Досліджено вплив розміру та кількості корегувальних вантажів (куль або циліндричних роликів) в автобалансирі на його балансувальну ємність та на тривалість перебігу перехідних процесів при автобалансуванні роторних систем. При цьому знайдені розміри та кількість корегувальних вантажів, при яких досягається найбільша балансувальна ємність автобалансира та найменша тривалість перехідних процесів

Ключові слова: автобалансир, автобалансування, куля, циліндричний ролик, балансувальна ємність, перехідні процеси, оптимізація

Исследовано влияние размера и количества корректирующих грузов (шаров или цилиндрических роликов) в автобалансире на его балансировочную емкость и на продолжительность протекания переходных процессов при автобалансировке роторных систем. При этом установлены размер и количество корректирующих грузов, при которых достигается наибольшая балансировочная емкость автобалансира и наименьшая продолжительность переходных процессов

Ключевые слова: автобалансир, автобалансировка, шар, цилиндрический ролик, балансировочная емкость, переходные процессы, оптимизация

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AN INCREASE OF THE BALANCING CAPACITY OF BALL OR ROLLER-TYPE AUTO-BALANCERS WITH REDUCTION OF TIME OF ACHIEVING AUTO-BALANCING

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1. Introduction

Passive auto-balancers are used [1–4] to balance on the running (working) fast-rotating rotors of the centrifugal machines under a constantly changing rotor unbalance. The motion of the rotor-auto-balance system becomes steady over time. The rotor is balanced by means of auto-balancers in the so-called main (steady) rather then secondary motions.

For designing auto-balancers, it is important to solve the following problems:

1) to determine of the conditions under which it is possible to achieve the rotor auto-balancing by one or several auto-balancers;

2) to choose the parameters of an auto-balancer to achieve:

– the highest balancing capacity of the auto-balancer, enough to rotor balancing [1–4];

– the highest balancing quality;

3) to choose the parameters of auto-balancers for the fastest balancing of the rotor.

2. Literature review and problem statement

The designs and the action principle of the so-called classical (ring, pendulum, ball, roller) auto-balancers with solid corrective weights which are intended for balancing mainly the drums of washing machines are described in [1]. In classical auto-balancers the centers of mass of corrective weights move in circles whose planes are perpendicular to the longitudinal axle of the rotor with the centers being on this axis. Most fully, the designs of various types of classical auto-balancers that are intended to balance various fast-rotating rotors are described in [2]. Non-classical auto-balancers with corrective weights of a special form that can rotate in a certain way around a point on the longitudinal axle of the rotor are described in [3].

Most fully, the conditions of achieving of the auto-balancing to balance rotors by passive auto-balancers are defined in the works: [4] – for two-ball auto-balancers within a flat model of the rotor, a rotor model with a fixed point, a rotor model on two elastic supports; [3] – for the classical and non-classical auto-balancers within the above mentioned rotor models.

In [4] the formulas for calculating of the balancing capacity of ball auto-balancers separately for even and odd quantities of balls. The balancing capacity is studied for the maximum radius of the balls (at their unchanging number). The findings were the following: the maximum capacity of an auto-balancer is achieved if there is only one ball; if there are six balls or more, they are excessive, for they reduce the balancing capacity of the auto-balancer.

In [2] similar researches for roller-type auto-balancers are conducted. The findings are the following: the use of rollers makes it possible to increase the balancing capacity of the auto-balancer in 1.5 times in comparison with the ball auto-balancer of the same dimensions; the highest capacity of the auto-balancer is achieved with one or three rollers of a particular radius in the auto-balancer.

The common faults of works [2, 4] are:

– the formulae to calculate the balancing capacity of an auto-balancer (as a total sum value) are different for even and odd quantities of the balls, which makes them inconvenient for using and analyzing;

– there is no technical (engineering) check of the correctness of stating the research problem about the extremum of the balancing capacity of an auto-balancer on the radius of the corrective weights;

– the obtained result relate only to the balancing capacity of an auto-balancer and disregard the requirements to raise the accuracy and to reduce the time of achieving of the balancing.

The influence of various factors on the accuracy of balancing is studied in [2, 4–6]. In [5] the main reasons of a decrease in the accuracy of balancing rotors by passive auto-balancers are called to be as follows: the eccentricity of the racetrack; the forces of resistance to swing of the corrective weights on the track; external vibrations (or revolting forces).

In [4] the influence of the racetrack eccentricity and the forces of resistance to swing of the corrective weights on the accuracy of balancing the rotor by ball auto-balancer are estimated. An equilibrium equation for the corrective weights on the track in steady motion is constituted. It is shown that the lower the eccentricity and the forces of dry friction, the smaller the balancing error. It is suggested that the forces of resistance to swing can be reduced by increasing the radius of the balls. In [2] similar results for roller-type auto-balancers are obtained. In [6], the Hertz contact mechanics and hysteresis losses are taken into consideration to specify the zones of stagnation of balls around the auto-balancing position.

In [7-10], differential equations on the motion of the rotor-balancer system are used to study the influence of the forces of resistance to swing (along with other factors) on the dynamics of the system. In [7] it is determined that under certain conditions the forces of dry friction can excite oscillating motions of the balls. In [8], it is shown that slight external revolting forces do not necessarily produce vibrations of the balls around the auto-balancing position because of the inaction of the corrective weights under the effect of the forces of resistance to swing. In [9], it is shown that revolting forces can influence the accuracy of balancing. In [10], it is revealed that concussions between the balls also influence the dynamics of the rotor-balance system.

If conclusions are derived from [2, 4–10] to specify the design of auto-balancers, it is possible to suggest that a higher accuracy of balancing is achieved with: corrective weights of a larger diameter; a maximum precision in the racetrack design, and if corrective weights made of the hardest solid material.

