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Розглянуто модель взаємозв'язку показників надійності і основних значимих параметрів двокаскадного термоелектричного охолоджуючого пристрою заданої конструкції, який працює в режимі найбільшої енергетичної ефективності при послідовному з'єднанні каскадів. Одержані співвідношення, які дозволяють визначити основні параметри і показники надійності при різних співвідношенні елементів в каскадах, робочому діапазоні перепадів температур для проектування охолоджувачів підвищеної надійності

Ключові слова: термоелектричні охолоджуючі пристрої, показники надійності, перепад температури, енергетична ефективність

Рассмотрена модель взаимосвязи показателей надежности и основных значимых параметров двухкаскадного термоэлектрического охлаждающего устройства заданной конструкции, работающего в режиме наибольшей энергетической эффективности при последовательном электрическом соединении каскадов. Получены соотношения, позволяющие определить основные параметры и показатели надежности при различном соотношении элементов в каскадах, рабочем диапазоне перепадов температур для проектирования охлаждающих устройств повышенной надежности

Ключевые слова: термоэлектрические охлаждающие устройства, показатели надежности, перепад температуры, энергетическая эффективность

DEVELOPMENT OF A MODEL FOR PREDICTING THE RELIABILITY INDICATORS IN THE DESIGN OF CASCADE THERMOELECTRIC COOLERS

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1. Introduction

The application of cascade thermoelectric devices (CTED) is predetermined not only by the need to achieve a maximally possible level of cooling, but also to improve cost efficiency. In some cases, a designer has a number of chosen structures of CTED at his disposal, constructed based on the standardized modules. It is necessary to determine maximally possible energy effectiveness at the assigned temperature differential, to select such current mode in the operation of CTED that it would match the maximum of energy effectiveness of CTED of the assigned design (regime E_{max}). The relevance of providing the maximum of energy effectiveness is caused by the need to reduce the mass-and-size indicators in the systems that provide thermal modes of the thermally loaded elements.

2. Literature review and problem statement

Expansion in the scope of application of thermoelectric coolers [1, 2] leads to more stringent requirements to energy and reliability indicators. Since the refrigeration capacity of

coolers depends first of all on the thermoelectric effectiveness of material of thermoelements, considerable efforts of designers are concentrated in this direction [3, 4]. The most impressive results are achieved in the field of nano-technologies [5, 6], however, there is still a long way to go before the implementation of industrial production of such materials. Furthermore, improving the effectiveness of thermoelectric materials does not fully solve the given problem, since the reliability of functioning of coolers is not a less important indicator [7, 8]. The capability to resist mechanical impacts is one of the components of coolers reliability [9]. A transition to the planar technologies in the production of thermoelectric coolers [10, 11] and the corresponding reduction in the mass-and-size indicators does not resolve the task either. New problems occur that are related to the influence of resistances of thermoelements and the increased thermal conductivity [12]. Moreover, when we take into account the existing market for thermoelectric coolers [13], then it becomes obvious that it is necessary to search for the ways of improving the energy and reliability indicators of the existing thermoelectric modules. The models of interrelation between the indicators of thermoelectric effectiveness and reliability, presented in [14], as well as the influence of re-

gimes on the indicators of reliability [15], allow us to argue about the expediency of the given developments. For this purpose, it is necessary at the assigned temperature range to determine basic significant parameters, namely, relative operating currents and relative temperature differentials in the cascades, and then to estimate reliability indicators of energy-efficient CTED.

3. The aim and tasks of the study

The aim of present work is to develop a model that would make it possible to evaluate the efficiency of functioning and predicting the reliability indicators of a two-stage TED of the chosen design.

To achieve this objective, it is necessary to solve the following tasks:

- to develop a model of interrelation between reliability indicators of CTED and the design and energy indicators under the mode of the highest energy effectiveness;

- to analyze the model to identify conditions for improving the efficiency of CTED of different designs.

4. Development and analysis of the model of a cascade thermoelectric cooler

4.1. Development of a reliability-oriented model of CTED

In order to solve the set problem, we shall use known relationships [16]. The refrigeration capacity Q_0 of a two-cascade CTED can be written in the form

$$Q_0 = n_1 I_{\max 1}^2 R_1 (2B_1 - B_1^2 - \Theta_1), \quad (1)$$

where $I_{\max 1}$ is the maximum operating current, A; $I_{\max 1} = e_1 T_0 / R_1$; n_1 is the quantity of thermoelements in the first cascade, pieces; T_0 is the temperature of the heat-absorbing joint of the first cascade, K; e_1 is the coefficient of thermal EMF of the branch of thermoelements of the first cascade, V/K; R_1 is the electrical resistance of the branch of thermoelement of the first cascade, Ohm; B_1 is the relative operating current of the first cascade, rel. un., $B_1 = I / I_{\max 1}$; Θ_1 is the relative temperature drop in the first cascade, rel. un.

$$\Theta_1 = (T_1 - T_0) / \Delta T_{\max 1},$$

where T_1 is the intermediate temperature, K; $\Delta T_{\max 1}$ is the maximum temperature differential in the first cascade, K.

Sequential electrical connection of cascades defines the equality of operating currents in the cascades, which can be written in the form

$$I_{\max 1} B_1 = I_{\max 2} B_2, \quad (2)$$

where B_2 is the relative operating current of the second cascade, rel. un.,

$$B_2 = I / I_{\max 2};$$

$$I_{\max 2} = e_2 T_1 / R_2;$$

where e_2 is the coefficient of thermal EMF of the branch of thermoelements of the second cascade, V/K; R_2 is the electri-

cal resistance of the branch of thermoelement of the second cascade, Ohm.

A general temperature differential on a two-stage CTED can be written in the form

$$\Delta T = \Delta T_1 + \Delta T_2 = \Delta T_{\max 1} \Theta_1 + \Delta T_{\max 2} \Theta_2, \quad (3)$$

where ΔT_1 is the temperature differential in the first cascade, K, $\Delta T_1 = T_1 - T_0$; ΔT_2 is the temperature differential in the second cascade, K, $\Delta T_2 = T - T_1$; Θ_2 is the relative temperature differential in the second cascade, rel. un.,

$$\Theta_2 = (T - T_1) / \Delta T_{\max 2},$$

where $\Delta T_{\max 2}$ is the maximum temperature differential in the second cascade, K.

