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FORMATION OF SECURITIES PORTFOLIO UNDER CONDITIONS OF UNCERTAINTY

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Розглянуто задачу формування портфеля цінних паперів. За результатами аналізу відомих підходів до розв'язання задачі запропонована математична модель, в якій параметри задачі задані нечітко. Задача формування портфеля вирішена в припущенні про найгіршу щільність розподілу випадкових значень вартостей активів. Запропонований метод відшукування найгіршої щільності розподілу для випадку, коли математичне очікування і дисперсія випадкової вартості задані нечіткими числами. Обгрунтований вибір раціонального значення математичного очікування доходності портфеля

Ключові слова: портфель цінних паперів, нечіткі вартості активів, мінімаксна модель, імовірнісний дробово-лінійний критерій

Рассмотрена задача формирования портфеля ценных бумаг. По результатам анализа известных подходов к решению задачи предложена математическая модель, в которой параметры задачи заданы нечетко. Задача формирования портфеля решена в предположении о наихудшей плотности распределения случайных значений стоимостей активов. Предложен метод отыскания наихудшей плотности распределения для случая, когда математическое ожидание и дисперсия случайной стоимости заданы нечеткими числами. Обоснован выбор рационального значения математического ожидания доходности портфеля

Ключевые слова: портфель ценных бумаг, нечеткие стоимости активов, минимаксная модель, вероятностный дробно-линейный критерий

1. Introduction

The task on the formation of securities portfolio belongs to a broad class of problems on rational allocation of resources [1–4]. A mathematical statement of similar tasks leads to the standard technique: to find a set of variables that assigns the required allocation of resources, which renders an extreme value to nonlinear criterion and satisfies linear constraints. Problems of this type are solved by the known methods of mathematical programming [4–6]. A specific character of the resource allocation problems in contemporary setting manifests itself in taking into account the uncertain character of initial data. Traditional approaches to the statement and solution of such problems employ the theoretical and probabilistic interpretation of uncertainty in the parameters of the problem, which leads to the models of stochastic programming [7–11]. A fundamental drawback of these models is the insufficient accuracy under conditions of small sample of initial data when there is no possibility to adequately describe the densities of distribution of random parameters in the problem. In this case, it is natural to use the methods that realize a minimax approach taking into account the "worst" densities of distributions of random parameters [12, 13]. This approach provides for obtaining very careful, warranting solution, which is not always acceptable. An alternative approach can be based on the description of uncertain parameters of a problem in the terms of theory of fuzzy sets [14–18]. The appropriate mathematical apparatus for solving the optimization problems has been examined insufficiently, which indicates that the problem explored in present work is relevant.

2. Literature review and problem statement

A traditional theoretical and probabilistic statement of the problem on the formation of securities portfolio is in the following.

Assume there is a certain market of assets and R_j is the random effectiveness of the jth form of assets (random profit from the realization of portfolio that consists entirely of the assets of the jth form), $j=(1,2,\ldots,n)$, x_j is the share of the jth form of assets in the portfolio.

Then

$$R=\sum_{i=1}^{n} R_j\, x_j$$

is the random effectiveness of portfolio, and

$$m=\sum_{j=1}^{n} E\left[R_j\right]x_j=\sum_{j=1}^{n} m_j x_j = MX$$

is its average effectiveness (or mean profit), which corresponds to strategy

$$X=\left(x_1,x_2,\ldots,x_n\right)^T.$$

Here $m_j=E\left[R_j\right]$ is the mean value of random effectiveness of investing in the jth form of assets, $E[\cdot]$ is the symbol of operator of the calculation of mathematical expectation,

$$M=\left(m_1,m_2,\ldots,m_n\right).$$

A dispersion of effectiveness in the portfolio (risk) is calculated by the following formula

$$D(x)=M\left[(m-R)^2\right]=$$
$$=M\left[\left(\sum_{j=1}^{n} m_j x_j-\sum_{j=1}^{n} R_j x_j\right)^2\right]=M\left[\left(\sum_{j=1}^{n}\left(m_j-R_j\right)x_j\right)^2\right]=$$
$$=\sum_{i=1}^{n}\sum_{j=1}^{n} v_{ij}x_i x_j = X^T V X,$$

where

$$v_{ij}=M\left[\left(m_i-R_i\right)\left(m_j-R_j\right)\right],\quad V=\left(v_{ij}\right).$$

