*За допомогою числової моделі, розробленої на базі запропонованих фізичної та математичної моделей, виконано моделювання процесу сушіння флютингу інфрачервоним випромінюванням. Верифікація числової моделі показала збіжність результатів числового моделювання температури поверхні флютингу, тривалості та швидкості сушіння з даними фізичного експерименту. За маси квадратного метра сухого флютингу від 0,112 кг/м² до 0,2 кг/м² відхилення результатів становило до 5 %*

*Ключові слова: папір для гофрування, флютинг, інфрачервоне випромінювання, вологовміст, тривалість сушіння, числове моделювання*

*С помощью численной модели, разработанной на основе предложенных физической и математической моделей, выполнено моделирование процесса сушки флютинга инфракрасным излучением. Верификация числовой модели показала сходимость результатов числового моделирования температуры поверхности флютинга, продолжительности и скорости сушки данным физического эксперимента. При массе квадратного метра сухого флютинга от 0,112 кг/м² до 0,2 кг/м² отклонения результатов составило до 5 %*

*Ключевые слова: бумага для гофрирования, флютинг, инфракрасное излучение, влагосодержание, продолжительность сушки, численное моделирование*

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# NUMERICAL MODELING OF PHYSICAL FIELDS IN THE PROCESS OF DRYING OF PAPER FOR CORRUGATING BY THE INFRARED RADIATION

**A. Karvatskii**
Doctor of Technical Sciences, Professor*
E-mail: anton@rst.kpi.ua
**V. Marchevsky**
PhD, Professor**
E-mail: marchevsky@mail.ru
**O. Novokhat**
PhD, Senior Lecturer**
E-mail: novokhatoleh@gmail.com
*Department of chemical, polymer and silica engineering***
**Department of Machines and Apparatus of Chemical and Oil Refinery Productions***
***National Technical University of Ukraine «Igor Sikorsky Kyiv Polytechnic Institute»
Peremohy ave., 37, Kyiv, Ukraine, 03056

## 1. Introduction

In connection with the growing volume of production and demand for corrugated cardboard [1], there has been a significant increase in the production of paper for corrugating – fluting.

The most energy-intensive and metal-intensive stage in the production of fluting is a drying process on the drying cylinders of paper machine [2]. However, this method has several disadvantages. First of all, it is the high cost and metal consumption of drying equipment. There may also occur the sticking of glue on the surface of drying cylinders after gluing the fluting. Moreover, in the course of water vapor production for the drying cylinders, there are emissions of fuel combustion products. In addition, there are design limitations on the way to intensify the process of drying.

The specified shortcomings are a substantial reason to search for alternative techniques of drying and to develop new, or modernize the existing, designs of drying plants that would ensure energy efficiency, reduce the cost of equipment and decrease anthropogenic impact on the environment.

One of the promising ways of drying is the radiation method using infrared (IR) radiation that has a capacity to penetrate the volume of material, thereby improving efficiency of the drying process [3]. The fluting is rapidly heated from the heat that is released in its volume when exposed to the IR radiation, reducing the duration of its heating and drying in general. This makes it possible to reduce the number of drying cylinders in a paper machine and, consequently, the cost of water vapor. In addition, when compared to drying cylinders, the equipment that employs IR emitters differs by lower metal consumption and cost. The lack of direct contact between a fluting and IR emitters (heaters) makes it possible to apply the drying process immediately after its gluing. In this case, under condition of using "clean" electricity (for example, nuclear power), the IR heaters do not emit combustion products of fossil fuel into the environment.

Development of industrial plants for drying the fluting requires the knowledge of kinetic regularities in this process, first of all, changes in moisture content and temperature of fluting during its drying. This requires examining the kinetics of drying process of fluting by the radiation method and developing a procedure for calculating its basic parameters required for designing and constructing the appropriate drying equipment.

Therefore, a relevant task is to develop mathematical and numerical models for the process of IR drying of fluting to determine kinetic parameters of the process, which are required to design new equipment with the minimization of the number of physical experiments.

## 2. Literature review and problem statement

Theoretical foundations of the process of radiation-convection drying of colloidal capillary-porous bodies, including paper, in the form of a system of integral-differential equations are described in [4]. However, for the practical application of fundamental provisions of the theory of radiation-convection drying, each particular case requires additional identification and substantiation of radiation model of the environment. It is also necessary to determine the character of borders reflection, conditions of uniqueness, in particular boundary conditions, and numerical method for solving the stated problem, etc. This significantly complicates the practical application of this theory.

The existing mathematical models for the drying process of fibrous material, including paper or cardboard, for the most part describe a contact or a contact-convective heat transfer method.

Thus, for example, [5] describes a mathematical model for drying the fluting by a contact method on the drying cylinders. The model is based on the balance of interrelation between mass and energy.

Similar mathematical model for drying the paper on a multi-cylinder paper machine by the contact method is described in [6].

Common way of drying the paper is a contact-convective one, in particularly sanitary-hygienic paper on a tissue cylinder. Mathematical model that describes this method of drying is given in [7].

Less common are other types of drying. Thus, authors of [8] describe modeling the filtration method for drying of tissue paper with a decrease in water consumption at a paper processing plant.

The described mathematical models of the paper or cardboard drying process with the use of IR radiation do not make it possible to compute the drying process of fluting due to the absence of required kinetic laws.

Thus, authors of [9] describe modeling the drying process by a radiation method. However, the model lacks the distribution law of IR radiation by thickness of the material.

[10] described radiation, convection, and radiation-convective drying methods for coated paper. However, there were many empirical dependences employed for the examined paper during modeling. This complicates the application of the presented model to describe the drying for other types of paper. It was not specified how temperature of paper is calculated during drying.

There are a large number of mathematical models that describe the radiation method of drying the products in food industry.

There is a known model of the radiation drying of carrot slices [11]. The article defines the impact of emitters' power on the speed of drying. It is shown that the duration of drying is determined by using the resulting regression equation. However, this technique does not allow defining the kinetic patterns in the drying of other materials, in particular fluting.

