Визначено умови настання автобалансування для жорсткого осесиметричного ротора на двох ізотропних пружних опорах, що балансується будь-якою кількістю пасивних автобалансирів будь-якого типу. Застосовано емпіричний критерій настання автобалансування. Розглянуто випадки: динамічного балансування ротора декількома автобалансирами в декількох площинах корекції; статичного балансування ротора одним і більше автобалансирами в одній площині корекції

Ключові слова: двохопорний ротор, пасивний автобалансир, автобалансування, критерій настання автобалансування, просторовий рух ротора

Определены условия наступления автобалансировки для жесткого осесимметричного ротора на двух изотропных упругих опорах, балансируемого любым количеством пассивных автобалансиров любого типа. Применен эмпирический критерий наступления автобалансировки. Рассмотрены случаи: динамической балансировки ротора несколькими автобалансирами в нескольких плоскостях коррекции; статической балансировки ротора одним и более автобалансирами в одной плоскости коррекции

Ключевые слова: двухопорный ротор, пассивный автобалансир, автобалансировка, критерий наступления автобалансировки, пространственное движение ротора

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APPLICATION OF THE EMPIRICAL CRITERION FOR THE OCCURRENCE OF AUTO-BALANCING FOR AXISYMMETRIC ROTOR ON TWO ISOTROPIC ELASTIC SUPPORTS

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1. Introduction

For balancing of high-speed rotors in motion, in the process of exploitation, passive auto-balancers are applied [1, 2].

For application of passive auto-balancers, it is necessary to know whether it is principality possible and on what rotation speeds to balance by them in motion the rigid or flexible rotor installed on the certain supports. Analytically the problems of determining the conditions for the occurrence of auto-balancing are solved for the concrete type auto-balancers [1, 3–10]. Therefore, the received results have special case character.

The solution of similar tasks becomes considerably complicated in cases of rotor balancing by:

– multi-corrective weight auto-balancers;

– multi-row ball or roller balancers;

– several auto-balancers in several correction planes, etc.

These difficulties make it possible to overcome the empirical criteria [2, 11, 12]. The conditions for the occurrence of auto-balancing received with their application are applicable for any type auto-balancers.

2. Literature review and problem statement

The conditions for the occurrence of auto-balancing at static balancing of a two-support rotor by one auto-balancer were found in the cases:

– isotropic supports and two-ball balancer [3];

– anisotropic supports and two-ball balancer [4];

– isotropic supports and multi-ball balancer [5].

The conditions for the occurrence of auto-balancing at dynamic balancing of a two-support rotor by two auto-balancers were found in the cases:

– isotropic supports and two-ball balancers which mass is much less than the mass of the rotor [6];

– isotropic supports and two-ball balancers [7];

– anisotropic supports and two-ball balancers [8];

– the rotor installed in the massive case withheld by elastic supports, and multi-ball balancers with identical balls [9];

– the flexible rotor on isotropic supports and multi-ball balancers with identical balls [10].

The conditions were received by research of the stability of the so-called basic motions. These are steady-state motions of a rotor-auto-balancer system on which an auto-balancing occurs.

The search for conditions for the occurrence of auto-balancing by allocation of the steady-state motions of the rotor-auto-balancer system and research of their stability is a complex and cumbersome mathematical problem. The approach is not effective in the cases of balancing the rotor by:

– multi-corrective weight auto-balancers;
– several auto-balancers (excess quantity);
– multi-row ball or roller balancers etc.

The second drawback of this approach is that the conditions for the occurrence of auto-balancing are obtained for the specific type of auto-balancer. For another type of auto-balancer, it is necessary to receive these conditions anew.

Paper [2] proposed the engineering (empirical) criterion for the occurrence of auto-balancing with the balancing of the rotor by an auto-balancer of any type in one correction plane. In accordance with the criterion, the occurrence of auto-balancing depends on the reaction of the rotor to an elementary imbalance applied in the correction plane. The auto-balancing will occurr when and only when, on average at one rotation of the rotor, the sagging of rotor in the plane of correction will be directed opposite to this imbalance. It is assumed that the mass of the auto-balancer is much less than the mass of the rotor. By applying the criterion, analytical conditions for the occurrence of auto-balancing were obtained when balancing of rotor that performs planar, spherical, spatial motions by one auto-balancer of any type.

Paper [11] proposed the empirical criterion of stability of the main motion in the case of rotor balancing (both elastic and rigid) by several auto-balancers of a particular type. Its effectiveness was demonstrated when determining the stability conditions of the main motions at balancing of artificial Earth satellites, stabilized by rotation, by one or two auto-balancers. This criterion is the most effective for the analysis of stability of the main motions and their families. But there is a caveat. In the studies, the type of auto-balancers is considered. That is why the studies remain cumbersome while the obtained results are applicable only to a particular type of auto-balancers.

