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Дослідження проведено у рамках дискретної моделі гнучкого двохопорного ротора, що балансується двома пасивними автобалансирами, розташованими біля опор. Отримано систему диференціальних рівнянь, що описують процес автобалансування. Встановлено, що основні рухи, за умови їх існування, є стійкими на зарезонансних швидкостях обертання ротора. Проведено оцінку перебігу перехідних процесів за коренями характеристичного рівняння

Ключові слова: гнучкий ротор, автобалансир, автобалансування, основні рухи, стійкість, перехідні процеси

Исследования проведены в рамках дискретной модели гибкого двухопорного ротора, балансируемого двумя пассивными автобалансирами, расположенными возле опор. Получена система дифференциальных уравнений, описывающая процесс автобалансировки. Установлено, что основные движения, при условии их существования, устойчивы на зарезонансных скоростях вращения ротора. Проведена оценка протекания переходных процессов по корням характеристического уравнения

Ключевые слова: гибкий ротор, автобалансир, автобалансировка, основные движения, устойчивость, переходные процессы

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RESEARCH OF STABILITY AND TRANSITION PROCESSES OF THE FLEXIBLE DOUBLE-SUPPORT ROTOR WITH AUTO-BALANCERS NEAR SUPPORT

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1. Introduction

Rotors of many gas turbine engines, turbine units, centrifugal machines and so forth work at high velocities of rotation. Thereof they behave as flexible [1, 2]. The form and rotor unbalance of the flexible rotor depend on the current velocity of rotation, change from temperature, wear of the rotor and so forth. Therefore, the balance of such rotors during operation is worthwhile to be constantly corrected by passive auto-balancers [3–13]. In the latter corrective weights (pendulums, balls or rollers) under certain conditions come to positions in which they counterbalance the rotor. Then carry out steady state motions which it is accepted to call the main. There are also secondary motions on which auto-balancing does not come and auto-balancers increase vibrations of the rotor.

For balancing of a wide class of flexible rotors by passive auto-balancers in practice it is necessary to have a certain method which operability is theoretically proved. At justification of the method the conditions of existence and stability of the main motions (conditions of occur of auto-balancing) are defined and transition processes are estimated.

The most perspective method of balancing of flexible rotors on two pliable supports is the method in which two auto-balancers are located as close as possible to supports [3]. Below operability of this method is proved.

2. Literature review and problem statement

In the work [3] the method of auto-balancing of flexible rotors on two pliable supports in which the rotor is balanced with two auto-balancers located as close as possible to supports is offered. Let's notice that when balancing the flexible rotor in two [4] and more [5] correction planes which are not matching supports, reactions of supports (vibration in supports) are not eliminated. Also with the growth of angular rotor velocity the auto-balancing serially comes ore disappears. In the new method deflections of the rotor are not eliminated, but vibrations in supports are eliminated. It is expected, that the auto-balancing will come on a broader range of the rotor velocities, than at rotor balancing in not supporting planes.

In the work [6] the discrete model of the flexible rotor on two pliable supports with two multi-ball (multi-roller, multi-pendulum) auto-balancers located near supports is constructed. The system of differential equations of motion of the auto-balanced rotor system and the system of differential equations describing the process of auto-balancing are received. The latter is made with reference to the coordinates of discrete masses of the rotor and total rotor unbalances (from rotor and auto-balancers) in two correction planes (supports). The conditions of existence of main motions are established. It is shown that when approaching to critical velocities the balancing capacity of auto-balancers in view of significant increase in the total rotor unbalances given to two correction planes can't be enough.

For further justification of the method of balancing of flexible rotors it is necessary to investigate the stability of main motions, to estimate transition processes and the influence of parameters of the auto-balanced system on their passing.

The system of differential equations received in the work [6] describing the process of occur of auto-balancing has a high order and it is difficult to investigate it analytically.

It should be noted that the empirical stability criterion of main motions [7] and empirical (engineering) criterion of occur of auto-balancing [8] are the most effective methods of definition of conditions of occur of auto-balancing. But these criteria allow to determine only resonant or critical rotor velocities through which upon transition the auto-balancing comes or disappear. For research of transition processes they are inapplicable.

