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*Проведено комплексну оцінку зв'язків критеріїв оптимальності процесу буріння свердловин (мінімумів собівартості 1 м проходки і питомих витрат енергії) за допомогою методу Фаррара-Глобера. Визначено, що спостерігається повна мультиколінеарність між досліджуваними критеріями при зміні осьової сили на долото і частоти його обертання. Запропоновано дуалістичний підхід до вирішення задачі оптимального управління процесом буріння і формування критерію оптимальності на засадах енергоінформаційного підходу*

*Ключові слова: оптимальне управління, процес буріння, критерії оптимальності, взаємозв'язки, метод Фаррара-Глобера*

*Проведена комплексная оценка связей критериев оптимальности процесса бурения скважин (минимумов себестоимости 1 м проходки и удельных расходов энергии) с помощью метода Фаррара-Глобера. Определено, что наблюдается полная мультиколлинеарность между исследуемыми критериями при изменении осевой силы на долото и частоты его вращения. Предложен дуалистический подход к решению задачи оптимального управления процессом бурения и формирования критерия оптимальности на основе энергоинформационного подхода*

*Ключевые слова: оптимальное управление, процесс бурения, критерии оптимальности, взаимосвязи, метод Фаррара-Глобера*

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## ANALYSIS OF INTERRELATIONS BETWEEN THE CRITERIA OF OPTIMAL CONTROL OVER THE PROCESS OF DRILLING THE WELLS

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### 1. Introduction

One of the key technologies in the extraction of hydrocarbons is the process of drilling wells. This is an irreproducible non-stationary non-linear stochastic-chaotic process that evolves over time under the influence of disturbances, which

requires making optimal control decisions under conditions of a priori and current uncertainty about the parameters and structure of the object. The object is related to the class of MI-MO (multiple input-multiple output).

The basic optimality criterion of the well drilling process is the cost per meter – C-criterion. The models of C-criterion

in the parameter space of a drilling mode (axial force on bit  $F$ , bit rotation frequency  $\omega$  and the consumption of washing fluid  $Q$ ) are typically characterized by a unimodal form [1]:

$$C(x) = \frac{1}{T} \int_0^T C_T(u, f, t) dt \xrightarrow{x \in S} \min; \quad N \leq N_{\text{add}},$$

where  $T$  is the duration of drilling a well,  $t \in T$ ;  $C_T$  is the current value of the cost per meter of drilling;  $u$  are the controlling actions (drilling mode parameters);  $f$  are the controlled and uncontrolled disturbances (strength, hardness, abrasivity, rock drillability, ductility of rocks, etc.; reservoir pressures, friction in a column of drill pipes in a well, etc.);

$$S = \left\{ \begin{array}{l} (F_i, \omega_i, Q_i)_{i=1,2,\dots,M}; F \in \{F_{\min}, F_{\max}\}; \\ \omega \in \{\omega_{\min}, \omega_{\max}\}; Q \in \{Q_{\min}, Q_{\max}\} \end{array} \right\},$$

$M$  is the number of levels of depth in wellbore  $H$ ,  $H = \text{const}$  is the design depth of wellbore

$$H = \sum_{i=1}^M h_i;$$

$h_i$  is the footage per bit in the  $i$ -th run;  $N$  is the power spent for the destruction of rock.

It should be noted that the current value of the cost per meter of drilling also depends on the price and durability of rock cutting tools, drilling depth, time spent on the lowering-lifting and auxiliary operations; energy consumed by the drives of a rig. The impact of each factor on the cost of drilling is quite significant and it should be taken into account when optimizing the process of control over drilling on-line and selecting the criterion of optimal management.

The task of optimizing the process of control over drilling is complicated by the fact that the models employed to calculate the cost per meter of drilling include the duration of drilling with one bit and the footage per bit. However, they can be defined only upon completing the bit run, bits, which lasts for several tens of hours.

That is why such additional criteria are used as the maximum of run drilling speed

$$v_p(x) \xrightarrow{x \in S} \max; N \leq N_{\text{add}}$$

or the maximum of footage per bit

$$h_i(x) \xrightarrow{x \in S} \max; N \leq N_{\text{add}}.$$

When drilling in low depths, there is a significant difference between the performance indicators obtained when using the criteria of minimal cost per meter of drilling and maximal run drilling speed and maximal footage per bit. In the large depths, however, this difference is very small and it can be neglected.

The use of different optimization criteria, which change one by one in a certain sequence depending on the depth of a well, complicates the process of determining the cost per meter of drilling and estimating the total expenditures on drilling a well.

However, to measure the cost per meter of drilling a well in real time is impossible since the conditions of drilling are not stable. Therefore, it is a relevant scientific-applied task to identify relations between the cost per meter of drilling and other indicators, for example, specific summarized energy cost, which can be controlled on-line using modern technical means.

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## 2. Literature review and problem statement

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The problem of automated control over the process of drilling oil and gas wells has been the object of constant attention from foreign researchers. 2009 saw successful implementation of the SCADA Drill system to control the process of drilling by the company Shell, the ultimate goal of which was to expand capabilities of the system for different purposes of drilling [2, 3]. Specialists from Schlumberger developed a module for the ROPO optimization of deepening a well. It operates in real time and determines optimal values for the bit speed rotation and the load on the bit in a set of complex of restrictions for reaching the maximum speed of drilling [4]. The Schlumberger company developed several programs to accelerate drilling and control the trajectory of drilling. They aim at improving productivity as well as overall management of the process of drilling a well [5].

In 2008, a drilling control automated system was tested on the platform Statfjord C in the Norwegian zone of the North Sea [6]. This technology aims to reduce the non-productive time during drilling operations by entering the operational data into the system directly on the drilling equipment, automation of auxiliary operations in the management of drilling and identification of emergency situations.

