Розроблена математична модель та прикладна програма для моделювання виникнення регенеративних коливань при точінні у 3D просторі. Програма призначена для цілеспрямованого пошуку найкращих параметрів гармонічного закону зміни частоти обертання шпинделя при програмуванні на токарному верстаті з числовим програмним управлінням. Експериментально перевірено ефективність створеної програми

Ключові слова: математична модель процесу точіння, регенеративні коливання, управління швидкістю обертання шпинделя

Разработана математическая модель и прикладная программа для моделирования возникновения регенеративных колебаний при точении в 3D пространстве. Программа предназначена для целенаправленного поиска наилучших параметров гармонического закона изменения частоты вращения шпинделя при программировании на токарном станке с числовым программным управлением. Экспериментально проверена эффективность созданной программы

Ключевые слова: математическая модель процесса точения, регенеративные колебания, управление скоростью вращения шпинделя

UDC 621.91.01
DOI: 10.15587/1729-4061.2017.95204

PROGRAMMING SPINDLE SPEED VARIATION IN TURNING

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1. Introduction

Any process of cutting is carried out in the elastic technological machining system (TMS) and it is quasi-stationary, which determines its tendency to fluctuate. The presence of natural feedback from the process in combination with the machining «by chatter marks» gives rise to the so-called regenerative oscillations. The existence of these oscillations exerts negative effect on the quality of cutting process and may even lead to the loss of its stability. Suppression of regenerative oscillations was traditionally carried out by two techniques: by an increase in hardness and an increase in damping of elastic system or by reduction in the regime of cutting. These methods, however, were not characterized by proper effectiveness. The first one required the use of special mechanisms and rather labor-intensive procedure for their regulation. The second one led to the reduction in the machining efficiency. New possibilities for solving this issue appeared only in the contemporary machine tools with computer numerical control (CNC), equipped with drives for stepless regulation of the spindle frequency. The availability of such regulation made it possible to add machining with the variable cutting speed to the methods for suppressing the regenerative oscillations. Such an operating mode of machine tool in the turning operation is achieved due to a special change in the spindle speed of main motion engine. In contemporary turning machining centers with CNC, this regime is programmed by the option «Spindle Speed Variation» (SSV). Nevertheless, specific recommendations regarding the selection of parameters for such control are lacking [1]. Therefore, the task to provide for effectiveness in the application of the SSV option for suppressing the regenerative oscillations during turning is a relevant one.

2. Literature review and problem statement

The SSV control implies a change in the spindle speed according to the harmonic law. It is obvious that the amplitude of change and the period must depend on the characteristics of the cutting process and the TMS parameters. Thus, there is a need to create adequate mathematical model for the cutting process during turning.

A cutting process during turning is predominantly described by the single-mass dynamic models. In this case, they examine the motion along one coordinate only – normal to the processed surface. Thus, one of the first studies [2] on this issue present results of modeling the machining with a variable speed of the spindle by the method of approximate analytical solutions for the TMS motion equations in a rather narrow, specially selected, range of change in the parameters of machining. Results of this study are of qualitative nature and, first of all, demonstrate the prospects for applying such a mode of machining. Article [3], based on the model for cutting process with regard to the delayed argument, proposes to employ a heuristic criterion for selecting the frequency of change in speed taking into account a minimum of power consumption for the process of cutting. Directions of further studies [4, 5] are related to determining the optimum relationships between the amplitude and frequency of SSV. However, these studies give only certain recommendations, based on the dynamic phenomena of the drive and thermal limitations of the main motion engine. Furthermore, all the papers point to complexity in the creation of universal strategy for selecting the optimum combination of the SSV amplitude and frequency.

A new direction that should be noted separately for the elimination of vibrations at cutting, which implies the suppression of TMS oscillations with the aid of closed servo systems with feedback through the vibration sensors [6, 7]. This method implies a periodic action on the cutting process in the reversed phase with disturbing oscillations. It is clear that the quality of such control depends both on the performance speed of control servo systems and on the adequacy of mathematical model for the cutting process.

Among the reasons that lead to the occurrence of self-oscillations, the three most significant are as follows: the nonlinearity of dependence between the cutting force and the displacement of mechanical system [8], delay of the cutting force in comparison with the cutting depth (time constant of chip formation) [9] or the existence of connection that is caused by machining «by chatter marks» [10, 11]. An analysis of the known studies reveals that, when constructing mathematical models that describe dynamic phenomena during cutting, the complete totality of these factors is not considered. Consequently, the problem that remains vital is the construction of adequate model, which would represent the cutting process with regenerative oscillations and would make it possible to conduct the search for the most rational (or optimal) parameters in the SSV control.