A condition of the thermal joining of cascades can be written in the form

$$\frac{n_1}{n_2} = \frac{I_{\max 2}^2 R_2 (2B_2 - B_2^2 - \Theta_2)}{I_{\max 1}^2 R_1 \left[2B_1 \left(1 + \frac{\Delta T_{\max 1}}{T_0} \Theta_1 \right) + B_1^2 - \Theta_1 \right]}, \quad (4)$$

where n is the quantity of thermoelements in the second cascade, pieces

Refrigeration coefficient of a two-stage CTED can be written in the form

$$E^{N=2} = \frac{Q_0}{W_1 + W_2}, \quad (5)$$

where W_1 is the power of consumption of the first cascade, W,

$$W_1 = 2n_1 I_{\max 1}^2 R_1 B_1 \left(B_1 + \frac{\Delta T_{\max 1}}{T_0} \Theta_1 \right), \quad (6)$$

where W_1 is the power of consumption of the second cascade, W,

$$W_2 = 2n_2 I_{\max 2}^2 R_2 B_2 \left(B_2 + \frac{\Delta T_{\max 2}}{T_1} \Theta_2 \right). \quad (7)$$

By using relations (1)–(7), refrigeration coefficient can be written in the form

$$E^{N=2} = \frac{2aB_1b - aB_1^2c + 2a^2B_1^3 \frac{\Delta T_{\max 1}}{T_0} - a \frac{\Delta T}{\Delta T_{\max 2}}}{2B_1^2X - 2B_1^3Y + 2B_1H \frac{\Delta T}{\Delta T_{\max 2}}}, \quad (8)$$

where

$$a = \frac{n_1 I_{\max 1}^2 R_1}{n_2 I_{\max 2}^2 R_2};$$

$$b = \frac{\Delta T_{\max 1}}{\Delta T_{\max 2}} + \frac{I_{\max 1}}{I_{\max 2}};$$

$$c = \frac{\Delta T_{\max 1}}{\Delta T_{\max 2}} + 2a \left(1 + 2 \frac{\Delta T_{\max 1}}{T_0} \right) + \frac{I_{\max 1}^2}{I_{\max 2}^2};$$

$$\begin{aligned}
 X &= \left(\frac{\Delta T_{\max 1}}{\Delta T_{\max 2}} + a \right) \left(a + \frac{I_{\max 1}^2}{I_{\max 2}^2} \right) - \\
 &- 2a \frac{\Delta T_{\max 1}}{T_0} \frac{I_{\max 1}}{I_{\max 2}} \frac{\Delta T_{\max 2}}{T_1} \frac{\Delta T}{\Delta T_{\max 2}} + \\
 &+ 2 \left(\frac{I_{\max 1}}{I_{\max 2}} \frac{\Delta T_{\max 2}}{T_1} \frac{\Delta T_{\max 1}}{\Delta T_{\max 2}} - a \frac{\Delta T_{\max 1}}{T_0} \right); \\
 Y &= \left(a + \frac{I_{\max 1}^2}{I_{\max 2}^2} \right) \left(\frac{I_{\max 1}}{I_{\max 2}} \frac{\Delta T_{\max 2}}{T_1} \frac{\Delta T_{\max 1}}{\Delta T_{\max 2}} + a \frac{\Delta T_{\max 1}}{T_0} \right); \\
 H &= a \left(\frac{I_{\max 1}}{I_{\max 2}} \frac{\Delta T_{\max 2}}{T_1} + \frac{\Delta T_{\max 1}}{T_0} \right).
 \end{aligned}$$

Functional dependence

$$E^{N=2} = f(B_1),$$

has a maximum for different designs of TED (n_1/n_2) and temperature differentials $\Delta T = 60$ K; 70 K; 80 K; 90 K at $T = 300$ K, $n_1 = 9$, $l_2/s_2 = l_1/s_1 = 10$.

With an increase in temperature ΔT , the optimum magnitude of relative operating current B_1 shifts toward larger values.

From condition $\frac{dE^N}{dB_1} = 0$ we shall obtain a relation for determining the optimum magnitude of relative operating current B_1 , corresponding to the maximum of refrigeration coefficient E^N of the TED of assigned design (n_1/n_2) and to the temperature differential ΔT :

$$\begin{aligned}
 B_1^4 \left(Yc - 2aX \frac{\Delta T_{\max 1}}{T_0} \right) - 4B_1^3 \left(Yb + aH \frac{\Delta T_{\max 1}}{T_0} \frac{\Delta T}{\Delta T_{\max 2}} \right) + \\
 + B_1^2 \left(2Xb + Hc \frac{\Delta T}{\Delta T_{\max 2}} + 3Y \frac{\Delta T}{\Delta T_{\max 2}} \right) - \\
 - 2B_1X \frac{\Delta T}{\Delta T_{\max 2}} - \left(\frac{\Delta T}{\Delta T_{\max 2}} \right)^2 = 0.
 \end{aligned} \tag{9}$$

Presented relation (9) makes it possible to determine the magnitude of optimum relative operating current B_1 that provides for the maximum refrigeration coefficient E^N at the given values of ratio n_1/n_2 and temperature differential ΔT .

Next, we determine relative temperature differentials in cascades θ_1 и θ_2 , using a successive approximation method, taking into account the temperature dependence of parameters (one-two approximations suffice):

$$\theta_1 = \frac{B_1^2 \left(a + \frac{I_{\max 1}^2}{I_{\max 2}^2} \right) - 2B_1 \left(\frac{I_{\max 1}}{I_{\max 2}} - a \right) + \frac{\Delta T}{\Delta T_{\max 2}}}{\frac{\Delta T_{\max 1}}{\Delta T_{\max 2}} + a - 2aB_1 \frac{\Delta T_{\max 1}}{T_0}}; \tag{10}$$

$$\theta_2 = \frac{\Delta T}{\Delta T_{\max 2}} - \frac{\Delta T_{\max 1}}{\Delta T_{\max 2}} \theta_1 \tag{11}$$

and, according to expression (1), refrigeration capacity Q_{01} for the assigned design (n_1/n_2) of TED in regime E_{\max} at the assigned ΔT .