In the paper [12] the empirical criterion of stability of the main motion was modernized for obtaining conditions for the occurrence of auto-balancing, suitable for any type of auto-balancers. Application of the new criterion and its efficiency were illustrated on the problem of balancing of a rigid axisymmetric rotor with a fixed point and an isotropic elastic support by several auto-balancers (excess quantity).

In this paper, the empirical criterion for the occurrence of auto-balancing is applied to an axisymmetric rotor on two isotropic elastic supports. As the result, the conditions of the static or dynamic balancing of the rotor by any quantity of passive auto-balancers of any type are defined. The task is relevant for extractors, centrifuges, separators, etc. The rotors of such machines can be loaded with a large amount of the treated raw material, which creates large imbalances. Therefore, it is expedient to balance them with several auto-balancers, including multi-row balancers.

3. The purpose and tasks of the research

The purpose of this work is to obtain the conditions under which several passive automatic balancers of any type will balance statically or dynamically a rigid axisymmetric rotor on two isotropic elastic supports.

To achieve this purpose, it is necessary to solve the following research tasks:

– to construct the physical and mathematical model of a rigid axisymmetric rotor on two isotropic elastic supports with elementary unbalances, applied in the future suspension points of the auto-balancers;
– to receive the functional that determines the conditions for the occurrence of auto-balancing with application of the empirical criterion for the occurrence of auto-balancing;
– to find the conditions for the occurrence of auto-balancing in the case of dynamic balancing of the rotor (in two and more correction planes);
– to find the conditions for the occurrence of auto-balancing in the case of static balancing of the rotor (in one correction plane).

4. Methods of searching the conditions for the occurrence of auto-balancing

The empirical criterion for the occurrence of auto-balancing is used [12]. The criterion is intended to answer the question – whether it is possible in principle and under what conditions to balance automatically the concrete rotor by n passive auto-balancers of any type. According to the criterion, for the occurrence of auto-balancing it is necessary and sufficient that at any elementary imbalances the condition is satisfied:

$$\frac{1}{T}\int_0^T\left(\sum_{j=1}^{n}\vec{s}_j(t)\cdot\vec{r}_j(t)\right)dt<0, \qquad (1)$$

where t is the time; $\vec{s}_j$ is the elementary rotor unbalance lying in the j-th correction plane and applied at the corresponding point j on the longitudinal axle of the rotor $/j=\overline{1,n}/$; $\vec{r}_j$ is the vector of deviation of the point j from its position in the motionless rotor, caused by elementary rotor unbalances $\vec{s}_1,...,\vec{s}_n$; T is the period in case the motion is periodic or another characteristic time interval (time of one or several rotations of rotor, time interval considerably larger than 1, etc.).

The criterion is applied in the following sequence:

1) a physical-mechanical model of a rotor with elementary rotor unbalances applied at the future suspension points of auto-balancers, is described;
2) differential equations of motion of the unbalanced rotor are derived;
3) steady-state motion of a rotor, which corresponds to the applied elementary imbalances, is searched for as a particular solution of the heterogeneous system of equations of motion;
4) a functional of the criterion for the occurrence of auto-balancing is built;
5) conditions for the occurrence of auto-balancing are determined from the condition of negativity of the functional.

Let us note that, as a rule, the functional of the criterion is a quadratic form of the elementary imbalances. The negative definiteness of this form can be investigated by using Sylvester's criterion. The result is the conditions of two types. The first ones impose limitations on the mass-inertia characteristics of a rotor. The second ones are the range of angular rates of rotation of the rotor, on which auto-balancing will occur provided the first conditions are satisfied.

5. Results of the researches to determine the conditions for the occurrence of auto-balancing for an axisymmetric rotor on two isotropic supports

5. 1. Description of a physical-mechanical model of the rotor

The scheme explaining the way of definition of motion of a rotor on a rigid weightless shaft and elastic supports is given in Fig. 1. In Fig. 1, *a*, the position of a motionless rotor is shown. For the description of its motion, we use the immovable right rectangular coordinate system Kxyz. It begins in a center of mass of a motionless rotor. The axis z is directed on a rotor axis of rotation, and axes x and y are directed perpendicular to this axis (Fig. 1, *a*). Similar axes, Guvw are rigidly connected with the rotor. In the initial position, the Guvw axes coincide with the axes Kxyz.

Coordinates x, y set the translatory motion of the rotor together with the center of mass – point G (Fig. 1, *b*). Rezal's angles α, β define the turn of the longitudinal axis of the rotor around the point G (Fig. 1, *c*). We consider that the rotor rotates around the longitudinal axis with constant angular speed ω. Then, the rotor angle of rotation around this axis φ=ωt, where t is the time.

