ABSTRACT AND REFERENCES

MATHEMATICS AND CYBERNETICS – APPLIED ASPECTS

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METHODOLOGY OF PROBABILISTIC ANALYSIS OF STATE DYNAMICS OF MULTIDIMENSIONAL SEMIMARKOV DYNAMIC SYSTEMS (p. 6-13)

Yelyzaveta Meleshko
Central Ukrainian National Technical University, Kropyvnytskyi, Ukraine
ORCID: http://orcid.org/0000-0001-8791-0063

Lev Raskin
National Technical University "Kharkiv Polytechnic Institute", Kharkiv, Ukraine
ORCID: http://orcid.org/0000-0002-9015-4016

Serhii Semenov
National Technical University "Kharkiv Polytechnic Institute", Kharkiv, Ukraine
ORCID: http://orcid.org/0000-0003-4472-9234

Oksana Sira
National Technical University "Kharkiv Polytechnic Institute", Kharkiv, Ukraine
ORCID: http://orcid.org/0000-0002-4869-2371

The problem of probabilistic analysis of a complex dynamic system, which in the process of functioning passes from one state to another at random times, is considered. The methodology for calculating the conditional probabilities of the system getting into a given state at a given time t, provided that at the initial time the system was in any of the possible states is proposed. The initial data for analysis are a set of experimentally obtained values of the duration of the system stay in each of the states before transition to another state. Approximation of the resulting histograms using the Erlang distribution gives a set of distribution densities of the duration of the system stay in possible states before transition to other states. At the same time, the choice of the proper Erlang distribution order provides an adequate description of the semi-Markov processes occurring in the system. The mathematical model that relates the obtained distribution densities to the functions determining the probabilistic dynamics of the system is proposed. The model describes a random process of system transitions from any possible initial state to any other state during a given time interval. Using the model, a system of integral equations for the desired functions describing the probabilistic transition process is obtained. To solve these equations, the Laplace transform is used. As a result of solving the system of integral equations, functions are obtained that specify the probability distribution of the system states at any time t. The same functions also describe the asymptotic probability distribution of states. An illustrative example of solving the problem for the case when the distribution densities of the lengths of the system stay in possible states are described by the second-order Erlang distributions is given. The solution procedure is described in detail for the most natural special case, when the initial state is H_0.

Keywords: dynamic system with many possible states, random transition process, integral dynamic equations, Laplace transforms.

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FINDING OF BOUNDED SOLUTIONS TO LINEAR IMPULSIVE SYSTEMS (p. 14-20)

Farkhod Asrorov
Taras Shevchenko National University of Kyiv, Kyiv, Ukraine
ORCID: http://orcid.org/0000-0002-3917-4724

Valentyn Sobchuk
Lesya Ukrainka Eastern European National University, Lutsk, Ukraine
ORCID: http://orcid.org/0000-0002-4002-8206

Olexandr Kurylko
Taras Shevchenko National University of Kyiv, Kyiv, Ukraine
ORCID: http://orcid.org/0000-0002-1711-7083

The problem of the existence of bounded on the entire real axis solutions to linear nonhomogeneous systems of differential equations undergoing impulsive perturbations at the fixed moments of time is investigated. Sufficient conditions for the hyperbolicity of solutions to the homogeneous multidimensional impulsive system are obtained. The derived conditions are applied to the study of the bounded solutions to the nonhomogeneous impulsive system. Sufficient conditions for the existence of a unique bounded solution to the nonhomogeneous system in the case of weak regularity of the corresponding homogeneous system are formulated. The advantage of such an approach is that the established conditions can be effectively tested for specific classes of impulse-perturbed systems, since they are formulated in terms of coefficients of initial problems. The obtained conditions allow applying classical solution methods of differential equations for the propositions on solvability and continuous dependence of solutions on parameters of the impulsive systems.