Some issues on the automation of the drilling process, in particular interrelations with other sectors of industry [7], were discussed at the international conference in Amsterdam. Automation experts studied the role of bits in obtaining the information about the process of deepening a well [8], the results of testing the drilling process automation systems on the fields of Argentina [9]. Further development of the drilling process automation [10], the prospects of developing the control over drilling in real time [11] were considered and examined by scientists in Norway, Argentina, Austria, Mexico, Great Britain, the USA and other countries.

Significant contribution to the studies into this problem was made by the Ukrainian scientists. Optimal control over the process of drilling with one controlling action (axial force applied to a bit) was explored in article [12]. Paper [13] proposed a fuzzy model for monitoring the cost of drilling oil and gas wells. The optimal consumption of a washing fluid for drilling the wells of diameter 215.9 mm was examined in article [14]. Development of methods for the signal identification of rock drillability in real time was outlined in paper [15].

The development of models for managing the process of drilling deep wells based on fuzzy logic was proposed in [16].

At the same time, still insufficiently developed are the scientific and methodological provisions for assessing the multicollinearity of basic criteria for the optimal control over the process of drilling the wells, which are the cost per meter of drilling and specific energy consumption, as well as the substantiation of applying the energy-informational approach to manage the process of drilling in real time.

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## 3. The aim and tasks of the study

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The aim of present work is the substantiation for using in order to control the process of drilling the wells a criterion

of “minimum specific energy consumption” based on the analysis of interrelations between this criterion and the cost per meter of drilling.

To achieve the set aim, the following tasks were formulated:

- to analyze interrelations between such criteria of optimal control over the process of drilling the wells as the cost per meter of drilling and specific energy consumption;
- to establish the degree of completeness of multicollinearity among the examined criteria;
- to compile the recommendations for the criterion of optimal control over the process of drilling the wells with regard to the energy-informational approach.

**4. Materials and methods of research**

The following methods, approaches and techniques for the study of complex control objects form the methodological basis of present work:

- theoretical foundations of analysis of the multicollinearity of independent variables and its impact on the estimation of parameters of mathematical models for the objects of control;
- criteria and algorithms that are employed to identify the multicollinearity;
- methods of describing the informational and technological processes of drilling the oil and gas wells.

Methodological apparatus is the energy-informational approach and the theory of random processes, based on which we substantiated the choice of rational criterion for the optimal control over the process of drilling the oil and gas wells.

In this paper, we used a totality of methods and techniques:

- the Farrar-Glauber Test – to determine the degree of multicollinearity;
- the Curve and Expert method and technology – for examining the informational models;
- graphic method to visualize the resulting theoretical material.

**5. Analysis of multicollinearity among the criteria of optimal control over the process of drilling the wells**

In order to analyze, we shall use results of experimental studies [17] carried out when drilling the wells by the drilling machine 2SBSH-200N with controlled mode parameters. The type of drive of the rotary table is TP-DTP, technical performance – up to 90 m/h, mean stability of roller cutting bits – 391 m (footage per bit), the type of control system is “Rezhim 2NM”.

The ranges of change in the drilling mode parameters in the course of active experiment were as follows:

$$2 < F < 300, \text{ kN}; 0,2 < \omega < 2,4, \text{ s}^{-1}; 0,05 < Q < 0,45, \text{ m}^3/\text{s}.$$

The category of rock strength is  $f=6\div 8$  by the scale of Prof. Protodyakonov.

Within the framework of interrelations between the criteria of optimum control based on the identification of the phenomenon of multicollinearity, let us first consider the sta-

tistical totality of observations of factors – C and w during a change in the axial force on bit F. We shall introduce the following designations:

$$F \rightarrow Y; C \rightarrow X_1; w \rightarrow X_2.$$

Compute the mean values and standard deviations of variables  $X_1, X_2$ . For this purpose, we shall use formula [18]:

$$\bar{X}_j = \frac{\sum X_{ij}}{n}, \delta_i = \sqrt{\frac{\sum (X_{ij} - \bar{X}_j)^2}{n}}, \tag{1}$$

where  $\bar{X}_j$  is the mean value of the j-th variable;  $X_{ij}$  is the individual value of the j-th variable; j is the number of variable (j=1, 2); i is the number of point of observation (axial force on the bit);  $\delta_i$  is the standard deviation of the j-th variable; n is the number of observations (n=16).

We shall consider normalized values of variables C and w, which are given in Table 1.

Table 1

Normalized variables

No.	Force (F) on the bit, kN	Cost per meter of drilling C	Specific energy consumption w
1	18.25	0.600	0.580
2	37.50	0.550	0.480
3	56.25	0.495	0.405
4	75.00	0.470	0.365
5	93.75	0.430	0.320
6	112.50	0.410	0.310
7	131.25	0.395	0.305
8	150.00	0.380	0.300
9	168.75	0.390	0.308
10	187.50	0.395	0.325
11	206.25	0.410	0.338
12	225.00	0.425	0.370
13	243.75	0.430	0.375
14	262.50	0.430	0.430
15	281.25	0.470	0.520
16	300.00	0.510	0.630

Let us check the existence of multicollinearity between the cost per meter of drilling C and specific energy consumption w. For this purpose, we shall apply the Farrar-Glauber algorithm [18–21]. This algorithm has three types of statistical criteria, according to which the multicollinearity is checked from the entire array of independent variables ( $\chi^2$ ), of each independent variable with the rest of the variables (F-criterion) and of each pair of independent variables (t-criterion).

All the computations are conducted in the MS Excel software. Let us perform interim calculations and enter the data in Table 2, 3.

