $\varepsilon_1$ and $\varepsilon_2$, refrigerating capacity $Q_{01}$ and $C_1$ of two-cascade TED in regime $E_{max}$ on ratio $n_1/n_2$ at T=300 K; ΔT=60 K; $n_1$=9; $(l/s)_i$=10: 1 – $E=f(n_1/n_2)$; 2 – $\varepsilon_1=f(n_1/n_2)$; 3 – $\varepsilon_2=f(n_1/n_2)$; 4 – $Q_{01}=f(n_1/n_2)$; 5 – $C_1=f(n_1/n_2)$

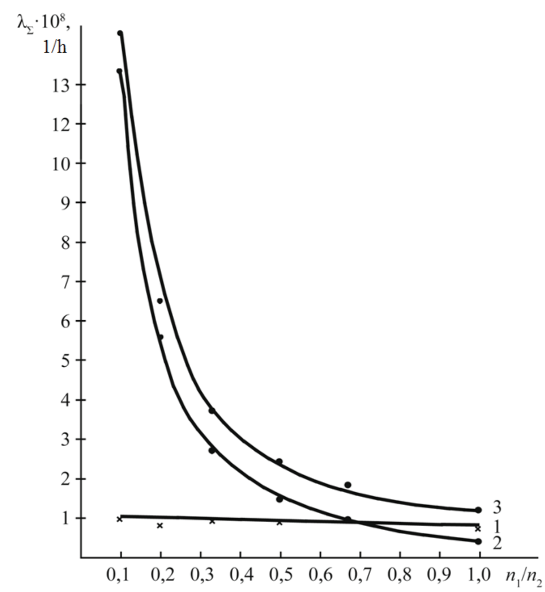


Fig. 3. Dependence of the total failure rate $\lambda_\Sigma$ and each cascade $\lambda_1$ and $\lambda_2$ individually of two-cascade TED in mode $E_{max}$ on ratio $n_1/n_2$ at T=300 K; ΔT=60 K; $n_1$=9; $(l/s)_i$=10, $\lambda_0=3\cdot10^{-8}$ 1/h: 1 – $\lambda_1=f(n_1/n_2)$; 2 – $\lambda_2=f(n_1/n_2)$; 3 – $\lambda_\Sigma=f(n_1/n_2)$

With the decrease in ratio $n_1/n_2$ at the assigned value of temperature drop ΔT=70 K:

– magnitude of intermediate temperature $T_1$ decreases (Fig. 5, pos. 1);

– relative operating current of the first cascade $B_1$ and the second cascade $B_2$ increases (Fig. 5, pos. 2, 3);

– relative temperature drop of the first cascade $\Theta_1$ decreases (Fig. 5, pos. 4);

– relative temperature drop of the second cascade $\Theta_2$ increases (Fig. 5, pos. 5);

– magnitude of operating current I increases (Fig. 5, pos. 6);

– refrigerating coefficient E has the maximum at $n_1/n_2$= =0,37 (Fig. 6, pos. 1);

– refrigerating coefficient of the first cascade $\varepsilon_1$ increases (Fig. 6, pos. 2), of the second cascade $\varepsilon_2$ – decreases (Fig. 6, pos. 3);

– refrigerating capacity $Q_{01}$ (Fig. 6, pos. 4) and its relative magnitude $C_1$ (Fig. 6, pos. 5) increase;

– total number of failure rate $\lambda_\Sigma$ increases (Fig. 7, pos. 3);

– failure rates of the first $\lambda_1$ and the second $\lambda_2$ cascades also increase (Fig. 7, pos. 1, 2);

– total power of energy consumption $W_\Sigma$ increases (Fig. 7, pos. 4);

– total probability of failure-free operation $P_\Sigma$ decreases (Fig. 8, pos. 3);

– probability of the failure-free operation of the first $P_1$ and the second cascade $P_2$ decreases (Fig. 8, pos. 1, 2).