3. The aim and tasks of the study

The aim of present study is to develop a mathematical model for the turning process taking into account oscillations in a 3D space, which will be applicable to determine parameters (amplitude and period) of the spindle speed variable.

To achieve the set aim, the following tasks are to be solved:

– to create a structural scheme for the turning process in a 3-D space taking into account the machining «by chatter marks» with the aid of delayed arguments;

– to create an applied program for the simulation of the cutting process with the reproduction of regenerative oscillations;

– conduct experimental studies of the turning machining with the SSV control according to the specified parameters.

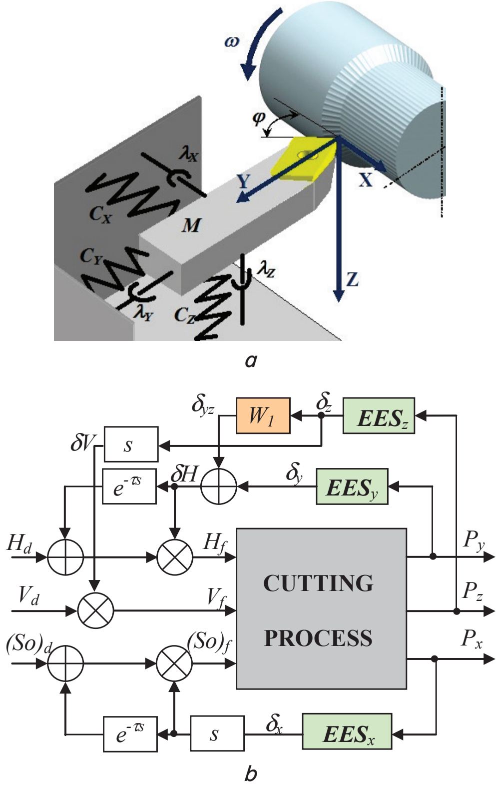
4. Methods and results of examining the turning process with a variable speed of the spindle

4. 1. Mathematical model

The existence of adequate mathematical model for the cutting process will make it possible to conduct a study on the impact of SSV parameters on the stability of this process. In this case, there occurs a possibility to select such values of the SSV parameters, which will ensure the required dynamic characteristics and subsequently to purposefully influence dynamics of the shape-forming process.

In order to compile a model, we shall use a dynamic scheme, in which elastic TMS is represented in the form of a single-mass system (reduced mass M) with three degrees of mobility along the Z, Y and X coordinates. Mass has elastic and damping links with the basis. These connections are characterized by rigidity C_Z, C_Y and C_X, as well as by viscous friction (frictional force is proportional to velocity) with the proportionality coefficients λ_Z, λ_Y and λ_X (Fig. 1, *a*).

In order to construct a mathematical model, it is convenient to use a structural scheme of the process that shows all the most important feedbacks. If we accept as input magnitudes parameters of the cutting process: assigned depth H_d, feed per revolution $(So)_d$ and cutting velocity V_d, and as the output magnitudes – constituents P_x, P_y and P_z of the cutting force, then the cutting process can be represented by the schematic, shown in Fig. 1, *b*.



a

b

Fig. 1. Model of the turning process: *a* — dynamic scheme; *b* — structural scheme; s — the Laplace operator; τ — period of one spindle revolution of the machine tool

Equivalent elastic system (EES) is represented by the blocks, which reflect its reaction to the action of the cutting force components along coordinate axes. That is why the EES reaction in the form of elastic deformation is also represented along the coordinate axes by components δ_Z, δ_Y and δ_X, respectively.

The rate of change in the component δ_X of the EES_x deformation is directed along the X axis and therefore it influences actual feed $(So)_f$, and, taking into account machining by chatter marks, this influence is represented by equation:

$$\left(So\right)_f = \left(So\right)_d - \frac{d\delta_x}{dt}\left(1 - e^{-\tau s}\right). \qquad (1)$$

Similarly, the rate of change in the component δ_Z of deformation EES_z affects the actual cutting speed:

$$V_f = V_d - \frac{d\delta_z}{dt}. \qquad (2)$$

Deformation EES_y exorts direct impact on the actual cutting depth H_f, and the influence of deformation EES_z can be determined based on the geometric relationships for shaping a cylindrical surface [11]. Ultimately, with regard to machining «by chatter marks», we have the following relation equation:

$$H_f = H_d - \left(\delta_y + \sqrt{R^2 + \delta_z^2} - R\right) \cdot \left(1 - e^{-\tau s}\right), \qquad (3)$$

where $R = D/2$ is the radius of the part.