In this case, if the random values of effectiveness for the assets of different type are not correlated, then

$$v_{ij}=D_j,\quad j=1,2,\ldots,n.$$

Now the optimization problem of portfolio is stated as follows: to find a set $X=\left(x_1,x_2,\ldots,x_n\right)$, which minimizes the risk dispersion of investing the means (1) and provides for the assigned level m_{set} of mean profit.

A mathematical model of the problem is as follows: to find non-negative vector X that minimizes (1) and satisfies constraints

$$\sum_{j=1}^{n} m_j x_j = MX = m_{set}, \tag{2}$$

$$\sum_{j=1}^{n} x_j = IX = 1,\ I=\left(1,1,\ldots,1\right),\ x_j\ge 0. \tag{3}$$

The obtained problem is the canonical problem of quadratic programming. Solving it by the method of Lagrange indeterminate multipliers yields the following result

$$X^*=V^{-1}C^T\left(CV^{-1}C^T\right)^{-1}d,\quad C=\begin{pmatrix} I\\ M\end{pmatrix},\quad d=\begin{pmatrix}1\\ m_{set}\end{pmatrix}. \tag{4}$$

Formula (4) yields the desired vector X that determines structure of the portfolio.

For many real optimization problems, in which numerical values of criterion depend on random parameters, it is expedient to refrain from the optimization on the average [14]. In this case, in the problem on the formation of securities portfolio, the appropriate mathematical model takes another form. Statement of the problem: to find distribution $\{x_j\}$ that satisfies (2), (3) and maximizes probability on that the random summary profit from realization of the portfolio will exceed certain assigned threshold – R_0. Should we, in view of the central limit theorem in the probability theory, consider that the summary profit is the normally distributed random variable, then this probability will be described by relationship:

$$P\left(R\ge R_0\right)=\int_{R_0}^{\infty}\frac{1}{\sqrt{2\pi}\sigma_\Sigma}\exp\left\{-\frac{\left(R-\mu_1\right)^2}{2\sigma_\Sigma^2}\right\}dR, \tag{5}$$

$$\mu_1=\sum_{i=1}^{n} m_j x_j,\quad \sigma_\Sigma^2=\sum_{j=1}^{n}\sigma_j^2 x_j^2.$$

Now the problem on the formation of structure of the portfolio is reduced to the following: to find vector X that maximizes (5) and satisfies (2), (3).

Let us transform (5)

$$P\left(R\ge R_0\right)=\int_{R_0}^{\infty}\frac{1}{\sqrt{2\pi}\sigma_\Sigma}\exp\left\{-\frac{\left(R-\mu_1\right)^2}{2\sigma_\Sigma^2}\right\}dR=$$
$$=\int_{\frac{R_0-\mu_1}{\sigma_\Sigma}}^{\infty}\frac{1}{\sqrt{2\pi}}e^{\frac{u^2}{2}}du\,. \tag{6}$$

It is clear that the maximization (6) is equivalent to the problem on maximization

$$L(x)=\frac{(R_0-\mu_1)^2}{\sum\limits_{\sigma_j^2x_j^2=1}^{n}\sigma_j^2x_j^2}=\frac{\left[R_0\sum\limits_{j=1}^{n}x_j-\sum\limits_{j=1}^{n}m_jx_j\right]^2}{\sum\limits_{j=1}^{n}\sigma_j^2x_j^2}=$$

$$=\frac{\left(\sum\limits_{j=1}^{n}(R_0-m_j)x_j\right)^2}{\sum\limits_{j=1}^{n}\sigma_j^2x_j^2}=\frac{\left(\sum\limits_{j=1}^{n}d_jx_j\right)^2}{\sum\limits_{j=1}^{n}\sigma_j^2x_j^2}. \quad (7)$$

The obtained problem of fraction-quadratic programming with linear constraints is solved using a procedure for continuous improvement in strategy [4].