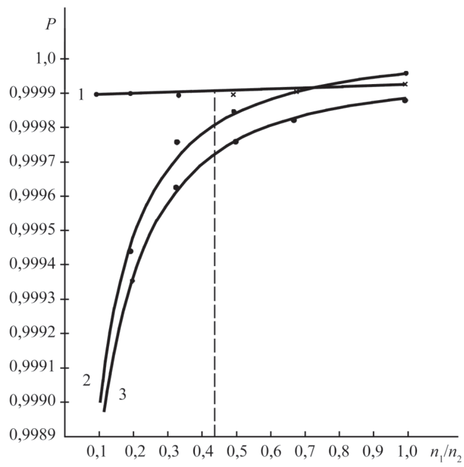


Fig. 4. Dependence of total probability of failure-free operation $P_\Sigma$ and for each cascade individually $P_1$ and $P_2$ of two-cascade TED of different designs in regime $E_{max}$ on ratio $n_1/n_2$ at T=300 K; ΔT=60 K; $n_1$=9; $(l/s)_i$=10; $\lambda_0=3\cdot10^{-8}$ 1/h; $t=10^4$ h: 1 – $P_1=f(n_1/n_2)$; 2 – $P_2=f(n_1/n_2)$; 3 – $P_\Sigma=f(n_1/n_2)$

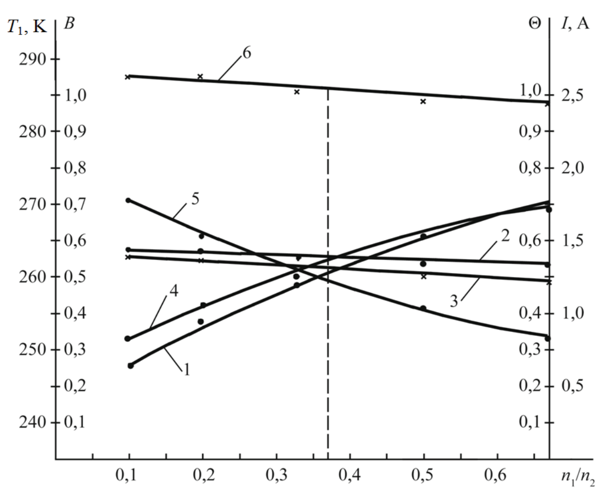


Fig. 5. Dependence of intermediate temperature $T_1$, relative operating currents $B_1$ and $B_2$, relative temperature drops $\Theta_1$ and $\Theta_2$ in the cascades and the magnitude of operating current I of two-cascade TED in regime $E_{max}$ on ratio $n_1/n_2$ at T=300 K; ΔT=70 K; $n_1$=9; $(l/s)_i$=10: 1 – $T_1=f(n_1/n_2)$; 2 – $B_1=f(n_1/n_2)$; 3 – $B_2=f(n_1/n_2)$; 4 – $\Theta_1=f(n_1/n_2)$; 5 – $\Theta_2=f(n_1/n_2)$; 6 – $I=f(n_1/n_2)$

With the decrease in ratio $n_1/n_2$ at the assigned value of temperature drop ΔT=80 K:

– magnitude of intermediate temperature $T_1$ decreases (Fig. 9, pos. 1);

– relative operating current of the first cascade $B_1$ and the second cascade $B_2$ increases (Fig. 9, pos. 2, 3);

– relative temperature drop of the first cascade $\Theta_1$ decreases (Fig. 5, pos. 4), of the second cascade $\Theta_2$ – increases (Fig. 9, pos. 5);

– magnitude of operating current I increases (Fig. 9, pos. 6);

– refrigerating coefficient E has the maximum at $n_1/n_2$= =0,23 (Fig. 10, pos. 1), in this case, refrigerating coefficients of the first cascade $\varepsilon_1$ and the second cascade $\varepsilon_2$ are equal to each other: $\varepsilon_1=\varepsilon_2$=0,22 (Fig. 10, pos. 2, 3);

– refrigerating capacity $Q_{01}$ (Fig. 10, pos. 4) and its relative magnitude $C_1$ (Fig. 10, pos. 5) increase;

– total magnitude of failure rate $\lambda_\Sigma$ increases (Fig. 11, pos. 3), in this case, the failure rates of the first $\lambda_1$ and the second $\lambda_2$ cascades also increase (Fig. 11, pos. 1, 2);

– total power of energy consumption $W_\Sigma$ increases (Fig. 11, pos. 4);

– total probability of failure-free operation $P_\Sigma$ decreases (Fig. 12, pos. 3), in this case, probability of the failure-free operation of the first ($P_1$) and the second ($P_2$) cascades also decreases (Fig. 12, pos. 1, 2).