More bulky and therefore less suitable is the method of definition of conditions of occur of auto-balancing based on the method of synchronization of dynamic systems [9]. For example, the conditions of occur of auto-balancing at rotor balancing, by two-ball auto-balancers in one [10] and two [11] correction planes were determined during its application. In the work [12] the latter task was investigated at insignificant anisotropy of supports. This method does not allow investigating a stability of families of steady-state motions. When accounting forces of resistance it becomes too bulky for application.

In the works [13, 14] transition processes in multi-ball auto-balancers with the minimum quantity of the special parameters setting the main motions of the rotor system – Lagrange coordinates of the rotor and special combinations of the corners describing the position of balls were analytically investigated. The theory of stability of stationary motions of nonlinear autonomous systems was used. In the work [13] researches were conducted within the flat model, and in the work [14] – within the space model of the rigid rotor. In the developed approach the final mass of corrective weights is considered. Therefore, differential equations of motion of the auto-balanced system are too bulky and hardly amenable to calculus.

In the work [15] stability of dynamic balancing of the rigid rotor placed in the body on two pliable supports, by two passive auto-balancers was investigated. Research is conducted with the minimum quantity of the dynamic variables describing the process of auto-balancing of the rotor: two generalized complex coordinates of the rotor; two complex total rotor unbalances and auto-balancer.

So, the system of differential equations received in the work [6] describing the process of occur of auto-balancing can be investigated analytically only at additional assumptions.

3. Purpose and research problems

The purpose of researches is research of stability of main motions of the flexible two-support rotor at its balancing by two auto-balancers located near supports and assessment of features of passing of the transition processes arising at the specified auto-balancing.

For achievement of the goal it is necessary to solve the following research problems:

1) to prove the assumptions of passing of the process of auto-balancing allowing to investigate analytically differential equations, describing this process;

2) to receive stability conditions of main motions;

3) to establish the features of passing of the transition processes arising when balancing the flexible rotor and to estimate the influence of parameters of the flexible rotor on the duration of passing of transition processes.

4. Methods of research of stability of main motions when balancing the flexible rotor by two passive auto-balancers

Researches are conducted within the discrete model of the flexible two-support rotor with passive auto-balancers with many corrective weights (Fig. 1) described in the work [6].

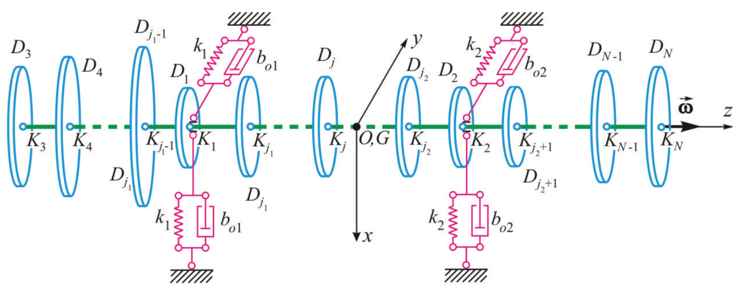

Fig. 1. Discrete model of flexible two-support rotor

The system of differential equations received in the work [6] describing the process of auto-balancing of the rotor within the specified model is used:

$$\mathrm{Left}_j = M_{\Sigma j} D_\tau^2 \Xi_{zj} - L_j^T K(Z - \Xi_{z1} L_1 - \Xi_{z2} L_2) + b_{oj} D_\tau \Xi_{zj} + k_j \Xi_{zj} + D_\tau^2 S_{z,\Sigma j} + \omega^2 L_j^T K (K - \omega^2 M)^{-1} S_{0,z} = 0,$$

$$\overline{\mathrm{Left}_j} = 0, \quad /j = 1, 2/,$$

$$\text{Left}=MD_\tau^2 Z+K(Z-\Xi_{z1}L_1-\Xi_{z2}L_2)-\omega^2 S_{0,z}=0,\ \overline{\text{Left}}=0, \qquad (1)$$

$$\text{Left}_{2N+j}=\ddot{S}_{z,\Sigma j}+b_j/(m_j\kappa_j)\cdot\dot{S}_{z,\Sigma j}+$$
$$+n_j m_j/(2\kappa_j)\cdot(D_\tau^2\Xi_{zj}-p_j e^{i\vartheta_j}\bar{D}_\tau^2\bar{\Xi}_{zj})=0,$$

$$\overline{\text{Left}}_{2N+j}=0,\ /j=1,2/. \qquad (2)$$

In the equations above:

$$M_{\Sigma j}=M_j+m_{0j}+n_j m_j,\quad /j=1,2/,$$

$$M_{\Sigma j}=M_j+m_{0j},\ /j=\overline{3,N}/$$

is the total mass of j disk, where M_j, $/j=\overline{1,N}/$ – mass of j disk; $m_{0,j}$, $/j=\overline{1,N}/$ – the dot mass forming static unbalance of $s_{0,j}$ in j disk; m_j, n_j, /j=1,2/– respectively, the mass and quantity of corrective weights in j auto-balancer; $D_\tau\bullet=\dot{\bullet}+i\omega\cdot\bullet$ – the differential operator, where i – conventional unit, ω – angular rotor velocity; $\Xi_{zj}=(x_j+iy_j)e^{-i\omega t}$, $/j=\overline{1,N}/$ – Lagrange coordinates of the centers of mass of disks in the moving coordinate system, where x_j, y_j – Lagrange coordinates of the centers of mass of disks in the fixed coordinate system;

$$L_1=(1-\tilde{z}_3,1-\tilde{z}_4,...,1-\tilde{z}_N)^T,\quad L_2=(\tilde{z}_3,\tilde{z}_4,...,\tilde{z}_N)^T$$

is vectors, in which $\tilde{z}_j=(z_j-z_1)/(z_2-z_1)$, $/j=\overline{3,N}/$, where z_j, $/j=\overline{1,N}/$ – longitudinal coordinate of the center of mass of j disk;

$K=(k_{ij})$, $/i,j=\overline{3,N}/$ – stiffness matrix, where k_{ij}, $/i,j=\overline{3,N}/$ – rigidity coefficients (the latter are defined as the size of static vertical force which needs to be put in the shaft K_i point in order that its bend was resulted by the single shift of the point K_j); $Z=(\Xi_{z3},\Xi_{z4},...,\Xi_{zN})^T$ – the vector of displacements of the rotor shaft in non-reference points; k_j, b_{oj}, /j=1,2/– respectively, coefficients of rigidity and viscosity of supports;

$$S_{z,\Sigma j}=S_{zj}+L_j^T S_{0,z}+L_j^T MZ_0,\ /j=1,2/ \qquad (3)$$

is the total rotor unbalances of the flexible rotor given to two correction planes (the given total rotor unbalances);

$$Z_0=(\Xi_{z30},\Xi_{z40},...,\Xi_{zN0})^T=\omega^2(K-\omega^2 M)^{-1}S_{0,z} \qquad (4)$$

is the vector made of shaft deflections in the planes of not support disks of the pseudo-rigid rotor; $S_{zj}=m_j r_j e^{-i\omega t}\sum_{i=0}^{n_j}e^{i\phi_{i,j}}$, $/j=1,2/$ – total rotor unbalances in correction planes, where $\phi_{i,j}$, $/i=\overline{1,n_j}/$ – the angles defining positions of corrective weights;

$$M=\text{diag}(M_{\Sigma 3},M_{\Sigma 4},...,M_{\Sigma N});$$

$S_{0,z}=(S_{0,z3},S_{0,z4},...,S_{0,zN})^T$ – vector, where $S_{0,zj}=s_{0,j}e^{i(\phi_{0,j}-\omega t)}$, $/j=\overline{3,N}/$ – static unbalances of not support disks; $\kappa_j=1;7/5;3/2$, $/j=1,2/$ – respectively, for pendulums, balls and cylindrical rollers;

$$p_j=\sqrt{p_{1j}^2+p_{2j}^2},/j=1,2/;$$

$$p_{1j}=\left(\sum_{i=1}^{n_j}\cos 2\tilde{\psi}_{i,j}\right)/n_j,\ p_{2j}=\left(\sum_{i=1}^{n_j}\sin 2\tilde{\psi}_{i,j}\right)/n_j,$$

$$\vartheta_j=\arccos(p_{1j}/p_j)/2;$$

$\tilde{\psi}_{i,j}$, $/i=\overline{1,n_j},j=1,2/$ – the angles setting positions of corrective weights on main motion.

It is considered that the flexible rotor is rather rigid, that is its resonant velocities are much less than first critical velocity.

It is supposed that after the flexible rotor reached the cruiser velocity of rotation (close to the first critical or higher) the rotor shaft at first quickly enough caves in and further behaves as pseudo-rigid.

