

*Побудовано модель оптимізації структури вітрової електричної станції (ВЕС), яка ґрунтується на сформульованій задачі цілочисельного програмування. Визначено правило поділу множини рішень на підмножини та критерій обчислення оцінки верхньої границі кожної підмножини, що дало змогу застосувати метод гілок та границь. В процесі розв'язання оптимізаційної задачі розроблено структуру програмної системи, інформаційне і програмне забезпечення та наведено результати оптимізації вітрової електричної системи*

*Ключові слова: вітрова електрична станція, задача цілочисельного програмування, метод гілок та границь*

*Построена модель оптимизации структуры ветровой электростанции, которая базируется на сформулированной задаче целочисленного программирования. Определено правило разделения множества решений на подмножества и критерий вычисления оценки верхней границы каждого подмножества, что позволило применить метод ветвей и границ. В процессе решения оптимизационной задачи разработана структура программной системы, информационное и программное обеспечение и представлены результаты оптимизации ветровой электрической системы*

*Ключевые слова: ветровая электрическая станция, задача целочисленного программирования, метод ветвей и границ*

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# OPTIMIZATION OF THE STRUCTURE OF WIND POWER STATION WITH THE USE OF THE BRANCH AND BOUND METHOD

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## 1. Introduction

Every year, the problem of supplying energy resources and of environmental pollution is becoming increasingly acute. One of the ways of partial solution of this problem is the use of environmentally friendly methods and means of power generation [1]. These areas include wind power industry [2]. However, effective use of wind power requires solving an optimization problem, which is determined by special features of modern wind power stations, when each wind power plant has its own effectiveness; separate groups of wind plants must be united into a system and various tasks concerning optimization of parameters and processes of power generation must be fulfilled. Accordingly, in the process of operation, the problem of efficient use of wind power stations (WPS) arises, which lies, on one hand, in the generation of the assigned, in advance required capacity for a certain period. Meanwhile, on the other hand, it is necessary to use only those wind mills that have the best parameter of effectiveness, which will make it possible to provide the lowest cost per unit of electricity. From these positions, the task of optimizing the structure of a wind power station is a relevant scientific study.

## 2. Literature review and problem statement

In the energy sector, different kinds of optimization problems very often have to be dealt with. A number of literary sources present the results of studying the concepts, models and methods for the solution of optimization problems in the energy sector. In particular, the models of influence of reliability factor on optimum composition of a generation system are presented in papers [3, 4], and the models of optimization of electric power plants are analyzed in [5]. Simulation of joint operation of a power system and a group of WPS with regard to the wind conduct is explored in [6]. Method of aligning the fluctuations of wind power in a hybrid energy system is described in [7] and others. The possibilities of using the branch and bound method (BBM) for solving optimizing problems in the energy sector are presented in [8]. All these models, methods and tools are designed for the solution of problems of optimization and management of effective power systems and focused on the input parameters for such systems, rather than optimization of the structure of the WPS.

The methods listed in papers [9, 10] do not imply long-term management of the composition of wind power stations

because they are designed for a single short-term determination of the WPS composition.

The task of optimization of the structure of wind power stations using the method of dynamic programming based on the Bellman principle and boolean programming [11] and genetic algorithms [12, 13] requires significant amount of computation. This special feature makes it difficult to solve the problems of large dimensionality. Accordingly, the task of enhancing efficiency of solving the problems of optimization of the structure of the WPS of great dimensionality appears.

In this paper, the authors proposed to apply the branch and bound method to the solution of the problem of optimization of the WPS structure, which allows us to effectively solve the problem of large dimensionality.

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### 3. The aim and tasks of the study

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The aim of the work is to enhance the efficiency of the WPS by solving the optimization problem of integer programming (PIP) using the branch and bound method.

To achieve the set goal, the following tasks are to be solved:

- to build a model of the problem of integer programming for the optimization of the WPS structure, which will include efficiency of a wind power station as optimization criterion;
- to apply the BBM to the solution of the stated problem of integer programming (IP) that determines the definition of the rule of division of a set of solutions into subsets and the assessment criterion for each subset of solutions;
- to develop software tools for solving the problem of optimization of the WPS structure, information support and to conduct research.