In accordance with [16], for a two-stage TED, the magnitude of relative failure rate can be written in the form

$$\begin{aligned}
 \frac{\lambda_{\Sigma}}{\lambda_0} &= \frac{n_1 B_1^2 (\theta_1 + C_1) \left(B_1 + \frac{\Delta T_{\max 1}}{T_0} \theta_1 \right)^2}{\left(1 + \frac{\Delta T_{\max 1}}{T_0} \theta_1 \right)^2} K_{T_1} + \\
 &+ \frac{n_2 B_2^2 (\theta_2 + C_2) \left(B_2 + \frac{\Delta T_{\max 2}}{T_1} \theta_2 \right)^2}{\left(1 + \frac{\Delta T_{\max 2}}{T_1} \theta_2 \right)^2} K_{T_2},
 \end{aligned} \tag{12}$$

where λ_0 is the nominal failure rate, 1/h; C_1, C_2 are the relative thermal load of the first and second cascades, rel. un.,

$$C_1 = \frac{Q_{01}}{n_1 I_{\max 1}^2 R_1};$$

$$C_2 = \frac{Q_0 + W_1}{n_2 I_{\max 2}^2 R_2},$$

K_{T_1} and K_{T_2} is the coefficient of significance taking into account the effect of reduced temperature [16]. The analytical model obtained provides the possibility to analyze a relation between the relative failure rate and the energy and design indicators of a thermoelectric cooler in the working range of functioning temperature.

4. 2. Analysis of results of modeling the reliability and energy indicators

The calculated data on the basic parameters are given in Tables 1–4 for $l_2/s_2 = l_1/s_1 = 10$; $T = 300$ K; $\Delta T = 60$ K; 70 K; 80 K; 90 K; $n_1 = 9$; $n_1/n_2 = 1.0$; 0.67; 0.5; 0.33; 0.2; 0.1 and the averaged value of effectiveness of thermoelectric modules $\bar{z}_m = 2.5 \cdot 10^{-3}$ 1/K; $\lambda_0 = 3 \cdot 10^{-8}$ 1/h; $t = 10^4$ h.

At the decrease in ratio n_1/n_2 at the assigned value of temperature differential $\Delta T = 60$ K:

- the magnitude of intermediate temperature T_1 decreases (Fig. 1, pos. 1);
- relative operating current of the first cascade B_1 increases (Fig. 1, pos. 2);
- relative operating current of the second cascade B_2 increases (Fig. 1, pos. 3);
- relative drop in temperature of the first cascade θ_1 decreases (Fig. 1, pos. 4);
- relative drop in temperature of the second cascade θ_2 increases (Fig. 1, pos. 5);
- the magnitude of operating current I increases (Fig. 1, pos. 6);
- refrigeration coefficient of TED has an absolute maximum at $n_1/n_2 = 0.44$ (Fig. 2, pos. 1);
- refrigeration coefficient of the first cascade ϵ_1 increases (Fig. 2, pos. 2), and of the second cascade ϵ_2 decreases (Fig. 2, pos. 3);
- the point of intersection of dependence charts of the refrigeration coefficients of cascades ϵ_1 and ϵ_2 corresponds to $n_1/n_2 = 0.44$ (Fig. 2);
- refrigeration capacity Q_{01} (Fig. 2, pos. 4) and its relative magnitude C_1 (Fig. 2, pos. 5) increase;

– the total magnitude of failure rate λ_2 increases (Fig. 3, pos. 3);
 – the failure rates of the first λ_1 and the second λ_2 cascades also increase (Fig. 3, pos. 1, 2);

– total probability of failure-free operation P_Σ decreases (Fig. 4, pos. 3);
 – the probability of failure-free operation of the first (P_1) and the second (P_2) cascade decreases (Fig. 4, pos. 1, 2).

Table 1

Results of calculation of the basic parameters and indicators of reliability of two-cascade TED of different designs under the mode E_{max} at $\Delta T=60$ K

n_1/n_2	B_1	B_2	I, A	T_1, K	θ_1	θ_2	ϵ_1	ϵ_2	E	Q_{01}, W	C_1	W_1, W	W_2, W	W_Σ, W	U_Σ, V	$\lambda_2 \cdot 10^{-8}, 1/h$	P
1.0	0.41	0.38	1.91	279.8	0.61	0.22	0.1	1.13	0.051	0.09	0.05	0.89	0.86	1.75	0.92	1.18	0.999882
0.67	0.43	0.405	2.0	272.8	0.51	0.31	0.35	0.83	0.133	0.32	0.17	0.92	1.49	2.4	1.21	1.79	0.99982
0.5	0.44	0.42	2.10	268.3	0.44	0.37	0.50	0.64	0.151	0.46	0.24	0.93	2.15	3.1	1.5	2.4	0.99976
0.45	0.447	0.43	2.13	267.2	0.427	0.40	0.54	0.59	0.148	0.50	0.27	0.94	2.44	3.4	1.6	2.9	0.99971
0.33	0.45	0.44	2.14	262.5	0.35	0.46	0.70	0.45	0.146	0.64	0.34	0.92	3.45	4.37	2.0	3.7	0.99963
0.2	0.45	0.45	2.13	257.0	0.27	0.55	0.92	0.27	0.114	0.79	0.42	0.86	6.05	6.9	3.2	6.5	0.999352
0.1	0.455	0.46	2.2	253.4	0.21	0.62	1.05	0.14	0.069	0.91	0.49	0.86	12.4	13.2	6.0	13.3	0.99867

Table 2

Results of calculation of the basic parameters and indicators of reliability of two-cascade TED of different designs under the mode E_{max} at $\Delta T=70$ K

n_1/n_2	B_1	B_2	I, A	T_1, K	θ_1	θ_2	ϵ_1	ϵ_2	E	Q_{01}, W	C_1	W_1, W	W_2, W	W_Σ, W	U_Σ, V	$\lambda_2 \cdot 10^{-8}, 1/h$	P
0.67	0.53	0.49	2.43	270.3	0.69	0.34	0.12	0.68	0.044	0.15	0.086	1.28	2.114	3.4	1.4	4.3	0.99957
0.50	0.54	0.50	2.45	264.6	0.60	0.41	0.26	0.53	0.0767	0.33	0.19	1.26	3.0	4.26	1.7	5.5	0.99945
0.37	0.55	0.53	2.51	260.0	0.55	0.50	0.40	0.40	0.083	0.45	0.25	1.26	4.3	5.6	2.2	7.8	0.99922
0.33	0.547	0.53	2.52	258.7	0.50	0.52	0.40	0.36	0.0823	0.50	0.29	1.26	4.85	6.11	2.42	9.0	0.99910
0.20	0.57	0.55	2.64	253.6	0.42	0.61	0.51	0.23	0.0673	0.66	0.39	1.28	8.5	9.8	3.7	15.8	0.99842
0.10	0.573	0.56	2.66	248.2	0.33	0.71	0.66	0.12	0.0433	0.82	0.49	1.25	17.7	18.9	7.1	31.9	0.99682