Stability of main motions of the pseudo-rigid rotor is investigated using the Lagrange coordinates offered in the work [6] – Ξ_{zj}, $S_{\Sigma j}$, / j=1,2/, setting, respectively, the coordinates of the centers of mass of support disks and the given total rotor unbalances.

On the basis of the assumptions from the system (1), (2) the simplified system of differential equations describing the process of auto-balancing of the flexible rotor within the model is obtained.

It is shown that the received system of equations accurate within designations matches the system of differential equations describing the process of dynamic balancing of the rigid rotor on pliable supports by two auto-balancers [15].

5. Results of research of stability of main motions of flexible rotors at their balancing by passive auto-balancers

5. 1. Receiving the equations for research of stability of main motions of the flexible rotor

The coordinates of the centers of mass of not support and support disks for the pseudo-rigid rotor are connected as follows

$$Z=Z_0+L_1\Xi_{z1}+L_2\Xi_{z2}.$$

Let's substitute Z in the equations (1):

$$\text{Left}_j=M_{\Sigma j}D_\tau^2\Xi_{zj}-L_j^T KZ_0+b_{oj}D_\tau\Xi_{zj}+k_j\Xi_{zj}+$$
$$+D_\tau^2 S_{\Sigma j}+\omega^2 L_j^T K(K-\omega^2 M)^{-1}S_{0,z}=0,$$

$$\overline{\text{Left}}_j=0,\quad /j=\overline{1,2}/, \qquad (5)$$

$$\text{Left}=(K-\omega^2 M)Z_0+$$
$$+ML_1D_\tau^2\Xi_{z1}+ML_2D_\tau^2\Xi_{z2}-\omega^2 S_{0,z}=0,$$

$$\overline{\text{Left}}=0, \qquad (6)$$

From the equation (6) at $\det(K-\omega^2 M)\neq 0$ the vector Z_0 can be expressed:

$$Z_0=-(K-\omega^2 M)^{-1}[M(L_1D_\tau^2\Xi_{z1}+L_2D_\tau^2\Xi_{z2})-\omega^2 S_{0,z}]. \qquad (7)$$

From (4), (7) it follows that for the pseudo-rigid rotor equality takes place

$$(K-\omega^2 M)^{-1}M(L_1D_\tau^2\Xi_{z1}+L_2D_\tau^2\Xi_{z2})=0. \qquad (8)$$

Let's substitute (7) in the equation (5). Considering (8) we will receive:

$$\mathrm{Left}_j = (M_{\Sigma j} + L_j^T M L_j) D_\tau^2 \Xi_{zj} + L_j^T M L_{3-j} D_\tau^2 \Xi_{z,3-j} +$$
$$+ b_{oj} D_\tau \Xi_{zj} + k_j \Xi_{zj} + D_\tau^2 S_{\Sigma j} = 0,\ \overline{\mathrm{Left}}_j = 0,$$
$$/j = 1,2/. \qquad (9)$$

The equations (2), (9) describe the process of auto-balancing of the flexible rotor relative to the variables Ξ_{zj}, $S_{\Sigma j}$, /j=1,2/.

Let's write down the equation (9) relative to the coordinates of the center of mass of rotor – Ξ_z and positions of the axis of rotation of the rotor – Δ_z. As

$$\Xi_{zj} = \Xi_z - i z_j \Delta_z,\ /\ j = 1,2/, \qquad (10)$$

the equations (9) will take the form

$$\mathrm{Left}_j = [M_{\Sigma j} + L_j^T M (L_1 + L_2)] D_\tau^2 \Xi_z - i[z_j M_j +$$
$$+ L_j^T M (z_1 L_1 + z_2 L_2)] D_\tau^2 \Delta_z + b_{oj} D_\tau \Xi_z - i b_{oj} z_j D_\tau \Delta_z +$$
$$+ k_j \Xi_z - i k_j z_j \Delta_z + D_\tau^2 S_{\Sigma j} = 0,\ \overline{\mathrm{Left}}_j = 0,\ /\ j = 1,2\ /. \qquad (11)$$