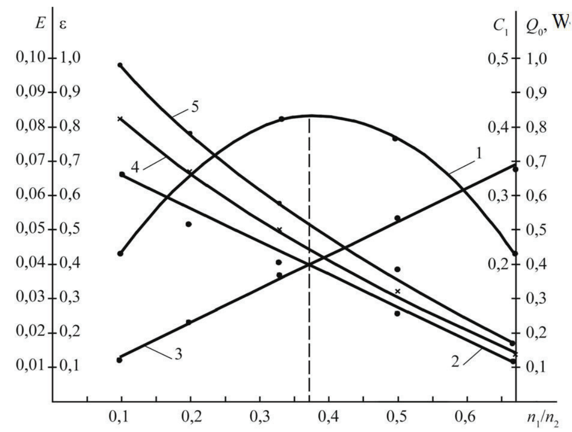


Fig. 6. Dependence of general refrigerating coefficient E and by the cascades $\varepsilon_1$ and $\varepsilon_2$, refrigerating capacity $Q_{01}$ and $C_1$ of two-cascade TED in regime $E_{max}$ on ratio $n_1/n_2$ at T=300 K; ΔT=70 K; $n_1$=9; $(l/s)_i$=10: 1 – $E=f(n_1/n_2)$; 2 – $\varepsilon_1=f(n_1/n_2)$; 3 – $\varepsilon_2=f(n_1/n_2)$; 4 – $Q_{01}=f(n_1/n_2)$; 5 – $C_1=f(n_1/n_2)$

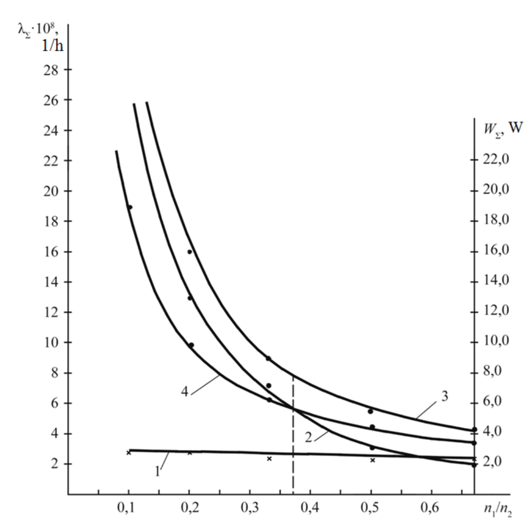


Fig. 7. Dependence of the total failure rate $\lambda_\Sigma$ and each cascade $\lambda_1$ and $\lambda_2$ individually of two-cascade TED in regime $E_{max}$ on ratio $n_1/n_2$ at T=300 K; ΔT=70 K; $n_1$=9; $(l/s)_i$=10; $\lambda_0=3\cdot10^{-8}$ 1/h: 1 – $\lambda_1=f(n_1/n_2)$; 2 – $\lambda_2=f(n_1/n_2)$; 3 – $\lambda_\Sigma=f(n_1/n_2)$

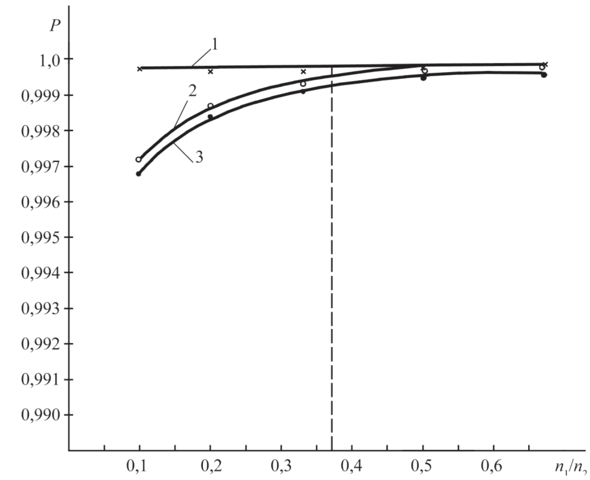


Fig. 8. Dependence of total probability of failure-free operation $P_\Sigma$ and for each cascade individually $P_1$ and $P_2$ of two-cascade TED of different designs in regime $E_{max}$ on ratio $n_1/n_2$ at T=300 K; ΔT=70 K; $n_1$=9; $(l/s)_i$=10; $\lambda_0=3\cdot10^{-8}$ 1/h; $t=10^4$ h: 1 – $P_1=f(n_1/n_2)$; 2 – $P_2=f(n_1/n_2)$; 3 – $P_\Sigma=f(n_1/n_2)$