The influence of the parameters of an auto-balancer on the velocity of achieving auto-balancing is described in many studies, including:

[11] – about a flat model of the rotor with an two-ball auto-balancer;

[3, 12–14] – about a flat model of the rotor with an multi-ball auto-balancer;

[15] – about a rotor model with a viscoelastic fixation of the corps with a fixed point and one auto-balancer;

[16] – about a spatial model of the rotor on two supports and balanced by one auto auto-balancer;

[17] – about a spatial model of the rotor placed in a heavy corps with viscoelastic fixation and balanced by one auto-balancer;

[18] – about a spatial model of the rotor on two supports and balanced by two auto-balancers;

[19] – about a discrete model of a flexible two-support rotor, balanced by two auto-balancers that are placed near the supports.

In these works the velocity of achieving auto-balancing is estimated by the roots of a characteristic equation that describes the stability of the main motion or a family of the main motions.

For rotor machines with one auto-balancer with a lot of corrective weights, the root with the smallest value of the negative real part is obtained by the following:

$$\lambda = -[\tilde{b} - \sqrt{\tilde{b}^2 - (1-p)mn\tilde{b}_{cr}^2}\,]/2, \tag{1}$$

where $\tilde{b}$ is a the dimensionless parameter characterizing the scope of the forces of resistance relative to the motion of the corrective weights;

$$p = \sqrt{\sum\nolimits_{i,k=1}^{n} \cos 2(\tilde{\psi}_i - \tilde{\psi}_k)}\,/\,n \in [0;1]; \tag{2}$$

it is a the parameter that is determined by the positions of the corrective weights in an unperturbed main motion; $\tilde{\psi}_i, / i = \overline{1,n}\,/$ are the angles that set the positions of the corrective weights in the main motion; n is the number of the corrective weights in the auto-balancer, m is the mass of one corrective weight; $\tilde{b}_{cr}$ is a dimensionless parameter that is determined by mass-inertial characteristics of the rotor system irrespective of the parameters of the auto-balancer.

The root (1) shows that the arrangement of the balls in the main motion significantly affects the velocity of achieving auto-balancing.

If the rotor model has on two supports and two auto-balancers (both for rigid [18] and for flexible [19] rotors), the duration of the transition processes depends on the positions of the corrective weights in the main motions of each auto-balancer:

$$p_j = \sqrt{\sum\nolimits_{i,k=1}^{n_j} \cos 2(\tilde{\psi}_{i,j} - \tilde{\psi}_{k,j})}\,/\,n_j \in [0;1],\ \ /j = 1,2/$$

and generally it is impossible to write the root (1) in an explicit form, but the tendencies with which the parameters p_j, m_j, n_j, /j=1,2/ affect the duration of the transition processes remain the same.

The expression of the root (1) shows that at p=1 it is equal to 0 which theoretically can extend the time of achieving auto-balancing occurrence to infinity.

In [12–14], for corrective weights of an infinitesimal radius, it is proved that (at a fixed rotor unbalance in the plane of the auto-balancer correction) the parameter p changes within $0<p_{min}\le p\le p_{max}<1$, with the transition of the corrective weights from one main motion to another in the multiple parameter family. However, a rotor unbalance can change from 0 to a certain maximum value. So there are always such rotor unbalances that will be balanced by auto-balancers for an infinitely long time. In practice, however, auto-balancers quickly track and balance any change in the rotor unbalance because corrective weights are always finite in size. This is not taken into account when calculating the parameter p.

Thus, it is still essential to solve the complex problem of selecting the parameters of real ball or roller-type balancers (with finite corrective weights) that can maximize the balancing capacity of the auto-balancer and minimize the time of achieving auto-balancing.

3. The purpose and objectives of the study

The purpose of the work is to research of the influence of the size of corrective weights and their quantity in an auto-balancer on the balancing capacity of the auto-balancer and the duration of the transition processes in the auto-balancer.

To achieve this purpose it is necessary to solve the following research problems:

1) to receive a general formula for estimating the balancing capacity of the auto-balancer that would be suitable for both even and odd quantities of corrective weights;

2) to establish a technically correct task of optimizing the parameters of the auto-balancer and to determine the size and the quantity of the corrective weights that would maximize the balancing capacity;

3) to define the limits of changes in the parameter p, taking into account the final sizes of the corrective weights, the radius and the quantity of the corrective weights in the auto-balancer to minimize the duration of the transition processes.

4. Methods of researching the dependence of the balancing capacity of an auto-balancer and the duration of the transition processes on the parameters of the corrective weights

Rotor systems with typical one-row auto-balancers having corrective weights in the form of identical balls or cylindrical rollers are investigated.

To obtain the general formula of determining the balancing capacity of an auto-balancer applicable to both even and odd quantities of corrective weights, the study will make use of the known trigonometric identities.

The balancing capacity of the auto-balancer will be considered as a function of many variables. The influence of each parameter on the value of this function will be investigated. The balancing capacity of the auto-balancer will be investigated for extremum quantities and radius of corrective weights in auto-balancers. The obtained results will be evaluated from the engineering point of view, and abstract mathematical results will be rejected.

From the form of the root (1), it follows that the duration of the transition processes (the velocity of achieving auto-balancing) can be estimated by the function

$$a=(1-p)mn. \quad (3)$$

The duration of the transition processes is shorter if we have bigger a(p,n,m). Therefore, auto-balancing will start faster at a reduced parameter p and at increased mass m and quantity n of corrective weights in the auto-balancer.

While determining the limits of the parameter p, it will be taken into account that corrective weights have a finite radius and cannot move one through another.