In the problems with a small sample of the initial data, an assumption about the normality of their distribution is excessively binding. In this case, [13, 14] propose a minimax technique for solving the problem under assumption about the "worst" density of distribution of its random parameters. In accordance with this technique, solving a problem is realized in two stages. First, for the examined set $\{x_j\}$ that satisfies (2), (3), one finds the "worst" distribution density of random magnitude of summary profit with the known mathematical expectation and dispersion. In this case, the probability of non-attainment of the assigned threshold R_0 must be maximal. Next, taking into account the obtained "worst" density, one solves the problem about the calculation of assets distribution that minimizes the probability of not exceeding R_0. When searching for the "worst" density distribution, mathematical expectation μ_1 and dispersion μ_2 of the random variable of profit are assumed to be known.

The problem on finding the "worst" density distribution is stated as follows: to find function $x^*(t)$ that maximizes the functional

$$P(x(t))=\int\limits_0^{R_0}x(t)dt \quad (8)$$

and satisfies the constraints

$$\int\limits_0^{\infty}x(t)dt=1, \quad (9)$$

$$\int\limits_0^{\infty}tx(t)dt=\mu_1, \quad (10)$$

$$\int\limits_0^{\infty}t^2x(t)dt=\mu_2=\mu_1^2+\sigma^2. \quad (11)$$

The obtained problem is the canonical problem of continuous linear programming. As shown in [12], the "worst" density $x^*(t)$ is to be searched for in the form of weighted linear combination of delta functions, that is,

$$x^*(t)=\sum\limits_{j=1}^{n}x_j\delta(t-t_j). \quad (12)$$

In this case, the problem is reduced to determining the unknown values (x_j, t_j), $j=1,2,\dots,n$ that maximizes (8) and satisfies constraints (9)–(11). Substituting (12) into (8)–(11), we shall obtain the following problem of nonlinear programming: to find sets $X^*=\{x_j\}$ and $T=\{t_j\}$ that maximize

$$P(X,T)=\sum\limits_{j=1}^{n}c(t_j)x_j,$$

and satisfy the constraints

$$\sum\limits_{j=1}^{n}x_j=1,$$

$$\sum\limits_{j=1}^{n}t_jx_j=\mu_1,$$

$$\sum\limits_{j=1}^{n}t_j^2x_j=\mu_2.$$

Solution of the problem is iterative and, as shown in [12], it yields the desired function

$$x^*(t)=\frac{\mu_2-\mu_1^2}{\mu_2-2\mu_1T_0+T_0^2}\delta(t-T_0)+$$
$$+\frac{\mu_1^2-2\mu_1T_0+T_0^2}{\mu_2-2\mu_1T_0+T_0^2}\delta\left(t-\frac{\mu_2-\mu_1T_0}{\mu_1-T_0}\right).$$

In this case, maximum probability that the random variable fits interval $[0, R_0]$ is equal to

$$P(x^*(t))=\frac{\mu_2-\mu_1^2}{\mu_2-2\mu_1R_0+R_0^2}=\frac{\sigma^2}{\sigma^2+(\mu_1-R_0)^2}=$$
$$=\frac{1}{1+\frac{(\mu_1-R_0)^2}{\sigma^2}}=\frac{1}{1+\frac{\left(\sum\limits_{j=1}^{n}x_jm_j-R_0\right)^2}{\sum\limits_{j=1}^{n}\sigma_j^2x_j^2}}. \quad (13)$$

The obtained relationship at the second stage determines an objective function of problem on the formation of portfolio $\{x_j\}$ for the "worst" distribution density of random profit. The problem comes down to finding a distribution, which minimizes (13) and which satisfies portfolio constraints.