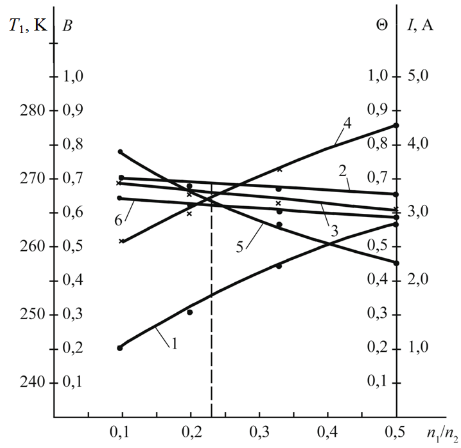


Fig. 9. Dependence of intermediate temperature $T_1$, relative operating currents $B_1$ and $B_2$, relative temperature drops $\Theta_1$ and $\Theta_2$ in the cascades and the magnitude of operating current I of two-cascade TED in regime $E_{max}$ on ratio $n_1/n_2$ at T=300 K; ΔT=80 K; $n_1$=9; $(l/s)_i$=10: 1 – $T_1=f(n_1/n_2)$; 2 – $B_1=f(n_1/n_2)$; 3 – $B_2=f(n_1/n_2)$; 4 – $\Theta_1=f(n_1/n_2)$; 5 – $\Theta_2=f(n_1/n_2)$; 6 – $I=f(n_1/n_2)$

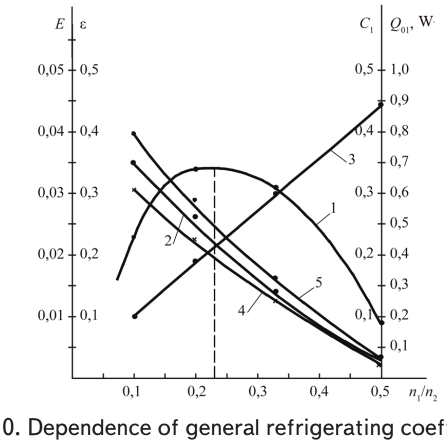


Fig. 10. Dependence of general refrigerating coefficient E and by the cascades $\varepsilon_1$ and $\varepsilon_2$, refrigerating capacity $Q_{01}$ and $C_1$ of two-cascade TED in regime $E_{max}$ on ratio $n_1/n_2$ at T=300 K; ΔT=80 K; $n_1$=9; $(l/s)_i$=10: 1 – $E=f(n_1/n_2)$; 2 – $\varepsilon_1=f(n_1/n_2)$; 3 – $\varepsilon_2=f(n_1/n_2)$; 4 – $Q_{01}=f(n_1/n_2)$; 5 – $C_1=f(n_1/n_2)$

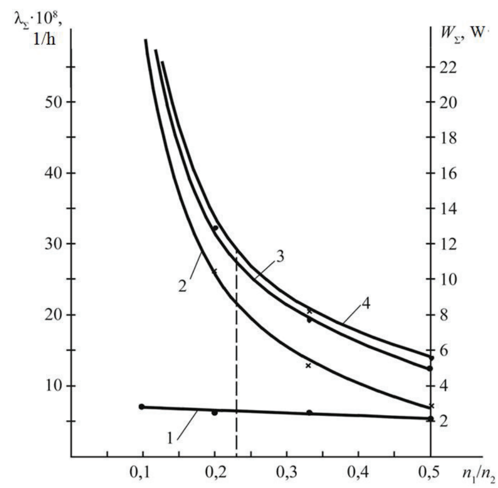


Fig. 11. Dependence of general total failure rate $\lambda_\Sigma$ and of each cascade $\lambda_1$ and $\lambda_2$ individually of two-cascade TED in regime $E_{max}$ on ratio $n_1/n_2$ at T=300 K; ΔT=80 K; $n_1$=9; $(l/s)_i$=10; $\lambda_0=3\cdot10^{-8}$ 1/h: 1– $\lambda_1=f(n_1/n_2)$; 2 – $\lambda_2=f(n_1/n_2)$; 3 – $\lambda_\Sigma=f(n_1/n_2)$; 4 – $W_\Sigma=f(n_1/n_2)$