The cutting process can be represented with the help of dependences known from the theory of cutting:

$$P_z = C_{Pz} H_f^{x_{pz}} \left(So\right)_f^{y_{pz}} \left(V\right)_f^{n} k, \qquad (4)$$

where C_{Pz}, k are the empirical coefficients, x_{pz}, y_{pz}, n are the exponents.

The dynamic characteristic of the cutting process, which in the first approximation is determined by the delay of cutting force P from the change in thickness of the cut-off allowance layer [9, 11], is expressed with the help of the chip formation time constant. By using known relationship $P_{xy} = 0.6 \cdot P_z$ and geometric arrangement of the cutting force components [10], we obtain the following dependences for calculating components P_y and P_x of the cutting force:

$$P_y = \sqrt{\frac{P_{xy}^2}{1 + \tan^2(\varphi + \eta)}}, \qquad (5)$$

$$P_x = P_y \cdot \tan(\varphi + \eta), \qquad (6)$$

where φ is the main angle in the cutter's plane, η is the chip exit angle ($\approx 5°$).

The motion of a single-mass system along each coordinate is described by three differential equations, each of the second order:

$$\begin{cases} \dfrac{d^2\delta_x}{dt^2} M + \lambda_X \dfrac{d\delta_x}{dt} + C_X \delta_x = P_x, \\ \dfrac{d^2\delta_y}{dt^2} M + \lambda_Y \dfrac{d\delta_y}{dt} + C_Y \delta_y = P_y, \\ \dfrac{d^2\delta_z}{dt^2} M + \lambda_Z \dfrac{d\delta_z}{dt} + C_Z \delta_z = P_z. \end{cases} \qquad (7)$$

Mathematical model of the cutting process in a closed elastic TMS, compiled by dependences (1)–(7), fully matches the functional scheme in Fig. 1, *b*.

4. 2. Modeling of the turning process

Taking into account that the above-described mathematical model is nonlinear and, with regard to the time constant of chip formation, has seventh order, modeling can be realized only by numerical methods on computer. For the convenience of using known numerical methods of integrating the differential nonlinear equations (for example, Runge-Kutta method of fourth order), it is necessary to represent system (7) with the help of state variables. Let us adopt the following state variables of dynamic model:

$$x_1 = \delta_X, \quad x_2 = s\delta_X, \quad x_3 = \delta_Y, \quad x_4 = s\delta_Y, \quad x_5 = \delta_Z, \quad x_6 = s\delta_Z,$$

where $s = d/dt$.

Then system (7) can be represented in the matrix form, which is the most acceptable for employing numerical methods:

$$\begin{vmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \end{vmatrix} = \begin{vmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ a_1 & a_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & a_3 & a_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & a_5 & a_6 \end{vmatrix} \cdot \begin{vmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{vmatrix} + \begin{vmatrix} 0 & 0 & 0 \\ b_1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & b_2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & b_3 \end{vmatrix} \cdot \begin{vmatrix} P_x \\ P_y \\ P_z \end{vmatrix}, \qquad (8)$$

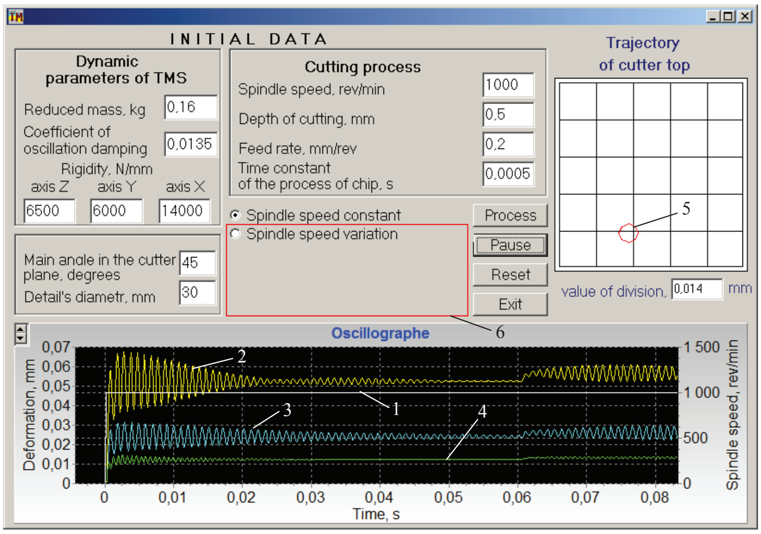
where $a_1 = -C_X/M$, $a_2 = -\lambda_X/M$, $a_3 = -C_Y/M$, $a_4 = -\lambda_Y/M$, $a_5 = -C_Z/M$, $a_6 = -\lambda_Z/M$, $b_1 = 1/M$, $b_2 = 1/M$, $b_3 = 1/M$ are the matrix coefficients.