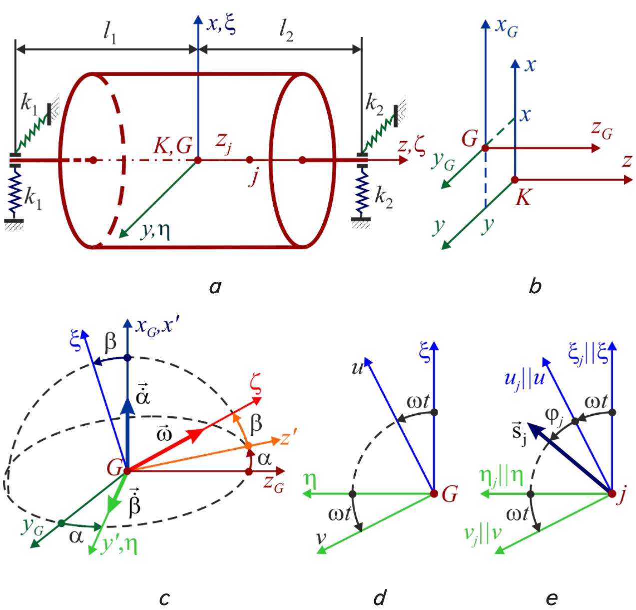

Fig. 1. The model of an unbalanced axisymmetric rotor on two isotropic elastic supports: *a* – the scheme of the rotor with points of application of elementary unbalances to the longitudinal axis of the rotor; *b* – forward displacement of the rotor together with center of mass G; *c* – turns of the rotor around the center of mass G on Rezal's angles α, β; *d* – the turn of the rotor around the longitudinal axis at an angle ωt; *e* – the turn of the elementary unbalance $\vec{s}_j$ together with the rotor

It is supposed that the rotor is axisymmetric. Then, the axial moments of inertia of the rotor relative to the main central axes ξ_G, η_G, ζ_G parallel to the axes ξ, η, ζ (and the u_G, v_G, w_G axes parallel to the axes u, v, w) are respectively equal to A, A, C.

5. 2. Differential equations of motion of the rotor

With application of the Lagrange's equations of the second kind or the main theorem of dynamics, it is possible to receive the following differential equations of motion of the rotor [2]:

$$M\ddot{x}+k_{11}x+k_{14}\beta=\omega^2\sum\nolimits_{j=1}^{n}s_j\cos(\omega t+\phi_j),$$

$$M\ddot{y}+k_{11}y-k_{14}\alpha=\omega^2\sum\nolimits_{j=1}^{n}s_j\sin(\omega t+\phi_j),$$

$$A\ddot{\alpha}+C\omega\dot{\beta}+k_{33}\alpha-k_{14}y=-\omega^2\sum\nolimits_{j=1}^{n}s_j z_j\sin(\omega t+\phi_j),$$

$$A\ddot{\beta}-C\omega\dot{\alpha}+k_{33}\beta+k_{14}x=\omega^2\sum\nolimits_{j=1}^{n}s_j z_j\cos(\omega t+\phi_j), \qquad (2)$$

where

$$k_{11}=k_1+k_2,\quad k_{14}=k_2l_2-k_1l_1,\quad k_{33}=k_1l_1^2+k_2l_2^2. \qquad (3)$$

Let us introduce complex variables:

$$z=x+iy,$$

$$\psi=\alpha+i\beta. \qquad (4)$$

Let us multiply the second equation in (2) by imaginary unit i and add to the first one. Let us multiply the fourth equation in (2) by imaginary unit i and add to the third one. We will obtain the following differential equation of motion of the rotor in the complex form:

$$M\ddot{z}+k_{11}z-ik_{14}\psi=S\omega^2e^{i\omega t},$$

$$A\ddot{\psi}-iC\omega\dot{\psi}+k_{33}\psi+ik_{14}z=iI\omega^2e^{i\omega t}, \qquad (5)$$

where

$$S=\sum\nolimits_{j=1}^{n}s_je^{i\phi_j},$$

$$I=\sum\nolimits_{j=1}^{n}s_jz_je^{i\phi_j}. \qquad (6)$$

Let us note that:

$$S_x=\sum\nolimits_{j=1}^{n}s_j\cos\phi_j,\quad S_y=\sum\nolimits_{j=1}^{n}s_j\sin\phi_j,$$

$$I_{uw}=\sum\nolimits_{j=1}^{n}s_jz_j\cos\phi_j,\ I_{vw}=\sum\nolimits_{j=1}^{n}s_jz_j\sin\phi_j, \qquad (7)$$

the static moments and the products of inertia of the rotor, which define, respectively, its static and moment unbalances.

If there are two or more correction planes, the static moments and products of inertia are independent. If all auto-balancers have a common correction plane ($z_j=z$, $/\,j=\overline{1,n}\,/$), then:

$$I_{uw}=z\sum\nolimits_{j=1}^{n}s_j\cos\phi_j=zS_x,\quad I_{vw}=z\sum\nolimits_{j=1}^{n}s_jz_j\sin\phi_j=zS_y. \qquad (8)$$

In this case, the static moments and products of inertia are dependent.

