На підставі розв'язання задачі про течію рідини в циліндричному каналі отримані залежності для визначення витрати рідини через шпаринне ущільнення. Визначено вплив випадкової зміни параметрів шпаринних ущільнень на їх витратні характеристики та на об'ємний коефіцієнт корисної дії (ККД) відцентрового насоса. Показано, що при експлуатації насоса його ККД може відчутно зменшуватися

Ключові слова: відцентровий насос, шпаринне ущільнення, витрата, об'ємний ККД, випадкові параметри

На основании решения задачи о течении жидкости в цилиндрическом канале получены зависимости для определения расхода жидкости через щелевое уплотнение. Определено влияние случайного изменения параметров щелевых уплотнений на их расходные характеристики и на объемный коэффициент полезного действия (КПД) центробежного насоса. Показано, что при эксплуатации насоса его КПД может существенно уменьшаться

Ключевые слова: центробежный насос, щелевое уплотнение, расход, объемный КПД, случайные параметры

## 1. Introduction

Annular seals are the most widespread as seals of running part of centrifugal pumps due to the simplicity, low cost and reliability of work in the wide range of pressure drop and rotating frequencies. However, the leakages through the annular seals of impellers and system of autodump of axial forces can be up to 10 % of pump output and it can be about hundreds of kilowatts for powerful multistage feed pumps. That is why it is necessary to take into account the leakages of the pumped over environment through the annular seals under calculation of centrifugal pump efficiency. Because the latter determine the volume efficiency of the pump.

Leakages through the annular seal, as well as forces and moments, acting on a rotor from the side of the pumped over environment, depend on the geometrical parameters of seal, character and velocity of movement of sealing surfaces and mode of liquid flow in a gap. Values of volume efficiency, calculated on the design stage, can substantially differ from those that are actually due to tolerances accepted in pump building and also the possible change of gap geometry under pump exploitation. This fact can destroy all attempts of designers to increase pump efficiency. So, this development of a methodology of centrifugal pump efficiency calculation taking into account the change of basic parameters of annular seals is a new and actual problem of nowadays. Solvation of this problem will allow to determine the efficiency of a centrifugal pump with necessary confidence probability and create more effective machines of this class.

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# RESEARCH OF INFLUENCE OF RANDOM CHANGE OF ANNULAR SEAL PARAMETERS ON EFFICIENCY OF CENTRIFUGAL PUMP

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## 2. Literature review and problem statement

Well-known Navier-Stokes and Reynolds equations of motion of viscid incompressible liquid are the basis of calculation of flow-rate characteristics of annular seals. Basic analytical dependences for determination of flow-rate characteristics of annular seals are got in [1] and later specified by the same author in [2]. The different configurations of seal gaps and sealing surfaces are used to reduce leakages of the pumped over liquid through annular seal. Researches of hydrodynamic characteristics of annular seals with the different forms of sealing surfaces are presented in [3, 4]. So, in [3] numeral-analytical method of research of flow-rate through an interstage annular seal with the set homogeneous roughness of sealing surfaces is given. In [4] the influence of sawtooth pattern yields axial grooves in the stator on leakage is examined. Influence of turbulence model on pressure distribution in the gap of seal is investigated in [5]. Experimental researches of influence of inside flow whirling on distribution of velocities in a gap with smooth surfaces are made in [6]. Experimental researches of leakage for the model of short floating seal are considered in [7]. These data can be used for an estimation of correctness of the got analytical expressions. Lately most researches are conducted numerically, by means of methods of CFD analysis. In [8] the comparison of experimental data and the results of numerical analysis of liquid motion in the annular seal taking into account the whirling of flow in the inlet of gap is given. In [9] the good coincidence of flow rate characteristics of smooth

annular seal taken by 3d CFD analyses with existent experimental data is shown. CFD modeling of "short" and "long" annular seal with taking into account shaft precession and determination and analysis of hydrodynamic characteristics of seals is carried out in [10]. In [11] numeral research of the fields of velocities and pressure in seals with different roughnesses of sealing surfaces and in labyrinth seals is made. Numeral methods allow not only to increase exactness of calculation but also to solve the problem of optimization of leakage through annular seal, as it is done, for example, in [12]. However, the results of both analytical and numeral calculations agree well with experimental data only for the set (calculation) parameters and areas of work of annular seals. Most researchers solve a problem in the deterministic statement, not taking into account the random change of parameters of seals and probabilistic nature of its geometrical characteristics. In actual fact, accepted in pump building tolerances on the sizes of sealing surfaces, as a rule, are comparable with the size of gap in annular seal, and relative variation of gap values can be  $\pm(10...30)$  %, that reduce to zero any efforts to increase the exactness of calculation.