At first we will add the equations (11), and then we will multiply the first by z_1, and the second by z_2, and also we will add (at the same time we will consider that

$$\mathrm{Left}_1 = M D_\tau^2 \Xi_z + b_x D_\tau \Xi_z - i b_{y\alpha} D_\tau \Delta_z +$$
$$+ k_x \Xi_z - i k_{y\alpha} \Delta_z + D_\tau^2 S_{\Sigma 1} + D_\tau^2 S_{\Sigma 2} = 0,\ \overline{\mathrm{Left}}_1 = 0,$$

$$\mathrm{Left}_2 = A D_\tau^2 \Delta_z + i b_{y\alpha} D_\tau \Xi_z + b_\alpha D_\tau \Delta_z +$$
$$+ i k_{y\alpha} \Xi_z + k_\alpha \Delta_z + i z_1 D_\tau^2 S_{\Sigma 1} + i z_2 D_\tau^2 S_{\Sigma 2} = 0,\ \overline{\mathrm{Left}}_2 = 0, \qquad (12)$$

where

$$M = \sum_{i=1}^N M_i,\ A = \sum_{i=1}^N M_i z_i^2$$

is respectively, the mass and cross moment of inertia of the pseudo-rigid rotor;

$$b_x = b_{o1} + b_{o2},\ k_x = k_1 + k_2,\ b_{y\alpha} =$$
$$= z_1 b_{o1} + z_2 b_{o2},\ k_{y\alpha} = z_1 k_1 + z_2 k_2,$$

$$b_\alpha = z_1^2 b_{o1} + z_2^2 b_2,\ k_\alpha = z_1^2 k_1 + z_2^2 k_2.$$

The equations (2), (12) describe the process of auto-balancing of the flexible rotor relative to the variables Ξ_z, Δ_z, $S_{\Sigma j}$, /j=1,2/.

The equations (12) are similar to the corresponding equations of the work [15] in which stability of main motions at dynamic balancing of the rigid rotor in the massive body is investigated. The only difference is that here rotor is absolutely long (the longitudinal moment of inertia is equal to zero – $C \to 0$) and the role of the total rotor unbalances S_{zj} is played by the given total rotor unbalances $S_{z,\Sigma j}$.

From [15] it follows that the frequency and the characteristic equations of the flexible rotor have the form:

$$\Delta_\omega = M A \omega^4 - (A k_x + M k_\alpha)\omega^2 + k_x k_\alpha - k_{y\alpha}^2 = 0, \qquad (13)$$

$$X\bar{X} + \tilde{m}_1(\bar{X} Y_{11} + X \bar{Y}_{11}) + \tilde{m}_2(\bar{X} Y_{22} + X \bar{Y}_{22}) +$$
$$+ \tilde{m}_1^2 Y_{11} \bar{Y}_{11} \Sigma_1 + \tilde{m}_2^2 Y_{22} \bar{Y}_{22} \Sigma_2 + \tilde{m}_1 \tilde{m}_2 [X + \bar{X} + Y_{11} \bar{Y}_{22} +$$
$$+ \bar{Y}_{11} Y_{22} - 2 Y_{12} \bar{Y}_{12} p_1 p_2 \cos(\vartheta_2 - \vartheta_1)] + \tilde{m}_1 \tilde{m}_2 [\tilde{m}_1 (Y_{11} + \bar{Y}_{11}) \Sigma_1 +$$
$$+ \tilde{m}_2 (Y_{22} + \bar{Y}_{22}) \Sigma_2] + \tilde{m}_1^2 \tilde{m}_2^2 \Sigma_1 \Sigma_2 = 0, \qquad (14)$$

where

$$X = \lambda^2 (\lambda + \tilde{b}_1)(\lambda + \tilde{b}_2)(a_{11} a_{33} - a_{13}^2) / [\Lambda^8 (\tilde{z}_1 - \tilde{z}_2)^2],$$

$$Y_{ij} = -\sqrt{\lambda^2 (\lambda + \tilde{b}_i)(\lambda + \tilde{b}_j)} \times$$
$$\times [a_{11} \tilde{z}_i \tilde{z}_j - a_{13} (\tilde{z}_i + \tilde{z}_j) + a_{33}] / [\Lambda^4 (\tilde{z}_1 - \tilde{z}_2)^2],$$

$$/i, j = 1,2/,$$

$$\tilde{m}_j = m_j n_j / (2 \kappa_j M),\ \tilde{b}_j = b_j / (m_j \kappa_j \omega_0),$$

$$\Sigma_j = 1 - p^2,\ / j = 1,2/,\ \Lambda = \lambda + i\tilde{\omega},$$

$$a_{11} = \Lambda^2 + \tilde{b}_x \Lambda + \tilde{k}_x,\ a_{13} = \tilde{b}_{y\alpha} \Lambda + \tilde{k}_{y\alpha},\ a_{33} = \Lambda^2 + \tilde{b}_\alpha \Lambda + 1.$$