Table 3

Results of calculation of the basic parameters and indicators of reliability of two-cascade TED of different designs under the mode E_{max} at $\Delta T=80$ K

n_1/n_2	B_1	B_2	I, A	T_1, K	θ_1	θ_2	ϵ_1	ϵ_2	E	Q_{01}, W	C_1	W_1, W	W_2, W	W_Σ, W	U_Σ, V	$\lambda_2 \cdot 10^{-8}, 1/h$	P
0.50	0.65	0.60	2.9	263.4	0.85	0.45	0.03	0.44	0.0089	0.051	0.033	1.71	4.0	5.71	1.97	12.3	0.99877
0.33	0.67	0.63	3.0	257.1	0.73	0.56	0.14	0.31	0.030	0.25	0.16	1.75	6.55	8.3	2.75	19.7	0.9980
0.20	0.68	0.65	3.1	250.2	0.60	0.67	0.26	0.19	0.034	0.45	0.29	1.72	11.5	13.2	4.3	32.5	0.99676
0.10	0.70	0.69	3.2	245.2	0.51	0.78	0.35	0.10	0.023	0.61	0.40	1.75	24.9	26.7	8.4	71.9	0.99283

Table 4

Results of calculation of the basic parameters and indicators of reliability of two-cascade TED of different designs under the mode E_{max} at $\Delta T=90$ K

n_1/n_2	B_1	B_2	I, A	T_1, K	θ_1	θ_2	ϵ_1	ϵ_2	E	Q_{01}, W	C_1	W_1, W	W_2, W	W_Σ, W	U_Σ, V	$\lambda_2 \cdot 10^{-8}, 1/h$	P
0.20	0.83	0.77	3.6	249.4	0.89	0.71	0.05	0.16	0.0067	0.12	0.085	2.3	15.1	17.4	4.9	64.9	0.99353
0.10	0.84	0.81	3.7	243.6	0.77	0.83	0.12	0.08	0.0081	0.28	0.21	2.3	32.4	34.7	9.4	140	0.9861

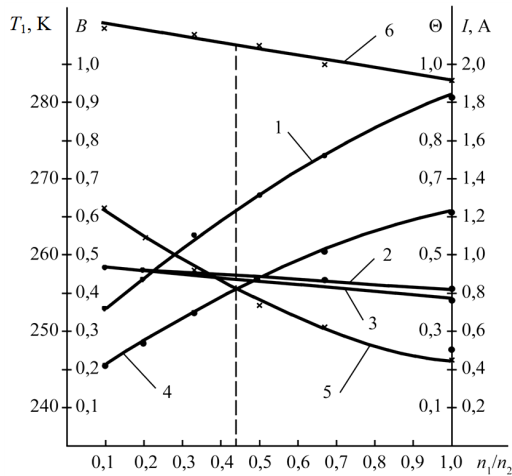


Fig. 1. Dependence of the intermediate temperature T_1 , relative operating currents B_1 and B_2 , relative temperature differentials θ_1 and θ_2 in the cascades and the magnitude of operating current I of two-cascade thermoelectric coolers under the mode E_{max} on the ratio n_1/n_2 at $T=300$ K; $\Delta T=60$ K; $n_i=9$; $(I/s)_i=10$: 1 - $T_1=f(n_1/n_2)$; 2 - $B_1=f(n_1/n_2)$; 3 - $B_2=f(n_1/n_2)$; 4 - $\theta_1=f(n_1/n_2)$; 5 - $\theta_2=f(n_1/n_2)$; 6 - $I=f(n_1/n_2)$

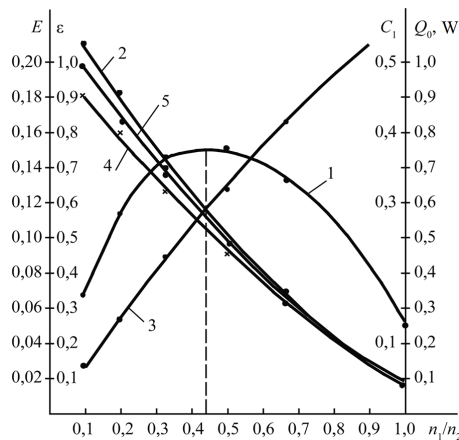


Fig. 2. Dependence of general refrigeration coefficient E and on the cascades ϵ_1 and ϵ_2 , refrigeration capacity Q_{01} and C_1 of two-stage thermoelectric coolers under the mode E_{max} on the ratio n_1/n_2 at $T=300$ K; $\Delta T=60$ K; $n_i=9$; $(I/s)_i=10$: 1 - $E=f(n_1/n_2)$; 2 - $\epsilon_1=f(n_1/n_2)$; 3 - $\epsilon_2=f(n_1/n_2)$; 4 - $Q_{01}=f(n_1/n_2)$; 5 - $C_1=f(n_1/n_2)$

At the decrease in ratio n_1/n_2 at the assigned value of temperature differential $\Delta T=70$ K:

- the magnitude of intermediate temperature T_1 decreases (Fig. 5, pos. 1);
- relative operating current of the first cascade B_1 and the second cascade B_2 increases (Fig. 5, pos. 2 and 3);
- relative drop in temperature of the first cascade θ_1 decreases (Fig. 5, pos. 4);
- relative drop in temperature of the second cascade θ_2 increases (Fig. 5, pos. 5);
- the magnitude of operating current I increases (Fig. 5, pos. 6);
- refrigeration coefficient E has a maximum at $n_1/n_2=0.37$ (Fig. 6, pos. 1);
- refrigeration coefficient of the first cascade ϵ_1 increases (Fig. 6, pos. 2), and of the second cascade ϵ_2 - decreases (Fig. 6, pos. 3);

- refrigeration capacity Q_{01} (Fig. 6, pos. 4) and its relative magnitude C_1 (Fig. 6, pos. 5) increase;
- the total magnitude of failure rate λ_Σ increases (Fig. 7, pos. 3);
- failure rates of the first λ_1 and the second λ_2 cascades also increase (Fig. 7, pos. 1, 2);
- total power of energy consumption W increases, (Fig. 7, pos. 4);
- the total probability of failure-free operation P decreases (Fig. 8, pos. 3);
- the probability of failure-free operation of the first P_1 and the second cascade P_2 decreases (Fig. 8, pos. 1, 2).