5. 3. The steady-state motion of the rotor, which corresponds to the applied elementary unbalances

We look for the partial solution of a system of differential equations (5) in the form:

$$z = De^{i\omega t}, \quad \psi = Ee^{i\omega t}. \tag{9}$$

Substitution of (9) into (5), after reduction by $e^{i\omega t}$ gives the following system of the algebraic equations for the definition of D, E:

$$\begin{pmatrix} k_{11} - M\omega^2 & -ik_{14} \\ ik_{14} & k_{33} - (A-C)\omega^2 \end{pmatrix}\begin{pmatrix} D \\ E \end{pmatrix} = \begin{pmatrix} S \\ iI \end{pmatrix}\omega^2. \tag{10}$$

The determinant of this system is the frequency equation and has the form:

$$\Delta(\omega) = (k_{11} - M\omega^2)[k_{33} - (A-C)\omega^2] - k_{14}^2 = \\ = M(A-C)\omega^4 - [(A-C)k_{11} + Mk_{33}]\omega^2 + k_{11}k_{33} - k_{14}^2. \tag{11}$$

Let us introduce designations:

$$\Delta_D(\omega) = \omega^2\{[k_{33} - (A-C)\omega^2]S - Ik_{14}\},$$

$$\Delta_E(\omega) = i\omega^2[(k_{11} - M\omega^2)I - Sk_{14}]. \tag{12}$$

Then, the solution of the system of equations (10) has the form:

$$D = \Delta_D(\omega)/\Delta(\omega), \; E = \Delta_E(\omega)/\Delta(\omega). \tag{13}$$

We investigate the solutions of the equations (11).

Let us note that:

$$k_{11}k_{33} - k_{14}^2 = (k_1 + k_2)(k_1 l_1^2 + k_2 l_2^2) - \\ -(k_2 l_2 - k_1 l_1)^2 = k_1 k_2 (l_1 + l_2)^2 > 0. \tag{14}$$

1. In the case of a long rotor, A>C and the frequency equation (11) gives two resonance frequencies, such that:

$$\omega_1^2 = \frac{(A-C)k_{11} + Mk_{33} - \sqrt{[(A-C)k_{11} + Mk_{33}]^2 - 4M(A-C)(k_{11}k_{33} - k_{14}^2)}}{2M(A-C)},$$

$$\omega_2^2 = \frac{(A-C)k_{11} + Mk_{33} + \sqrt{[(A-C)k_{11} + Mk_{33}]^2 - 4M(A-C)(k_{11}k_{33} - k_{14}^2)}}{2M(A-C)}. \tag{15}$$

Let us introduce the partial frequencies:

$$\omega_{p1} = \sqrt{k_{11}/M}, \; \omega_{p2} = \sqrt{k_{33}/(A-C)}. \tag{16}$$

For real rotor systems $\omega_{p1} < \omega_{p2}$ and therefore:

$$\omega_1 < \omega_{p1} < \omega_{p2} < \omega_2. \tag{17}$$

In the case under consideration, the determinant (11) can be presented in the form:

$$\Delta(\omega) = M(A-C)(\omega_{p1}^2 - \omega^2)(\omega_{p2}^2 - \omega^2) - k_{14}^2 = \\ = M(A-C)(\omega_1^2 - \omega^2)(\omega_2^2 - \omega^2). \tag{18}$$

On above resonance speeds of the rotor rotation ($\omega > \omega_2$), the determinant $\Delta(\omega) > 0$.

2. In the case of a spherical rotor, A=C and the determinant (11) takes the form

$$\Delta(\omega) = -Mk_{33}\omega^2 + k_{11}k_{33} - k_{14}^2 = 0,$$

from which we find the only resonance frequency:

$$\omega_1 = \sqrt{(k_{11}k_{33} - k_{14}^2)/Mk_{33}}. \tag{19}$$

In the case under consideration, the determinant (11) can be presented in the form:

$$\Delta(\omega) = Mk_{33}(\omega_1^2 - \omega^2). \tag{20}$$

On above resonance speeds of rotor rotation ($\omega > \omega_1$), the determinant $\Delta(\omega) < 0$.

3. In the case of a short rotor, A<C and the equation (11) gives one resonance frequency, such that:

$$\omega_1^2 = \frac{-(C-A)k_{11} + Mk_{33} + \sqrt{[-(C-A)k_{11} + Mk_{33}]^2 + 4M(C-A)(k_{11}k_{33} - k_{14}^2)}}{2M(C-A)}. \tag{21}$$

In the case under consideration, the determinant (11) can be presented in the form:

$$\Delta(\omega) = M(C-A)(\omega_{p1}^2 - \omega^2)(\omega_{p2}^2 + \omega^2) - k_{14}^2 = \\ = M(C-A)(\omega_1^2 - \omega^2)(\omega_2^2 + \omega^2), \tag{22}$$

where

$$\omega_{p2} = \sqrt{k_{33}/(C-A)},$$

$$\omega_2^2 = \frac{(C-A)k_{11} - Mk_{33} + \sqrt{[-(C-A)k_{11} + Mk_{33}]^2 + 4M(C-A)(k_{11}k_{33} - k_{14}^2)}}{2M(C-A)}. \tag{23}$$

On above resonance speeds of rotor rotation ($\omega > \omega_1$), the determinant $\Delta(\omega) < 0$.