[12] described modeling the drying process of potato slices. A comparison of the results of calculation and experiments is presented. A change in moisture content of the material during drying is proposed to define by logarithmic formulas. However, their diversity indicates the need to confirm the possibility of applying these formulas for other types of material.

A mathematical description of kinetic regularities in the drying process of pomegranate seeds is given in [13]. Nevertheless, the method for calculating the radiation distribution by thickness of the material is not specified. There is no a procedure for determining the coefficients of heat and mass transfer, either.

Therefore, it is a relevant task to develop a mathematical model for the radiation method of fluting drying to define important parameters of this process. First of all, these are the temperature and moisture content of fluting, velocity and duration of drying. The accuracy of the obtained data must be of precision sufficient for the engineering calculations of equipment with IR heaters.

## 3. The aim and tasks of the study

The aim of present work is the numerical modeling of physical fields in the process of radiation drying of fluting and the verification of adequacy of results of the numerical analysis to the data of physical experiments.

To achieve the set aim, the following tasks had to be solved:

– to formulate a physical model based on the analysis of physical processes that occur during radiation drying of fluting;

– to develop, based on the physical representations of the process, a mathematical and a numerical model of the process of drying the fluting by IR radiation;

– to verify the developed numerical model of the process of radiation drying of fluting by the data of physical experiments.

## 4. Physical model of the process of radiation drying of fluting

The medium between an IR emitter and fluting is accepted as absorbing and emitting and comprises air and water vapor.

Wet fluting is a selective or "grey" body that has reflective, absorbing and transmitting capacity. The specified physical properties change with the change in the ratio of a square meter of the fluting mass and the amount of moisture in it. At the thickness of standard fluting (up to 0.2 mm), almost all of the IR radiation is absorbed in it. Therefore, we can assume that the lower side of fluting is opaque to radiation.

When passing through a layer of fluting, the IR radiation causes the thermal energy release in it, the amount of which decreases in the direction of heat flux. In this connection, there is a gradient of temperature that is directed away from the more heated surface layers of fluting to those less heated inside it. Due to the presence of a temperature gradient, some amount of heat is transferred by thermal conductivity in the direction of the IR radiation passage. However, under conditions of low temperature difference between the surfaces and small thickness of fluting, the amount of this heat is negligible.

Heat from the surfaces of fluting is released into the environment due to convective heat transfer from the surfaces and is removed along with the formed vapor.

When the wet fluting is heated, the process of drying starts. Its intensity is the largest in the surface layers, where the amount of the released heat is maximal, and it decreases into the depth of a layer of the material. At the surface of fluting, there forms a layer of saturated water vapor.

The driving force behind the process of drying in the first period is the difference between the pressure of the saturated water vapor in the boundary layer at the surface of fluting and the partial pressure of vapor in the environment. The driving force in the second period of drying is the difference between the magnitudes in the current and resulting equilibrium moisture content [14].

During drying, evaporation of moisture can occur not only from the surfaces of fluting, but also from its deeper layers.

Fluting as a capillary-porous body contains free moisture, moisture in micro capillaries and fibers, and adsorption-associated moisture [4]. In the first period of drying, the free moisture evaporates, and the rate of the process is limited by the intensity of heat flow. In the second period of drying, the rate is initially limited by the diffusion of moisture into the area of evaporation (removal of moisture in micro capillaries), and by the end of drying it is limited by the destruction of adsorption bond between moisture and the fluting and its evaporation.

Prior to the start of drying, moisture content and temperature are the same throughout the entire thickness of fluting. During drying, in the surface layers where there is the largest amount of adsorbed heat from the IR radiation, the temperature value is the highest while the moisture content is the lowest. Since, during the IR radiation passage through fluting, the amount of the released heat decreases, then the temperature, accordingly, falls while the moisture content increases. At the end of the drying process, the moisture content reaches an equilibrium value and levels along thickness of the layer of fluting.

In a paper machine, IR heaters are expedient to install on a free run of the paper web between drying cylinders, or replacing them. Under these conditions, inside the paper web and at its boundary with the surrounding environment, the heat is transferred due to the radiation, conductive and convective constituents. The radiation component occurs under the action of IR heaters, conductive – due to heat transfer inside the material by thermal conductivity and convective – due to the contact between a surface of the paper and a gaseous heat carrier. Thus, the heat exchange in a paper web (fluting) is complex – radiation-conductive (RCH). That is why the material of fluting can be assigned to the class of media, semi-transparent to the IR radiation.

## 5. Mathematical model of the process of radiation drying of fluting

In a general case, the equation of RCH for a semi-transparent medium, by which we accept fluting and a gaseous environment over it in the process of IR drying, is in the divergent form [15, 16]:

$$c_p \rho \frac{\partial T}{\partial \tau} = -\mathrm{div}\left(\mathbf{q}_\lambda + \mathbf{q}_r\right) + q_v, \qquad (1)$$

where $c_p$ is the specific mass isobar heat capacity, J/(kg·K); ρ is the density, kg/m$^3$; T is the absolute temperature, K; τ is the time, s;

$$\mathbf{q}_\lambda = -\lambda \mathrm{grad} T = -\lambda \nabla T$$

is the density vector of heat flux, which is defined by the Fourier law, W/m$^2$; λ is the coefficient of thermal conductivity, W/(m·K); ∇ is the Hamiltonian, m$^{-1}$; $\mathbf{q}_r$ is the density vector of radiation heat flux, W/m$^2$; $q_v$ is the internal source of heat, for example, due to the evaporation and moisture transportation, W/m$^3$.