Table 2

Interim calculations

No.	Y	X <sub>1</sub>	X <sub>2</sub>	(X <sub>1i</sub> -X <sub>1mean</sub> ) <sup>2</sup>	(X <sub>2i</sub> -X <sub>2mean</sub> ) <sup>2</sup>
1	18.25	0.600	0.580	0.0227	0.0333
2	37.50	0.550	0.480	0.0101	0.0068
3	56.25	0.495	0.405	0.0021	0.0001
4	75.00	0.470	0.365	0.0004	0.0011
5	93.75	0.430	0.320	0.0004	0.0060
6	112.50	0.410	0.310	0.0016	0.0077
7	131.25	0.395	0.305	0.0030	0.0086
8	150.00	0.380	0.300	0.0048	0.0095
9	168.75	0.390	0.308	0.0035	0.0080
10	187.50	0.395	0.325	0.0030	0.0053
11	206.25	0.410	0.338	0.0016	0.0035
12	225.00	0.425	0.370	0.0006	0.0008
13	243.75	0.430	0.375	0.0004	0.0005
14	262.50	0.430	0.430	0.0004	0.0011
15	281.25	0.470	0.520	0.0004	0.0150
16	300.00	0.510	0.630	0.0037	0.0540
Total	2549.5	7.19	6.361	0.0585	0.1611

Table 3

Interim calculations (continued)

Indicator	X <sub>1</sub>	X <sub>2</sub>
Mean value	0.449	0.397
Standard deviation	0.062	0.103

We shall normalize variables X<sub>1</sub> and X<sub>2</sub> by using the "STANDARDIZE" function in MS Excel. For this purpose, let us apply formula [18]:

$$X_{ij}^* = \frac{X_{ij} - \bar{X}_j}{\sqrt{n\sigma_{X_j}^2}}, \tag{2}$$

where n is the number of observations in the sample (i=1,2,...,n); n=16; m is the number of independent variables (m=2); X<sub>j</sub> is the arithmetic mean of the j-th independent variable; σ<sub>X<sub>j</sub></sub><sup>2</sup> is the dispersion of the j-th independent variable; X<sub>ij</sub><sup>\*</sup> are the normalized independent variables that are components of matrix X\*: X<sub>ij</sub><sup>\*</sup> ∈ X\*.

Thus, we received

$$X^* = \begin{pmatrix} 2,4121 & 1,7602 \\ 1,6114 & 0,7954 \\ 0,7306 & 0,0718 \\ 0,3303 & -0,3142 \\ -0,3103 & -0,7483 \\ -0,6305 & -0,8448 \\ -0,8707 & -0,8931 \\ -1,1109 & -0,9413 \\ -0,9508 & -0,8641 \\ -0,8707 & -0,7001 \\ -0,6305 & -0,5747 \\ -0,3903 & -0,2659 \\ -0,3103 & -0,2177 \\ -0,3103 & 0,3130 \\ 0,3303 & 1,1813 \\ 0,9708 & 2,2426 \end{pmatrix}$$

The next step of the algorithm is to build a transposed matrix (X\*)<sup>T</sup>, whose elements are the normalized independent variables X<sub>ij</sub><sup>\*</sup>, and the computation of correlation matrix, that is, of matrix of moments of the normalized system of normal equations [18]:

$$r = (X^*)^T = \begin{pmatrix} 1 & r_{12} & \dots & r_{1m} \\ r_{12} & 1 & \dots & r_{2m} \\ \dots & \dots & \dots & \dots \\ r_{m1} & r_{m2} & \dots & 1 \end{pmatrix}, \tag{3}$$

where (X\*)<sup>T</sup> is the matrix, transposed to matrix X\* whose elements characterize the density of bond between one independent variable and another; r<sub>ij</sub> = r<sub>X<sub>1</sub>X<sub>2</sub></sub> are the paired correlation coefficients.

Let us multiply matrices (X\*)<sup>T</sup> and X\* using the "MMULT" function to obtain:

$$(X^*)^T X^* = \begin{pmatrix} 15,000 & 12,499 \\ 12,499 & 15,000 \end{pmatrix}$$

Find correlation matrix r. To do this, each element of matrix (X\*)<sup>T</sup>X\* should be multiplied by  $\frac{1}{n-1} = \frac{1}{16-1} = \frac{1}{15}$ :

$$r = \begin{pmatrix} 1,000 & 0,833 \\ 0,833 & 1,000 \end{pmatrix}$$

Find the determinant of correlation matrix r using the "MDETERM" function to obtain:

$$\det r = 0,305.$$

Since det r approaches zero, then there is the multicollinearity in the array of explanatory variables.

Define the estimated value of the Pearson criterion χ<sup>2</sup> by formula [18]:

$$\chi^2 = -\left\{n-1 - \frac{1}{6}(2m+5)\right\} \ln(\det r), \tag{4}$$

$$\ln(\det r) = -1,185,$$

$$\chi^2 = -\left\{16-1 - \frac{1}{6}(2 \cdot 2+5)\right\} \cdot (-1,185) = 16,003.$$

At the degree of freedom

$$k = \frac{1}{2}m(m-1) = \frac{1}{2} \cdot 2 \cdot (2-1) = 1$$

and the level of significance α=0,05 criterion χ<sub>table</sub><sup>2</sup> = 3.8. Since χ<sup>2</sup> > χ<sub>table</sub><sup>2</sup> (16.003 > 3.8), we conclude that there is the multicollinearity in the array of examined variables.

Next we shall compute F – the Fischer criterion by determining the matrix of C-errors, which is inverse to the correlation matrix r, by using the "MINVERSE" function [18]:

$$C = r^{-1} = ((X^*)^T X^*)^{-1}. \tag{5}$$

Hence

$$C = \begin{pmatrix} 3,272 & -2,726 \\ -2,726 & 3,272 \end{pmatrix}$$

Using the diagonal elements of matrix C, compute the F-Fischer criterion for independent variables [18]:

$$F = (C_{kk} - 1) \left( \frac{n - m}{m - 1} \right), \tag{6}$$

where  $C_{kk}$  are the diagonal elements of matrix of C-errors,

$$F = (3,272 - 1) \left( \frac{16 - 2}{2 - 1} \right) = 31,808.$$

For the level of significance  $\alpha = 0,05$  and the degrees of freedom  $k_1 = m - 1 = 2 - 1 = 1$  and  $k_2 = n - m = 16 - 2 = 14$ , using statistical tables, we find critical value of the Fisher criterion  $F_{table} = 4.60$ . We shall compare the tabular value  $F_{table}$  to the estimated value.  $F > F_{table}$  ( $31.808 > 4.60$ ) and this means that variables  $X_1$  and  $X_2$  are multicollinear.