5. 4. Construction of the functional of criterion for the occurrence of auto-balancing

Let us introduce the complex displacements of the points j:

$$x_j = x + z_j\beta, \; y_j = y - z_j\alpha,$$

$$\rho_j = z - iz_j\psi = (D - iz_jE)e^{i\omega t}. \tag{24}$$

Projections on the x, y axis of the j-th elementary imbalance vector:

$$s_{jx} = s_j\cos(\omega t + \phi_j),$$

$$s_{jy} = s_j\sin(\omega t + \phi_j). \tag{25}$$

Let us introduce a complex representation of imbalance:

$$s_j = s_{jx} + is_{jy} = s_j\cos(\omega t + \phi_j) + \\ + is_j\sin(\omega t + \phi_j) = s_j e^{i(\omega t + \phi_j)}. \tag{26}$$

Taking into account (6), (24), (26), the integrand of the functional (1) is reduced to the form:

$$\sum_{j=1}^{n}\vec{s}_j(t)\cdot\vec{r}_j(t)=\sum_{j=1}^{n}(s_j\bar{\rho}_j+\bar{s}_j\rho_j)/2=$$
$$=\sum_{j=1}^{n}[s_je^{i(\omega t+\phi_j)}(\bar{D}+iz_j\bar{E})e^{-i\omega t}+s_je^{-i(\omega t+\phi_j)}(D-iz_jE)e^{i\omega t}]/2=$$
$$=\frac{\bar{D}}{2}S+i\frac{\bar{E}}{2}I+\frac{D}{2}\bar{S}-i\frac{E}{2}\bar{I}=\frac{1}{2}(\bar{D}S+D\bar{S})+\frac{1}{2}i(\bar{E}I-E\bar{I}).$$

Since

$$\frac{1}{2}(\bar{D}S+D\bar{S})=\frac{\omega^2}{2\Delta(\omega)}\{2[k_{33}-(A-C)\omega^2]\bar{S}S-k_{14}(\bar{I}S+I\bar{S})\},$$

$$\frac{1}{2}i(\bar{E}I-E\bar{I})=\frac{\omega^2}{2\Delta(\omega)}[2(k_{11}-M\omega^2)\bar{I}I-k_{14}(S\bar{I}+\bar{S}I)],$$

the integrand takes the form:

$$\sum_{j=1}^{n}\vec{s}_j(t)\cdot\vec{r}_j(t)=$$
$$=\frac{\omega^2}{\Delta(\omega)}\{[k_{33}-(A-C)\omega^2]\bar{S}S-k_{14}(\bar{I}S+I\bar{S})+(k_{11}-M\omega^2)\bar{I}I\}.$$

Since

$$S\bar{S}=S_x^2+S_y^2,\ I\bar{I}=I_{uw}^2+I_{vw}^2,$$

$$\bar{I}S+I\bar{S}=(I_{uw}-iI_{vw})(S_u+iS_v)+$$
$$+(I_{uw}+iI_{vw})(S_u-iS_v)=2(I_{uw}S_u+I_{vw}S_v),$$

the integrand of the functional is reduced to the form:

$$\sum_{j=1}^{n}\vec{s}_j(t)\cdot\vec{r}_j(t)=\frac{\omega^2}{\Delta(\omega)}\{[k_{33}-(A-C)\omega^2](S_x^2+S_y^2)-$$
$$-2k_{14}(I_{uw}S_u+I_{vw}S_v)+(k_{11}-M\omega^2)(I_{uw}^2+I_{vw}^2)\}. \tag{27}$$

Let us introduce the matrix and the vector:

$$K=\begin{pmatrix} k_{11}-M\omega^2 & 0 & -k_{14} & 0\\ 0 & k_{11}-M\omega^2 & 0 & -k_{14}\\ -k_{14} & 0 & k_{33}-(A-C)\omega^2 & 0\\ 0 & -k_{14} & 0 & k_{33}-(A-C)\omega^2\end{pmatrix},$$

$$J=\begin{pmatrix} I_{uw}\\ I_{vw}\\ S_u\\ S_v\end{pmatrix}. \tag{28}$$

Then

$$\sum_{j=1}^{n}\vec{s}_j(t)\cdot\vec{r}_j(t)=\frac{\omega^2J^TKJ}{\Delta(\omega)}$$

and the criterion for the occurrence of auto-balancing (1) takes the form:

$$\frac{\omega}{2\pi}\int_0^{2\pi/\omega}\left(\sum_{i=1}^{n}s_j(t)\cdot r_j(t)\right)dt=\frac{\omega^2J^TKJ}{\Delta(\omega)}<0, \tag{29}$$

where T=2π/ω is the time of one turn of the rotor.