It is clear that the problem on minimization (13) is equivalent to the problem of maximization

$$L(X)=\frac{\left(\sum\limits_{j=1}^{n}x_jm_j-R_0\right)^2}{\sum\limits_{j=1}^{n}\sigma_j^2x_j^2},$$

which coincides with (7).

Let us replace the maximization problem of fractional-quadratic functional with the maximization problem of linear-fractional functional. Since

$$\sum\limits_{j=1}^{n}\sigma_j^2x_j^2<\left(\sum\limits_{j=1}^{n}\sigma_jx_j\right)^2,$$

then $L(X)=\frac{\left(\sum\limits_{j=1}^{n}d_jx_j\right)^2}{\sum\limits_{j=1}^{n}\sigma_j^2x_j^2}>\frac{\left(\sum\limits_{j=1}^{n}d_jx_j\right)^2}{\left(\sum\limits_{j=1}^{n}\sigma_jx_j\right)^2}.$

Now, instead of maximization problem (7), we shall solve the maximization problem of the obtained minorant. Next, since the involution is a monotonic transform, let us proceed from the maximization problem L(X) to the maximization problem equivalent to it

$$\left[L(X)\right]^{\frac{1}{2}}=\frac{\sum_{j=1}^{n}d_jx_j}{\sum_{j=1}^{n}\sigma_jx_j}=\frac{\sum_{j=1}^{n}\left(R_0-m_j\right)x_j}{\sum_{j=1}^{n}\sigma_jx_j}. \qquad (14)$$

Obtained objective function is fractional-linear. The corresponding problem of linear-fractional programming is easily solved by reducing to the conventional problem of linear programming.

A brief analysis of the known approaches to solving the problem on the formation of securities portfolio, which we conducted, allows us to draw the following conclusions. A method for solving this problem and its realization procedure are determined by the nature of uncertainty relative to the real cost of assets, as well as by the level of confidence in adequacy of its accepted analytical descriptions. Traditional approaches to the solution of problem on the formation of securities portfolio employ the technologies of statistical analysis [15] and are based on the theoretical-probabilistic tools [16, 17]. However, under conditions of high volatility in the market situation, an analysis of the existing uncertainty is forcedly limited by the small samples of initial data, which leads to unsatisfactory accuracy in the calculation of statistical characteristics of the model. This circumstance renders promising the solution of problem on taking account of the occurring real uncertainty by using, to describe this model, the least demanding mathematical apparatus – the theory of fuzzy sets [18–22].

3. The aim and tasks of the study

The aim of present study is to develop a method for solving the problem on the formation of securities portfolio under conditions when initial data are fuzzily assigned. An appropriate mathematical model adequately describes a real uncertainty of the stated problem, which provides for obtaining more reliable results.

To achieve the set aim, the following tasks were formulated:

– to select a membership function for describing the fuzzy parameters of the problem;

– to select a criterion for portfolio effectiveness;

– to construct an analytical expression for the criterion of portfolio effectiveness in the terms of fuzzy mathematics;

– to devise a method and computational technique to solve optimization problem about the formation of portfolio under conditions of fuzzy initial data.

4. Key results. A method for the formation of securities portfolio under conditions of fuzzy initial data

Let us assume that under conditions of scarcity of statistical data about the shares profitability, the values of their mathematical expectations m_j and dispersions σ_j^2, j=1,2,...,n, are fuzzily assigned.

To describe fuzzy values m_j and σ_j, j=1,2,...,n, we shall introduce sets of the Gaussian membership functions

$$\mu(m_j)=\exp\left\{-\frac{(m_j-\bar{m}_j)^2}{2D_{mj}}\right\},$$

$$\mu(\sigma_j)=\exp\left\{-\frac{(\sigma_j-\bar{\sigma}_j)^2}{2D_{\sigma j}}\right\},\ j=1,2,...,n.$$

Let us state the problem of finding set

$$X=\left(x_1,x_2,...,x_n\right),$$

which maximizes (14) and satisfies constraints (2), (3).