Divergence of the radiation heat flux is determined by equation [16]:

$$\mathrm{div}\mathbf{q}_r = \int_{\nu=0}^{\infty} K_\nu \left[ \int_{\Omega=4\pi} \vec{I}_\nu d\Omega - 4\pi n_\nu^2 I_{0\nu}(T) \right] d\nu, \qquad (2)$$

where $K_\nu$ and $n_\nu$ are the spectral absorption coefficient (m$^{-1}$) and index of refraction, respectively; Ω is the solid angle, sr; $I_{0\nu}$ is the Planck function, W·s/(m$^2$· sr; $\vec{I}_\nu$ is the vector of spectral intensity of radiation (W·s/(m$^2$·sr)) for direction S in the solid angle dΩ; $S_0$ corresponds to the boundary of the region; ν is the radiation frequency, Hz.

The vector of spectral intensity of radiation is determined by the equation of energy transfer [16]:

$$\vec{I}_\nu(s) = \vec{I}_\nu(s_0)\exp\left(-\int_{s0}^{s} K_\nu ds\right) + \int_{s0}^{s} n_\nu^2 I_{0\nu} K_\nu \exp\left(-\int_{s'}^{s} K_\nu ds''\right) ds'. \qquad (3)$$

For unambiguous equation of RCH (1), it is necessary to record the appropriate initial and boundary conditions.

Equation (1) refers to the class of integral-differential equations, which is why its solution in a three-dimensional statement by numerical methods is a non-trivial task [15, 16]. However, technological modes and equipment for the radiation drying of fluting allow us to consider this complex process as a one-dimensional one [3, 14]. In this case, the accuracy of such approximation is sufficient to perform engineering calculations when designing the equipment.

Let us consider main assumptions in the one-dimensional problem to formulate the basic interrelations of non-stationary heat exchange in a layer of fluting based on precise equations for determining the radiation component of heat exchange [17]:

– temperature field and other fields that are derivatives from it, are one dimensional and vary only in the direction of the Oz axis by thickness of the layer of fluting (Fig. 1);

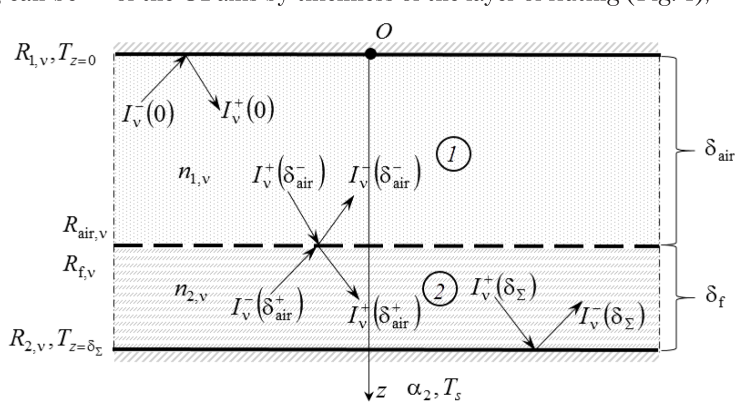


Fig. 1. Schematic of estimated region for the formulation of one-dimensional non-stationary mathematical model of RCH regarding the drying of fluting by IRR: 1 — a layer of air with water vapor; 2 — a layer of wet fluting

– the modeling region consists of two layers – the upper and the lower. The upper layer is a mixture of air with water vapor (gas), and the lower one is a wet fluting. Wet fluting is a capillary-porous body (from here on denoted as the system) that undergoes the process of radiation drying. That is, the system is a two-phase one;

– thermal-physical properties of wet fluting (mass isobar heat capacity, density and coefficient of thermal conductivity) are effective values and depend on temperature;

– the layer of gas is considered to be immobile because temperature of fluting is higher or equal to the surrounding gaseous heat carrier (hence the convective component as the source of heat supply of heat is not taken into account);

– a two-phase system from the point of view of radiation properties is selective, semi-transparent, non-scattering, as well as absorbing and radiating. The upper boundary of the system (IR emitter) is opaque, and a boundary between the layers is translucent. The lower side of fluting is assumed to be opaque for the simplification;

– all the boundaries of the system are diffuse, that is, a reflection coefficient does not depend on the direction;

– radiation properties (absorption rate, index of refraction, the degree of blackness and reflection coefficients of the system's boundaries) are selective (depend on the radiation frequency).

Taking into account the described assumptions, equation of RCH of type (1) for the system gas – porous body for the direction of axis Oz takes the form:

$$c_{pi\,eff}(T)\rho_{i\,eff}(T)\frac{\partial T}{\partial \tau} = \lambda_{i\,eff}(T)\frac{\partial^2 T}{\partial z^2} - \frac{\partial \mathbf{q}_{ri}}{\partial z} + q_{vi}, \qquad (4)$$

where i=1, 2 is the phase index (for i=1 – a mixture of air with water vapor, for i=2 – wet fluting (Fig. 1)); $c_{pi\,eff}(T)$ $c_{pi\,eff}(T)$ is the effective mass isobar heat capacity of the ith phase, J/(kg·K); $\rho_{i\,eff}(T)$ is the effective density of the i-th phase, kg/m$^3$; $\lambda_{i\,eff}(T)$ is the effective coefficient of thermal conductivity of the i-th phase, W/(kg·K); $\frac{\partial \mathbf{q}_r}{\partial z}$ is the divergence of density of radiation heat flux density, W/m$^3$; $\mathbf{q}_r$ is the density vector of radiation heat flux, W/m$^2$; T is the absolute temperature, K; τ is the time, s; z is the coordinate, m;

$$q_{vi} = \begin{cases} 0, & i=1; \\ -\rho_{2f}(T)\, r\dfrac{du}{d\tau}, & i=2, \end{cases}$$

is the volumetric density of source of heat due to the evaporation and moisture transport of fluting, W/m$^3$; r is the mass heat of the water vaporization, J/kg; $\frac{du}{d\tau}$ is the relative rate of evaporation and moisture transport, s$^{-1}$ (kg/(kg·s)) [4].