Using matrix C, we shall compute partial coefficients of correlation by formula [18]:

$$r_{12} = \frac{-c_{12}}{\sqrt{c_{11}c_{22}}}, \tag{7}$$

$$r_{12} = \frac{-(-2,726)}{\sqrt{3,272 \cdot 3,272}} = 0,833.$$

Therefore, the resulting correlation coefficient shows that there is the multicollinearity between the variables since  $r_{12}$  is close to 1.

Based on the found partial coefficient of correlation, we find the estimated value of the Student t-criterion by formula [18]:

$$t_{12} = \frac{r_{12} \sqrt{n - m}}{\sqrt{1 - r_{12}^2}}, \tag{8}$$

$$t_{12} = \frac{0,833 \sqrt{16 - 2}}{\sqrt{1 - 0,833^2}} = 5,639.$$

The computed value of t-criterion shall be compared to the tabular value ( $t_{table} = 2.145$ ) when the level of significance is  $\alpha = 0,05$  and the degree of freedom is  $k_2 = n - m = 14$ . Since  $t_{12} > t_{table}$  ( $5.639 > 2.145$ ), it can be argued that there is the multicollinearity in variables  $X_1$  and  $X_2$  at a change in the axial force to the bit from 18.25 to 300 kN.

The existence of multicollinearity between criteria C and w is confirmed by the information models, built in the Curve Expert programming environment by the results of experimental studies (Table 4, Fig. 1, 2).

Table 4

Source data for constructing information models  
 $C=f(F)$ ;  $w=f(F)$

No.	Q	C	w	No.	Q	C	w
1	18.25	0.6	0.58	9	168.75	0.39	0.308
2	37.5	0.55	0.48	10	187.5	0.395	0.325
3	56.25	0.495	0.405	11	206.25	0.41	0.338
4	75	0.47	0.365	12	225	0.425	0.37
5	93.75	0.43	0.32	13	243.75	0.43	0.375
6	112.5	0.41	0.31	14	262.5	0.43	0.43
7	131.25	0.395	0.305	15	281.25	0.47	0.52
8	150	0.38	0.3	16	300	0.51	0.63

Next, we analyze the existence of multicollinearity between the cost per meter of drilling and specific energy consumption w at a change in the bit rotation frequency  $\omega$  from 0.25 to 2.375 s<sup>-1</sup> (Table 5).

Let us check the existence of multicollinearity between the cost per meter of drilling C and specific energy consumption w. To check it, we shall again apply the Farrar-Glauber algorithm. All the computations are in the MS Excel software.

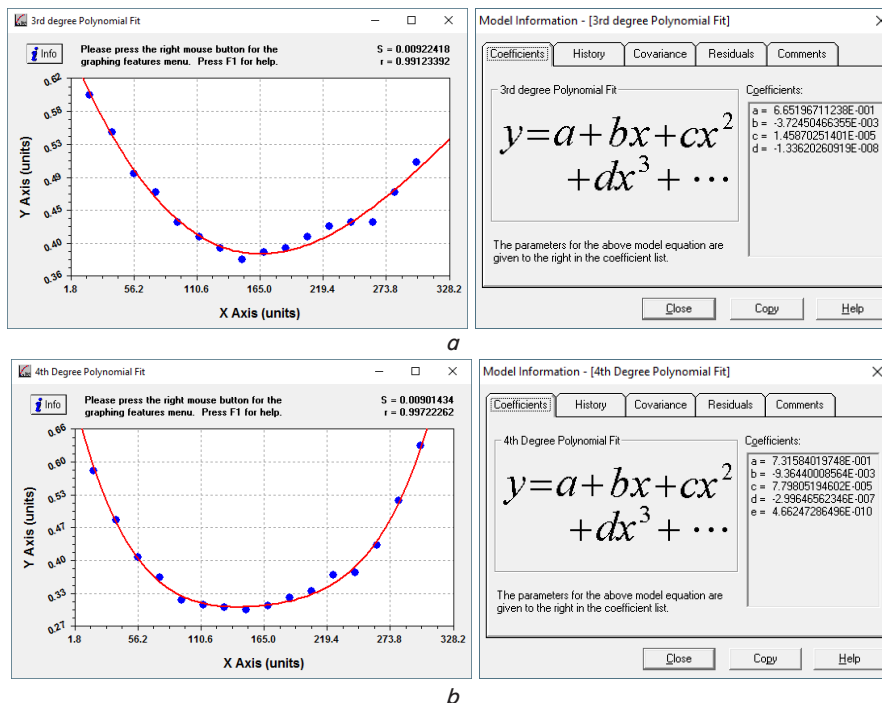


Fig. 1. Information models: a – the cost per meter of drilling  $C=f(F)$ ; b – specific energy consumption  $w=f(F)$

Table 5

Normalized variables

No.	Rotation frequency $\omega$ , s <sup>-1</sup>	Cost per meter of drilling C	Specific energy consumption w
1	0.250	0.804	0.865
2	0.375	0.782	0.705
3	0.500	0.701	0.607
4	0.625	0.642	0.531
5	0.750	0.589	0.463
6	0.875	0.538	0.402
7	1.000	0.500	0.350
8	1.125	0.469	0.308
9	1.250	0.447	0.281
10	1.375	0.429	0.267
11	1.500	0.421	0.260
12	1.625	0.430	0.265
13	1.750	0.453	0.278
14	1.875	0.484	0.299
15	2.000	0.526	0.325
16	2.125	0.573	0.360
17	2.250	0.637	0.391
18	2.375	0.701	0.430

Perform interim calculations and enter the data in Table 6, 7.