Contemporary approach to the construction of mathematical models requires its representation in the form of software. In this case, there appears a possibility to verify (qualitative) adequacy of the created model and to obtain results of simulation in the digital form. Subsequently they can be directly used when solving practical tasks in the technology of machine building. Therefore, the mathematical model developed forms the basis of the algorithm, by which the created applied program operates.

Simulation is carried out by numerical method with a constant integration step (0.000025 s) in the function of data array by the angular coordinate of the billet's contour and by the number of its revolution. Such a structure of algorithm makes it possible to consider the impact of machining «by chatter marks». In this case, error from elastic deformations, which was formed in the billet's first revolution, is received by TMS as a change in the assigned parameters of the cutting mode in the next revolution, etc. Mathematical model is represented through the state variables of system [7], which makes it possible to carry out its integration by the Runge-Kutta method of fourth order.

Fig. 2 shows an interface of the applied software during simulation of the turning process. The initial parameters of this process are given in the appropriate windows of the interface. We simulate machining at constant speed of the spindle – 1000 r/min – line 1 in the oscillograph. TMS, after the end of transient process, in time 0.06 s, which corresponds to one revolution of the billet, registered the nondamping self-oscillations: along the Z axis – line 2, along the Y axis – line 3 and along the X axis – line 4.

In the YZ cross section (Fig. 1, *a*), such oscillations trigger the motion of cutter top along the ellipsis (line 5), which confirms qualitative adequacy of the developed mathematical model. In order to simulate the turning at the variable speed of the spindle, it is sufficient to select the appropriate option in the interface. In this case, window 6 provides for a possibility to assign the amplitude and period for changing the spindle frequency.



INITIAL DATA

Dynamic parameters of TMS
Reduced mass, kg: 0,16
Coefficient of oscillation damping: 0,0135
Rigidity, N/mm: axis Z 6500; axis Y 6000; axis X 14000

Cutting process
Spindle speed, rev/min: 1000
Depth of cutting, mm: 0,5
Feed rate, mm/rev: 0,2
Time constant of the process of chip, s: 0,0005

● Spindle speed constant
○ Spindle speed variation

Main angle in the cutter plane, degrees: 45
Detail's diametr, mm: 30

Trajectory of cutter top
value of division, 0,014 mm

Fig. 2. Interface of the applied software

4. 3. Experimental studies of turning at the SSV control

The applied program developed makes it possible to perform simulation of the cutting process at variable frequency of the spindle. Thus, with the aid of the created program, we conducted preliminary search for the best period of change in the spindle speed at the amplitude of 15 % from the assigned one. We considered the best period being the one when the time of oscillation damping to the magnitude of amplitude (1 μm), assigned in the program, over the time of one revolution of the billet, was minimal. Obtained results (Fig. 3) demonstrated that for the selected range of change in the cutting depth (H=0.5 mm, 0.75 mm and 1 mm), the best one is the period of 1.2 s. Such particular parameters were set on the CNC rack of the machine tool ST-30 made by HAAS (USA). The rest of the parameters, assigned in the windows of interface of the applied program (Fig. 2), also match the conditions for conducting the experiment.

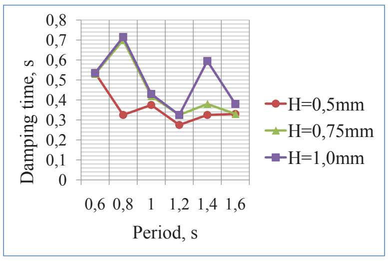


Fig. 3. Chart of dependence of the time of oscillation damping on the period of change in the frequency of spindle

In the course of experiments (Fig. 4), we processed billets 1, fixed in the cartridge, of diameter 30 mm at the length of console 200 mm, by tool 2 with vibration sensor 3 installed on it. The machining was conducted both at constant frequency of the spindle (1000 rev/min) and when applying the SSV option (amplitude 15 %, period 1.2 s). Vibrations of TMS at machining were recorded on the personal computer with the aid of the applied package LabView.