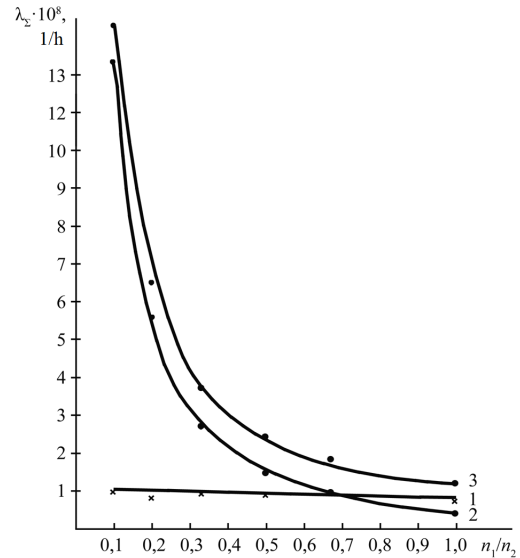


Fig. 3. Dependence of the total failure rate λ_Σ and each cascade λ_1 and λ_2 separately of two-stage thermoelectric coolers under the mode E_{max} on the ratio n_1/n_2 at $T=300$ K; $\Delta T=60$ K; $n_i=9$; $(I/s)_i=10$; $\lambda_0=3 \cdot 10^{-8}$ 1/h: 1 - $\lambda_1=f(n_1/n_2)$; 2 - $\lambda_2=f(n_1/n_2)$; 3 - $\lambda_\Sigma=f(n_1/n_2)$

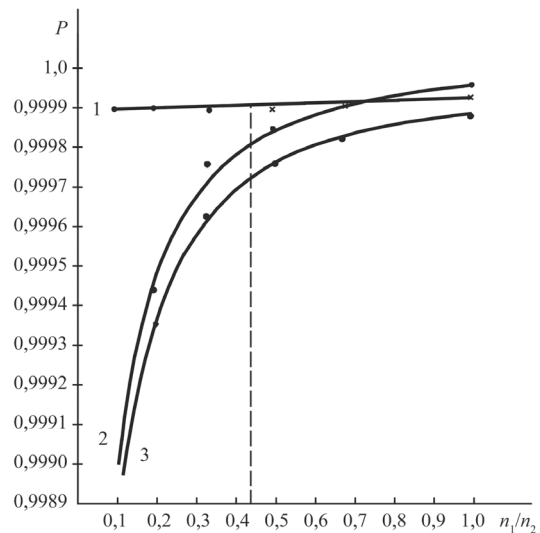


Fig. 4. Dependence of the total probability of failure-free rate P_Σ and by each cascade P_1 and P_2 separately of two-stage thermoelectric coolers of different designs under the mode E_{max} on the ratio n_1/n_2 at $T=300$ K; $\Delta T=60$ K; $n_i=9$; $(I/s)_i=10$; $\lambda_0=3 \cdot 10^{-8}$ 1/h; $t=10^4$ h: 1 - $P_1=f(n_1/n_2)$; 2 - $P_2=f(n_1/n_2)$; 3 - $P_\Sigma=f(n_1/n_2)$

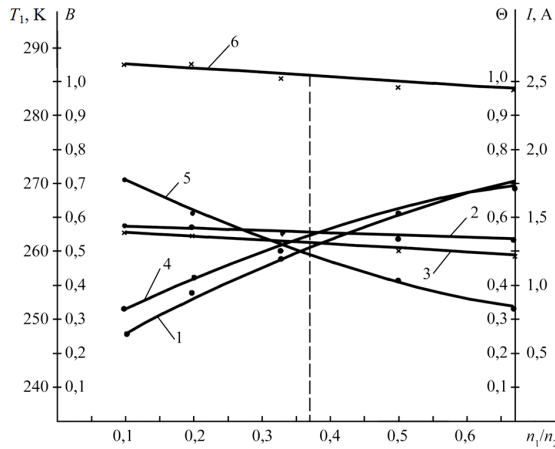


Fig. 5. Dependence of the intermediate temperature T_1 , relative operating currents B_1 and B_2 , relative temperature differentials θ_1 and θ_2 in the cascades and the magnitude of operating current I of two-cascade thermoelectric coolers under the mode E_{max} on the ratio n_1/n_2 at $T=300$ K; $\Delta T=70$ K; $n_1=9$; $(l/s)_i=10$: 1 - $T_1=f(n_1/n_2)$; 2 - $B_1=f(n_1/n_2)$; 3 - $B_2=f(n_1/n_2)$; 4 - $\theta_1=f(n_1/n_2)$; 5 - $\theta_2=f(n_1/n_2)$; 6 - $I=f(n_1/n_2)$

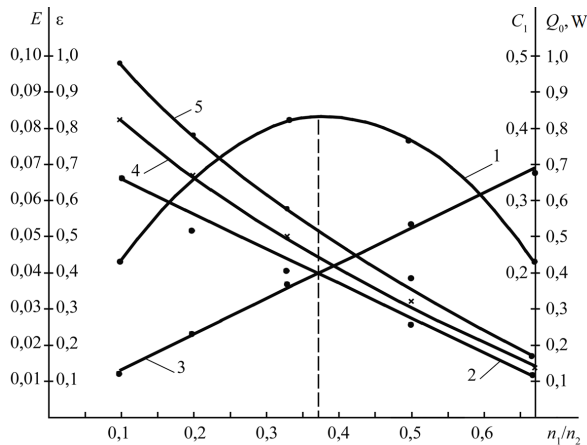


Fig. 6. Dependence of general refrigeration coefficient E and by the cascades ϵ_1 and ϵ_2 , refrigeration capacity Q_{o1} and C_1 of two-cascade thermoelectric coolers under the mode E_{max} on the ratio n_1/n_2 at $T=300$ K; $\Delta T=70$ K; $n_1=9$; $(l/s)_i=10$: 1 - $E=f(n_1/n_2)$; 2 - $\epsilon_1=f(n_1/n_2)$; 3 - $\epsilon_2=f(n_1/n_2)$; 4 - $Q_{o1}=f(n_1/n_2)$; 5 - $C_1=f(n_1/n_2)$

At the decrease in ratio n_1/n_2 at the assigned value of temperature differential $\Delta T=80$ K:

- the magnitude of intermediate temperature T_1 decreases (Fig. 9, pos. 1);
- relative operating current of the first cascade B_1 and of the second cascade B_2 increases (Fig. 9, pos. 2, 3);
- relative drop in temperature of the first cascade θ_1 decreases (Fig. 5, pos. 4), and of the second cascade θ_2 increases (Fig. 9, pos. 5);
- the magnitude of operating current I increases (Fig. 9, pos. 6);
- refrigeration coefficient E has a maximum at $n_1/n_2=0.23$ (Fig. 10, pos. 1), in this case the refrigeration coefficients of the first cascade ϵ_1 and of the second cascade ϵ_2 are equal to each other: $\epsilon_1=\epsilon_2=0.22$ (Fig. 10, pos. 2, 3);