The analysis of theoretical and experimental works devoted to the calculation of flow-rate characteristics of annular seals has shown that works in that the random character of change of annular seal parameters is taken into account are practically absent. Thus, the determination of influence of random change of annular seal parameters on its flow-rate characteristics and pump efficiency is an important practical problem.

### 3. Object and problems of research

Determination of influence of random change of annular seals parameters on their flow-rate characteristics and volume efficiency of multistage centrifugal pump is the goal of this work.

To achieve this goal such problems must be solved:

 determination of flow-rate components through the annular seal as functions of basic geometrical and regime parameters;

 determination of probabilistic characteristics of centrifugal pump efficiency.

# 4. Determination of flow-rate components through annular seal

The flow of viscid incompressible liquid in cylindric gap is described by the system of Reynolds equations [3], and after the estimation of members can be presented by two equations:

$$\frac{\partial p}{\partial y} = 0, \quad \rho \left( \frac{\partial w}{\partial t} + w \frac{\partial w}{\partial z} \right) = -\frac{\partial p}{\partial z} + \mu \frac{\partial^2 w}{\partial y^2} - \rho \frac{\partial}{\partial y} \overline{v'w'}. \tag{1}$$

The changes of flow parameters on a circle direction are conditioned only by variables on the coordinate x by the maximum values of pressure and velocities.

As a liquid fills a circular gap fully, then to the system of equations (1) the equation of continuity can be added:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0.$$
 (2)

To determinate the leakages through the annular seal such calculation scheme of annular channel was used [3]: axis of the sleeve is fixed, axis of the shaft rotates around sleeve axis with precession frequency  $\Omega$  and simultaneously makes small radial ( $e_0 = e_m \text{cosvt}$ ) and angular ( $\vartheta_x = \vartheta_{x0} \cos vt$ ,  $\vartheta_y = \vartheta_{y0} \cos vt$ ) harmonic oscillations in two perpendicular planes yOx and xOz. Except the own rotation of shaft with frequency  $\omega_1$  the rotation of external wall of the seal around the axis of shaft or immobile axis of sleeve with frequency  $\omega_2$  is also taken into account. Calculation is made in the movable system of coordinates xyz, where the axis of  $O_2y$  is directed on the lines of centers  $OO_1$  in the middle cross section of channel and rotates together with the line of centers with a constant frequency of precession  $\Omega$ .

Gap in any crossing can be expressed as:

$$\mathbf{h} = \mathbf{H} \left( 1 - \varepsilon \cos \phi - \left[ \theta_0 + \theta_x \cos \phi + \theta_y \sin \phi \right] \overline{z} \right)$$

or in dimensionless form

 $\overline{h} = \frac{h}{H} = y_*(1 - \alpha \overline{z}),$ 

where

$$H = R - r, \quad \varepsilon = \frac{e_0}{H}, \quad \theta_0 = \frac{\vartheta l}{2H},$$
$$\theta_{x,y} = \frac{\vartheta_{x,y} l}{2H}, \quad \overline{z} = \frac{2z}{l}, \quad y_* = 1 - \varepsilon \cos\phi,$$
$$\alpha = \frac{\theta}{y_*} = \theta_0 \frac{\vartheta}{y_*},$$
$$\vartheta = \frac{\vartheta_0 + \vartheta_x \cos\phi + \vartheta_y \sin\phi}{\vartheta_0} - \text{dimensionless values.}$$