To solve the stated problem of fuzzy mathematical programming, we shall apply one of the general methods for solving similar problems, described in [23].

Let us write down membership functions of fuzzy numbers, which are in the numerator and denominator of relationship (14). Let us introduce

$$u=\sum_{j=1}^{n}d_jx_j,\quad v=\sum_{j=1}^{n}\sigma_jx_j.$$

Then

$$\mu(u)=\exp\left\{-\frac{\left[u-\sum_{j=1}^{n}(\bar{m}_j-R_0)x_j\right]^2}{2\sum_{j=1}^{n}D_{mj}x_j^2}\right\}=\exp\left\{-\frac{\left[u-\bar{u}\right]^2}{2\sigma_u^2}\right\},$$

$$\mu(v)=\exp\left\{-\frac{\left(v-\sum_{j=1}^{n}\bar{\sigma}_jx_j\right)^2}{2\sum_{j=1}^{n}D_{\sigma_j}x_j^2}\right\}=\exp\left\{-\frac{\left[v-\bar{v}\right]^2}{2\sigma_v^2}\right\}.$$

Now, we shall write down a membership function of fuzzy number $z=\frac{u}{v}$. In this case, we obtain

$$\mu(z)=\exp\left\{-\frac{\left(z-\frac{\bar{u}}{\bar{v}}\right)^2}{2\frac{\sigma_u^2\bar{v}^2+\sigma_v^2\bar{u}^2}{\bar{v}^4}}\right\}. \qquad (15)$$

Thus, the problem on forming the portfolio under conditions of fuzzy initial data relative to the mathematical expectation and dispersion of the “worst” law of distribution of random profit from the sale of shares is reduced to the following: to find set X={x_j} that maximizes (15) and satisfies (2), (3).

Let us assign membership level α and solve equation $\mu(z)=\alpha$.

Hence

$$\left(z-\frac{\bar{u}}{\bar{v}}\right)^2=-2\frac{\sigma_u^2\bar{v}^2+\sigma_v^2\bar{u}^2}{\bar{v}^4}\ln\alpha,$$

$$z=\frac{\overline{u}}{\overline{v}}\pm\left(\frac{\sigma_u^2\overline{v}^2+\sigma_v^2\overline{u}^2}{\overline{v}^4}\ln\frac{1}{\alpha^2}\right)^{\frac{1}{2}}.$$

By selecting the least of these roots, we shall obtain

$$z=\frac{\overline{u}}{\overline{v}}-\frac{\left[\left(\sigma_u^2\overline{v}^2+\sigma_v^2\overline{u}^2\right)\ln\frac{1}{\alpha^2}\right]^{\frac{1}{2}}}{\overline{v}^2}=$$

$$=\frac{\sum_{j=1}^{n}\left(\overline{m}_j-R_0\right)x_j}{\sum_{j=1}^{n}\overline{\sigma}_j x_j}-\frac{\left[\left[\left(\sum_{j=1}^{n}D_{m_j}x_j^2\right)\left(\sum_{j=1}^{n}\overline{\sigma}_j x_j\right)^2+\left(\sum_{j=1}^{n}D_{\sigma j}x_j^2\right)\left(\sum_{j=1}^{n}\left(\overline{m}_j-R_0\right)x_j\right)^2\right]\ln\frac{1}{\alpha^2}\right]^{\frac{1}{2}}}{\left(\sum_{j=1}^{n}\overline{\sigma}_j x_j\right)^2}. \quad (16)$$

Thus, the problem on the formation of portfolio under conditions of fuzzy initial data is reduced to the following: to find set $X=\{x_j\}$ that maximizes (16) and satisfies (2), (3). The complexity of analytical expression of objective function (16) limits the possibility of applying the methods of optimization of first or second orders. However, the methods of zero order (for example, a Nelder-Mead method) can be effectively employed to obtain the solution.