Table 6

Interim calculations

No.	Y	X <sub>1</sub>	X <sub>2</sub>	(X <sub>1i</sub> -X <sub>1mean</sub> ) <sup>2</sup>	(X <sub>2i</sub> -X <sub>2mean</sub> ) <sup>2</sup>
1	0.250	0.804	0.865	0.0583	0.2067
2	0.375	0.782	0.705	0.0482	0.0868
3	0.500	0.701	0.607	0.0192	0.0387
4	0.625	0.642	0.531	0.0063	0.0145
5	0.750	0.589	0.463	0.0007	0.0028
6	0.875	0.538	0.402	0.0006	0.0001
7	1.000	0.500	0.350	0.0039	0.0036
8	1.125	0.469	0.308	0.0088	0.0105
9	1.250	0.447	0.281	0.0134	0.0167
10	1.375	0.429	0.267	0.0178	0.0206
11	1.500	0.421	0.260	0.0200	0.0226
12	1.625	0.430	0.265	0.0176	0.0211
13	1.750	0.453	0.278	0.0120	0.0175
14	1.875	0.484	0.299	0.0062	0.0124
15	2.000	0.526	0.325	0.0013	0.0073
16	2.125	0.573	0.360	0.0001	0.0025
17	2.250	0.637	0.391	0.0055	0.0004
18	2.375	0.701	0.430	0.0192	0.0004
Total	23.625	10.126	7.387	0.2590	0.4852

Table 7

Interim calculations (continued)

Indicator	X <sub>1</sub>	X <sub>2</sub>
Mean value	0.5626	0.4104
Mean deviation	0.1234	0.1689

Let us normalize examined variables X<sub>1</sub> and X<sub>2</sub> by using the "STANDARDIZE" function in MS Excel.

$$X^* = \begin{pmatrix} 1,9560 & 2,6909 \\ 1,7778 & 1,1638 \\ 1,1216 & 0,7139 \\ 0,6436 & -0,3142 \\ 0,2142 & -0,3114 \\ -0,1989 & -0,0497 \\ -0,5068 & -0,3574 \\ -0,7579 & -0,6060 \\ -0,9361 & -0,7659 \\ -1,0820 & -0,8487 \\ -1,1468 & -0,8902 \\ -1,0739 & -0,8606 \\ -0,8875 & -0,7836 \\ -0,6364 & -0,6593 \\ -0,2961 & -0,5054 \\ 0,0846 & -0,2983 \\ 0,6031 & -0,1148 \\ 1,1216 & 0,1161 \end{pmatrix}$$

Multiply matrices (X\*)<sup>T</sup> and X\* by using the "MMULT" function to obtain:

$$(X^*)^T X^* = \begin{pmatrix} 17,000 & 15,726 \\ 15,726 & 17,000 \end{pmatrix}$$

Find correlation matrix r. To do this, each element of matrix (X\*)<sup>T</sup>X\* should be multiplied by  $\frac{1}{n-1} = \frac{1}{18-1} = \frac{1}{17}$ :

$$r = \begin{pmatrix} 1,000 & 0,925 \\ 0,925 & 1,000 \end{pmatrix}$$

Find the determinant of correlation matrix r using the "MDETERM" function to receive:

$$\det r = 0,114.$$

Since det r approaches zero, then there is the multicollinearity in the array of variables X<sub>1</sub> and X<sub>2</sub>.

Determine the estimated value of the Pearson criterion  $\chi^2$  by formula (4):

$$\ln(\det r) = -1,936,$$

$$\chi^2 = -\left\{18-1-\frac{1}{6}(2 \cdot 2+5)\right\} \cdot (-1,936) = 30,011.$$

At the degree of freedom

$$k = \frac{1}{2}m(m-1) = \frac{1}{2} \cdot 2 \cdot (2-1) = 1$$

and the level of significance  $\alpha=0,05$ , criterion  $\chi_{table}^2 = 3.8$ . Since  $\chi^2 > \chi_{table}^2$  (30.011 > 3.8), then we conclude that there is the multicollinearity in the array of variables X<sub>1</sub> and X<sub>2</sub>.

We shall determine matrix C, which is inverse to the correlation matrix r, by using the "MINVERSE" function:

$$C = \begin{pmatrix} 6,932 & -6,412 \\ -6,412 & 6,932 \end{pmatrix}$$

Using the diagonal elements of matrix C, we compute the F-Fischer criterion for independent variables by formula (6):

$$F = (6,932 - 1) \left( \frac{18 - 2}{2 - 1} \right) = 94,913.$$

For the level of significance  $\alpha = 0,05$  and the degrees of freedom  $k_1 = m - 1 = 2 - 1 = 1$  and  $k_2 = n - m = 18 - 2 = 16$ , using statistical tables of the F-distribution, we shall find critical value of the Fischer criterion  $F_{table} = 4.49$ . Tabular value  $F_{table}$  shall be compared to the estimated value.  $F > F_{table}$  ( $94.913 > 4.49$ ) and this means that variables  $X_1$  and  $X_2$  are multicollinear.

Using matrix C, we compute partial correlation coefficients by formula (7):

$$r_{12} = \frac{-(-6,412)}{\sqrt{6,932 \cdot 6,932}} = 0,925.$$

Therefore, the obtained correlation coefficient shows that there is the multicollinearity between the variables since  $r_{12}$  is close to 1.

Based on the obtained partial correlation coefficient, we find the estimated value of the Student t-criterion by formula (8):

$$t_{12} = \frac{0,925\sqrt{18 - 2}}{\sqrt{1 - 0,925^2}} = 9,742.$$

The computed value of the t-criterion shall be compared to tabular value ( $t_{table} = 1.746$ ) when the level of significance is  $\alpha = 0,05$  and the degree of freedom is  $k_2 = n - m = 18 - 2 = 16$ . Since  $t_{12} > t_{table}$  ( $9.742 > 1.746$ ), it can be argued that there is the multicollinearity in variables

$X_1$  and  $X_2$  at a change in the bit rotation frequency in the range of  $0.25 - 2.375 \text{ s}^{-1}$ .