From the equation (13) we find resonant velocities

$$\omega_{рез1,2} = \sqrt{[k_x/M + k_\alpha/A \pm \sqrt{(k_x/M - k_\alpha/A)^2 + 4k_{y\alpha}^2/(MA)}]/2}. \qquad (15)$$

For the roots of the characteristic equation (14) accurate within small of the first order inclusive relative to the resistance forces in supports and when performing the conditions $k_{y\alpha} \sim 0$, $b_{y\alpha} \sim 0$ we receive the following ratios and expressions

$$\lambda_{\overline{1,4}} \in \left(\min\{\lambda_{\overline{1,4}}^{(1)}, \lambda_{\overline{1,4}}^{(2)}\};\ \max\{\lambda_{\overline{1,4}}^{(1)}, \lambda_{\overline{1,4}}^{(2)}\}\right), \qquad (16)$$

$$\lambda_{5,6} \approx -\sqrt{A / k_\alpha}\, b_1 / m_1 + O(m_1, m_2, b_\alpha^2),\ \lambda_{7,8} \approx$$
$$\approx -\sqrt{A / k_\alpha}\, b_2 / m_2 + O(m_1, m_2, b_\alpha^2),$$

$$\lambda_{9,10} \approx -b_\alpha / (2\sqrt{A k_\alpha}) - i(\omega \sqrt{A / k_\alpha} \pm 1) +$$
$$+ O(m_1, m_2, b_\alpha^2),\quad \lambda_{11,12} = \bar{\lambda}_{9,10},$$

$$\lambda_{13,14} = -\sqrt{A / k_\alpha}[b_x / (2M) + k_x b_\alpha / (A k_x + k_\alpha M) +$$
$$+ i(\omega \pm \sqrt{k_x / M})] + O(m_1, m_2, b_\alpha^2),\ \lambda_{15.16} = \bar{\lambda}_{13,14}, \qquad (17)$$

where

$$\lambda_{\overline{1,4}}^{(q)} = -[c_1 P_{1,2}^{(1,q)} + c_2 P_{1,2}^{(2,q)} \pm$$
$$\pm \sqrt{(c_1 P_{1,2}^{(1,q)} - c_2 P_{1,2}^{(2,q)})^2 + 4 c_1 c_2 P_{1,2}^{(1,q)} P_{1,2}^{(2,q)} g_{12}^2 / (g_{11} g_{22})}] / 2 +$$
$$+ O(m_1^2, m_2^2, b_\alpha^2);$$

$$c_j = \tilde{m}_j \tilde{\omega}^4 g_{jj} / [\tilde{b}_j (A\omega^2 - k_x)(M\omega^2 - k_\alpha)],\ / j = \overline{1,2}/;$$

$$g_{ij} = \omega^2 (A + M z_i z_j) / k_\alpha - z_i z_j k_x / k_\alpha - 1,$$

$$P_{1,2}^{(j,q)} = 1 \mp (-1)^{(j-1)q} p_j.$$

The roots $\lambda_{\overline{1,4}}$ are steady (have negative real part), in the area $\omega \in (\max\{\omega_{рез1,2}\}; \infty)$, and the roots $\lambda_{\overline{5,16}}$ – on the entire range of mass-inertial parameters. Therefore, main motions, if any, are steady on the above resonance rotor velocities.

From (3) it follows that in the vicinity of any critical velocity which is the solution of the equation $\det(K-\omega^2 M)=0$, the given total rotor unbalances significantly increase. At the same time the conditions of existence of main motions [3] can't be satisfied – there will not be enough balancing capacity of auto-balancers:

$$\left|S_{0,zj}+L_j^T S_{0,z}+L_j^T M Z_0\right| \le S_{j\max}, \quad /j=1,2/,$$

where $S_{1\max}, S_{2\max}$ – balancing tanks of auto-balancers.

From (16), (17) it follows that transition processes when balancing the flexible rotor are divided into:

– fast (the roots $\lambda_{\overline{5,16}}$), at which fast relative motions of corrective weights stop and the motion of rotor corresponding to the current given total rotor unbalances of the pseudo-rigid rotor and auto-balancers is established;

– slow at which corrective weights come to the auto-balancing positions.