A question that remains open is the rational selection of value m_{set}. It is clear that with an increase in m_{set}, the risk of portfolio grows. In connection with this, no any general considerations can be acknowledged as convincing in the substantiation of one or another value of m_{set} Let us examine possible procedure for the substantiation of rational choice of m_{set}.

We shall obtain the relationship, which connects dispersion of the portfolio with its structure. In accordance with formula (1)

$$D=X^TVX. \quad (17)$$

Let us substitute vector X*, determined by (4), which corresponds to the assigned value of m_{set}, into (17). Then

$$D=d^T\left(CV^{-1}C^T\right)^{-1}CV^{-1}VV^{-1}C^T\left(CV^{-1}C^T\right)^{-1}d=$$
$$=d^T\left(CV^{-1}C^T\right)^{-1}d.$$

Upon replacing

$$\left(CV^{-1}C^T\right)^{-1}=A,$$

we shall receive

$$D=\left(1\ \ m_{set}\right)A\begin{pmatrix}1\\ m_{set}\end{pmatrix}.$$

Since

$$A=\left(CV^{-1}C^T\right)^{-1}=\begin{pmatrix}a_{11} & a_{12}\\ a_{21} & a_{22}\end{pmatrix}=$$

$$=\left[\begin{pmatrix}1 & 1\ldots1\\ m_1 & m_2\ldots m_n\end{pmatrix}\begin{pmatrix}\frac{1}{D_1} & & & 0\\ & \frac{1}{D_2} & & \\ & & \ddots & \\ 0 & & & \frac{1}{D_n}\end{pmatrix}\begin{pmatrix}1 & m_1\\ 1 & m_2\\ \vdots & \vdots\\ 1 & m_n\end{pmatrix}\right]^{-1}=$$

$$=\left[\begin{pmatrix}\frac{1}{D_1} & \frac{1}{D_2} & \cdots & \frac{1}{D_n}\\ \frac{m_1}{D_1} & \frac{m_2}{D_2} & \cdots & \frac{m_n}{D_n}\end{pmatrix}\begin{pmatrix}1 & m_1\\ 1 & m_2\\ \vdots & \vdots\\ 1 & m_n\end{pmatrix}\right]^{-1}=$$

$$=\begin{bmatrix}\sum_{j=1}^{n}\frac{1}{D_j} & \sum_{j=1}^{n}\frac{m_j}{D_j}\\ \sum_{j=1}^{n}\frac{m_j}{D_j} & \sum_{j=1}^{n}\frac{m_j^2}{D_j}\end{bmatrix}^{-1}=$$

$$=\frac{1}{\left(\sum_{j=1}^{n}\frac{1}{D_j}\right)\left(\sum_{j=1}^{n}\frac{m_j^2}{D_j}\right)-\left(\sum_{j=1}^{n}\frac{m_j}{D_j}\right)^2}\begin{pmatrix}\sum_{j=1}^{n}\frac{m_j^2}{D_j} & -\sum_{j=1}^{n}\frac{m_j}{D_j}\\ -\sum_{j=1}^{n}\frac{m_j}{D_j} & \sum_{j=1}^{n}\frac{1}{D_j}\end{pmatrix},$$

then

$$D=\sigma^2=(1\ \ m_{set})\begin{pmatrix}a_{11} & a_{12}\\ a_{21} & a_{22}\end{pmatrix}\begin{pmatrix}1\\ m_{set}\end{pmatrix}=$$
$$=\left(a_{11}+m_{set}a_{21}a_{12}+m_{set}a_{22}\right)\begin{pmatrix}1\\ m_{set}\end{pmatrix}=$$
$$=a_{11}+a_{21}m_{set}+a_{12}m_{set}+a_{22}m_{set}^2=$$
$$=a_{11}+2a_{12}m_{set}+a_{22}m_{set}^2=a_0+a_1m_{set}+a_2m_{set}^2.$$

Thus, it is demonstrated that the dispersion of optimum portfolio is the quadratic function of m_{set}. The obtained relationship makes it possible to solve the problem on the rational formation of portfolio by the substantiated assignment of value m_{set}. In this case, it is clear that different strategies on the formation of portfolio can be realized.