5. Results of researching the dependence of the balancing capacity of an auto-balancer and the duration of course of the transition processes on the parameters of the corrective weights

5. 1. Justification of using at least three corrective weights in an auto-balancer

By now, the theory of auto-balancing a rotor by means of a two-ball auto-balancer has been developed quite well. It is based on the theory of stability of isolated motions.

In practice, however, it is typical to use auto-balancers with many corrective weights, which makes it possible:

– to increase the balancing capacity of the auto-balancer or to reduce its dimensions;

– to minimize the reaction time of the corrective weights to changes in the static unbalance of the rotor.

Let's illustrate the latter by an example of balancing an elementary static unbalance of an auto-balancer with two and three corrective weights (Fig. 1).

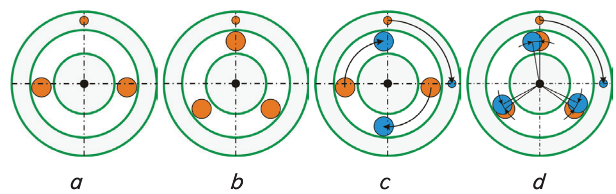


Fig. 1. The positions of corrective weights while balancing an elementary static unbalance of a rotor with the help of an auto-balancer having two and three corrective weights: *a*, *b* show the positions of the corrective weights under the initial rotor unbalance; *c*, *d* illustrate the positions of the corrective weights under a shift of the rotor unbalance by 90°

In an auto-balancer with two corrective weights (Fig. 1, *a*), the latter always take the only possible position in which they balance the static unbalance of the rotor, and in an auto-balancer with three corrective weights (Fig. 1, *b*), the latter always take one of the possible positions from a one-parameter family in which they balance the static unbalance of the rotor.

If the elementary static unbalance of the rotor shifts by 90°, the corrective weights will take the positions shown in Fig. 1, *c* and Fig. 1, *d*, that is:

– the two corrective weights will turn around the center by 90° (Fig. 1, *c*) and will cover considerable distances on the track;

– the three corrective weights will make elementary shifts (Fig. 1, *d*) and will stop in one of the auto-balancing positions from the one-parameter family of such positions.

Thus, in an auto-balancer with three corrective weights, the process of auto-balancing happens much faster. With more corrective weights (n>3) in the auto-balancer, there is an (n–2)-parameter family of auto-balancing positions, and auto-balancing can happen even faster.

5. 2. Calculation of the balancing capacity of an auto-balancer

Let's consider an auto-balancer with n corrective weights (Fig. 2). Let us define that:

– the radius and the mass of the corrective weights are equal to r and m, respectively;

– the radius of the racetrack of the corrective weights is equal to R;

– the corrective weights are densely pressed to each other and symmetrically located relative to the vertical axis.

Let's introduce the angle α:

$$\alpha = \arcsin[r/(R-r)] = \angle O_j O O_{j+1}/2, \quad /j=\overline{1,n-1}/. \quad (4)$$

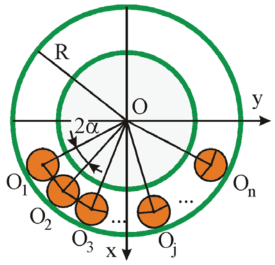


Fig. 2 Determination of the balancing capacity of the auto-balancer

Then:

$$\angle O_j Ox = \angle O_1 Ox + 2\alpha(j-1) =$$
$$= -\alpha(n-1) + 2\alpha(j-1) = -\alpha(n+1-2j), \quad /j=\overline{1,n}/$$

and the balancing capacity of the auto-balancer is equal to:

$$S_{AB} = m(R-r)\sum_{j=1}^{n}\cos\angle O_j Ox =$$
$$= m(R-r)\left\{\sum_{j=1}^{n}\cos[\alpha(n+1-2j)]\sin\alpha\right\}/\sin\alpha.$$

After applying the trigonometric identity and summation we receive:

$$S_{AB} = m(R-r)\left\{\sum_{j=1}^{n}\sin[\alpha(n+2-2j)] - \sin[\alpha(n-2j)]\right\}/\sin\alpha =$$
$$= m(R-r)\sin n\alpha/\sin\alpha. \quad (5)$$

Taking into account (4) equation (5) takes the following form:

$$S_{AB} = m(R-r)^2/r\cdot\sin\{n\arcsin[r/(R-r)]\}.$$

The mass of a ball or a cylindrical roller, respectively, is equal to:

$$m^{(b)} = 4\pi r^3\gamma/3, \quad m^{(r)} = \pi r^2 h\gamma, \quad (6)$$

where γ is the density of the material of which the corrective weights are made, h is the height of a cylindrical roller. Then, the balancing capacity of the ball or roller-type auto-balancer, respectively, is equal to:

$$S_{AB}^{(b)} = \frac{4\pi}{3}\gamma r^2 (R-r)^2 \sin\left(n\arcsin\frac{r}{R-r}\right) =$$
$$= \frac{4\pi}{3}\gamma R^4\rho^2(1-\rho)^2\sin\left(n\arcsin\frac{\rho}{1-\rho}\right);$$
$$S_{AB}^{(r)} = \pi\gamma h r(R-r)^2\sin\left(n\arcsin\frac{r}{R-r}\right) =$$
$$= \pi\gamma R^4\eta\rho(1-\rho)^2\sin\left(n\arcsin\frac{\rho}{1-\rho}\right), \quad (7)$$

where ρ=r/R, η=h/R – dimensionless parameters.

Thus,

$$S_{AB}^{(b)} = S_{AB}^{(b)}(\gamma,R,\rho,n), \quad S_{AB}^{(r)} = S_{AB}^{(r)}(\gamma,R,\eta,\rho,n),$$

which means that the balancing capacities of the ball and roller-type auto-balancers, respectively, are analytical functions of four or five variables.