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### 4. Optimization of the structure of a wind power station

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In general, a wind power station includes wind power plants (WPP), which usually have different structural and technological parameters and are connected in a single system. In the process of generating electricity it is not necessary to involve all wind power plants, but only those that provide the necessary total capacity with the highest value of efficiency coefficients. Solution of such an optimization task allows us to generate electricity with the least cost for a long period of time.

#### 4.1. Construction of a model of optimization problem

Let us state the optimization problem, given that capacity of the components of a system and effectiveness of each source are known (Table 1) [14]. In the process of solving the problem, it is necessary to maintain generation capacity of not less than the assigned magnitude (20 000 kW) while providing the best total value of efficiency [15] of a power system based on the wind power plants.

Therefore, effectiveness of the system of wind power stations is determined by the coefficients of Table 1. This parameter is an integral magnitude and includes the parameters relating to the cost of production of one KW of electricity. Thus, from the system of wind power stations, it is necessary to select such WS that have the best efficiency parameters. Accordingly, the criterion of optimality of the problem of integer programming will be the total efficiency of the WPS, which are included in the system and produce

electricity. Based on the criterion of optimality, we will build the objective function (OF), which must be maximized and which will take the following form [16, 17]:

$$\max F(\bar{x}) = \sum_{i=1}^n f_i^{\text{ef}} x_i, \quad (1)$$

where  $f_i^{\text{ef}}$  is the parameter of efficiency for the  $i$ -th WS;  $x_i$  is the parameter, which takes the value of 0 or 1, in the first case the WS is turned off and in the second case it is included in the WPS, or any other integer number which defines the number of the included WS of this type;  $n$  is the total number of WS.

For example, in the PIP problem that must be solved, we will assume that  $n=40$  (although there is no constraint to the number of WS, and their number could reach several hundred). Accordingly, taking into account parameters of the real example, objective function (1) may be presented in the form:

$$\max F(\bar{x}) = \sum_{i=1}^{40} f_i^{\text{ef}} x_i.$$

Formalization of the PIP problem for the specified example involves the determination of OF and the constraint. The constraint must take into account the requirement for the total generated electrical power of the system (20 000 kW), in particular:

$$\sum_{i=1}^n w_i x_i \leq W,$$

where  $W$  is the total required capacity of the power system;  $w_i$  is the capacity of the  $i$ -th WS.

In this case, taking into account the requirements of the example, we must add the following constraint:

$$\sum_{i=1}^{40} w_i x_i \leq 20000.$$

However, one should take into account that in order to consider integer parameters  $x_i$  and the existing number of WPP of each type, it is necessary to add another  $n$  of constraints. Each constraint specifies the value of parameter  $x_i$  that must be considered in the problem of integer programming, which must be more than or equal to zero, but less than or equal to the number of WPP of this type. We will write these constraints in this form:

$$0 \leq x_i \leq N, \quad i = \overline{1, n}, \quad (2)$$

where  $N$  is the number of WPP of the  $i$ -th type.

Constraint (2) includes the requirement for the value of the design parameter, which must be more than or equal to zero, which is determined by the condition of canonicity of linear programming problems. Therefore, constraints (2) are bilateral.

In the final case, the model of the problem of determining the optimum structure of the wind power system is stated as follows: find the values of design parameters  $x_i$  that provide the maximum value of objective function:

$$\max F(\bar{x}) = \sum_{i=1}^{40} f_i^{\text{ef}} x_i, \quad (3)$$

and the following constraints hold:

$$\sum_{i=1}^{40} w_i x_i = 20000, \tag{4}$$

$$x_i \leq N, \quad i = \overline{1,40}, \quad x_i \geq 0. \tag{5}$$

Therefore, the stated optimization problem (3)–(5) is the basis of mathematical support for the software system for determining the optimal WPS structure.