- refrigeration capacity Q_{o1} (Fig. 10, pos. 4) and its relative magnitude C_1 (Fig. 10, pos. 5) increase;
- the total magnitude of failure rate λ_Σ increases (Fig. 11, pos. 3), in this case the failure rates of the first λ_1 and of the second λ_2 cascades also increase (Fig. 11, pos. 1, 2), but not equally;
- total power of energy consumption W_Σ increases (Fig. 11, pos. 4);
- the total probability of failure-free operation P_Σ decreases (Fig. 12, pos. 3), in this case the probability of failure-free operation of the first (P_1) and of the second (P_2) cascades also decreases (Fig. 12, pos. 1, 2).

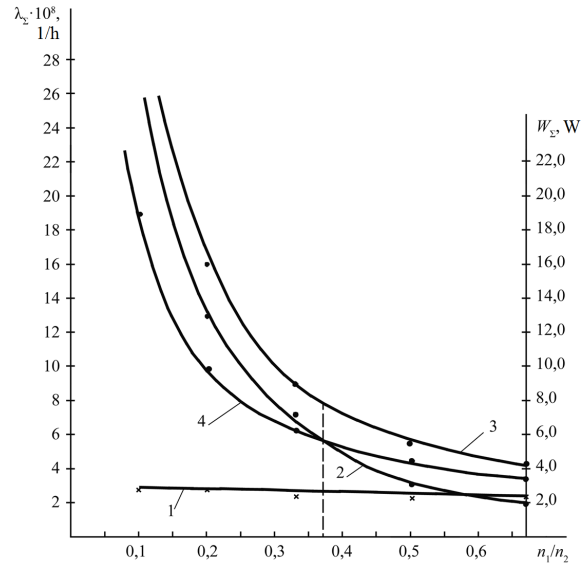


Fig. 7. Dependence of the total failure rate λ_Σ and of each cascade λ_1 and λ_2 separately of two-cascade thermoelectric coolers under the mode E_{max} on the ratio n_1/n_2 at $T=300$ K; $\Delta T=70$ K; $n_1=9$; $(l/s)_i=10$; $\lambda_0=3 \cdot 10^{-8}$ 1/h: 1 - $\lambda_1=f(n_1/n_2)$; 2 - $\lambda_2=f(n_1/n_2)$; 3 - $\lambda_\Sigma=f(n_1/n_2)$

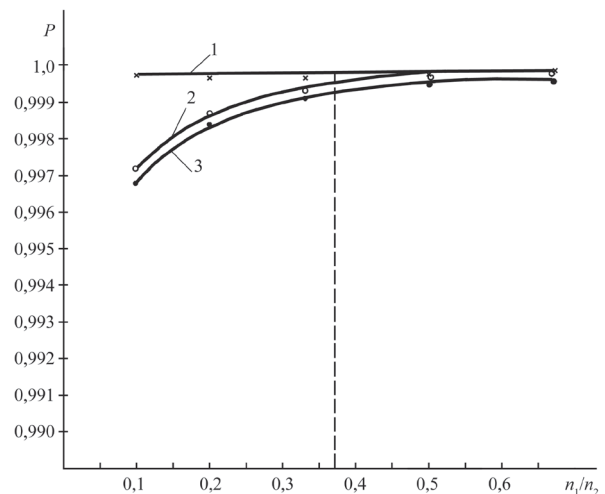


Fig. 8. Dependence of the total probability of failure-free operation P_Σ and of each cascade separately P_1 and P_2 of the two-cascade thermoelectric coolers of different designs under the mode E_{max} on the ratio n_1/n_2 at $T=300$ K; $\Delta T=70$ K; $n_1=9$; $(l/s)_i=10$; $\lambda_0=3 \cdot 10^{-8}$ 1/h; $t=10^4$ h: 1 - $P_1=f(n_1/n_2)$; 2 - $P_2=f(n_1/n_2)$; 3 - $P_\Sigma=f(n_1/n_2)$

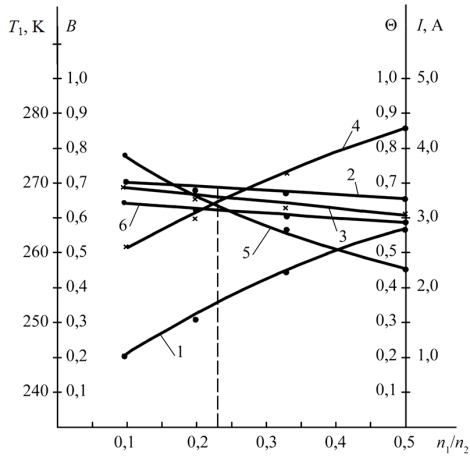


Fig. 9. Dependence of the intermediate temperature T_1 , relative operating currents B_1 and B_2 , relative temperature differentials θ_1 and θ_2 in the cascades and the magnitude of operating current I of two-cascade thermoelectric coolers under the mode E_{max} on the ratio n_1/n_2 at $T=300$ K; $\Delta T=80$ K; $n_1=9$; $(I/s)_i=10$: 1 – $T_1=f(n_1/n_2)$; 2 – $B_1=f(n_1/n_2)$; 3 – $B_2=f(n_1/n_2)$; 4 – $\theta_1=f(n_1/n_2)$; 5 – $\theta_2=f(n_1/n_2)$; 6 – $I=f(n_1/n_2)$

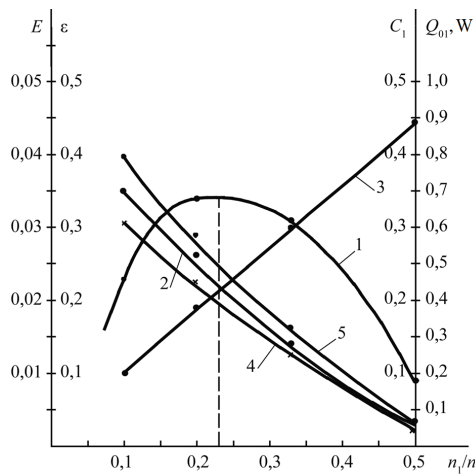


Fig. 10. Dependence of general refrigeration coefficient E and by the cascades ε_1 and ε_2 , refrigeration capacity Q_{01} and C_1 of two-cascade thermoelectric coolers under the mode E_{max} on the ratio n_1/n_2 at $T=300$ K; $\Delta T=80$ K; $n_1=9$; $(I/s)_i=10$: 1 – $E=f(n_1/n_2)$; 2 – $\varepsilon_1=f(n_1/n_2)$; 3 – $\varepsilon_2=f(n_1/n_2)$; 4 – $Q_{01}=f(n_1/n_2)$; 5 – $C_1=f(n_1/n_2)$