If in the ball and roller-type auto-balancers the racetracks have identical widths and the corrective weights fill them completely, then h=2r. Herewith,

$$S_{AB}^{(r)}/S_{AB}^{(b)} = 3h/(4r) = 1{,}5.$$

Thus, the roller-type auto-balancer has a balancing capacity that is 1.5 times higher in comparison with the ball auto-balancer.

5. 3. Research to determine the highest balancing capacity of an auto-balancer

The balancing capacities of both types of auto-balancers are monotonously increasing functions based on the parameters γ and R. Therefore, to obtain an auto-balancer of the highest balancing capacity, it is necessary to make corrective weights of a material of the biggest specific weight and to produce the racetrack of the largest possible radius.

The balancing capacity of the roller-type auto-balancer monotonously increases with an increase in η. Therefore, the highest balancing capacity is achieved when the cylindrical rollers have the maximum possible height.

Let us study the maximum balancing capacities with regard to the parameters n and ρ.

5. 3. 1. Choosing the quantity of the corrective weights for the condition of achieving the highest balancing capacity of an auto-balancer

Let us assume that the geometrical sizes of the racetrack and the corrective weights have been determined and the material to produce the corrective weights has been chosen. Then in formulae (7) the parameters γ, R, η, ρ are constants. Let's find the quantity of the corrective weights to maximize the balancing capacity of the auto-balancer.

From formulae (7), it is obvious that the balancing capacity of the auto-balancer is formally the highest when the expression

$$\sin\{\tilde{n}\arcsin[\rho/(1-\rho)]\}$$

is equal to 1 or

$$\tilde{n} = \pi/\{2\arcsin[\rho/(1-\rho)]\}. \quad (8)$$

Since n is an integral number, the quantity of balls should be determined by the formula $n=[\tilde{n}]$, where $[\tilde{n}]$ is a function of rounding $\tilde{n}$ to the nearest integral number.

If $\tilde{n}\notin N$, then, depending on the angle nα, the corrective weights can occupy slightly less or more than a half of the auto-balancer racetrack.

If $\tilde{n}\in N$, then $n=\tilde{n}$, the corrective weights fill exactly a half of the racetrack. Herewith, from (7) we receive the following:

$$S_{AB}^{(b)} = 4\pi/3\cdot R^4\gamma\rho^2(1-\rho)^2; \quad S_{AB}^{(r)} = \pi\gamma R^4\eta\rho(1-\rho)^2.$$

If equation (8) is solved relatively to ρ (at $\tilde{n}=n$), we receive:

$$\rho = \sin[\pi/(2n)]/\{1+\sin[\pi/(2n)]\},$$

we find the ratio between the radiuses of the corrective weights and the racetrack at which n corrective weights occupy exactly a half of the racetrack.

5. 3. 2. The influence of the radius of corrective weights on the auto-balancing capacity of an auto-balancer

This influence will be studied with regard to the parameter ρ. Let's introduce the dimensionless functions:

$$s_{AB}^{(r)}(\rho)=\rho(1-\rho)^2\sin\{n\arcsin[\rho/(1-\rho)]\},$$

$$s_{AB}^{(b)}(\rho)=\rho s_{AB}^{(r)}(\rho). \qquad (9)$$

The balancing capacities of auto-balancers are directly proportional to these functions. Therefore, we shall further investigate these functions to determine the extremum.

Functions (9) are studied at fixed quantities of the corrective weights. Table 1 shows the results: n is the quantity of corrective weights in an auto-balancer; $\rho^{(b)}$ and $\rho^{(r)}$ are the optimal values of the parameter ρ, respectively, for the ball and roller-type auto-balancers; $\phi^{(b)}$ and $\phi^{(r)}$ are the angles of the racetrack sector (in degrees) that is filled by the corrective weights; $s_{AB}^{(b)}$, and $s_{AB}^{(r)}$ are the values of the dimensionless functions that characterize the balancing capacity of the corresponding auto-balancer. A similar table, although without the angles of the sectors, was obtained in [2] on the basis of other formulae for calculating the balancing capacity of an auto-balancer.

Table 1

Dependence of the balancing capacity of the auto-balancer on the quantity of the corrective weights

n	The corrective weights are balls			The corrective weights are rollers		
	$\rho^{(b)}$	$\phi^{(b)}$	$s_{AB}^{(b)}$	$\rho^{(r)}$	$\phi^{(r)}$	$s_{AB}^{(r)}$
1	3/4	–	0,105	2/3	–	0,148
2	3/7	194	0,060	2/5	167	0,143
3	0,364	209	0,051	1/3	180	0,148
4	0,315	219	0,044	0,286	189	0,145
5	0,278	226	0,037	0,251	195	0,140
6	0,248	231	0,031	0,223	201	0,133
7	0,224	235	0,027	0,201	205	0,126
8	0,204	238	0,023	0,183	208	0,119

Fig. 3 shows examples of ball auto-balancers of the maximum balancing capacity (at the optimal radius of the ball) for various quantities of balls in the auto-balancer.

Table 1 and Fig. 3 show that the highest balancing capacity of the ball auto-balancer (with a small number of balls: n<5) is achieved when the balls take slightly more than a half of the racetrack space. With a large number of balls (n≥5), it is possible to observe that the balancing capacity of the auto-balancer increases when the number of the balls is reduced by the following quantity: one ball when n=5÷8 (Fig. 3, *e–g*), two balls when n=9÷11, more than two balls when n>11. For cylindrical rollers, it is possible to remove: one roller when n=8÷12, two rollers when n=13÷16, and more than two rollers when n>17. It is noteworthy that the possibility of such a notion as "excess" balls at n≥5 was first disclosed in [4]. That study, however, does not specify the number of such balls, nor does it present any analysis of the optimization task correctness from the technical point of view.