Table 1

Parameters of the system of wind power plants

No. WS	Number of WS	Generated capacity, kW	Effectiveness parameter
1	1	457	1,32
2	1	439	1,23
3	1	474	1,39
4	1	482	1,51
5	1	450	1,48
6	1	493	1,79
7	1	521	1,37
8	1	503	1,05
9	1	422	1,21
10	1	467	1,78
11	1	2455	1,62
12	1	2475	1,69
13	1	2460	1,37
14	1	2511	1,21
15	1	2386	1,05
16	1	2412	1,21
17	1	2321	1,78
18	1	2455	1,62
19	1	2475	1,69
20	1	2460	1,37
21	1	2511	1,21
22	1	2567	1,44
23	1	2491	1,81
24	1	2431	1,66
25	1	2388	1,91
26	1	2429	1,56
27	1	2248	1,39
28	1	2345	1,73
29	1	2567	1,21
30	1	2888	1,68
31	1	2931	1,35
32	1	2944	1,63
33	1	2897	1,64
34	1	3030	1,55
35	1	2980	1,56
36	1	2950	1,21
37	1	2970	1,13
38	1	2960	1,61
39	1	2980	1,33
40	1	2950	1,20

### 4.2. Features of application of the branch and bound method for solving the problems of integer programming

It is commonly known that the branch and bound method [18] belongs to the group of combinatorial methods of discrete programming and is one of the most common methods of this group. In the process of application of the BBM to solving practical problems, it is necessary to solve two subtasks, in particular:

- to determine the rule of division of a set of solutions into subsets;
- to devise a computational criterion for assessment of the lower (upper) bound of each subset.

In this case, to realize the first rule, we will use the stated problem of linear programming (PLP) as a possible solution. At the first step, it will be a task that is described by expressions (3)–(5). For the following steps we will use the PLP that are necessary to state for the design variable, which will have a real value that is nonzero and different from an integer [18]. As a computational criterion for the bound for each PLP, we will use the value of objective function, which may be calculated with the use of dependence (3).

Then the process of solving the problem of determining the optimum structure for a wind power system with the use of the BBM may be represented using a schematic, which is shown in Fig. 1. The figure shows the initial PLP of the first step (3)–(5) that must be solved using modifications of the simplex-method [19]. After that, we will analyze the obtained values of variables  $x_i$ . As a rule, at the first step it is practically impossible to get the integer solution (IS) of the problem, so using the method described in [20], we state two PLP (second step), which we solve again. We will continue the division process until we get the integer solution to the PLP. In the final case, we get a set of integer solutions of linear programming problems, from which we choose such solution of the PLP, where the assessment of the OF value has the highest value. The obtained values of design parameters  $x_i$  determine which WPP should be currently included ( $x_i \neq 0$ ), and which should not ( $x_i = 0$ ).

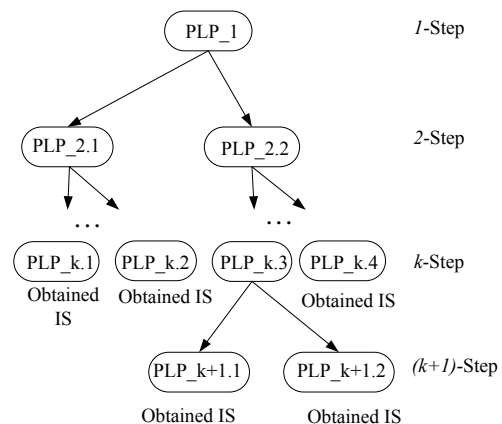


Fig. 1. Schematic of division into subsets

The peculiarity of the algorithm of the BBM application to solving practical problems is its natural parallelism (Fig. 2). With these positions, considering multi-nucleus architectures of modern microprocessors and multi-processor systems, it is possible to significantly reduce the time of solving the problem of determining the optimal structure of the WPS, which is a relevant issue for the problems of high dimensionality.