Results are shown in Fig. 5. They indicate a considerable decrease in the level of oscillations in the range of frequencies to 1000 Hz when applying the SSV option (Fig. 5, *d*). In this case, we observe the corresponding reduction in roughness of the processed surface (Fig. 5, *c*).

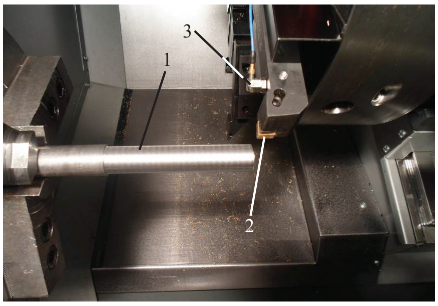
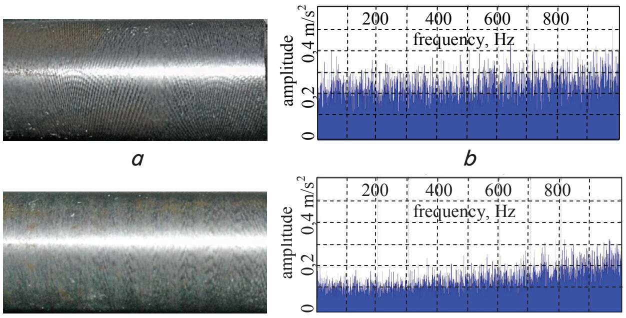


Fig. 4. Working zone of machine tool ST-30 when conducting the experiment

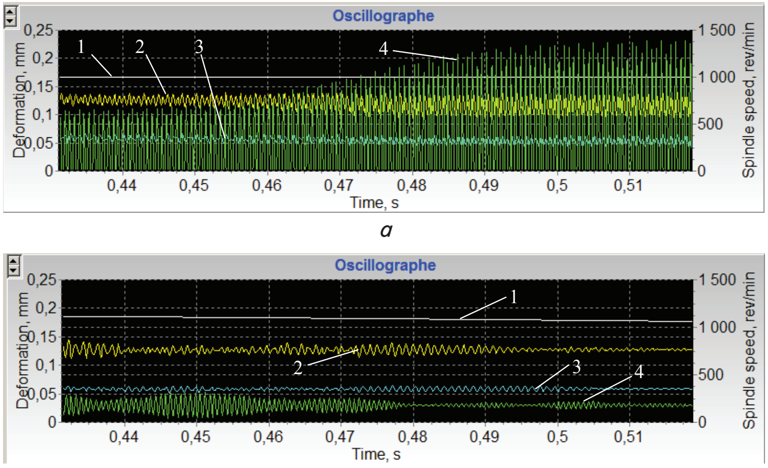


a *b*

c *d*

Fig. 5. Results of the experiments: machining at constant frequency of the spindle (*a* — part, *b* — oscillation spectrum); machining with SSV (*c* — part, *d* — oscillation spectrum)

Applying control over the spindle speed in the turning operation makes it possible to considerably enlarge the region of steady cutting due to the suppression of regenerative oscillations, which typically lead to the loss of stability. Fig. 6 shows oscillograms of the TMS oscillations, obtained in the simulation of cutting process with depth 1.3 mm in the created applied program:



a

b

Fig. 6. Oscillations of TMS: *a* — machining at constant frequency of the spindle; *b* — SSV; 1 — frequency of the spindle; 2 — oscillations of TMS along the Z axis; 3 — oscillations of TMS along the Y axis; 4 — oscillations of TMS along the X axis

Oscillograms (Fig. 6) demonstrate that over the identical time (0.515 s), the system at constant frequency of the spindle is approaching the loss of stability. Let us note that, in spite of large rigidity along the X axis (Fig. 2), in this particular direction the regenerative oscillations lead to the self-excitation of the system. Switching on the SSV option with the parameters, specified earlier, leads to the elimination of regenerative oscillations and to the possibility of machining with the assigned cutting depth.

5. SSV Discussion of results of examining the SSV programming

Any control based on the a priori information, first of all, depends on the adequacy of given information. Programming of SSV on the machine tool with CNC is related to such a control. That is why a fundamental difference between the present study and those conducted earlier is in the application of the created model for the turning process by taking into account oscillations in a 3D space. The model accounts for the influence of natural feedbacks between the elastic TMS and cutting process on the change in depth, feed and cutting speed. For the first time, the effect of machining by chatter marks is introduced to the model by two directions – by depth and by feed. In this case, error from the TMS elastic deformations, which was formed in the first revolution of the billet, is perceived by TMS as a change in the assigned parameters for the cutting mode in the next revolution, etc.