Equation (4) in the effective values of thermal-physical magnitudes takes into account the content of water vapor in the air or moisture in a porous fluting by the rule of additivity according to their relative humidity.

Initial conditions for (4), τ=0:

$$T = T(z),\ 0 \le z \le \delta_{air} + \delta_f = \delta_\Sigma, \qquad (5)$$

where $\delta_{air}$, $\delta_f$, $\delta_\Sigma$ are the thickness of layers of wet air, wet fluting and the total thickness of layers, respectively, m.

Boundary conditions for (4), τ>0:

– on the upper opaque boundary IR heater – wet air (*z*=0) we accept boundary conditions of the first kind:

$$T\big|_{z=0} = T_1 = T\big(W(\tau)\big), \qquad (6)$$

– where W(τ) is the dependence of change in the electric power of radiation heater on the time, W. At this boundary we also assign the degree of blackness $\varepsilon_{1,\nu} = f_{\varepsilon 1}(\nu)$ or reflection coefficient $R_{1,\nu} = 1-\varepsilon_{1,\nu}$ to determine the radiation component of equation (2);

– at the translucent boundary between the layer of gas and fluting we consider boundary conditions of absolute contact or of the fourth kind:

$$\begin{cases} \{T\} = 0; \\ \{\mathbf{n}\cdot\mathbf{q}_\lambda\} = 0, \end{cases} \qquad (7)$$

where $\{T\} = T^+ - T^-$ is the temperature on the left and on the right in the vicinity of translucent boundary;

$$\{\mathbf{n}\cdot\mathbf{q}_\lambda\} = \mathbf{n}^-\cdot\mathbf{q}_\lambda^+ - \mathbf{n}^+\cdot\mathbf{q}_\lambda^-;$$

$$\mathbf{q}_\lambda^+ = -\lambda_{2\,eff}(T)\nabla T,\quad \mathbf{q}_\lambda^- = -\lambda_{1\,eff}(T)\nabla T$$

is the density vector of heat flux in the vicinity of translucent boundary form the sides of wet fluting and air, respectively, W/m$^2$; $\nabla = \frac{\partial}{\partial z}$ is the Hamiltonian in the one-dimensional case, m$^{-1}$; $\mathbf{n}^-$, $\mathbf{n}^+$ are the vectors of external normal to the translucent boundary from the side of wet fluting and air, respectively. At this boundary, it is also necessary to define and assign the reflection coefficient to the layer of gas $R_{air,\nu} = f_{air,\nu}(\nu)$ and the reflection coefficient to the layer of fluting $R_{f,\nu} = f_{f,\nu}(\nu)$;

– at the bottom opaque boundary, which is the lower side of fluting – environment ($z=\delta_\Sigma$), we consider boundary conditions of the third kind:

$$\mathbf{n}\cdot\left(-\lambda_{2\,eff}(T)\nabla T\right)\Big|_{z=\delta_\Sigma} = \alpha_2(T - T_s), \qquad (8)$$

where $\alpha_2$ is the coefficient of heat transfer, W/(m$^2$·K); $T_s$ is the ambient temperature, K. At this boundary, we also assign the degree of blackness $\varepsilon_{2,\nu} = f_{\varepsilon 2}(\nu)$ or reflection coefficient $R_{2,\nu} = 1-\varepsilon_{2,\nu}$.

The density of radiation flow in the one-dimensional system, which consists of two layers, different in their properties, of selective-emitting and absorptive media, is described by a system of transport equations [17]:

$$\begin{cases} \mu_i\dfrac{\partial \vec{I}_\nu^+(z,\mu_i)}{\partial z} = -K_{i,\nu}\left[\vec{I}_\nu^+(z,\mu_i) - n_{i,\nu}^2 I_{b\nu}(T(z))\right]; \\ \mu_i\dfrac{\partial \vec{I}_\nu^-(z,\mu_i)}{\partial z} = K_{i,\nu}\left[\vec{I}_\nu^-(z,\mu_i) - n_{i,\nu}^2 I_{b\nu}(T(z))\right], \end{cases} \qquad (9)$$

where i=1, 2 refers to the wet air and wet fluting, respectively; $\vec{I}_\nu^+, \vec{I}_\nu^-$ are the vectors of spectral intensity of radiation in the positive and negative directions, respectively, W·s/(m$^2$·sr); $I_{b\nu}(T(z))$ is the Planck function, W·s/(m$^2$·sr); μ=|cos φ|; φ is the angle between a vector of the intensity of radiation and axis z, rad; $n_\nu$ is the spectral index of refraction; $K_\nu$ is the spectral absorption coefficient, m$^{-1}$; ν is the radiation frequency, Hz.

In (9) and hereinafter, the absorption coefficients of media are also effective values. For example, for a wet fluting this coefficient is determined by formula:

$$K_{2,\nu} = K_{f,\nu}(1-w) + K_{w,\nu}w, \qquad (10)$$

where $K_{f,\nu}$, $K_{w,\nu}$ are the absorption coefficients of dry fluting and water, respectively, $m^{-1}$; w is the volumetric share of moisture in fluting.

The density vector of radiation flow in any part of the system of wet air or fluting is determined by relation [18]:

$$\mathbf{q}_r(z) = 2\pi \int_{\nu=0}^{\nu_{tr}} \left\{ \int_0^1 \left[ \vec{I}_\nu^+(z,\mu_i) - \vec{I}_\nu^-(z,\mu_i) \right] \mu_i d\mu_i \right\} d\nu, \qquad (11)$$

where $\nu_{tr}$ is the limit of thermal radiation transmittance in the IR region of the spectrum, Hz.