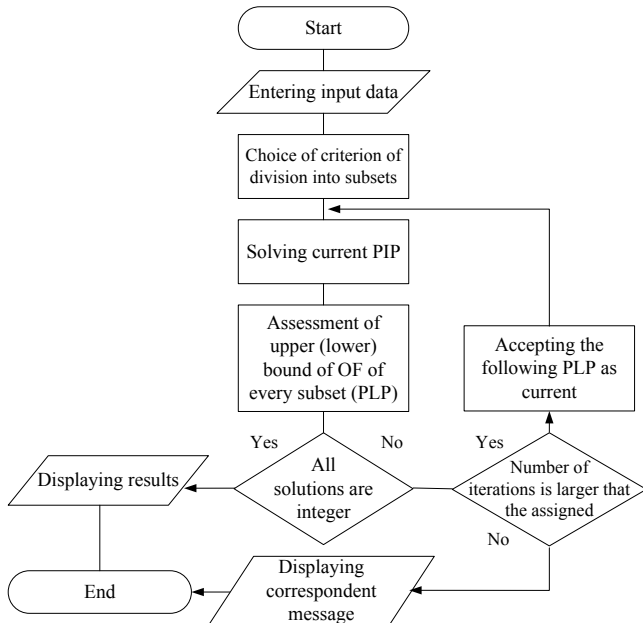


Fig. 2. Block-diagram of the algorithm for applying the branch and bound method

**4. 3. Features of development of software system of solving design problems using the branch and bound methods**

To solve the problem of integer programming [18, 19] using the branch and bound method [19], a software system was developed. The developed structure of the software system includes the following subsystems, which are implemented in the form of program modules:

- subsystem of control that enables a user to actively use the software tool for solving the PIP;
- subsystem of introduction and correctness control of input data that provides control of the input data;
- subsystem of the file work, which provides data reading from a file, saving intermediate and output data in files, which holds for PIP of great dimensionality;
- subsystem of displaying output results, which represents the data in a convenient format for a user;
- subsystem of aid, where the features of using and effective solution of the PIP using the BBM;
- subsystem of establishing the control point, which allows dividing a complex integer problem into parts and solving it step by step by establishing control points (saving data arrays of large dimension and recover solving the PIP from the given stage);
- subsystem of the BBM provides solution of the PIP using the BBM, simplex-method and large numbers method [19].

Using the modular principle of construction of the software system enables you to efficiently modify and improve it in the future.

In particular, the example of the menu with input data for a LP problem, which is being solved with the use of the simplex method, is shown in Fig. 3.

The developed software system is focused on working with large arrays of information, in particular the tasks of great dimensionality. At the same time, there are high requirements for the high-speed performance of a software system. After reading the input data for the LP problem by a software system, it is necessary to save information in the

structure with the shortest possible time of access to them. After analyzing existing approaches and data storage models and organization templates [20], it was proposed to use the list data structures, in particular doubly connected lists.

Moreover, in the process of data storage, we used the XML format that enables us to organize effectively data exchange with the existing software systems.

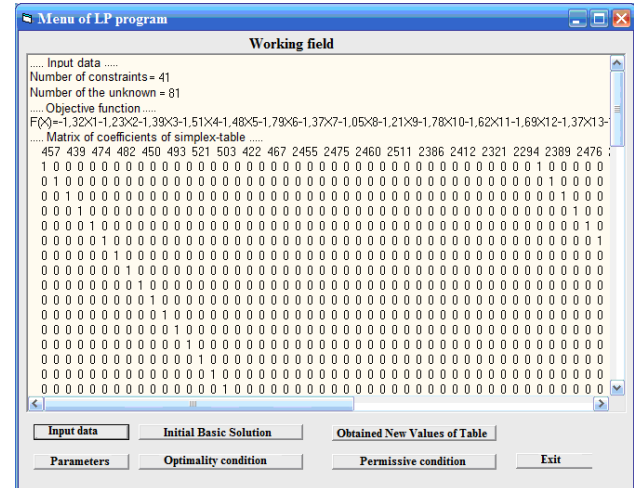


Fig. 3. Example of a menu with the input data for the LP problem, which is solved using the simplex method

**4. 4. Features of the solution of the stated problem of integer programming**

The developed software system was used for solving the problem of integer programming to determine the optimal structural composition of a wind power system.

Before moving on to direct practical solution of PLP, described by formulas (3)–(5), you need to write it in canonical form by adding artificial variables up to forty constraints (5), in particular:

$$x_i + x_{40+i} = N, \quad i=1,40. \tag{6}$$

At the next step, we will apply the simplex method to the solution of the problem, which includes expressions (3)–(6) [19]. As a result of solving the problem of linear programming, we see that all variables  $x_i$  take integer values, which are equal to zero or one, except for variable  $x_i$ , which accepts fractional value (Fig. 4), while the OF value is equal to 23.95.