Using experimental data (Table 8), we shall construct information model for the dependences  $C=f(\omega)$  and  $w=f(\omega)$  in the Curve Expert programming environment (Fig. 2). One can see that the information models 4th Degree Polynomial Fit and 3rd Degree Polynomial Fit describe experimental data with correlation coefficient  $r = 0.998$  and standard approximation error  $S = 0.007$  for model  $C=f(\omega)$ , and  $S = 0.009$  for model  $w=f(\omega)$ .

Table 8

Source data for constructing information models  $C=f(\omega); w=f(\omega)$

No.	$\omega$	C	w	No.	$\omega$	C	w
1	0,25	0,804	0,865	10	1,375	0,429	0,267
2	0,375	0,782	0,705	11	1,5	0,421	0,26
3	0,5	0,701	0,607	12	1,625	0,43	0,265
4	0,625	0,642	0,531	13	1,75	0,453	0,278
5	0,75	0,589	0,463	14	1,875	0,484	0,299
6	0,875	0,538	0,402	15	2	0,526	0,325
7	1	0,5	0,35	16	2,125	0,573	0,36
8	1,125	0,469	0,308	17	2,25	0,637	0,391
9	1,25	0,447	0,281	18	2,375	0,701	0,43

Next, we consider the multicollinearity of the examined variables C and w when the third controlling action changes – consumption of a washing solution. The normalized values of variables are given in Table 9.

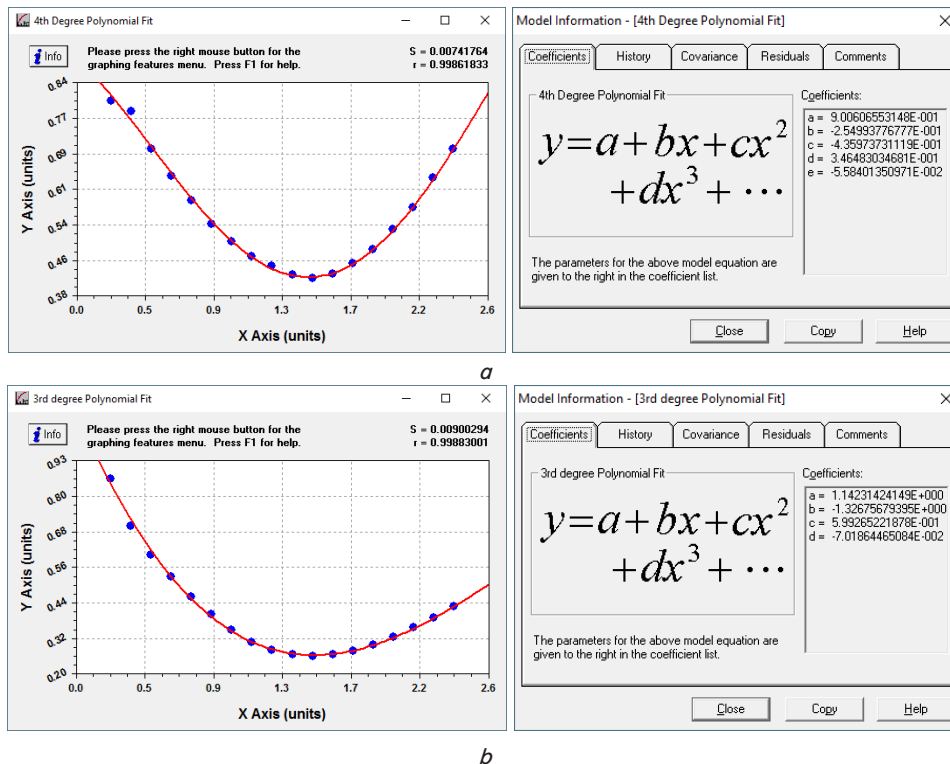


Fig. 2. Information models: a – the cost per meter of drilling  $C=f(\omega)$ ; b – specific energy consumption  $w=f(\omega)$

Table 9

Normalized variables

No.	Consumption of washing solution Q, m <sup>3</sup> /h	Cost per meter of drilling, C	Specific energy consumption, w
1	0.025	0.943	0.344
2	0.050	0.801	0.237
3	0.075	0.700	0.162
4	0.100	0.621	0.126
5	0.125	0.578	0.108
6	0.150	0.552	0.101
7	0.175	0.531	0.110
8	0.200	0.525	0.135
9	0.225	0.522	0.162
10	0.250	0.532	0.195
11	0.275	0.541	0.229
12	0.300	0.550	0.273
13	0.325	0.574	0.312
14	0.350	0.593	0.354
15	0.375	0.628	0.399
16	0.400	0.669	0.450

Perform interim calculations and enter the data in Table 10, 11.

Table 10

Interim calculations

No.	Y	X <sub>1</sub>	X <sub>2</sub>	(X <sub>1i</sub> -X <sub>1mean</sub> ) <sup>2</sup>	(X <sub>2i</sub> -X <sub>2mean</sub> ) <sup>2</sup>
1	0.025	0.943	0.344	0.1068	0.0128
2	0.050	0.801	0.237	0.0341	0.0000
3	0.075	0.700	0.162	0.0070	0.0048
4	0.100	0.621	0.126	0.0000	0.0110
5	0.125	0.578	0.108	0.0015	0.0151
6	0.150	0.552	0.101	0.0041	0.0169
7	0.175	0.531	0.110	0.0073	0.0147
8	0.200	0.525	0.135	0.0083	0.0092
9	0.225	0.522	0.162	0.0089	0.0048
10	0.250	0.532	0.195	0.0071	0.0013
11	0.275	0.541	0.229	0.0057	0.0000
12	0.300	0.550	0.273	0.0044	0.0018
13	0.325	0.574	0.312	0.0018	0.0066
14	0.350	0.593	0.354	0.0005	0.0151
15	0.375	0.628	0.399	0.0001	0.0282
16	0.400	0.669	0.450	0.0028	0.0479
Total	3.400	9.860	3.697	0.2004	0.1902