Upon differentiating the integrand expression (11 by parameter z, we obtain a formula for determining $\frac{\partial \mathbf{q}_r}{\partial z}$, which is included in equation (4) through partial derivatives:

$$\frac{\partial q_r(z)}{\partial z} = 2\pi \int_{\nu=0}^{\nu_{tr}} \left\{ \int_0^1 \left[ \frac{\partial \vec{I}_\nu^+(z,\mu_i)}{\partial z} - \frac{\partial \vec{I}_\nu^-(z,\mu_i)}{\partial z} \right] \mu_i d\mu_i \right\} d\nu. \qquad (12)$$

Since the density vector of radiation flow $\mathbf{q}_r(z)$ coincides with the Oz axis, then in (12) and hereafter in the text, instead of the vector we write its component by z.

To exclude partial derivatives in the integrand expression (12), it is necessary to substitute the appropriate expressions from the system of transport equations (9) that yields as a result:

$$\frac{\partial q_r(z)}{\partial z} = = -2\pi K_{i,\nu} \int_{\nu=0}^{\nu_{tr}} \left\{ \int_0^1 \left[ \vec{I}_\nu^+(z,\mu_i) - \vec{I}_\nu^-(z,\mu_i) \right] d\mu_i - 2n_{i,\nu}^2 I_{b\nu}(T(z)) \right\} d\nu. \qquad (13)$$

To determine magnitudes $\vec{I}_\nu^+, \vec{I}_\nu^-$, which are included in (12), (13), it is necessary first to record boundary conditions for the radiation on the opaque and translucent boundaries. Under conditions of the accepted model of diffuse reflection, boundary conditions take the form [17]:

$$\begin{cases} I_\nu^+(0) = n_{1,\nu}^2 (1-R_{1,\nu}) I_{b\nu}(T_{z=0}) + 2R_{1,\nu} \int_0^1 \vec{I}_\nu^-(0,\mu_1)\mu_1 d\mu_1; \\ I_\nu^-(\delta_{air}^-) = 2R_{air,\nu} \int_0^1 \vec{I}_\nu^+(\delta_{air}^-,\mu_1)\mu_1 d\mu_1 + 2(1-R_{f,\nu}) \int_0^1 \vec{I}_\nu^-(\delta_{air}^+,\mu_2)\mu_2 d\mu_2, \end{cases} \qquad (14)$$

$$\begin{cases} I_\nu^+(\delta_{air}^+) = 2R_{f,\nu} \int_0^1 \vec{I}_\nu^-(\delta_{air}^+,\mu_2)\mu_2 d\mu_2 + 2(1-R_{air,\nu}) \int_0^1 \vec{I}_\nu^+(\delta_{air}^-,\mu_1)\mu_1 d\mu_1; \\ I_\nu^-(\delta_\Sigma) = n_{2,\nu}^2 (1-R_{2,\nu}) I_{b\nu}(T_{z=\delta_\Sigma}) + 2R_{2,\nu} \int_0^1 \vec{I}_\nu^+(\delta_\Sigma,\mu_1)\mu_1 d\mu_1, \end{cases} \qquad (15)$$

where $T_{z=0}$, $T_{z=\delta_\Sigma}$ are the temperatures of respective opaque boundaries of the system gas – capillary-porous body, K.

In the systems of equations (14) and (15), $I_\nu^+(0)$ and $I_\nu^-(\delta_\Sigma)$ are related to the non-transparent boundaries of the system gas – capillary-porous body, and $I_\nu^\pm(\delta_{air}^\pm)$ – to the translucent boundary between the layers of wet air and fluting. Spectral values of magnitude $R_{air,\nu}$ are defined from the Fresnel formulas [18]. In this case, it is assumed that $n_{1,\nu}\approx 1$, similar for the gas medium. Spectral magnitude $R_{f,\nu}$ is found from the condition of preserving the balance of radiation energy on the translucent boundary:

$$(1-R_{air,\nu})n_{1,\nu}^2 = (1-R_{f,\nu})n_{2,\nu}^2 \to 1-R_{air,\nu} = (1-R_{f,\nu})n_{2,\nu}^2, \qquad (16)$$

where $n_{1,\nu}\approx 1$.

The value of degree of blackness $\varepsilon_{f,\nu}=1-R_{f,\nu}$ on the translucent boundary is determined by using Fresnel formulas [18] under conditions $n_{2,\nu}>n_{1,\nu}$, $n_{2,\nu}>n_{f,\nu}$ $n_{2,\nu}=n_{f,\nu}$, $n_{1,\nu}\approx 1$:

– perpendicular component of the reflection coefficient:

$$R_{s,\nu}(\mu) = \frac{\left[\mu - \sqrt{n_{2,\nu}^2 - (1-\mu^2)}\right]^2}{\left[\mu + \sqrt{n_{2,\nu}^2 - (1-\mu^2)}\right]^2}; \qquad (17)$$

– parallel component of the reflection coefficient:

$$R_{p,\nu}(\mu) = \frac{\left[n_{2,\nu}^2\mu - \sqrt{n_{2,\nu}^2 - (1-\mu^2)}\right]^2}{\left[n_{2,\nu}^2\mu + \sqrt{n_{2,\nu}^2 - (1-\mu^2)}\right]^2}; \qquad (18)$$

– mean value of the reflection coefficient:

$$R_\nu(\mu) = \frac{1}{2}\left(R_{s,\nu}(\mu) + R_{p,\nu}(\mu)\right). \qquad (19)$$

Then the selective hemispherical degree of blackness of the surface of wet air – fluting in the direction of wet air will be determined by formula:

$$\varepsilon_{air,\nu} = 2\int_0^1 (1-R_\nu(\mu))\mu d\mu, \; R_{air,\nu} = 1-\varepsilon_{air,\nu}. \qquad (20)$$

The degree of blackness of the surface of wet air – fluting in the direction of fluting is defined based on (16):

$$1-R_{air,\nu} = (1-R_{f,\nu})n_{2,\nu}^2 \to \varepsilon_{air,\nu} = \varepsilon_{f,\nu}n_{2,\nu}^2. \qquad (21)$$

Hence

$$\varepsilon_{f,\nu} = \frac{\varepsilon_{air,\nu}}{n_{2,\nu}^2}. \qquad (22)$$

That is, provided $n_{2,\nu}>1$, inequality $\varepsilon_{f,\nu}<\varepsilon_{air,\nu}$ holds.