Then we apply the BBM to solving the problem of integer programming by stating two problems of linear programming (step 2, Fig. 1). Each of these problems is a modification of  $x_i$  that is described by expressions (3)–(6).

Therefore, the first PIP of the 2<sup>nd</sup> step includes an expression for OF (3), constraints (4), (5) and additional constraint for variable  $x_{21}$ , in particular  $x_{21} = 0$ .

The second PIP of the 2<sup>nd</sup> step also includes expression for OF (3), constraints (4), (5) and additional constraint for the same variable in the form of inequality  $x_{21} \geq 1$ .

In the process of solving problems of linear programming of the second step, it is necessary to consider expression (6). The obtained results of solving the first and the second PIP of the 2<sup>nd</sup> step are shown in Fig. 5. From the obtained results we can see that for the first PIP of the 2<sup>nd</sup> step we have fully integer solution with the OF value of 21.55. In this case, the variables are equal to zero except for the following  $x_3-x_7, x_{10}, x_{12}, x_{17}-x_{19}, x_{23}, x_{25}, x_{28}$ , which are equal to one. This

integer solution is obtained for the case when  $x_{21}=0$ . For the case when  $x_{21} \geq 1$ , we do not have any fully integer solution (second PIP of the 2<sup>nd</sup> step). The OF value in this case is equal to 21.08, whereas the variable  $x_{33}=0,06$ . Therefore, calculations must be continued by constructing new PIP of the 3<sup>rd</sup> step based on the second PIP of the 2<sup>nd</sup> step and the further solution.

.....The obtained solution is optimal.....

X1	1	6,214629E+07
X61	0,9999999	2,867531E-02
X2	1	1
X3	1	1
X4	1	1
X5	1	1
X6	1	1
X7	1	1
X8	1	1
X9	1	1
X10	1	1
X52	1	0
X53	1	4,237699E+08
X54	1	0
X55	1	0
X56	1	0
X16	1	1
X17	1	1
X18	1	1
X60	1	0
X21	0,4379469	0,4379469
X62	0,5620531	1
X63	1	0
X23	1	1
X65	1	0
X25	1	1
X67	1	0
X68	1	0
X28	1	1
X70	1	0
X71	1	0
X72	1	0
X73	1	0
X74	1	0
X75	1	0
X76	1	0
X77	1	0
X78	1	0
X79	1	0
X80	1	0

Fig. 4. Results of solving the problem (3)–(6): values of required variables

Similarly, we build two problems of the third step from the second PIP of the 2<sup>nd</sup> step. The first PIP of the 3<sup>rd</sup> step will include expressions for the second PIP of the 2<sup>nd</sup> step plus additional constraints of the following type  $x_{33} \geq 0$ .

The second PIP of the 3<sup>rd</sup> step will include expressions for the second PIP of the 2<sup>nd</sup> step and additional constraint  $x_{33} \leq 0$ .

The obtained results of solving problems of the 3<sup>rd</sup> step is shown in Fig. 5. From the obtained results we can see that we have received fully integer solutions for the problems. For the first one, the OF value is 15.66. In this case, the variables are equal to zero except for the following:  $x_6, x_{12}, x_{17}, x_{18}, x_{19}, x_{33}, x_{23}, x_{25}, x_{28}$ . For the second PIP, the OF value is 20.98. In this case, variables are equal to zero except for the following:  $x_3-x_7, x_{10}, x_{12}, x_{18}, x_{19}, x_{21}, x_{23}, x_{25}, x_{28}$ , which are equal to unity.

Therefore, comparing the obtained results of solutions to PLP, we see that the first PLP of the 2<sup>nd</sup> step has the highest value of objective function with the OF value of 21.55 and single values of the following variables  $x_3-x_7, x_{10}, x_{12}, x_{17}-x_{19}, x_{23}, x_{25}, x_{28}$ . Therefore, it is necessary to include wind power stations with numbers: 3, 4, 5, 6, 7, 10, 12, 17, 18, 19, 23, 25 and 28 into the structure of the power generating system. This will provide the highest value of efficiency of the system, which result into the smallest value of 1 kW of electricity.