Table 11

Interim calculations (continued)

Indicator	X <sub>1</sub>	X <sub>2</sub>
Mean value	0.6163	0.2988
Mean deviation	0.1156	0.0665

Let us normalize the examined variables X<sub>1</sub> and X<sub>2</sub> by using the "STANDARDIZE" function in MS Excel.

$$X^* = \begin{bmatrix} 2,8269 & 1,0030 \\ 1,5984 & 0,0527 \\ 0,7246 & -0,6134 \\ 0,0411 & -0,9331 \\ -0,3309 & -1,0929 \\ -0,5559 & -1,1551 \\ -0,7376 & -1,0752 \\ -0,7895 & -0,8531 \\ -0,8154 & -0,6134 \\ -0,7289 & -0,3203 \\ -0,6510 & -0,0183 \\ -0,5732 & -0,3725 \\ -0,3655 & 0,7188 \\ -0,2012 & 1,0918 \\ 0,1017 & 1,4915 \\ 0,4564 & 1,9444 \end{bmatrix}$$

Multiply matrices (X\*)<sup>T</sup> and X\* using the "MMULT" function to receive:

$$(X^*)^T X^* = \begin{bmatrix} 15,000 & 5,995 \\ 5,995 & 15,000 \end{bmatrix}$$

Find a correlation matrix r. To do this, each element of matrix (X\*)<sup>T</sup>X\* should be multiplied by  $\frac{1}{n-1} = \frac{1}{16-1} = \frac{1}{15}$ :

$$r = \begin{bmatrix} 1,000 & 0,399 \\ 0,399 & 1,000 \end{bmatrix}$$

Find the determinant of correlation matrix r by using the "MDETERM" function to obtain:

$$\det r = 0,84.$$

Since det r approaches 1, then there is the multicollinearity is lacking in the array of explanatory variables.

Determine the estimated value o the Pearson criterion  $\chi^2$  by formula (4):

$$\ln(\det r) = -0,174,$$

$$\chi^2 = -\left\{16 - 1 - \frac{1}{6}(2 \cdot 2 + 5)\right\} \cdot (-0,174) = 2,35.$$

At the degree of freedom

$$k = \frac{1}{2}m(m-1) = \frac{1}{2} \cdot 2 \cdot (2-1) = 1$$

and the level of significance  $\alpha = 0,05$  criterion  $\chi^2_{table} = 3.8$ . Since  $\chi^2 < \chi^2_{table}$  (2.35 < 3.8), we conclude that the multicollinearity does not exist in the array of explanatory variables.

Determine matrix C, inverse to correlation matrix r, by using the "MINVERSE" function:

$$C = \begin{bmatrix} 1,19 & -0,475 \\ -0,475 & 1,19 \end{bmatrix}$$



Using the diagonal elements of matrix C, we compute the F-Fisher criterion for independent variables by formula (6):

$$F = (1,19 - 1) \left( \frac{16 - 2}{2 - 1} \right) = 2,662.$$

For the level of significance  $\alpha = 0,05$  and the degrees of freedom  $k_1 = m - 1 = 2 - 1 = 1$  and  $k_2 = n - m = 16 - 2 = 14$ , by statistical tables of the F-distribution, we find critical value of the Fisher criterion  $F_{table} = 4.60$ . Tabular value  $F_{table}$  shall be compared to the estimated value.  $F < F_{table}$  ( $2.662 < 4.60$ ) and this means that variables  $X_1$  and  $X_2$  are not multicollinear.

Using matrix C, we compute partial correlation coefficients by formula (7):

$$r_{12} = \frac{-(-0,475)}{\sqrt{1,19 \cdot 1,19}} = 0,399.$$

Therefore, the obtained partial correlation coefficient shows that there is no multicollinearity between the variables since  $r_{12}$  is not close to 1.

Based on the found partial coefficient of correlation, we find the estimated value of the Student t-criterion by formula (8):

$$t_{12} = \frac{0,399\sqrt{16 - 2}}{\sqrt{1 - 0,399^2}} = 1,631.$$

Computed value of the t-criterion shall be compared to tabular value ( $t_{table} = 2.145$ ) when the level of significance is  $\alpha = 0,05$  and the degree of freedom is  $k_2 = n - m = 14$ . Since

$t_{12} < t_{table}$  ( $1.631 < 2.145$ ), then we can definitely state that there is no multicollinearity in variables  $X_1$  and  $X_2$ .

For the visual representation of the received result, let us consider information models  $C=f(Q), w=f(Q)$  obtained in the Curve Expert programming environment by experimental data (Fig. 3, Table 12).

Table 12

Source data for constructing information models  $C=f(Q); w=f(Q)$

No.	Q	C	w	No.	Q	C	w
1	0.025	0.943	0.344	9	0.225	0.522	0.162
2	0.05	0.801	0.237	10	0.25	0.532	0.195
3	0.075	0.7	0.162	11	0.275	0.541	0.229
4	0.1	0.621	0.126	12	0.3	0.55	0.273
5	0.125	0.578	0.108	13	0.325	0.574	0.312
6	0.15	0.552	0.101	14	0.35	0.593	0.354
7	0.175	0.531	0.11	15	0.375	0.628	0.399
8	0.2	0.525	0.135	16	0.4	0.669	0.45

An analysis of the information shown in Fig. 3, *a, b* reveals that the approximation of curves  $C=f(Q)$  and  $w=f(Q)$  was performed by the information models 4th Degree Polynomial Fit with a high correlation coefficient  $r=0.999$  and standard error  $S=0.002$ . However, the minima of these dependences match different values of controlling action  $Q$ : for chart  $C=f(Q) - 0.2$ , and for chart  $w=f(Q) - 0.1$ , which is the reason for the absence of phenomenon of the multicollinearity for the given process.