Averaging on the gap thickness the inertia forces, included in the equation (1), the equation of motion is possible to bring to the form:

$$\frac{\partial}{\partial \overline{z}} \left[ \frac{\overline{h}^{3}}{K} \left( \frac{\partial p}{\partial \overline{z}} + \frac{\rho l}{2h} \left[ \dot{q} + \kappa \omega_{a} \frac{\partial q}{\partial \phi} + \frac{2}{l} \left( \overline{w} \frac{\partial q}{\partial \overline{z}} + q \frac{\partial \overline{w}}{\partial \overline{z}} \right) \right] \right) \right] = \frac{\mu l^{2} k_{0}}{4H^{3}} (Q_{e} + Q_{\vartheta} \overline{z}),$$
(3)

where

$$\begin{split} k_{0} &= \frac{C}{8} R e_{0}^{1-n}, \ R e_{0} = \frac{2\rho q_{0}}{\mu}, \ K = \left(\frac{q}{q_{0}}\right)^{1-n}, \\ q_{0} &= w_{0} H = \frac{\Delta p H^{3}}{\mu l k_{0}} = \left(\frac{2\Delta p H^{2}}{\rho \zeta_{20}}\right)^{0.5} = \left(\frac{4\Delta p H^{3}}{\rho l C} \left(\frac{2\rho}{\mu}\right)^{n}\right)^{\frac{1}{2-n}}, \\ \zeta_{20} &= \frac{\lambda_{0} l}{2H}, \ \lambda_{0} = \frac{C}{R e_{0}^{n}} = \frac{8k_{0}}{R e_{0}}, \ \Delta \omega = \Omega - \kappa \omega_{a}, \\ Q_{e} &= -\left(\Delta \omega \epsilon \sin \phi + \dot{\epsilon} \cos \phi\right) = \frac{1}{H} \left(\Delta \omega y \sin \phi + \dot{y} \cos \phi\right), \\ \left(\epsilon = \frac{e_{0}}{H}, e_{0} = -y\right), \end{split}$$

$$Q_{\vartheta} = -\left[\left(\dot{\theta}_{y} + \Delta\omega\theta_{x}\right)\sin\phi + \left(\dot{\theta}_{x} - \Delta\omega\theta_{y}\right)\cos\phi\right]$$

where  $q_0$  – elementary flow-rate through a concentric annular channel with the constant value of the gap H,  $\zeta_{20}$  – a coefficient of hydraulic losses on a friction at length of such channel.

Averaging on the gap thickness the equation of continuity (2), for the short annular seal with a prevailing axial flow we get

$$\frac{\partial}{\partial \bar{z}} \left( \overline{w} h \right) = -\frac{1}{2} H \left( Q_e + Q_\vartheta \overline{z} \right).$$
(4)

By integrating the equation of continuity (4) over the length of the channel it is possible to determine the elementary flow-rate through the annular channel:

$$q_{*} - 0.5 lH \left( Q_{e} \bar{z} + 0.5 Q_{\theta} \bar{z}^{2} \right),$$
  
$$\overline{w}h = q = q_{*} - 0.5 lH \left( Q_{e} \bar{z} + 0.5 Q_{\theta} \bar{z}^{2} \right),$$
 (5)

where  $q_*$  – flow-rate in the middle cross section  $\overline{z} = 0$ . Flow-rate in any cross section of the channel:

$$q = q_{p} + q_{dz} + q_{g},$$

$$q_{p} = \frac{\Delta p h_{*}^{3}}{\mu l k_{0} K},$$

$$q_{dz} = q_{d} - \frac{l H}{2} \left( Q_{e} \overline{z} + \frac{Q_{\vartheta}}{2} \overline{z}^{2} \right) =$$

$$= \frac{l H}{2} \left[ Q_{e} \left( \alpha - \overline{z} \right) + \frac{Q_{\vartheta}}{2} \left( \frac{1}{3} - \overline{z}^{2} \right) \right].$$

Full flow-rate through annular seal in a set time moment can be derived by integrating elementary components of flow-rate over the circle:

$$\mathbf{Q} = \int_{0}^{2\pi} \mathbf{q} \mathbf{r} \mathbf{d} \boldsymbol{\phi}.$$