5. 5. Determining conditions for the occurrence of auto-balancing

5. 5. 1. Dynamic balancing of the rotor

In the case under consideration, two or more auto-balancers are located not in one correction plane. Therefore, I_{uw}, I_{vw}, S_u, S_v are independent. The necessary condition for the fulfillment of inequality (29) for any nonzero elementary imbalances is the sign definiteness of the quadratic form:

$$J^TKJ=[k_{33}-(A-C)\omega^2](S_x^2+S_y^2)-$$
$$-2k_{14}(I_{uw}S_u+I_{vw}S_v)+(k_{11}-M\omega^2)(I_{uw}^2+I_{vw}^2). \tag{30}$$

Let us apply Sylvester's criterion to the matrix K [2] and consider two possible cases.

1. The quadratic form (30) is positive definite if and only if, when:

$$\Delta_1=k_{11}-M\omega^2>0,$$

$$\Delta_2=(k_{11}-M\omega^2)^2>0,$$

$$\Delta_3=(k_{11}-M\omega^2)\Delta(\omega)>0,$$

$$\Delta_4=\Delta^2(\omega)>0.$$

These conditions are equivalent to such two conditions:

$$\Delta_1=k_{11}-M\omega^2>0,\ \Delta(\omega)>0.$$

Thus, if the quadratic form (30) is positive definite, then $J^TKJ>0,\ \Delta(\omega)>0.$ In this case, the criterion for the occurrence of auto-balancing (29) is not fulfilled.

2. The quadratic form (30) is negative definite if and only if, when:

$$\Delta_1=k_{11}-M\omega^2<0,$$

$$\Delta_2=(k_{11}-M\omega^2)^2>0,$$

$$\Delta_3=(k_{11}-M\omega^2)\Delta(\omega)<0,\ \ \Delta_4=\Delta^2(\omega)>0.$$

These conditions are equivalent to such two conditions:

$$\Delta_1=k_{11}-M\omega^2<0,\ \ \Delta(\omega)>0.$$

Thus, if the quadratic form (30) is negative definite, then $J^TKJ<0,\ \ \Delta(\omega)>0.$ In this case, the criterion for the occurrence of auto-balancing (29) is fulfilled.

In order that, under the condition $\Delta_1=k_{11}-M\omega^2<0,$ the condition $\Delta(\omega)>0$ be satisfied, it is necessary that the following condition be satisfied: $k_{33}-(A-C)\omega^2<0.$ It is only performed if the rotor is long and rotates at the frequency greater than the second partial one:

$$A>C,\ \omega>\omega_{p1}\ \ (\omega>\sqrt{k_{33}/(A-C)}).$$

At the same time, the condition $\Delta(\omega)>0$ will be satisfied only at above resonance rotational speeds of the rotor.

Thus, dynamic balancing of the rotor by two or more auto-balancers (located in two or more correction planes) is possible only for the long rotor at above resonance rotational speeds:

$$A > C,\ \omega > \omega_2. \tag{31}$$

The similar result for the case of balancing the rotor on two isotropic elastic supports by two two-ball balancers was obtained in [8]. For this, the differential equations of motion of the rotor-auto-balancer system were derived, the main motion of the system was singled out, its stability was investigated.

5. 5. 2. Static balancing of the rotor

In the case under consideration, one or more auto-balancers are located in one correction plane. Therefore, I_{uw}, I_{vw}, S_u, S_v are dependent. Taking into account (8), the quadratic form (30) takes the form:

$$J^T KJ = [k_{33} + z^2 k_{11} - 2k_{14} z - -(A + Mz^2 - C)\omega^2](S_u^2 + S_v^2).$$

The condition for the occurrence of auto-balancing (29) is transformed to the form:

$$\frac{\omega^2 J^T KJ}{\Delta(\omega)} = -[(A + Mz^2 - C)\omega^2 - k_{33} - -k_{11}z^2 + 2zk_{14}]\frac{(S_u^2 + S_v^2)\omega^2}{\Delta(\omega^2)} < 0. \tag{32}$$

Let us first consider the case of the fast-rotating rotor ($\omega >> 1$). At high speeds:

$$\Delta(\omega^2) \approx M(A - C)\omega^4,$$

$$\frac{\omega^2 J^T KJ}{\Delta(\omega)} \approx -\frac{A + Mz^2 - C}{M(A - C)}(S_u^2 + S_v^2). \tag{33}$$

From (32), (33), it follows that an auto-balancing:

– occurs at such two variants of the relations between the parameters:

1) $C>A+Mz^2$,

2) $C<A$; (34)

– does not occur at such relation:

$$A<C<A+Mz^2. \tag{35}$$

Thus, at the high speeds of the rotor rotation:

1) long rotors $C<A$ are automatically balanced for an arbitrary arrangement of the correction plane;

2) short rotors $C>A$ are automatically balanced if the distance z from the rotor center of mass to the correction plane does not exceed:

$$z_{max} = \sqrt{(C - A) / M}; \tag{36}$$

3) spherical rotors $C=A$ cannot be balanced.