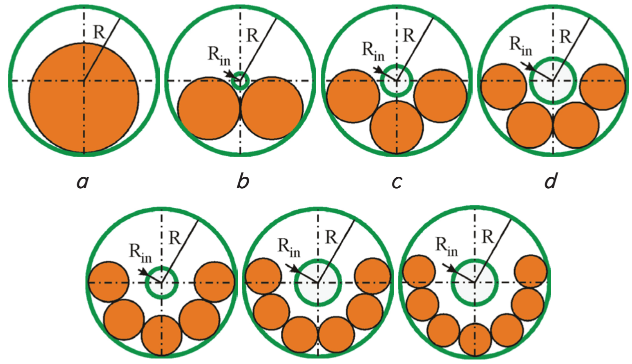


Fig. 3. The ball auto-balancers of the maximum capacity under a varied number of balls: *a* – one, *b* – two, *c* – three, *d* – four, *e* – five, *f* – six, *g* – seven

The absolute highest balancing capacity is achieved when there is one ball or one roller. From the practice point of view, this result is useless. It and the presence of "excess" corrective weights (at n≥5) can be explained by a technically incorrect definition of the task.

First, in the mathematical definition of the task, it is implicit that if it is an ball auto-alancer, the racetrack is a sphere, but if it is an roller-type balancer, the racetrack is a cylinder. In practice, auto-balancers are set on a rotor or a shaft and, therefore, their racetracks are torus-shaped. This limits the radii of the corrective weights from the top.

Second, when n≥5, there happens a false optimization. When the radius of the corrective weights is increased, the lower corrective weights (Fig. 3, *e–g* for the balls) boost the balancing capacity of the auto-balancer faster than it is reduced by the "excess" upper corrective weights.

A complete, rather than formal, optimization of the balancing capacity is carried out simultaneously for the quantity and the radius of the balls. To increase the balancing capacity, the ball radius increases continuously. When there appears a "excess" ball, it is removed immediately. The balancing capacity will grow faster in a "new" auto-balancer rather than in an auto-balancer with a "excess" ball.

Thus, when designing an auto-balancer, it is necessary to take into account the following:

1) for a steady performance of an auto-balancer and a faster achieving auto-balancing process, the auto-balancer should contain at least three correctiove weights;

2) to increase the balancing capacity of an auto-balancer, it is necessary:

– to use corrective weights of the largest possible radius;

– to choose the quantity of the correction weights so that they would take about a half of the racetrack, and their centers of mass would be under the horizontal axis passing through the center of the auto-balancer.

It is noteworthy that a sphere can contain a non-classical corrective weight in the form of a hemisphere having the radius of the sphere [3]. Such a corrective weight (one) can counterbalance a static unbalance in the cross plane crossing the rotor longitudinal axis in the center of the sphere.

The balancing capacity of the hemisphere is $\pi R^4\gamma/4$ [3]. The balancing capacity of one ball at $\rho=3/4$ is equal to $9\pi R^4\gamma/64$. Thus, the balancing capacity of the auto-balancer increases 16/9 times by replacing the ball with the hemisphere, which makes the auto-balancer completely functional.

In a cylinder, the highest balancing capacity is achieved with a corrective weight in the form of a semi-cylindrical sector. The balancing capacity of such a corrective weight is $S=2R^3h\gamma/3$ [3]. The balancing capacity of one roller at $\rho=2/3$ is equal to $4\gamma\pi R^3h/27$. Thus, the balancing capacity of the auto-balancer increases $9/(2\pi)\approx1.43$ times by replacing the roller with the semi-cylindrical sector. It is necessary to use two and more cylindrical sectors in order to make the sector auto-balancer functionally complete. These sectors all together should fill a half of the racetrack to maximize the balancing capacity.

5. 4. Increase of velocity of approach of the auto-balancing

We investigate expression (3) on to the largest value. For this purpose first, we will carry out assessment of the parameter p.

5. 4. 1. Determination of the largest value of the parameter p

In [17], with the assumption that the corrective weights do not impede each other's movement, it is shown that the largest value of the parameter p is equal to 1. It is reached when the corrective weights are broken into two groups and are placed in diametrically opposite positions. There are [n/2] kinds of such cases: one position is taken by j corrective weights (j=0,1,2, ..., [n/2]), and the other weights are in the diametrically opposite position. Each of these cases corresponds to a certain value of the static unbalance, and it may take infinitely long for auto-balancing to start.

In real auto-balancers, corrective weights can only touch each other. Therefore, the case when p=1 can occur only with two correctiove weights and under absence of a static unbalance of the rotor. In all other cases, the parameter p is less than 1. Let us determine the dependence of the largest value of the parameter p on the quantity and the size of the corrective weights for real auto-balancers (interpenetration of the corrective weights is not allowed).

First, we shall consider a marginal case when all corrective weights are on one side of the auto-balancer (the rotor unbalance is equal to the balancing capacity of the auto-balancer). In this case, the sum in (2) is equal to:

$$\sum_{i,k=1}^{n}\cos 2(\tilde{\psi}_i-\tilde{\psi}_k)=$$
$$=n+2\left[(n-1)\cos 4\alpha+\right.$$
$$\left.+(n-2)\cos 8\alpha+(n-3)\cos 12\alpha+\ldots+\cos 4(n-1)\alpha\right]=$$
$$=n+2\sum_{i=1}^{n-1}(n-j)\cos 4j\alpha=$$
$$-n+\left\{\sum_{i=1}^{n-1}(n-j)\left[\sin(2(2j+1)\alpha-\sin(2(2j-1)\alpha\right]\right\}/\sin 2\alpha=$$
$$=\left[\sum_{i=1}^{n}\sin 2(2j-1)\alpha\right]/\sin 2\alpha=$$
$$=\left\{\sum_{i=1}^{n}\left[\cos 4(j-1)\alpha-\cos 4j\alpha\right]\right\}/(2\sin^2 2\alpha)=$$
$$=\sin^2 2n\alpha/(\sin^2 2\alpha)$$

and from (2) we receive:

$$p=\sin 2n\alpha/(n\sin 2\alpha).$$

If the corrective weights fill exactly a half of the racetrack, then $\alpha=\pi/n$ and p=0.