In general, the flow-rate depends on time due to variable boundary data. Effective flow-rate, which has a practical value, is determined by averaging its momentary value over the period of oscillations  $T=2\pi/\nu$ :

$$\label{eq:Qm} \boldsymbol{Q}_{\mathrm{m}} = \frac{1}{T} \int\limits_{0}^{T} \boldsymbol{Q} dt = \frac{1}{2\pi} \int\limits_{0}^{2\pi} \boldsymbol{Q} d\big(\nu t\big).$$

Flow-rate of enforced flow for the laminar flow, averaged over the period of oscillation is:

$$\begin{aligned} \mathbf{Q}_{\mathrm{pm}} &= \left[ -\frac{12\mu l}{H\rho(\Delta\zeta - 2\alpha\zeta_{\mathrm{m}})} + \sqrt{\frac{2\Delta p H^2}{\rho(\Delta\zeta - 2\alpha\zeta_{\mathrm{m}})}} + \right. \\ &+ \frac{72\mu^2 l^2}{H^3} \sqrt{\frac{2}{\Delta p \left[ \rho(\Delta\zeta - 2\alpha\zeta_{\mathrm{m}}) \right]^3}} \right] 2\pi r + \\ &+ 0.5 \left[ \frac{72\mu^2 l^2}{H^3} \sqrt{\frac{72}{\Delta p \left[ \rho(\Delta\zeta - 2\alpha\zeta_{\mathrm{m}}) \right]^3}} - \frac{12\mu l}{\rho(\Delta\zeta - 2\alpha\zeta_{\mathrm{m}})} \right] \pi r \epsilon_{\mathrm{m}}^2; \end{aligned}$$

Pressure component of flow-rate for the turbulent flow is:

$$= Q_{p0} \left[ 1 + 0.153\varepsilon_m^2 - \frac{0.214}{\zeta_t} (\Delta \zeta \varepsilon_m + 2\zeta_m (\theta_{xm} - 0.43\theta_0 \varepsilon_m)) \varepsilon_m \right], \quad (6)$$

where  $Q_{p0}=2\pi r q_{p0}$  – full flow-rate with taking into account hydraulic resistance through the channel with fixed surfaces. Inertial component of flow rate:

$$\begin{split} Q_{gm} &= -\frac{\rho l H^3 \pi r K_t^{\frac{3}{4}}}{6 \mu k_0} (\theta_{xm} - 0.5 \epsilon_m) (\omega^2 + \Omega \Delta \omega) \epsilon_m - \\ &- \frac{\rho H^2 q_{po}}{\mu k_0 K_t^{\frac{1}{4}}} \frac{\zeta_m}{\zeta_t} \Delta \omega \epsilon_m \theta_{ym} - \\ &- \frac{4\rho H}{\mu k_0 l K_t^{\frac{5}{4}}} q_0^2 \pi r \Big[ \Big( \theta_0 + 0.357 \big( \theta_{xm} - \\ &- 0.15 \zeta_m \theta_0 \theta_{xm} + 0.624 \lambda_{00} (\Delta \zeta - 2 \theta_0 \zeta_m) \big) \Big) \epsilon_m + 2 \zeta_m \Big( \theta_{xm}^2 + \theta_{ym}^2 \Big) \Big]. \end{split}$$

Pressure component of flow-rate for automodel area of turbulent flow:

$$Q_{pm} = Q_{p0} \left[ 1 + \frac{1}{2} \left( \frac{3}{16} - \frac{3}{8} \frac{\Delta \zeta}{\zeta_0} \right) \varepsilon_m^2 - \frac{3}{8} \frac{\zeta_m}{\zeta_0} \theta_{xm} \varepsilon_m \right].$$
(7)

Inertial component:

$$\begin{split} \mathbf{Q}_{\mathrm{gm}} &= -\frac{\rho \mathbf{H}^{3}}{6\mu k_{0}} \sqrt{\frac{\zeta_{20}}{\zeta_{0}}} \pi r(\boldsymbol{\theta}_{\mathrm{xm}} + 1.73\boldsymbol{\theta}_{0}\boldsymbol{\varepsilon}_{\mathrm{m}})(\boldsymbol{\omega}^{2} + \boldsymbol{\Omega}\Delta\boldsymbol{\omega})\boldsymbol{\varepsilon}_{\mathrm{m}} - \\ &- \frac{4\rho \mathbf{H}}{\mu k_{0} \mathbf{l}} q_{0}^{2} \sqrt{\frac{\zeta_{20}}{\zeta_{0}}} \pi r \left(\boldsymbol{\theta}_{0} + 0.5 \left(\frac{\Delta\zeta}{2\zeta_{0}}\boldsymbol{\varepsilon}_{\mathrm{m}} + \frac{\zeta_{\mathrm{m}}}{\zeta_{0}}\boldsymbol{\theta}_{\mathrm{xm}}\right) \boldsymbol{\theta}_{\mathrm{xm}} + 0.5 \frac{\zeta_{\mathrm{m}}}{\zeta_{0}} \boldsymbol{\theta}_{\mathrm{ym}}^{2}\right). \end{split}$$

According to the got expressions an inertia component of flow-rate contains a time-independent constant value. This constant value is caused by dissimilarity of the velocities field in axial direction (the component  $q_p \frac{\partial w_p}{\partial \overline{z}}$  in the Reynolds' equation), namely by the change of axial velocity as a result of tapered form of the channel. Other components of the inertia flow-rate are determined by the radial and angular displacements and angular velocities of sealing surfaces.

According to the made calculations the flow-rate of expulsing flow is two orders of magnitude less than flow-rate component caused by a pressure flow.

The operability of annular seal is largely determined by the size of radial gap between shaft and sleeve. Sealing surfaces are subject to erosive wear under exploitation and that is why the radial gap increases with time. Even the insignificant changes of radial gap result in the change of calculation static and dynamic characteristics of seal. So, the prediction of wear of sealing surfaces and determination of flow-rate characteristics and critical frequencies taking into account changing in time value of gap is an actual and necessary problem. To solve this problem, it is necessary to know the intensiveness of sealing surfaces materials wear.

On the basis of experimental data from [13] the changing over time of radial gap in annular seal with taking into account erosive wear of the sealing surfaces can be presented as: - for the laminar flow

$$H = \frac{H_0}{\left(1 - k_i (2n_i - 1) \left(\frac{\Delta p}{12\mu l}\right)^{n_i} t\right)^{\frac{1}{2n_i - 1}}};$$
(8)

- for automodel area of turbulent flow

$$H = \frac{H_0}{\left(1 - k_i (0.5n_i - 1) \left(10 \left(\frac{\Delta p}{\rho l}\right)^{0.5}\right)^{n_i} t\right)^{\frac{1}{0.5n_i - 1}}},$$
(9)

where k<sub>i</sub> - wear characterizing material and terms of work,

$$n_{i} = \begin{cases} 2, & w < 60 \text{ m / s}, \\ 3, & w \ge 60 \text{ m / s}. \end{cases}$$

Further, the value of radial gap, that is calculated by the formulas (8), (9) is taken as the expected value of gap for the calculation moment of time.

Pump efficiency is determined by this formula:

$$\eta = \eta_{\rm m} \cdot \eta_{\rm v} \cdot \eta_{\rm h}, \tag{10}$$

where  $\eta_m$  – mechanical efficiency,  $\eta_v$  – volume efficiency,  $\eta_{\rm h}$  – hydraulic efficiency.

Leakages through annular seal do not affect the mechanical and hydraulic efficiency. So, change of volume efficiency caused by random changing of annular seals parameters and erosive wear of sealing surfaces will be examined. Volume efficiency is determined by the formula:

$$\eta_{v} = \frac{N'}{N_{h}} = \frac{N_{h} - N_{v}}{N_{h}} = \frac{\rho g Q_{k} H_{T} - \rho g q_{k} H_{T}}{\rho g Q_{k} H_{T}} = \frac{\rho g H_{T} (Q_{k} - q_{k})}{\rho g Q_{k} H_{T}} = \frac{\rho g H_{T} Q}{\rho g Q_{k} H_{T}} = \frac{Q}{Q_{k}} = \frac{Q}{Q + q_{k}},$$
(11)

where  $Q_k = Q + q_k$ ;  $q_k$  - leakage through annular seal of impeller;  $Q_k$  – amount of liquid at the output of the impeller;  $N_h = \rho g Q_k H_T$  – hydraulic capacity;  $H_T$  – theoretical pump head.