Let us consider different cases of rotors on the entire range of angular speeds of the rotor rotation.

1. The case of the long rotor $C<A$. Let us introduce the additional speed of the rotor rotation:

$$\tilde{\omega} = \sqrt{\frac{k_{33} + k_{11}z^2 - 2zk_{14}}{A + Mz^2 - C}}. \tag{37}$$

As $k_{11}k_{33} - k_{14}^2 > 0$, then

$$k_{33} + k_{11}z^2 - 2zk_{14} > k_{33} + k_{11}z^2 - -2z\sqrt{k_{11}}\sqrt{k_{33}} = (\sqrt{k_{33}} - z\sqrt{k_{11}})^2 \geq 0$$

and $\tilde{\omega}$ is the real positive number.

It is possible to show that for any z:

$$\omega_1 \leq \tilde{\omega} \leq \omega_2 \tag{38}$$

and the equality sign is satisfied only for two values of z (once at the left and once on the right hand).

The condition for the occurrence of auto-balancing (32) takes the form:

$$\frac{\omega^2 J^T KJ}{\Delta(\omega)} = = -\frac{(A + Mz^2 - C)(S_u^2 + S_v^2)\omega^2}{A - C} \cdot \frac{\omega^2 - \tilde{\omega}^2}{(\omega^2 - \omega_1^2)(\omega^2 - \omega_2^2)} < 0. \tag{39}$$

The auto-balancing will occurr at the rotation of the rotor with the angular speeds located: between the first resonant speed and additional speed; over the second resonant speed:

$$\omega \in (\omega_1, \tilde{\omega}) \cup (\omega_2, +\infty). \tag{40}$$

2. In the case of the spherical rotor A=C, and:

$$\omega_1 = \sqrt{\frac{k_{11}k_{33} - k_{14}^2}{Mk_{33}}},$$

$$\tilde{\omega} = \sqrt{\frac{k_{33} + k_{11}z^2 - 2zk_{14}}{Mz^2}},$$

$$\Delta(\omega) = Mk_{33}(\omega_1^2 - \omega^2). \tag{41}$$

Let us note that as:

$$\tilde{\omega}^2 - \omega_1^2 = \frac{(k_{33} - k_{14}z)^2}{Mk_{33}z^2} \geq 0, \text{ so } \tilde{\omega} \geq \omega_1.$$

The condition for the occurrence of auto-balancing (32) takes the form:

$$\frac{\omega^2 J^T KJ}{\Delta(\omega)} = \frac{\omega^2 z^2 (S_u^2 + S_v^2)}{k_{33}} \frac{\tilde{\omega}^2 - \omega^2}{\omega_1^2 - \omega^2} < 0. \tag{42}$$

The auto-balancing will occur between the first resonant speed of the rotor rotation and additional speed:

$$\omega \in (\omega_1, \tilde{\omega}). \tag{43}$$

It is seen from (41) that with increasing z, the speed $\tilde{\omega}$ increases without limit. Therefore, it is expedient to counterbalance the spherical rotor statically by one auto-balancer located as close as possible to the rotor center of mass.

3. The case of the short rotor A<C. We consider the possible cases, depending on the choice of z (cross-sections, where the auto-balancers will be installed).

a) z: $A+Mz^2>C$. With the account of (22), (37), the condition for the occurrence of auto-balancing (32) takes the form:

$$\frac{\omega^2 J^T KJ}{\Delta(\omega)}=\frac{(A+Mz^2-C)(S_u^2+S_v^2)\omega^2}{M(C-A)(\omega_2^2+\omega^2)}\cdot\frac{\tilde{\omega}^2-\omega^2}{\omega_1^2-\omega^2}<0. \quad (44)$$

The auto-balancing will occur between the first resonant velocity and additional velocity:

$$\omega\in(\omega_1,\tilde{\omega}). \quad (45)$$

It is seen from (37) that the auto-balancing region can be arbitrarily expanded at high speeds if

$$z\to\sqrt{(C-A)/M}.$$

b) z: $A+Mz^2<C$. Let us introduce the parameter:

$$\tilde{\omega}=\sqrt{\frac{k_{33}+k_{11}z^2-2zk_{14}}{C-A-Mz^2}}. \quad (46)$$

Then the condition for the occurrence of auto-balancing (32) will take the form:

$$\frac{\omega^2 J^T KJ}{\Delta(\omega)}=\frac{(C-A-Mz^2)(S_u^2+S_v^2)\omega^2}{M(C-A)(\omega^2+\omega_0^2)}\cdot\frac{\omega^2+\tilde{\omega}^2}{\omega_1^2-\omega^2}<0. \quad (47)$$

The auto-balancing will occur over the first resonant velocity:

$$\omega\in(\omega_1,+\infty). \quad (48)$$

The results of this subparagraph coincide with the results obtained in [2] in the case of the static balancing of the rotor by one auto-balancer.