At the decrease of ratio n_1/n_2 at the assigned value of temperature differential $\Delta T=90$ K:

- the magnitude of intermediate temperature T_1 decreases (Fig. 13, pos. 1);
- relative operating current of the first cascade B_1 and of the second cascade B_2 increases (Fig. 13, pos. 2, 3);
- relative drop in temperature of the first cascade θ_1 decreases (by Fig. 13, pos. 4), and that of the second cascade θ_2 increases (Fig. 13, pos. 5);
- the magnitude of operating current I increases (Fig. 13, pos. 6);
- refrigeration coefficient E has a maximum at $n_1/n_2=0.127$ (Fig. 14, pos. 1), in this case the refrigeration coefficients of the first cascade ε_1 and of the second cascade ε_2 are equal to each other: $\varepsilon_1=\varepsilon_2=0.10$ (Fig. 14, pos. 2, 3);

- refrigeration capacity Q_{01} (Fig. 14, pos. 4) and its relative magnitude C_1 (Fig. 14, pos. 5) increase;
- the total magnitude of failure rate λ_Σ increases (Fig. 15, pos. 3), in this case the failure rate of the first λ_1 and of the second λ_2 cascades increases (Fig. 15, pos. 1, 2);
- the total probability of failure-free operation P_Σ decreases (Fig. 15, pos. 7), in this case the probability of failure-free operation of the first (P_1) and of the second (P_2) cascades decreases (Fig. 15, pos. 5, 6).

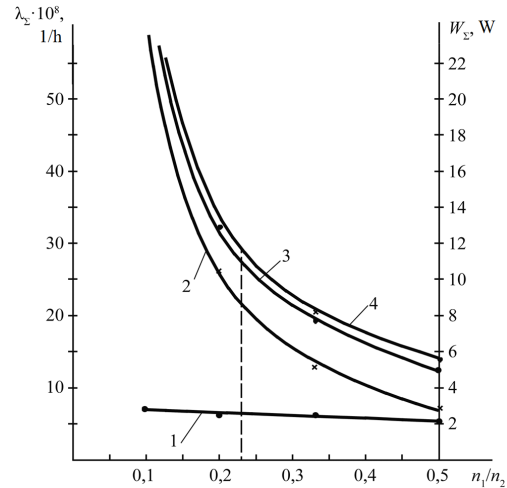


Fig. 11. Dependence of general total failure rate λ_Σ and of each cascade λ_1 and λ_2 separately of two-cascade thermoelectric coolers under the mode E_{max} on the ratio n_1/n_2 at $T=300$ K; $\Delta T=80$ K; $n_1=9$; $(I/s)_i=10$; $\lambda_0=3 \cdot 10^{-8}$ 1/h: 1 – $\lambda_1=f(n_1/n_2)$; 2 – $\lambda_2=f(n_1/n_2)$; 3 – $\lambda_\Sigma=f(n_1/n_2)$; 4 – $W_\Sigma=f(n_1/n_2)$

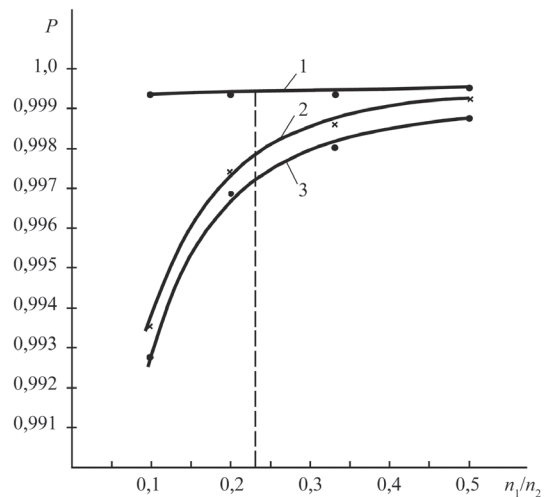


Fig. 12. Dependence of the total probability of failure-free operation P_Σ and for each cascade separately P_1 and P_2 of the two-cascade thermoelectric coolers of different designs under the mode E_{max} on the ratio n_1/n_2 at $T=300$ K; $\Delta T=80$ K; $n_1=9$; $(I/s)_i=10$, $\lambda_0=3 \cdot 10^{-8}$ 1/h; $\tau=10^4$ h: 1 – $P_1=f(n_1/n_2)$; 2 – $P_2=f(n_1/n_2)$; 3 – $P_\Sigma=f(n_1/n_2)$

The given qualitative description of the energy indicators of a cooler depending on the ratio of number of thermoelements in the cascades allows us to estimate the ways of designing the two-cascade thermoelectric devices with improved reliability.

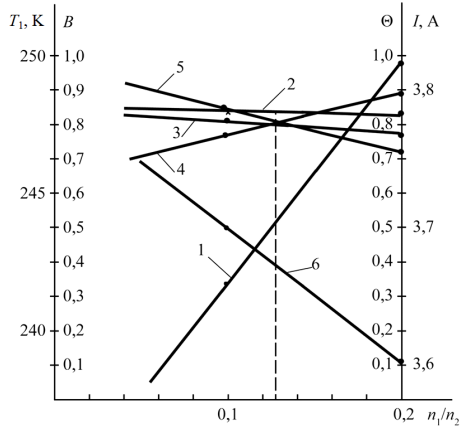


Fig. 13. Dependence of the intermediate temperature T_1 , relative operating currents B_1 and B_2 , relative temperature differentials θ_1 and θ_2 in the cascades and the magnitude of operating current I of two-cascade thermoelectric coolers under the mode E_{max} on the ratio n_1/n_2 at $T=300$ K; $\Delta T=90$ K; $n_1=9$; $(I/s)_i=10$: 1 - $T_1=f(n_1/n_2)$; 2 - $B_1=f(n_1/n_2)$; 3 - $B_2=f(n_1/n_2)$; 4 - $\theta_1=f(n_1/n_2)$; 5 - $\theta_2=f(n_1/n_2)$; 6 - $I=f(n_1/n_2)$