Taking into account such representation of the dynamic behavior of TMS when turning «by chatter marks», the mathematical model for this process becomes substantially nonlinear (seventh order), and modeling is realized by numerical methods on computer. In order to implement the studies conducted into practice, we proposed a procedure and original applied software.

Results of simulation of the TMS oscillations by three coordinates taking into account the effect of machining «by chatter marks» revealed ambiguity in the influence of machining parameters and elastic characteristics of TMS on the development of regenerative oscillations. In a general case, regenerative oscillations may lead to the self-excitation of the system along any coordinate. This to a certain extent explains complexity of eliminating the regenerative oscillations by traditional methods and proves the need for using 3D models to describing the process of their occurrence.

Experimental testing demonstrated effectiveness of applying the developed model for determining parameters for switching on the SSV option on the machine tool ST-30 made by HAAS (USA) in the process of turning.

In future, we plan to perform comprehensive studies for the purpose of improving adequacy of the model and developing a technique to search for the optimum parameters in the SSV programming.

6. Conclusions

1. A mathematical model that we developed for the turning process considers the TMS oscillations in a 3D space, along three axes simultaneously, as well as the influence of machining «by chatter marks» with the aid of functions of delayed argument. A method for numerical solution of the compiled mathematical model is proposed, organized in the form of the algorithm with matrix representation of the model by state variables and the Runge-Kutta procedure of integration of 4th order, which is implemented in the applied software.
2. It is proposed to determine the period of change in the spindle speed via simulation by the criterion of time minimum of oscillation damping to the established magnitude of amplitude along all three coordinate axes per one revolution of the billet. We experimentally confirmed adequacy of the developed mathematical model and effectiveness of the SSV method with specified parameters – the amplitude and period of change in the spindle frequency.
3. The developed mathematical model and created applied software make it possible to conduct modeling of the turning process with a SSV control and, thus, they can serve as the tool for a technologist-programmer to determine the best parameters for the law of variation in the spindle velocity – amplitude and frequency – in each particular case.

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UDC 621.91 : 621.9.015 : 539.2
DOI: 10.15587/1729-4061.2017.96403

SPECIAL FEATURES IN THE APPLICATION OF FRACTAL ANALYSIS FOR EXAMINING THE SURFACE MICRORELIEF FORMED AT FACE MILLING

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Метод фрактального аналізу застосований для опису площі поверхні мікрорельєфу, що формується торцевим фрезеруванням при різних значеннях подачі. Розглядаються найбільш сприятливі умови отримання фотозображень поверхні зразків та аналізуються залежності фрактальних параметрів рельєфа поверхні від просторових масштабів його досліджень. Отримані кількісні взаємозв'язки між Хаусдорфовими розмірностями для площі поверхні мікрорельєфа пластин і подачами заготовки

Ключові слова: фрактальний аналіз, шорсткість поверхні, торцеве фрезерування, площа поверхневого рельєфа

Метод фрактального анализа применен для описания площади поверхности микрорельефа, формирующегося торцевым фрезерованием при различных значениях подачи. Рассматриваются наиболее благоприятные условия получения фотоизображений поверхности образцов и анализируются зависимости фрактальных параметров рельефа поверхности от пространственных масштабов его исследований. Получены количественные взаимосвязи между Хаусдорфовыми размерностями для площади поверхности микрорельефа пластин и подачами заготовки

Ключевые слова: фрактальный анализ, шероховатость поверхности, торцевое фрезерование, площадь поверхностного рельефа

1. Introduction

Providing the assigned level of roughness of the surfaces of parts at machining is an important direction of machine engineering. Surface roughness has an essential effect on the performance properties of parts. Given this, it becomes clear why significant attention is paid to studying the microrelief of the surface of samples both theoretically and practically. Among the theoretical directions in the studies of the surface microrelief state, of special interest are the series of articles that deal with the application of sufficiently new, nontrivial mathematical methods for analysis of the surface relief state [1–7]. They include papers on the application of fractal representations to the description of the self-similar states, which are formed either naturally, for example, due to the reflection by the surface of crystallographic properties of the volume of material of the base layer [8, 9], or due to the periodic action on the surface of machnining tool [10, 11].