Now let us consider other critical cases. We shall consistently transfer the corrective weights to the diametrically opposite side one by one. It will increase the parameter p. Fig. 4 shows the dependence of the parameter p on the quantity of corrective weights $j=\overline{0,[n/2]}$ in the diametrically opposite group at n=20 (curve 1), n=30 (curve 2) and n=40 (curve 3) corrective weights in the auto-balancer.

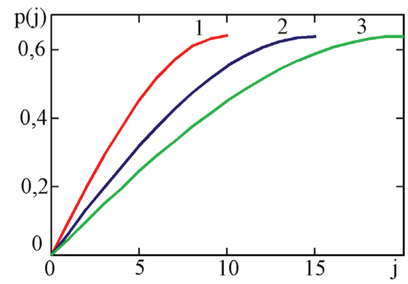


Fig. 4. Dependence of the parameter p on the quantity of corrective weights in the diametrically opposite position:
1 — 20 corrective weights in the auto-balancer;
2 — 30 corrective weights in the auto-balancer;
3 — 40 corrective weights in the auto-balancer

From Fig. 4, it follows that the largest value that is acquired by the parameter p is when the corrective weights are broken into two identical or almost identical groups at, respectively, an even number (n=2q) and an odd number (n=2q+1) of the corrective weights that are placed in diametrically opposite positions.

Let us assume that the auto-balancer contains an even number of corrective weights and they are located symmetrically relative to the vertical axis of the symmetry (Fig. 5) (it is possibly only in the absence of a static rotor unbalance).

Let us specify that the sum (2) has four groups of summands:

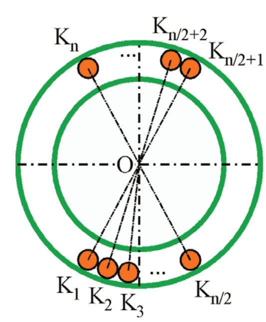


Fig. 5. Determination of the largest value of the parameter p

a) summands in which i=j, their number is equal to n and each of them is equal to 1, therefore $\Sigma_1=n$;

b) summands that contain corrective weights only from the bottom or only from the top group:

$$\Sigma_2=2\cdot 2\cdot\left[(q-1)\cos 4\alpha+(q-2)\cos 8\alpha+\right.$$
$$\left.+(q-3)\cos 12\alpha+\ldots+\cos 4(q-1)\alpha\right]=$$
$$=2\sum_{j=1}^{q}(n-2j)\cos 4j\alpha;$$

c) summands that contain one corrective weight from the bottom and the other corrective weight from the top group, but they are not diametrically opposite:

$$\Sigma_3 = 2\{[2+2(q-2)]\cos 4\alpha +$$
$$[4+2(q-4)]\cos 8\alpha + [6+2(q-6)]\cos 12\alpha + ... +$$
$$+[2(q-1)+2(q-2(q-1))]\cos 4(q-1)\alpha\} =$$
$$= 2\sum_{j=1}^{q}(n-2j)\cos 4j\alpha;$$

d) summands with diametrically opposite corrector weights (|i–j|=n/2)

$$\Sigma_4 = 2\cdot q = n.$$

From items a–d we receive:

$$\sum_{i,j=1}^{n}\cos 2(\tilde{\psi}_i-\tilde{\psi}_j) =$$
$$= 2n + 4\sum_{j=1}^{q}(n-2j)\cos 4j\alpha =$$
$$= 2n + 2\left\{\sum_{j=1}^{q}(n-2j)\left[\sin 2(2j+1)\alpha - \sin 2(2j-1)\alpha\right]\right\} / \sin 2\alpha =$$
$$= 2n + 2\left[-n\sin 2\alpha + 2\sum_{j=1}^{q}\sin 2(2j-1)\alpha\right] / \sin 2\alpha =$$
$$= 2\left\{\sum_{j=1}^{q}\left[\cos 4(j-1)\alpha - \cos 4j\alpha\right]\right\} / \sin^2 2\alpha =$$
$$= 2(1-\cos 2n\alpha)/\sin^2 2\alpha = (2\sin n\alpha/\sin 2\alpha)^2.$$

By entering the latter expression into (2), we receive:

$$p_{max} = 2\sin n\alpha / (n \sin 2\alpha). \quad (10)$$

If the corrective weights fill a half of the racetrack, then:

$$\alpha = \pi/(2n) \quad (11)$$

and

$$p_{max}(n) = 2/[n\sin(\pi/n)], \text{ when } n=2q. \quad (12)$$

Let us assume that the auto-balancer contains an odd quantity of corrective weights, and they are located symmetrically relative to the vertical axis. Herewith, in the lower position, there are q+1 corrective weights, and in the top position, their number is q (it is possible under a small static rotor unbalance, which is balanced almost by one corrective weight).