With the decrease in ratio $n_1/n_2$ at the assigned value of temperature drop ΔT=90 K:

– magnitude of intermediate temperature $T_1$ decreases (Fig. 13, pos. 1);

– relative operating current of the first cascade $B_1$ and the second cascade $B_2$ increases (Fig. 13, pos. 2, 3);

– relative temperature drop of the first cascade $\Theta_1$ decreases (Fig. 13, pos. 4), and of the second cascade $\Theta_2$ increases (Fig. 13, pos. 5);

– magnitude of operating current I increases (Fig. 13, pos. 6);

– refrigerating coefficient E has the maximum at $n_1/n_2$=0,127 (Fig. 14, pos. 1), in this case, refrigerating coefficients of the first cascade $\varepsilon_1$ and the second cascade $\varepsilon_2$ are equal to each other: $\varepsilon_1=\varepsilon_2$=0,10 (Fig. 14, pos. 2, 3);

– refrigerating capacity $Q_{01}$ (Fig. 14, pos. 4) and its relative magnitude $C_1$ (Fig. 14, pos. 5) increase;

– total magnitude of failure rate $\lambda_\Sigma$ increases (Fig. 15, pos. 3), in this case, failure rates of the first $\lambda_1$ and the second $\lambda_2$ cascades increase (Fig. 15, pos. 1, 2);

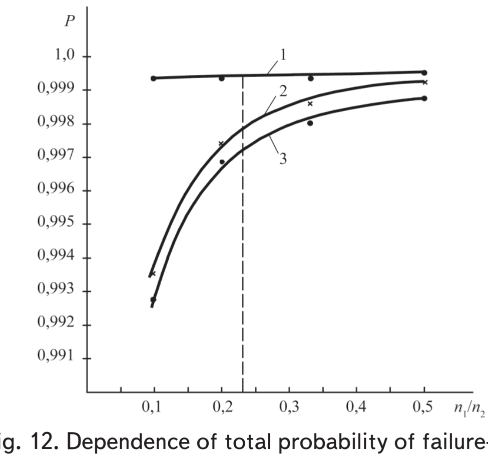


Fig. 12. Dependence of total probability of failure-free operation $P_\Sigma$ and for each cascade individually $P_1$ and $P_2$ of two-cascade TED of different designs in regime $E_{max}$ on ratio $n_1/n_2$ at T=300 K; ΔT=80 K; $n_1$=9; $(l/s)_i$=10, $\lambda_0=3\cdot10^{-8}$ 1/h; $t=10^4$ h: 1 – $P_1=f(n_1/n_2)$; 2 – $P_2=f(n_1/n_2)$; 3 – $P_\Sigma=f(n_1/n_2)$

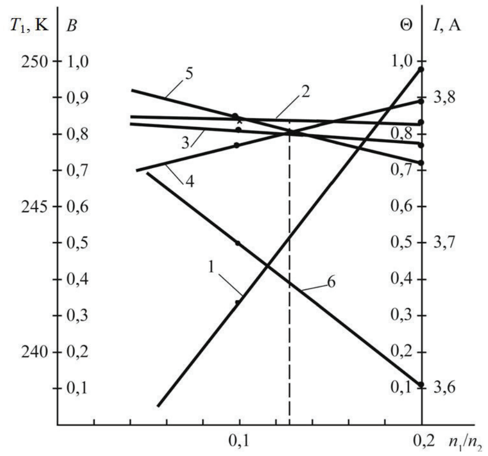


Fig. 13. Dependence of intermediate temperature $T_1$, relative operating currents $B_1$ and $B_2$, relative temperature drops $\Theta_1$ and $\Theta_2$ in the cascades and the magnitude of operating current I of two-cascade TED in regime $E_{max}$ on ratio $n_1/n_2$ at T=300 K; ΔT=90 K; $n_1$=9; $(l/s)_i$=10: 1 – $T_1=f(n_1/n_2)$; 2 – $B_1=f(n_1/n_2)$; 3 – $B_2=f(n_1/n_2)$; 4 – $\Theta_1=f(n_1/n_2)$; 5 – $\Theta_2=f(n_1/n_2)$; 6 – $I=f(n_1/n_2)$