Taper, eccentricity and angels of defect in the seals of the real machines depend on many random factors, that is why they are random values too. Thus, the determination of dynamic characteristics of seals must be examined from the point of view of probability theory.

To define the probability distribution of pump efficiency it is necessary to know multidimensional (compatible) distribution of the system of random values determining flow-rate through annular seal: H,  $\theta$ , e,  $\Delta p$ . An eccentricity, radial gap, taper of seal and pressure drop are independent random values, however the limits of change of eccentricity are determined by the value of radial gap. Therefore, the probability density function can be written down as:

$$f(\eta_v) = f(H, \vartheta, e, \Delta p) = f_1(H)f_2(\vartheta)f_3(e|H)f_4(\Delta p),$$

where  $f_3(e|H)$  – probability density function of eccentricity under H=const.

All directions are equivalent for the relative displacement of rotor and it can take on only positive values, that is why its probability density function can be described by the truncated law of Rayleigh

$$f_5(e) = \frac{7.716}{1 - e^{-3.86}} \frac{e}{H^2} \exp\left(-\frac{3.858e^2}{H^2}\right)$$
$$= 7.882 \frac{e}{H^2} \exp\left(-\frac{3.858e^2}{H^2}\right).$$

Pressure drop, middle radial gap and taper of annular seal substantially influence flow-rate through seal and are random values. In productive statistics for parameters that have bilateral tolerances, normal distribution is usually applied, modifying it accordingly by physical statistical problem specifications. Coming from it, probability density function of radial gap, angle of taper of seal and pressure drop can be expressed in the form:

$$f_{1}(H) = \frac{1}{\sigma_{H}\sqrt{2\pi}} \exp\left[-\frac{(H-m_{H})^{2}}{2\sigma_{H}^{2}}\right],$$
$$f_{2}(\vartheta) = \frac{1}{\sigma_{\vartheta}\sqrt{2\pi}} \exp\left[-\frac{(\vartheta-m_{\vartheta})^{2}}{2\sigma_{\vartheta}^{2}}\right],$$
$$f_{4}(\Delta p) = \frac{1}{\sigma_{\Delta p}\sqrt{2\pi}} \exp\left[-\frac{(\Delta p - m_{\Delta p})^{2}}{2\sigma_{\Delta p}^{2}}\right],$$

and limits of changes of these values (H\_{min}, H\_{max}, \vartheta\_{min}, \vartheta\_{max},  $p_{min}, p_{max}$ ) are determined by manufacturing tolerances (H is  $[2\cdot10^{-4}; 4\cdot10^{-4}]$ ) and by limits of possible pulsation of pressure. Distribution function of efficiency:

$$\begin{split} F(\eta) &= \int_{(\Omega)_{HibeAp}} f(H, \vartheta, e, \Delta p) dH d\vartheta ded \Delta p = \\ &= \int_{(\Omega)_{HibeAp}} f_1(H) f_2(\vartheta) f_3(e|H) f_4(\Delta p) dH d\vartheta ded \Delta p, \end{split}$$
(12)

where the area of integration  $(\Omega)_{H\theta e \Delta p}$  is determined by the inequality:

$$(\Omega)_{H \vartheta e \Delta p} < \frac{Q}{Q + q_{k^*}}.$$

Flow-rate through annular seal for automodel area of turbulent flow is determined by (7).

Taking into account dependences (7) and (11), the latter correlation can be solved with respect to pressure drop:

$$\Delta p = \frac{Q^2}{\eta_v^2} \frac{\rho l(1-\eta_v)}{400 H^3 \pi^2 r^2} \left(1+0.306 \frac{e^2}{H^2}\right)^{-2}.$$