Duration of passing of fast transition processes does not depend on the rotor unbalance of the pseudo-rigid rotor formed at cruiser velocity, the quantity and the current position of corrective weights in auto-balancers, and slow – on the resistance forces in supports.

Duration of passing of slow transition processes:

– has local minimum with respect to the forces of viscous resistance to the relative motion of corrective weights;

– is not an increasing function with respect to the masses of corrective weights, rigidity of supports, remoteness of the centers of support disks from the center of mass of the flexible rotor and reaches the smallest value at excess of defined characteristic values by these parameters;

– does not increase at the increase in the rotor velocity (at velocities higher than the first critical).

6. Discussion of the research results of stability of balancing of flexible rotors by passive auto-balancers

Research of stability of main motions, carried out within the discrete model of the flexible two-support rotor with two auto-balancers on the assumption that the mass of corrective weights and rotor unbalances of disks is smalls in relation to the mass of the flexible rotor, allow establishing the following:

– main motions, if any, are always steady on above resonance rotor velocities;

– for expansion of the area of existence of main motions it is necessary to use auto-balancers with bigger balancing capacity;

– transition processes are divided into: fast at which fast relative motions of corrective weights stop and the motion of rotor corresponding to the current given total rotor unbalances is established; slow at which corrective weights come to the auto-balancing positions;

– at the increase in forces of resistance to relative motion of corrective weights: duration of fast transition processes for corrective weights decreases (they come to the cruiser velocity of rotor quicker); duration of slow transition processes increases (corrective weights come to the auto-balancing positions more slowly);

– duration of passing of transition processes does not decrease at the reduction of: masses of corrective weights, rigidity of supports, remoteness of the centers of support disks from the center of mass of the flexible rotor;

– duration of passing of transition processes does not increase at the increase in the cruiser velocity of the rotor at velocities higher than the first critical (if at the same time the conditions of existence of main motions are not violated).

Advantages of the offered approach are the opportunity to analytically establish the conditions of occur of auto-balancing and to estimate the duration of passing of transition processes when balancing flexible rotors by passive auto-balancers with many corrective weights.

The shortcomings of the offered approach include the need of additional assumptions on the relative motion of the flexible rotor, trifle ratios between parameters and so forth.

The results of the work can be used at the design of auto-balancers for balancing on the run of rotors: turbine units, for the gas- and oil transportation industry, aircraft gas turbine engines, beaters of combine harvesters, etc.

7. Conclusions

The conducted researches within the discrete model of the flexible two-support rotor with two auto-balancers with the assumptions on the relative motion of the flexible rotor and ratios of trifle between parameters, allow establishing the following.

1. The simplified system of differential equations describing the process of auto-balancing of the two-support rotor with two auto-balancers near supports within its discrete model is obtained. The system is written down with respect to four dynamic variables – deviations of the centers of mass of support disks and the given total rotor unbalances from their values on main motions.

The received system accurate within designations matches the system of equations describing the process of dynamic balancing of the rigid rotor on pliable supports with two auto-balancers. But in the accepted rotor model: the longitudinal moment of inertia is equal to zero; the role of the total rotor unbalances is played by the given total rotor unbalances depending on the rotor velocity.

2. Main motions on condition of their existence are steady on above resonance rotor velocities. In the vicinity of any critical velocity the conditions of existence of main motions can be violated. For expansion of the area of stability of main motions it is necessary to increase the balancing capacity of auto-balancers.

3. It is established that:

a) transition processes when balancing the flexible rotor are divided into:

– fast at which fast relative motions of corrective weights stop and the motion of rotor corresponding to the current given total rotor unbalances of the pseudo-rigid rotor and auto-balancers is established;

– slow at which corrective weights come to the auto-balancing positions.

b) at the increase in forces of resistance to relative motion of corrective weights:

– duration of arrival of corrective weights to the auto-balancing positions increases;

– duration of exit of corrective weights to the cruiser velocity of rotor decreases;

c) duration of passing of transition processes:

– does not decrease at the reduction of: masses of corrective weights, rigidity of supports, remoteness of the centers of support disks from the center of mass of the flexible rotor;

– does not increase at the increase in the cruiser velocity of the rotor at velocities higher than the first critical (if at the same time the conditions of existence of main motions are not violated).

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