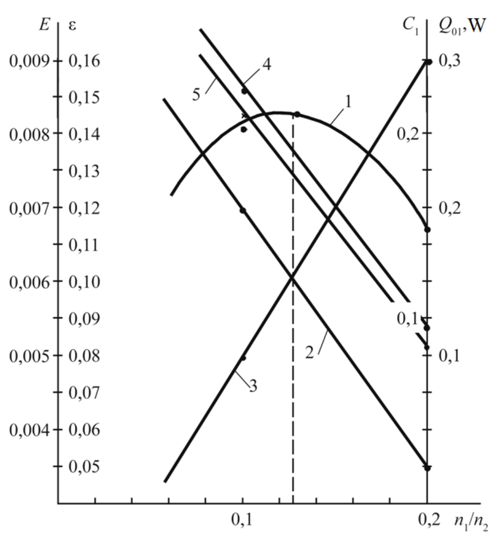


Fig. 14. Dependence of general refrigerating coefficient E and by the cascades $\varepsilon_1$ and $\varepsilon_2$, refrigerating capacity $Q_{01}$ and $C_1$ of two-cascade TED in regime $E_{max}$ on ratio $n_1/n_2$ at T=300 K; ΔT=90 K; $n_1$=9; $(l/s)_i$=10: 1 – $E=f(n_1/n_2)$; 2 – $\varepsilon_1=f(n_1/n_2)$; 3 – $\varepsilon_2=f(n_1/n_2)$; 4 – $Q_{01}=f(n_1/n_2)$; 5 – $C_1=f(n_1/n_2)$

– total probability of failure-free operation $P_\Sigma$ decreases (Fig. 15, pos. 7), in this case, the probability of failure-free operation of the first ($P_1$) and the second ($P_2$) cascades decreases (Fig. 15, pos. 5, 6).

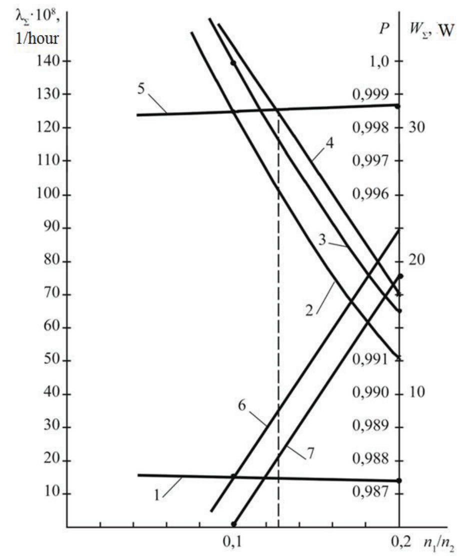


Fig. 15. Dependence of total failure rate $\lambda_\Sigma$ and probability of failure-free operation $P_\Sigma$ and each cascade individually $\lambda_1$ and $\lambda_2$ and $P_1$ and $P_2$ of two-cascade TED in regime $E_{max}$ on ratio $n_1/n_2$ at T=300 K; ΔT=90 K; $n_1$=9; $(l/s)_i$=10, $\lambda_0=3\cdot10^{-8}$ 1/h; t=$10^4$ h: 1 – $\lambda_1=f(n_1/n_2)$; 2 – $\lambda_2=f(n_1/n_2)$; 3 – $\lambda_\Sigma=f(n_1/n_2)$; 4 – $W_\Sigma=f(n_1/n_2)$; 5 – $P_1=f(n_1/n_2)$; 6 – $P_2=f(n_1/n_2)$; 7 – $P_\Sigma=f(n_1/n_2)$

## 5. Discussion of results of analysis of the two-cascade CTED simulation

An analysis of calculation data revealed that there is optimum ratio $n_1/n_2$, corresponding to the maximum of refrigerating coefficient E at the assigned temperature drop ΔT.