Similarly to the even quantity of corrective weights, the sum in (2) is broken into three groups of summands (there is no fourth group of summands for diametrically opposite corrective weights). The sums for the first two groups are the same. For the third group, we have:

$$\Sigma_3 = 4\left[q\cos 2\alpha + (q-1)\cos 6\alpha +\right.$$
$$\left.+(q-3)\cos 10\alpha + ... + \cos 2(2q-1)\alpha\right] =$$
$$= 2\sum_{j=1}^{q}(n+1-2j)\cos 2(2j-1)\alpha.$$

Thus, for an odd quantity of corrective weights we receive:

$$\sum_{i,j=1}^{n}\cos 2(\tilde{\psi}_i-\tilde{\psi}_j) =$$
$$= n + 2\sum_{j=1}^{q}\left[(n-2j)\cos 4j\alpha + (n+1-2j)\cos 2(2j-1)\alpha\right] =$$
$$= n + 2\sum_{j=1}^{n}(n-j)\cos 2j\alpha =$$
$$= n + \{\sum_{j=1}^{n}(n-j)\left[\sin(2j+1)\alpha - \sin(2j-1)\alpha\right]\} / \sin\alpha =$$
$$= \frac{1}{\sin\alpha}\sum_{j=1}^{n}\sin(2j-1)\alpha =$$
$$= \frac{1}{2\sin^2\alpha}\sum_{j=1}^{n}\left[\cos 2(j-1)\alpha - \cos 2j\alpha\right] = \frac{1-\cos n\alpha}{2\sin^2\alpha} = \frac{\sin^2 n\alpha}{\sin^2\alpha}.$$

By entering the latter expression into (2), we receive:

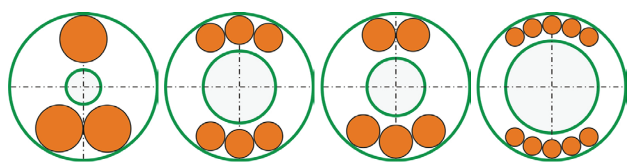
$$p_{max} = \sin n\alpha / (n\sin\alpha). \quad (13)$$

If the corrective weights fill a half of the racetrack, we receive:

$$p_{max}(n) = 1/\{n\sin[\pi/(2n)]\} \text{ when } n=2q+1. \quad (14)$$

Remark 1. It is possible to show that the right parts in (10) and (13) at n>2 are monotonously decreasing functions concerning the angle α (or a dimensionless parameter ρ) in their definition range.

Remark 2. Formulae (12) and (14) produce identical values of p_{max} if the quantity of corrective weights in them is equal, respectively, to n_1=2q+1 and n_2=2(2q+1), q=1, 2, 3... Fig. 6 shows examples of such pairs of auto-balancers for q=1 and q=2.



a *b* *c* *d*

Fig. 6. Examples of auto-balancers with odd and even quantities of corrective weights and the same value of the parameter p: *a* and *b* show three and six corrective weights in the auto-balancer; *c* and *d* show five and ten corrective weights in the auto-balancer

In (12) and (14), the function $p_{max}(n)$ is monotonously decreasing, and its smallest (extreme) value is equal to:

$$\lim_{n\to\infty} p_{max} = 2/\pi \approx 0{,}637.$$

Since $p_{max}(3)$=0.667, then the value of p_{max} can be sufficiently reduced by placing three corrective weights in the auto-balancer. Herewith, the parameter p_{max} exceeds its extreme value by 4.7 %.

5. 4. 2. Determination of the smallest value of the parameter p

In the absence of the static rotor unbalance and with more than two corrective weights in the auto-balancer, the value of p_{min} is always equal to 0.

Indeed, if n corrective weights are arranged in the tops of a regular n-angular shape, then the angles $\tilde{\psi}_i$ in (2) are equal to $\tilde{\psi}_i = 2\pi(i-1)/n, \ / i=\overline{1,n}\ /$ and

$$\sum_{i,j=1}^{n}\cos 2(\tilde{\psi}_i-\tilde{\psi}_j)=\sum_{i,k=1}^{n}\cos\left[4\pi(i-j)/n\right]=$$
$$=\left\{\sum_{i=1}^{n}\sum_{j=1}^{n}\left[\sin(2\pi(2i-2j+1)/n)-\sin(2\pi(2i-2j-1)/n)\right]\right\}/\left[2\sin(2\pi/n)\right]=$$
$$=\left\{\sum_{i=1}^{n}\left[\sin(2\pi(2i-1)/n)-\sin(2\pi(2i-2n-1)/n)\right]\right\}/\left[2\sin(2\pi/n)\right]=0.$$

When there appears and increases a static rotor unbalance, the value of p_{min} depends on the value of the static unbalance and on the quantity of the corrective weights. With an increase in the quantity of the corrective weights, the number of their positions at which $p_{min}=0$ increases significantly.

5. 4. 3. Determination of the shortest possible duration of transition processes depending on the radius and the quantity of corrective weights

Let us consider a case when there is no rotor unbalance (at an even quantity of corrective weights) or when the rotor is balanced by one corrective weight (at an odd quantity of corrective weights), i. e. when $p=p_{max}$.

Taking into account (3), (6), (10), and (13), we shall enter such dimensionless functions as $\tilde{a}^{(r)}(\rho)$, and $\tilde{a}^{(b)}(\rho)$, characterizing the duration of the transition processes, respectively, in roller-type and ball auto-balancers. For the odd quantity of corrective weights, they have the form of (15), but for the even quantity, they have the form of (16):

$$\tilde{a}^{(r)}(\rho)=$$
$$=\left\{n\rho-(1-\rho)\sin\left[n\arcsin(\rho/(1-\rho))\right]\right\}\rho,$$
$$\tilde{a}^{(b)}(\rho)=\rho\tilde{a}^{(r)}(\rho), \qquad (15)$$

$$\tilde{a}^{(r)}(\rho)=$$
$$=\left\{n\rho-(1-\rho)^2\sin\left[n\arcsin(\rho/(1-\rho))\right]/\sqrt{1-2\rho}\right\}\rho,$$
$$\tilde{a}^{(b)}(\rho)=\rho\tilde{a}^{(r)}(\rho). \qquad (16)$$

A larger value of these functions corresponds to a faster achieving of the auto-balancing.