Cautious strategies are matched with low values of m_{set}, which provide for the required gain at a relatively low risk (small dispersion of portfolio). On the other hand, realization of adventurous strategies, which correspond to large average gain, is connected with a high risk, which quadratically grows with an increase in m_{set}.

Regardless of the fact how fuzzy values m_j and σ_j, j=1,2,...,n are assigned, a natural approach towards the choice of rational value m_{set} implies the maximization of relationship

$$Q=\frac{m_{set}}{\sigma}=\frac{m_{set}}{\sqrt{a_0+a_1m_{set}+a_2m_{set}^2}}.$$

In order to find optimum m_{set}, we shall differentiate the obtained expression by m_{set}.

$$\frac{dQ}{dm_{set}} =$$

$$= \frac{\left(a_0 + a_1 m_{set} + a_2 m_{set}^2\right)^{\frac{1}{2}} - m_{set}\frac{1}{2}\left(a_0 + a_1 m_{set} + a_2 m_{set}^2\right)^{-\frac{1}{2}}\left(a_1 + 2a_2 m_{set}\right)}{\left(a_0 + a_1 m_{set} + a_2 m_{set}^2\right)} =$$

$$= \frac{2\left(a_0 + a_1 m_{set} + a_2 m_{set}^2\right) - m_{set}\left(a_1 + 2a_2 m_{set}\right)}{2\left(a_0 + a_1 m_{set} + a_2 m_{set}^2\right)^{\frac{3}{2}}} = 0.$$

Hence

$$2a_0 + 2a_1 m_{set} + 2a_2 m_{set}^2 - a_1 m_{set} - 2a_2 m_{set}^2 = 2a_0 + a_1 m_{set} = 0.$$

Then

$$m_{set}^* = \frac{-2a_0}{a_1}.$$

Obtained relationship makes it possible to calculate a compromise choice of the expedient value, average by the portfolio profitability.

5. Discussion of method for the formation of securities portfolio under conditions of uncertainty

Thus, in present work we propose a method for the formation of securities portfolio for the case when mathematical expectations and dispersions of random costs of shares are assigned by the fuzzy numbers with the known membership functions. The problem is stated in the terms of theory of optimum allocation of limited resource. A criterion for resource allocation effectiveness is the probability that the total portfolio profitability will exceed a permissible threshold.

The problem is solved under assumption about the "worst" distribution density in the cost of assets. The proposed procedure of finding the rational allocation of resources comes down to solving a problem of mathematical programming – the maximization of fractional-quadratic function at linear constraints and is realized by the method of optimization of zero order.

The required effectiveness of the portfolio, formed in this case, is achieved in the case when statistical characteristics of the cost of assets are stable. If this is not the case, then it is necessary to run a preliminary analysis of the dynamics in these characteristics with the application of the appropriate technoques for their prediction [24–26]. Furthermore, let us note that in the real situation of high uncertainty relative to the cost of assets, describing their mathematical expectation and dispersion by the means of fuzzy mathematics may prove to be inadequate. In this case, there remains one additional possibility – to employ apparatus of fuzzy mathematics [27]. In this case, a solution to the problem can be obtained by the formation of fuzzy models for the assigned inaccurate parameters [28].

6. Conclusions

1. We conducted analysis of the known methods for solving the problem on the formation of securities portfolio for the basic variants of knowledgeability relative to the uncertainty in the values of costs of the assets:

a) complete probabilistic certainty – distribution densities of the random costs of securities are known;

b) partial certainty – the moments of distributions of random costs are known.

2. A method for the formation of portfolio is proposed for the case when statistical characteristics in the distributions of costs of the assets are fuzzily assigned.

3. The desired solution to the problem is obtained by the optimization of criterion whose value is determined by a membership function of the fuzzy probability of exceeding a permissible threshold by the random value of the total portfolio profitability.

4. The procedure of obtaining a solution for the formation of portfolio is realized by the optimization method of zero order.

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