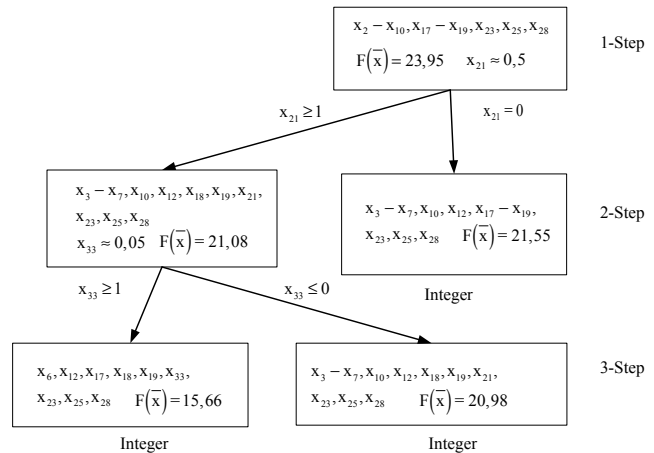


Fig. 5. Tree of solving the PIP problem using the branch and bound method

Thus, the BBM, applied to determining the optimal composition of the WES, makes it possible to solve problems of this type efficiently. The corresponding decision has the advantage and significant potential for solving similar problems of great dimensionality and minimizes the time of searching for the optimal solution using the technologies of parallel data processing. The disadvantages of such a solution may include complications of maintenance of software-hardware system that increases its costs.

The obtained results and the software system may be used in power generation systems of alternative power sector.

Further development of the developed software system is associated with the use of technologies of parallel data processing such as CUDA in the process of solving the problem of determining the optimal WPS structure.

## 5. Conclusions

1. The model for determining the optimal structure of a wind power system, based on the stated problem of integer programming, was developed. The constructed problem of integer programming uses the objective function that is the total of efficiencies of WPP, included into the WPS, and constraints that take into account the demand of the total generated electrical power of a system and conditions of integer solution of the problem. The peculiarity of the developed model is that the branch and bound method was used to solve the problem of integer programming in such statement.

2. The branch and bound method was applied to solve the problem of determining the optimal structure of a wind power station, which allows us to use parallel technologies in the process of finding the optimal solution.

3. We designed and implemented the software system for solving problems of integer programming using the BBM, based on the module principle and providing quick modification and improvement of the software. The information support of the software system, based on the use of the list data structures, was developed, which enables us to process large data arrays efficiently. The results of application of the developed software system, using the branch and bound method to determine the optimal structure for wind power system, were presented.