Functions (15) and (16) at any fixed $n>2$ are monotonously increasing in the area of $\rho\in[0; 0.5]$. Formally, ρ can change from 0 to $\rho_{max}(n)$ at which n corrective weights completely occupy the racetrack. It is possible to show that $\rho_{max}(n)=1/[1+\csc(\pi/n)]$ at $n\geq 2$.

Fig. 7 displays a dependence of the values of functions (15) and (16) on the quantity of corrective weights in the auto-balancer at the corresponding values of the parameter ρ from Table 1. All these values are less than the corresponding $\rho_{max}(n)$, and they formally ensure the highest balancing capacity of the auto-balancer. The graphs are correct in the points of n=2, 3, 4 for balls and n=2, 3,..., 7 for rollers. The values of functions (15) and (16) are overstated for the other n since the radius of the corrective weights from Table 1 is larger than the actual optimal radius (at which the auto-balancer has the highest balancing capacity, and there are no "excess" corrective weights).

From Fig. 7, it follows that the shortest duration of the transition processes is provided by three balls or five cylindrical rollers.

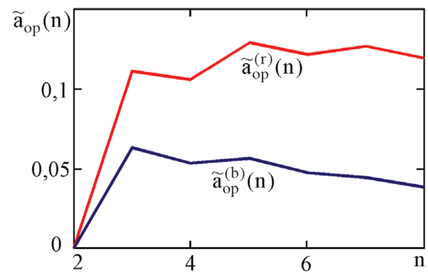


Fig. 7. Dependence of the functions that characterize the duration of the transition processes on the quantity of the corrective weights in the auto-balancer

6. Discussion of the research results about the influence of the parameters of corrective weights on the balancing capacity and the duration of the transition processes in an auto-balancer

1. The balancing capacity of ball and roller-type auto-balancers for any quantity of corrective weights is described by an analytical function that is subdued to analytical research.

2. To accelerate the process the achieving auto-balancing, it is necessary for the auto-balancer to contain at least three correctiove weights. To obtain the highest balancing capacity of an auto-balancer with a fixed volume, it is necessary for the corrective weights to fill almost a half of the racetrack.

The task of finding the radius of the corrective weights that would maximize the balancing capacity of the auto-balancer with a fixed diameter of the racetrack is not correct from the technical point of view. First, in the mathematical definition of the task, it is implicit that if it is an ball auto-balancer, the racetrack is a sphere, but if it is an roller-type auto-balancer, the racetrack is a cylinder. This leads to a practically useless result – a conclusion that the highest balancing capacity is achieved with one corrective weight. In practice, auto-balancers are set on a rotor or a shaft and, therefore, their racetracks are torus-shaped. This limits the radiuse of the corrective weights from the top. Second, when $n\geq 5$, there happens a false optimization. When the radius of the corrective weights is increased, the most remote corrective weights boost the balancing capacity of the auto-balancer faster than it is reduced by the "excess" corrective weights located closer to the horizontal line passing through the center of the auto-balancer. The only practically essential result of solving such a task is the requirement that corrective weights should have the largest possible radius.

It is noteworthy that the use of corrective weights of the largest radius increases the quality of balancing because it reduces the forces of resistance to swing.

3. The research on the duration of the transition processes for the smallest value has produced the following findings:

– the critical case (when p=1, it takes infinitely long for the process of auto-balancing to start) for real auto-balancers can occur only with two corrective weights in the auto-balancer and in the absence of a static unbalance;

– with an increase in the quantity of corrective weights in the auto-balancer, the maximum possible value of the parameter p decreases down to $2/\pi$;

– to accelerate the process the achieving auto-balbancing, it is necessary for the corrective weights to occupy nearly half of the racetrack;

– the shortest time of the auto-balbancing is achieved with three balls or five cylindrical rollers.

The advantage of the suggested approach of selecting the parameters of corrective weights for an auto-balancer is that it complies with the following three requirements:

– to increase the balance quality by reducing the forces of resistance to swing of the corrective weights;

– to achieve the highest balancing capacity with a fixed volume;

– to ensure the fastest possible the achieving auto-balbancing.

The limitations of the suggested approach are as follows. The obtained results are not checked in natural or virtual experiments. There are no estimates on an increase in the accuracy of auto-balancing or a reduced duration of transition processes for real rotors with auto-balancers.

The results of the study can be used in designing auto-balancers to balance fast-rotating rigid and flexible rotors on the run.

Further, it is planned to validate the obtained results by testing them on specific rotor machines with auto-balancers.

7. Conclusions

The study has revealed the optimum parameters of corrective weights in ball and roller-type auto-balancers. It has helped achieve coordination of the following three requirements: to increase the balance quality by reducing the forces of resistance to swing of the corrective weights; to achieve the highest balancing capacity with a fixed volume; and to ensure the fastest possible the achieving auto-balbancing. The main results are the following:

1. The balancing capacity of ball and roller-type auto-balancers for any quantity of corrective weights is described by an analytical function that is further subdued to analytical research.

2. To accelerate the process the achieving auto-balbancing, it is necessary for the auto-balancer to contain at least three correction weights. To obtain the highest balancing capacity of an auto-balancer with a fixed volume, it is necessary for the corrective weights to fill almost a half of the racetrack and to have the largest possible radius.

3. To accelerate the achieving auto-balbancing, it is necessary for the corrective weights to occupy nearly half of the racetrack. The shortest time of the auto-balbancing is achieved with three balls or five cylindrical rollers.

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