As  $\Delta p$  is a one-valued function  $\eta_v$  in the area of possible values  $\Delta p$ ,  $\eta_{v}$ , an integral (12) can be brought to the triple integral with the set limits of integration:

$$F(\eta_{v}) = \int_{\eta_{v_{max}}}^{\eta_{v_{max}}} d\eta_{v} \int_{p_{min}}^{p_{max}} \int_{H_{min}}^{H_{max}} \int_{0}^{H} f\left(e, H, \Delta p, \frac{Q^{2}}{\eta_{v}^{2}} \frac{\rho l(1-\eta_{v})}{400H^{3}\pi^{2}r^{2}} \left(1+0.306\frac{e^{2}}{H^{2}}\right)^{-2}\right) \left|\frac{\partial \Delta p}{\partial \eta_{v}}\right| dedHd\eta_{v},$$

then

$$f(\eta_{v}) = \int_{p_{min}}^{p_{max}} f_{4}(\Delta p) \int_{H_{min}}^{H_{max}} f_{1}(H) \int_{0}^{H} f_{3}(e|H) f_{4}\left(\frac{Q^{2}}{\eta_{v}^{2}} \frac{\rho l(1-\eta_{v})}{400H^{3}\pi^{2}r^{2}} \left(1+0.306\frac{e^{2}}{H^{2}}\right)^{-2}\right) dedHd\Delta p, (13)$$

under

 $0 \le Q_p \le Q_{p^*}$ ;  $f(Q_p)=0$ , and  $Q_p < 0$  and  $Q_p > Q_{p^*}$ .

By means of the got probability density function (13) it is possible to define basic momentum characteristics of volume pump efficiency (expected value, dispersion) for set parameters of annular seal and limits of their change.

### 6. A discussion of influence of random change of annular seal parameters on centrifugal pump efficiency

To examine the influence of random change of geometrical parameters and erosive wear of surfaces of annular seal the calculation of pump efficiency was conducted for the pump SD 800/32 – cantilever type with one impeller.

Because turbulent flows are dominant for annular seals in centrifugal pumps, the probabilistic characteristics of pump efficiency were determined only for turbulent flows and as functions of random values sizes H,  $\vartheta$ , e,  $\Delta p$  taking into account local hydraulic resistances. Expected value of volume efficiency for automodel area of turbulent flow (H=3·10<sup>-4</sup> m) is:  $m_{\eta \nu}$ =0.963. Under nominal mode of pump exploitation the value of

Under nominal mode of pump exploitation the value of the annular seal gap is H=3·10<sup>-4</sup> m and the volume efficiency of pump equals  $\eta_v$ =0.965. After 10 000 h pump exploitation, the gap according to (9) increases to H=3.93·10<sup>-4</sup> m and volume efficiency becomes  $\eta_v$ =0.952. This is in that case, when variation of the gap value is only 10 % of nominal. So, pump efficiency decreases under enlarging of relative variation of gap value. It means that pump efficiency with one stage can change more than by 1 % due to tolerances and erosive wear of sealing surfaces. It is possible to expect that such decreasing can be more significant for multi-stage pumps, at

that time as designers try to increase pump efficiency at least by the tenths of a percent.

Basic advantage of undertaken researches is an attempt of taking into account-the influence of random dispersions of basic geometrical and regime parameters of annular seals on their flow-rate characteristics.

The results of researches show that taking into account the influence of random change of annular seal parameters is useful to correct the centrifugal pump efficiency values. Presented methodology and got expressions can be used for correction of centrifugal pumps efficiency values both that are developed and those that are under exploitation.

Research is continuation of analysis of hydrodynamic characteristics of the system "rotor – bearings – seals" of centrifugal pump. In further the modernization of researches of influence of random variations of basic geometrical and regime parameters of annular seals on dynamic behavior of this system and on the oscillation state of pump are planned.

#### 7. Conclusions

1. On the basis of problem solving of liquid flow in a channel with the internal surface of gap that rotates and precesses, the components of flow-rate through annular seal are determined as functions of basic geometrical and regime parameters. In particular, it is shown that an inertia component of flow-rate contains a constant, time-independent term caused by discontinuity of the velocities field in axial direction and other terms are determined by the radial and angular displacements and angular velocities of sealing surfaces. It is also shown that expulsing flow-rate is two orders of magnitude less than flow-rate, caused by a pressure flow.

2. Probabilistic characteristics of centrifugal pump efficiency are determined taking into account influence of random variations of basic geometrical and regime parameters of annular seals under the assumption that all parameters are casual values. It is shown that pump efficiency can significantly decreases under pump exploitation and such decreasing is more appreciable for multistage pumps. With the increase of the amount of pump stages the reduction of efficiency will increase too.

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