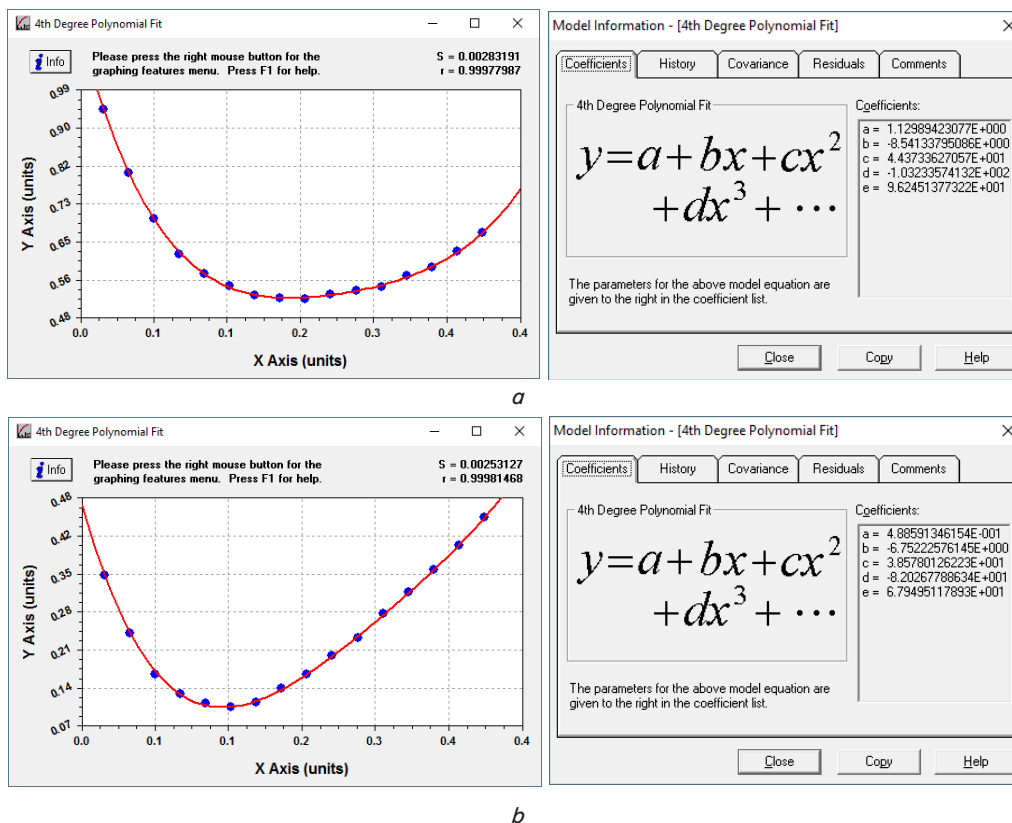


Fig. 3. Information models: *a* – the cost per meter of drilling  $C=f(Q)$ ; *b* – specific energy consumption  $w=f(Q)$

## 6. Discussion of results of examining the interrelations between the criteria of optimal control over the process of drilling the wells

The benefit of results of examining the interrelations between the criteria of optimal control over the process of drilling the wells, the cost per meter of drilling and specific energy consumption, is that they provide for a substantiated choice of criterion of the optimization of the process of drilling the wells when creating an automated control system. The established interrelations between the criteria of optimal control make it possible to pass over to using the indicator controlled in real time (specific energy consumption) instead of the uncontrolled one – the cost per meter of drilling and thereby provide solution to the problem of optimal control over the process of drilling the wells on-line. Another advantage of the results obtained is the fact that the close relation between the examined criteria is observed when the two basic controlling actions change – the axial force on the bit and the frequency of its rotation.

The research results can be used in the automated control systems of rotary drilling of oil and gas wells on offshore platforms and on land.

The above research is to be improved in the future in order to refine the relationship criteria of optimal control over the process of drilling the wells at the change, over a wide range, of washing fluid consumption under different drilling methods.

## 7. Conclusions

1. Based on an analysis of the interrelations of criteria of optimal control over the drilling process, it was found that when controlling this process by altering the axial force to a bit or frequency of its rotation, there is a complete multi-

collinearity between the examined criteria. This solves the problem of choice as a criterion of optimization of specific energy consumption and provides its control in real time in the system of automated control over the process of deepening the wells with two controlling actions.

2. We established the degree of completeness in the multicollinearity between the examined criteria:

– at the change of axial force to a bit  $F$ :

$$\det r=0,305; \chi^2 > \chi_{table}^2 (16.003 > 3.8);$$

$$F > F_{table} (31.808 > 4.60); t_{12} > t_{table} (5.639 > 2.145);$$

– at the change of rotation frequency  $\omega$ :

$$\det r=0,114; \chi^2 > \chi_{table}^2 (30.011 > 3.8);$$

$$F > F_{table} (94.913 > 4.49); t_{12} > t_{table} (9.742 > 1.746);$$

– at the change of washing fluid consumption  $Q$ :

$$\det r=0,84; \chi^2 < \chi_{table}^2 (2.35 < 3.8);$$

$$F > F_{table} (2.662 < 4.60); t_{12} > t_{table} (1.631 < 2.145).$$

3. We proposed a dualistic approach to solving the problem of optimal control over the process of drilling the wells in real time. This makes it possible, by applying the energy-informational approach, to directly process information on the specific energy consumption, and to provide intelligent support for the decision-making processes when a drilling master defines rational parameters of a drilling mode. Underlying the proposed approach are information models in the form of third- and fourth order polynomials that describe experimental data with correlation coefficients higher than 0.9 and standard errors lower than 0.01.

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