In order to obtain final equations for determining the density of heat flux of radiation and its divergence, it is necessary to solve the appropriate system of equations. This system includes (14), (15), a formal solution (9) for the boundary surfaces of the system and the current values of coordinate z [17].

Thus, the system of equations (4)–(22) is a complete mathematical formulation of the selective problem on radiation-conductive heat exchange that describes the drying process of fluting by the infrared radiation. In this case, expressions for $\mathbf{q}_r$ and $\frac{\partial \mathbf{q}_r}{\partial z}$ are accurate analytical formulas for determining the density of radiation flow and its divergence in the one-dimensional case.

The range of waves from the industrial emitters is close to the range of the largest absorbing capacity of water.

That is why it is possible to simplify a mathematical model (4)–(22) by using a transition from a selective to the "grey" model of environment.

For the transition from a model of selective environment to a simpler "gray" environment, it is necessary to perform the following transformations. All radiation properties of phases and boundaries of the system are averaged, for example, by Roseland [17, 19]. The Planck function in (9), (13)–(24) is replaced with expression $\frac{\sigma}{\pi}$. The value of σ in the expression is the Stefan-Boltzmann constant, W/(m$^2$·K$^4$).

Based on the scientific literature review, we found that in order to model the radiation drying of materials with physical properties that are close to fluting, it is possible to apply a simplified model based on the Burgers' equation [12, 20]. In this case, the results obtained describe the drying process with accurately sufficient for the engineering calculations.

Considering data [12, 20], it is possible to perform further simplification of the mathematical model (4)–(22) based on the Burgers' equation [4]. In this case, heat transfer from an IR emitter to the surface of fluting can be described by the Stefan-Boltzmann law, disregarding the layer of wet air above it. It is, however, necessary to employ the approximation of optically thin layer with regard to absorption [19].

The medium, which is a wet fluting, is assumed to be translucent "grey" and absorbing; the boundaries of its layer and the surface of an IR heater – diffuse. It is possible in this case to confine ourselves to considering only the layer of fluting's material, but under more complex conditions for external heat exchange than for the two-phase selective medium. It is also required to complement the model with a kinetic equation that describes a change in moisture content over time. At this approximation, mathematical statement of the problem, considering the kinetics of drying [4], includes the following system of equations:

$$\begin{cases} c_{p\,eff}(T)\rho_{eff}(T)\dfrac{\partial T}{\partial\tau}=\lambda_{eff}(T)\dfrac{\partial^2 T}{\partial z^2}-\dfrac{\partial \mathbf{q}_r(z)}{\partial z}-\rho_f(T)r\dfrac{du}{d\tau};\\ \mathbf{q}_r(z)=q_r(T_{z=0})e^{-\int_0^z K(T(z))dz};\\ q_r(T_{z=0})=\varepsilon_{pr}(K_{a\text{-}v})\sigma\left(T_{IR}^4-T_{z=0}^4\right);\\ \dfrac{du}{d\tau}=\dfrac{\beta}{g}\left[p_s\left(T_{z=0}^+\right)-p_p\left(T_{z=0}^-\right)\right]+k(u-u_m), \end{cases} \tag{23}$$

where $c_{p\,eff}(T)$ is the temperature dependence of mass isobar heat capacity of wet fluting, J/(kg·K); $\rho_{eff}(T)$ is the temperature dependence of effective density of wet fluting, kg/m$^3$; $\rho_f(T)$ is the temperature dependence of density of absolutely dry fluting, kg/m$^3$; T is the absolute temperature, K; τ is the time, s; $\lambda_{eff}(T)$ is the temperature dependence of effective coefficient of thermal conductivity of wet fluting, W/(m·K); z is the coordinate, m; $\mathbf{q}_r(z)$ is the density vector of radiation heat flux, W/m$^2$; r is the mass heat of water vaporization, J/kg; K(T(z)) is the temperature dependence of "gray" value of effective coefficient of absorption of wet fluting, m$^{-1}$; $\frac{du}{d\tau}$ is the drying speed, s$^{-1}$ (kg/(kg·s));

$$\varepsilon_{pr}(K_{a\text{-}v})=\frac{1}{\dfrac{1}{\varepsilon_{IR}}+\left(\dfrac{1}{\varepsilon_{z=0}}-1+\dfrac{3K_{a\text{-}v}\delta_{air}}{4}\right)\dfrac{F_{IR}}{F_f}},$$

is the resulting degree of blackness of surfaces of the IR heater of the layer of fluting, which takes into account the absorption of layer of wet air [16]; $\varepsilon_{IR}$ is the degree of blackness of radiating surface of an IR heater; $\varepsilon_{z=0}$ is the degree of blackness of the surface of fluting; $F_{IR}$, $F_f$ is the area of surfaces of IR-heater and fluting, respectively, m$^2$; $K_{a\text{-}v}$ is the "grey" value of absorption coefficient of wet air, m$^{-1}$; $\delta_{air}$ is the thickness of layer of wet air, m; σ is the Stefan-Boltzmann constant, W/(m$^2$·K$^4$); $T_{IR}$, $T_{z=0}$ is the absolute temperature of surfaces of IR heater and fluting, respectively, m; β is the moisture exchange coefficient, s/m; k is the drying constant, s$^{-1}$; g is the mass of square meter of dry fluting, kg/m$^2$; $p_s$, $p_p$ are the pressure of vapor saturation at the surface of a layer of fluting and the partial pressure of water vapor in the adjacent layer of environment (wet air), respectively, Pa; u, $u_m$ are the mean relative moisture content in the current moment and relative equilibrium in the layer of fluting, respectively.