## References

1. Perfilova, A. Otsenka obosnovannosti tselevykh parametrov "energeticheskoy strategii Ukrainy na period do 2035 hoda" na osnove kriteriya sushchestvennosti [Text] / A. Perfilova // Problemy ekonomiki. – 2015. – Issue 3. – P. 65–70.
2. Geletykha, G. Analiz osnovnykh polozheniy «Energeticheskoy strategii Ukrainy na period do 2030 goda» [Text] / G. Geletykha, T. Zheleznaia // Promyshlennaia teplotehnika. – 2006. – Vol. 28, Issue 5. – P. 82–92.
3. Ivanov, N. Optimizatsiia rezhimov raboty oborudovaniia teplovykh elektrostantsiy [Text] / N. Ivanov, V. Bespalov, N. Lopatin // Materialy 10 mezhdunarodnoy Konferentsii «Sovremennyye tekhniki i tekhnologii». – Tomsk, 2009. – P. 41–44.
4. Sydor, A. R. Recurrent expressions for reliability indicators of compound electropower systems [Text] / A. R. Sydor, V. M. Teslyuk, P. Y. Denysyuk // Technical Electrodynamics. – 2014. – Issue 4. – P. 47–49.
5. Ivanov, N. Matematicheskaya model optimizatsii kratkosrochnykh rezhimov raboty TETs v usloviakh konkurentnogo rynku [Text] / N. Ivanov, V. Bespalov, N. Lopatin // Izvestia Tomskogo politehnicheskogo universiteta. – 2008. – Vol. 313, Issue 4. – P. 37–40.
6. Kuznetsov, M. Osoblyvosti modeliuvaniia potuzhnosti vitrovykh elektrostantsiy, roztashovanyk na obmezheniy terytorii [Text] / M. Kuznetsov // Vidnovliuvana enerhyetika. – 2014. – Issue 4. – P. 57–61.
7. Yu, P. A New Method for Balancing the Fluctuation of Wind Power by a Hybrid Energy Storage System [Text] / P. Yu, W. Zhou, Y. Zhao, H. Sun, J. Liu, Y. Liu // Information Technology Journal. – 2012. – Vol. 1, Issue 1. – P. 58–66. doi: 10.3923/itj.2012.58.66
8. Ivanov, N. Programnyy kompleks dlia optimizatsii rezhimov raboty teplovykh elektrostantsiy i effektivnost ego primeneniia [Text] / N. Ivanov, V. Bespalov, N. Lopatin // Izvestia Tomskogo politehnicheskogo universiteta. – 2008. – Vol. 313, Issue 4. – P. 40–44.
9. Koutroulis, E. Methodology for optimal sizing of stand-alone photovoltaic/wind-generator systems using genetic algorithms [Text] / E. Koutroulis, D. Kolokotsa, A. Potirakis, K. Kalaitzakis // Solar Energy. – 2006. – Vol. 80, Issue 9. – P. 1072–1088. doi: 10.1016/j.solener.2005.11.002
10. Medykovskyy, M. O. The optimization of the wind turbine's structure using the method of dynamic programming [Text] / M. O. Medykovskyy, V. M. Teslyuk, O. B. Shunevych // Actual Problems of Economics. – 2014. – Issue 2 (152). – P. 508–515.
11. Medykovskyy, M. O. The use of dynamic programming for the problem of the uniform use of wind plants [Text] / M. O. Medykovskyy, V. M. Teslyuk, O. B. Shunevych // Technical Electrodynamics. – 2014. – Issue 4. – P. 135–137.
12. Yang, H. Optimal sizing method for stand-alone hybrid solar-wind system with LPSP technology by using genetic algorithm [Text] / H. Yang, W. Zhou, L. Lu, Z. Fang // Solar Energy. – 2008. – Vol. 82, Issue 4. – P. 354–367. doi: 10.1016/j.solener.2007.08.005
13. Hong, Y.-Y. Optimal Sizing of Hybrid Wind/PV/Diesel Generation in a Stand-Alone Power System Using Markov-Based Genetic Algorithm [Text] / Y.-Y. Hong, R.-C. Lian // IEEE Transactions on Power Delivery. – 2012. – Vol. 27, Issue 2. – P. 640–647. doi: 10.1109/tpwrd.2011.2177102
14. Medykovskyy, M. Doslidzhennia efektyvnosti metodiv vyznachenniia vagovykh koefitsientiv vazhlyvosti [Text] / M. Medykovskyy, O. Shunevych // Visnyk Khmelnytskogo natsionalnogo universytetu. – 2011. – Issue 5. – P. 176–182.
15. Denysyuk, P. Development and study of subsystem for solution of tasks of multicriterial optimization [Text] / P. Denysyuk, V. Teslyuk, T. A. AlOmari, T. Teslyuk // Proceedings of the 5th International Conference on Perspective Technologies and Methods in MEMS Design, MEMSTECH. – 2009. – P. 166–167.
16. Taha, Kh. Vvedenie v issledovanie operatsiy [Text] / Kh. Taha. – Moscow: Viliams, 2016. – 912 p.
17. Ma, X. A new branch and bound algorithm for integer quadratic programming problems [Text] / X. Ma, Y. Gao, X. Liu // J. Nonlinear Sci. – 2016. – P. 1153–1164.
18. Ponomarenko, V. Tsilichyselne programuvanniia v ekonomitsi [Text] / V. Ponomarenko, D. Golubnychyy, V. Tretiak. – Kharkiv: Vyd. KhNEU, 2005. – 204 p.
19. Golubeva, Y. A tool for simulating parallel branch-and-bound methods [Text] / Y. Golubeva, Y. Orlov, M. Posypkin // Open Engineering. – 2016. – Vol. 6, Issue 1. doi: 10.1515/eng-2016-0031
20. Kormen, T. Algoritmy. Postroenie i analiz [Text] / T. Kormen, Ch. Leyzerson, R. Rivest, K. Shtain. – 2-e izd. – Moscow: Izdatelskiy dom "Viliams", 2012. – 1296 p.