In the point of maximum refrigerating coefficient E, one may observe the equality of values of relative temperature drop $\Theta_1$, $\Theta_2$ and refrigerating coefficients $\varepsilon_1$ and $\varepsilon_2$ in the cascades. Results of calculations are given in Table 2.

With an increase in temperature drop ΔT for different designs of TED ($n_1/n_2$=1,0; 0,67; 0,5; 0,33; 0,2; 0,1):

– relative operating currents in the first ($B_1$) and the second ($B_2$) cascades increase;
– magnitude of operating current I also increases;
– intermediate temperature $T_1$ decreases;
– relative temperature drops in the first ($\Theta_1$) and the second ($\Theta_2$) cascades increase;
– refrigerating coefficient E decreases, in this case, refrigerating coefficient of the first cascade $\varepsilon_1$ and the second cascade $\varepsilon_2$ decrease;
– refrigerating capacity $Q_{01}$ and its relative value $C_1$ decrease;
– total power of energy consumption $W_\Sigma$ grows;
– summary voltage drop $U_\Sigma$ grows;
– total magnitude of failure rate $\lambda_\Sigma$ grows;
– total probability of failure-free operation $P_\Sigma$ decreases.

Table 2

Results of calculation of TED basic indicators for the range of temperature drop 60, 70, 80, 90 K

| ΔT, K | $n_1/n_2$ | $B_1$ | $B_2$ | I, A | $T_1$, K | $\Theta_1$ | $\Theta_2$ | $\varepsilon_1$ | $\varepsilon_2$ | E | $Q_{01}$, W | $C_1$ | $W_\Sigma$, W | $U_\Sigma$, W | $\frac{\lambda_\Sigma}{n_1\lambda_0}$ | $\lambda_\Sigma\cdot10^{-8}$, 1/h | P |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 60 | 0,44 | 0,45 | 0,43 | 2,1 | 267,2 | 0,43 | 0,43 | 0,58 | 0,58 | 0,151 | 0,50 | 0,27 | 3,4 | 1,6 | 0,11 | 2,9 | 0,99971 |
| 70 | 0,37 | 0,55 | 0,53 | 2,5 | 260,0 | 0,55 | 0,50 | 0,40 | 0,40 | 0,083 | 0,45 | 0,25 | 5,6 | 2,2 | 0,29 | 7,8 | 0,99922 |
| 80 | 0,23 | 0,69 | 0,66 | 3,1 | 253,0 | 0,63 | 0,63 | 0,22 | 0,22 | 0,034 | 0,38 | 0,24 | 11,6 | 3,7 | 1,0 | 27,0 | 0,9972 |
| 90 | 0,127 | 0,83 | 0,80 | 3,775 | 244,0 | 0,80 | 0,80 | 0,10 | 0,10 | 0,0083 | 0,24 | 0,18 | 31,0 | 8,2 | 4,26 | 115,0 | 0,9880 |

The results obtained are caused by the optimization of current regime of two-cascade thermoelectric coolers under mode of the largest energy effectiveness, which made it possible to decrease energy consumption and to increase the reliability indicators under varied operating conditions.

## 6. Conclusions

1. A model is proposed for the interrelation between failure rate of the two-cascade thermoelectric devices of different designs and the basic parameters of devices under regime of the largest energy effectiveness, which takes the thermal alignment of cascades into account. This provides for the possibility of determining the optimum ratio of thermoelements in the cascades, which corresponds to maximum refrigerating coefficient at the assigned temperature drop.

2. An analysis that we conducted makes it possible to estimate the effectiveness of functioning and to predict the reliability indicators of the two-cascade thermoelectric device of selected design under regime of the largest energy effectiveness under varied conditions of operation.

3. The developed current mode of the largest energy effectiveness of the two-cascade thermoelectric coolers of the assigned design makes it possible to reduce power consumption by 10–30 % and to increase the indicators of reliability by 20–40 % in comparison with traditional methods of constructing the cascade thermoelectric coolers based on the standardized modules.

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