A formula for determining the divergence of radiation flow can be received by differentiating, by parameter z, the integrand expression, which is an indicator of the power of exponent in the second equation of system (24).

$$\frac{\partial q_r(r)}{\partial z}=\frac{\partial}{\partial z}\left(q_r(T_{z=0})e^{-\int_0^z K(T(z))dz}\right)=$$

$$=\left|K(T(z))=\text{const}\right|=-q_r(T_{z=0})Ke^{-Kz}. \tag{24}$$

The equation of thermal conductivity in the system of equations (23) can also be written through a thermal diffusivity coefficient. Then, with regard to the latter and (24), the system of equations (23) takes the final form:

$$\begin{cases} \dfrac{\partial T}{\partial\tau}=a_{eff}(T)\dfrac{\partial^2 T}{\partial z^2}+\dfrac{q_r(T_{z=0})}{c_{p\,eff}(T)\rho_{eff}(T)}Ke^{-Kz}-\dfrac{\rho_f(T)r}{\rho_{eff}(T)c_{p\,eff}(T)}\dfrac{du}{d\tau};\\ q_r(T_{z=0})=\varepsilon_{pr}(K_{a\text{-}v})\sigma\left(T_{IR}^4-T_{z=0}^4\right);\\ \dfrac{du}{d\tau}=\dfrac{\beta}{g}\left[p_s\left(T_{z=0}^+\right)-p_p\left(T_{z=0}^-\right)\right]+k(u-u_m), \end{cases} \tag{25}$$

where

$$a_{eff}=\frac{\lambda_{eff}}{c_{p\,eff}\rho_{eff}}$$

is the effective thermal diffusivity of fluting, m$^2$/s.

For a wet fluting, effective "gray" value of absorption coefficient is expressed by the formula according to the known law of additivity:

$$K=K_f(1-w)+K_w w, \tag{26}$$

where $K_f$, $K_w$ are the absorption coefficients of dry fluting and water, m$^{-1}$; w is the volumetric share of moisture in fluting.

The rest of effective properties, which are included in the system of equations (25), are received similarly using the additivity property.

The system of equations (25) is a nonlinear one by the thermal-physical properties and source terms. Thus, for example, $q_r$ depends on the temperature at the surface of fluting $T_z$=0 in fourth power.

Initial conditions for (25), $\tau=0$:

$$T=T(z),\ \ 0\le z\le\delta_f, \tag{27}$$

where $\delta_f$ is the thickness of a layer of wet fluting, m.

Boundary conditions for (25), $\tau>0$:

– on the opaque boundary of an IR emitter, we consider boundary conditions of the first kind:

$$T_{IR}=T(W), \tag{28}$$

where W is the dependence of change in the electric power of a radiation heater on the time, W. At this boundary, we also assign a «gray» value of the degree of blackness of the working surface of IR emitter $\varepsilon_{IR}$;

– at the upper and lower boundaries of a layer of fluting, we assign boundary conditions of the third kind:

$$\mathbf{n}\cdot\left(-\lambda_{eff}(T)\nabla T\right)\Big|_{z=0}=\alpha_1\left(T-T_{s_1}\right); \tag{29}$$

$$\mathbf{n}\cdot\left(-\lambda_{eff}(T)\nabla T\right)\Big|_{z=\delta_f}=\alpha_2\left(T-T_{s_2}\right), \tag{30}$$

where $\mathbf{n}$ is the vector of external normal to the surfaces of sides of wet fluting; $\nabla=\frac{\partial}{\partial z}$ is the Hamiltonian in the one-dimensional case, $m^{-1}$; $\alpha_1$, $\alpha_2$ are the coefficients of heat transfer from the upper and lower sides of a layer of fluting, respectively, W/(m$^2$·K); $T_{S_1}$, $T_{S_2}$ are the ambient temperatures, above a layer of fluting from its upper and lower sides, respectively, K. At the upper boundary of a layer of fluting, we also assign the degree of blackness of the surface of fluting $\varepsilon_{z=0}$.

Mathematical statement of the problem on complex heat transfer taking into account the kinetics of the process (25)–(30) is a complete mathematical formulation of the nonlinear one-dimensional problem on drying the wet fluting by the radiation method.

## 6. Technique for the numerical solution of the problem on drying by the radiation method

In order to numerically solve the stated problem, described by equations (25)–(30), we employed the finite difference method [23].

Uniform grid is used for the discretization of computational domain

$$z=h(i-1),\ i=\overline{1,n},\ h=\frac{\delta_f}{n-1},$$

where n is the number of grid nodes. To obtain finite-difference analogs of the original differential equations (25), we use an absolutely stable implicit three-point scheme with the first order approximation by time and the second – by coordinate. Since the systems of obtained discrete equations for determining the temperature and relative moisture content are non-linear, then, in order to solve them, we apply the linearization by Newton's method by temperature.

Systems of linearized equations at each iteration step are resolved successively by economical method of three-point sweep relative to discrepancy vectors $\delta T_i^{k+1}$, $i=\overline{1,n}$ and $\delta u_i^{k+1}$, $i=\overline{1,n}$, respectively, and the desired temperature and relative humidity in the nodes of computational grid are determined by formulas:

$$\begin{cases} T_i^{k+1}=T_i^k+\delta T_i^{k+1},\ \ i=\overline{1,n}; \\ u_i^{k+1}=u_i^k+\delta u_i^{k+1},\ \ i=\overline{1,n}, \end{cases} \tag{31}$$

where k is the number of iteration at each step of the integration by time.

In this case, the system of discrete equations for relative humidity is solved first in the iteration loop of solving the problem (25)–(30). Thus, a system of equations of thermal conductivity and kinetics is resolved by the method of successive approximations. In order to improve convergence in the solution of a system of equations, it is possible to employ the method of top relaxation.