6. Discussion of the obtained conditions for the occurrence of auto-balancing

The dynamics of the rotor with the elementary unbalances applied at the future points of the suspension of auto-balancers, mounted on two isotropic elastic supports, is described by a system of four linear differential equations of motion.

The functional of the criterion for the occurrence of auto-balancing is a quadratic form of the static moments and the products of inertia of the rotor, which determine, respectively, its static and moment imbalances.

In the case of the axisymmetric rotor on two isotropic elastic supports, the dynamic auto-balancing (in two or more correction planes) is possible only in the case of a long rotor. There can be any quantity of auto-balancers (correction planes). The long rotor has two resonant rotational speeds. The auto-balancing occurs at above resonance speeds. This result was obtained for the first time for the excess number of auto-balancers. It coincides with the results obtained in [6] for the case of two two-ball balancers.

In the case of an axisymmetric rotor on two isotropic elastic supports, the static auto-balancing (in one correction plane) is possible at any quantity of auto-balancers in such cases.

If the rotor is long, then it has two resonant speeds and one additional speed, located between the resonant ones. The auto-balancing occurs between the first resonant speed of rotor rotation and additional speed, and over the second resonant speed.

If the rotor is spherical, then it has one resonant speed and the additional speed, which is higher than the resonant one. The auto-balancing occurs between resonant and additional speeds.

If the rotor is short, then the conditions for the occurrence of auto-balancing depend on the distance between the rotor center of mass and the correction plane. If this distance does not exceed the boundary size (36), then the rotor has the only resonant speed and the auto-balancing occurs at above resonance speeds. Otherwise, the rotor has one resonant and one additional speed, which is higher than the resonant one. The auto-balancing occurs between these speeds.

The additional speed is due to the installation of the auto-balancers on the rotor. Upon transition to it, the behavior of auto-balancers changes. At slightly lower rotor rotational speeds, the auto-balancers reduce the rotor imbalance, and at slightly higher ones – increase it.

These results for the excess quantity of auto-balancers are received for the first time. They coincide with the results received in [3] for the case of one two-ball balance.

The empirical criterion for the occurrence of auto-balancing makes it possible to obtain these conditions in the "zero approximation", since it does not take into account the type and mass of auto-balancers. More precise conditions (in the "first approximation") make it possible to obtain the empirical criterion for the stability of the main motion [11]. However, the calculations are bulkier, and the results are less general.

In the future, it is planned to obtain, with the help of the empirical criterion of the occurrence of auto-balancing, the conditions for balancing the rotor by any number of passive auto-balancers in the framework of:

– various flat models of the rigid rotor (modeling the balancing of flat rotors like disks of manual grinders, CD/DVD disks, impellers of axial fans, etc. by multi-row auto-balancers, auto-balancers with various corrective weights etc.);

– the models of the flexible rotor.

At the same time, a comparison of the results received with the use of the empirical criterion, with the known results received by other methods is planned.

7. Conclusions

The empirical criterion for the occurrence of auto-balancing is an effective method for determining the conditions under which auto-balancers of any type can balance the certain rotor. For the axisymmetric rotor on two isotropic elastic supports, using the method, the following is established.

1. The dynamics of the rotor with the elementary unbalances applied at the future points of the suspension of auto-balancers is described by a system of four linear differential equations of motion.

2. The functional of the criterion for the occurrence of auto-balancing is a quadratic form of the static moments and

the products of inertia of the rotor, which determine, respectively, its static and moment imbalances.

3. The dynamic auto-balancing of the rotor (in two or more correction planes by several passive auto-balancers) is possible only in the case of the long rotor. There can be any of auto-balancers. The long rotor has two resonant rotational speeds. The auto-balancing occurs at above resonance speeds.

4. The static auto-balancing of the rotor (in one correction plane) is possible at any quantity of auto-balancers in such cases.

If the rotor is long, then it has two resonant speeds and one additional speed, located between the resonant ones. The auto-balancing occurs between the first resonant speed of rotor rotation and additional speed, and over the second resonant speed.

If the rotor is spherical, then it has one resonant speed and the additional speed, which is higher than the resonant one. The auto-balancing occurs between resonant and additional speeds.

If the rotor is short, then the conditions for the occurrence of auto-balancing depend on the distance between the rotor center of mass and the correction plane. If this distance does not exceed the certain boundary size, then the rotor has the only resonant speed and the auto-balancing occurs at above resonance speeds. Otherwise, the rotor has one resonant and one additional speed, which is higher than the resonant one. The auto-balancing occurs between these speeds.

The additional speed is due to the installation of the auto-balancers on the rotor. Upon transition to it, the behavior of auto-balancers changes. At slightly lower rotor rotational speeds, the auto-balancers reduce the rotor imbalance, and at slightly higher ones – increase it.

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