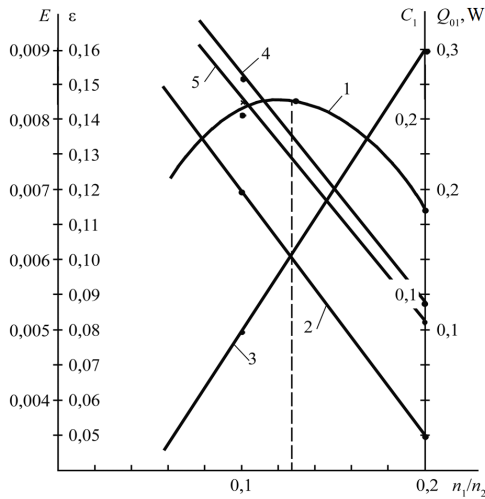


Fig. 14. Dependence of general refrigeration coefficient E and by the cascades ϵ_1 and ϵ_2 , refrigeration capacity Q_{01} and C_1 of two-cascade thermoelectric coolers under the mode E_{max} on the ratio n_1/n_2 at $T=300$ K; $\Delta T=90$ K; $n_1=9$; $(I/s)_i=10$: 1 - $E=f(n_1/n_2)$; 2 - $\epsilon_1=f(n_1/n_2)$; 3 - $\epsilon_2=f(n_1/n_2)$; 4 - $Q_{01}=f(n_1/n_2)$; 5 - $C_1=f(n_1/n_2)$

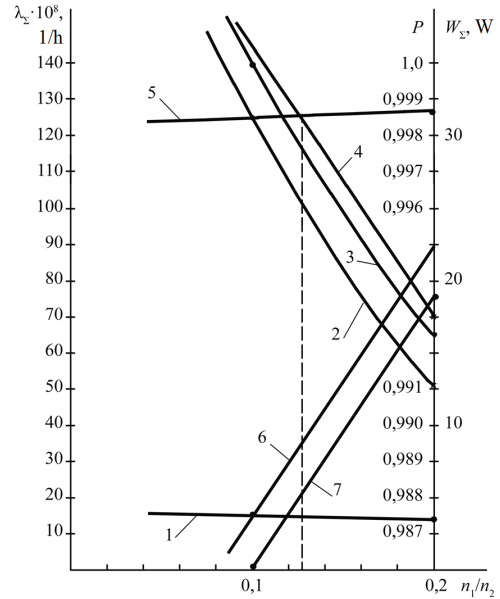


Fig. 15. Dependence of the total failure rate λ_Σ and the probability of failure-free operation P and of each cascade separately λ_1 and λ_2 and P_1 and P_2 of two-cascade thermoelectric coolers under the mode E_{max} on the ratio n_1/n_2 at $T=300$ K; $\Delta T=80$ K; $n_1=9$; $(I/s)_i=10$, $\lambda_0=3 \cdot 10^{-8}$ 1/h; $t=10^4$ h: 1 - $\lambda_1=f(n_1/n_2)$; 2 - $\lambda_2=f(n_1/n_2)$; 3 - $\lambda_\Sigma=f(n_1/n_2)$; 4 - $W_\Sigma=f(n_1/n_2)$; 5 - $P_1=f(n_1/n_2)$; 6 - $P_2=f(n_1/n_2)$; 7 - $P_\Sigma=f(n_1/n_2)$

5. Discussion of results of the analysis of relation between the number of elements and the energy and reliability indicators

An analysis of calculated data revealed that there is an optimum ratio n_1/n_2 , corresponding to the maximum of refrigeration coefficient E at the assigned temperature differential ΔT .

In the point of the maximum of refrigeration coefficient E we observe the equality of values of relative temperature differential θ_1 and θ_2 and refrigeration coefficients ϵ_1 and ϵ_2 in the cascades. Results of the calculations are given in Table 5.

Table 5

Results of the calculation of basic parameters and indicators of reliability of two-cascade TED of different designs under the mode E_{max} at different values of temperature differential

ΔT , K	n_1/n_2	B_1	B_2	I , A	T_1 , K	θ_1	θ_2	ϵ_1	ϵ_2	E	Q_{01} , W	C_1	W_Σ , W	U_Σ , V	$\lambda_\Sigma/n_1\lambda_0$	$\lambda_\Sigma \cdot 10^{-8}$, 1/h	P
60	0.44	0.45	0.43	2.1	267.2	0.43	0.43	0.58	0.58	0.151	0.50	0.27	3.4	1.6	0.11	2.9	0.99971
70	0.37	0.55	0.53	2.5	260.0	0.55	0.50	0.40	0.40	0.083	0.45	0.25	5.6	2.2	0.29	7.8	0.99922
80	0.23	0.69	0.66	3.1	253.0	0.63	0.63	0.22	0.22	0.034	0.38	0.24	11.6	3.7	1.0	27.0	0.9972
90	0.127	0.83	0.80	3.775	244.0	0.80	0.80	0.10	0.10	0.0083	0.24	0.18	31.0	8.2	4.26	115.0	0.9880

With an increase in the temperature differential ΔT for different designs of TED ($n_1/n_2=1.0; 0.67; 0.5; 0.33; 0.2; 0.1$):

- relative operating currents in the first (B_1) and the second (B_2) cascades increase;
- the magnitude of operating current I increases as well;
- the intermediate temperature T_1 decreases;
- relative temperature differentials in the first (θ_1) and the second (θ_2) cascades increase;
- refrigeration coefficient E decreases, in this case refrigeration coefficient of the first cascade ϵ_1 and of the second ϵ_2 decrease;
- refrigeration capacity Q_{01} and its relative magnitude C_1 decrease;
- the total power of energy consumption W_Σ grows;
- total voltage drop U_Σ grows;
- the total magnitude of failure rate λ_Σ grows;
- the total probability of failure-free operation P_Σ decreases.

6. Conclusions

1. We developed a model of the relation between indicators of reliability of a cascade thermoelectric cooler and the distribution of the number of thermoelements in the cascades of a thermoelectric cooler, temperature differential, the refrigeration capacity and thermal load. Its special feature is in providing for the possibility to design the structural and energy indicators of a cooler in accordance with a criterion of the minimum failure rate.

2. We carried out an analysis of the model under regime of the highest energy efficiency, which demonstrated a possibility to evaluate the operational efficiency of a cascade cooler, to predict the optimum values of refrigeration coefficient at the assigned temperature differential and the relation of the number of elements in cascades under varied conditions of operation.

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