A criterion for obtaining the numerical solution of the initial problem at each integration step by time is the fulfillment of condition:

$$\begin{cases} \left|\delta T_i^k\right|\le\varepsilon_T,\ \ i=\overline{1,n}; \\ \left|\delta u_i^k\right|\le\varepsilon_u,\ \ i=\overline{1,n}, \end{cases} \tag{32}$$

where $\varepsilon_T$, $\varepsilon_u$ are the errors, assigned in advance, in determining the temperature (K) and relative humidity, respectively.

Software realization of the numerical technique is implemented in the Mathcad programming language [24]. A description of the algorithm in the process of its development and application is given in [25]. By using the formulated mathematical model, whose numerical problem is described by equations (25)–(30), it is also possible to develop a software code in other programming languages.

## 7. Verification of numerical model for the process of radiation drying of fluting

A comparative analysis of results of the numerical modeling with data on physical experiments to the study of kinetics of drying of fluting by the IR radiation was conducted based on such parameters as the duration and speed of the drying process (at attaining the same values of moisture content) and the fluting surface temperature. The values of fluting drying speed were determined by the results of numerical and physical experiments (25).

To compare the results, we used the samples of fluting of mark B-1 with a square meter weight of 0.112, 0.125 and 0.140 kg/m$^2$, as well as the samples from non-standard fluting with increased specific weight of 0.2 kg/m$^2$.

As the source of radiation in the experiments, we used ceramic electric IR emitter of type ECH4 with nominal capacity of 1 kW and the temperature of working surface about 833 K [22]. The length of electromagnetic waves generated by the IR emitter is 3.3...3.7 μm. Results of comparison are given in Table 1.

Designations in Table 1: g – square meter weight of fluting, kg/m$^2$; q – density of heat flux at the surface of fluting, W/m$^2$; $\tau_1$ – drying time at the end of the first period, s; $\tau_\Sigma$ – drying time to resulting equilibrium moisture content, s; $T_1$, $T_2$ –absolute temperatures of the surface of fluting in the first period of drying and at the end of the second period (resulting temperature), K; $\frac{\partial u}{\partial\tau_1}$ – speed of drying in the first period of drying, $s^{-1}$ (kg/(kg·s)).

Table 1

Comparison of experimental data with the data of numerical analysis

| g, kg/m$^2$ | q, W/m$^2$ | $\tau_1$, s* | $\tau_2$, s* | $T_1$, K* | $T_2$, K* | $\frac{\partial u}{\partial \tau_1}$, s$^{-1}$* |
|---|---|---|---|---|---|---|
| 0.112 | 6835 | 56/58 | 189/187 | 363/367 | 496/495 | 0.0192/0.0190 |
| 0.125 | 6835 | 61/64 | 203/201 | 365/368 | 496/496 | 0.0178/0.0175 |
| 0.125 | 4300 | 116/117 | 244/246 | 342/345 | 460/462 | 0.0120/0.0119 |
| 0.125 | 3665 | 144/145 | 318/317 | 339/343 | 434/432 | 0.0090/0.0089 |
| 0.140 | 6835 | 65/67 | 212/214 | 367/370 | 497/499 | 0.0166/0.0163 |
| 0.200 | 6835 | 90/92 | 252/253 | 371/384 | 504/505 | 0.0112/0.0099 |

*Note: * – data of numerical analysis are given through a fraction*

As a result of comparing the data of numerical modeling with experimental data, we established that the discrepancy between them does not exceed 5 % on the interval of change in the weight of square meter of dry fluting from 0.112 kg/m$^2$ to 0.2 kg/m$^2$ and density of heat flux at the surface of fluting from 3665 W/m$^2$ to 6835 W/m$^2$.

## 8. Discussion of results of numerical modeling of physical fields in the process of drying of fluting

Numerical modeling of the drying process of fluting by the radiation method allows us to determine kinetic patterns and basic parameters required to intensify the process and design of drying equipment.

The numerical model developed makes it possible to determine duration of the process of drying the fluting to the degree of its dehydration assigned in advance. In this case, we also define the current degree of fluting dehydration, maximum value of fluting temperature and its thermal-physical parameters considering the moisture content. In addition, by using a numerical model, it is possible to calculate the capacity of IR heaters and their quantity. This makes it possible to design new drying equipment of the radiation type.

However, the mathematical and numeric models presented do not take into account the phenomenon of fluting shrinkage during drying that affects its thermal-physical properties. A change in the structure of fluting during the crystallization of glue that cements it is also not considered. But the results of verification of the numerical model indicate that the mentioned shortcomings do not essentially influence its accuracy.

The stated mathematical model and the developed numerical model of the radiation drying of capillary-porous materials of the fluting type can also be used for modeling the drying process of other types of paper, for example sanitary-hygienic [21].

Under high-temperature drying mode of fluting, deterioration in its physical-mechanical properties is possible. That is why we plan to complement the mathematical model developed by the kinetic patterns in the process of fluting destruction.

## 9. Conclusions

1. Based on the analysis of physical processes that occur during drying of fluting by the IR radiation, we formulated a physical model of the process. The physical model takes into account the absorbing and emissive capacity of the environment translucent for the IR radiation, diffuseness in the properties of medium boundaries, temperature dependence of physical properties and the kinetics of drying process.

2. We formulated a mathematical model and developed a numerical model of the process of radiation drying of fluting, which considers the kinetics of drying process, the translucency to the IR radiation of the material using the approximations of the Burgers' models, “gray” medium and diffuse reflection of boundaries. Software realization of the numerical method is implemented in the MathCAD programming language. The formulated models of drying process will also make it possible to develop a software code in other programming languages.

3. Verification of the numerical model of the radiation drying of fluting revealed a discrepancy between the results of numerical modeling and the data of physical experiment within the range of 5 %. A comparison of the results was conducted by the following parameters: surface temperature of fluting, duration and speed of fluting drying. The weight of square meter of dry fluting varied between 0.112 kg/m$^2$ and 0.2 kg/m$^2$. Heat flux density at the surface of fluting changed from 3665 W/m$^2$ to 6835 W/m$^2$.

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