The theory of systems with impulsive actions has wide possibilities for its application. Many evolution processes in physics, engineering, automatic control, biology, economics are exposed to short-term perturbations during their evolution. For example, processes with abrupt changes are observed in mechanics (spring movement under shock influence, functioning of the clock mechanism, change of rocket speed at separation stages), in radio engineering (generation of impulses of various forms). Similar processes are also observed in biology (heart beat, cell division), biotechnology (growing biocomposites), and control theory (industrial robots).

Therefore, qualitative investigation of impulsive systems in this work is a relevant challenge of the modern theory of mathematical modeling.

Keywords: differential equations, impulsive system, bounded solution, Green-Samoilenko function, regular solution.

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CONSTRUCTING AN ALGORITHM OF QUADRATIC TIME COMPLEXITY FOR FINDING THE MAXIMAL MATCHING (p. 21-28)

Andrii Morozov
Zhytomyr Polytechnic State University, Zhytomyr, Ukraine
ORCID: http://orcid.org/0000-0003-3167-0683

Tamara Loktikova
Zhytomyr Polytechnic State University, Zhytomyr, Ukraine
ORCID: http://orcid.org/0000-0002-3525-0179

Iurii Iefremov
Zhytomyr Polytechnic State University, Zhytomyr, Ukraine
ORCID: http://orcid.org/0000-0003-1119-849X

Anatolii Dykyi
Zhytomyr Polytechnic State University, Zhytomyr, Ukraine
ORCID: http://orcid.org/0000-0003-1420-4162

Pavlo Zabrodskyy
Zhytomyr National Agroecological University,
Zhytomyr, Ukraine
ORCID: http://orcid.org/0000-0002-3904-564X

By advancing the idea of finding width in bipartite graphs and basic definitions in matching theory, this paper shows that the task on establishing a maximal matching in an arbitrary graph can be reduced to its bipartite case. It has been proven that each current matching in an arbitrary graph is mutually consistent with the matching in a bipartite graph. It is demonstrated that each of the current solutions to the problem on establishing a maximum matching in an arbitrary graph is not lost when moving to the iterative scheme of establishing the maximum matching in a bipartite graph.

To find a prolonged path relative to the fixed matching of power k, it has been proposed a modification to known algorithm for finding paths from this a given vertex to all attainable vertices of the arbitrary graph. Performance of the proposed modification has been illustrated using an example.

Based on the ideas outlined, the proven statements, the proposed algorithms and their modifications, an algorithm has been constructed for finding the maximum matching with an improved time estimate compared to the known Edmons algorithm, which possesses a temporal assessment of complexity $O(n^4)$. The main drawback of the Edmons algorithm is the use of laborious procedure for compressing the odd-length cycles called "flowers", which renders the algorithm unsuitable for use in real-scale systems. Other known algorithms differ from the Edmons algorithm only by a better data storage and computational organization, while maintaining complex actions in detecting and packaging the odd-length cycles.

The proposed approach of moving from an arbitrary graph to a bipartite graph prevented the occurrence of odd-length cycles, which has made it possible to significantly improve the algorithm efficiency. Further performance improvement is possible by building parallel versions of the algorithm and the optimal arrangement of data storage.

Keywords: matching, maximal matching, bipartite graph, prolonged path, assignment problem.

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CONSTRUCTION OF A STOCHASTIC MODEL FOR A WATER SUPPLY NETWORK WITH HIDDEN LEAKS AND A METHOD FOR DETECTING AND CALCULATING THE LEAKS (p. 29-38)

Andrei Tevyashev
Kharkiv National University of Radio Electronics,
Kharkiv, Ukraine
ORCID: http://orcid.org/0000-0001-5261-9874

Olga Matviyenko
Kharkiv National University of Radio Electronics,
Kharkiv, Ukraine
ORCID: http://orcid.org/0000-0001-7492-7616

Glib Nikitenko
Municipal Enterprise «KharkivVodokanal»,
Kharkiv, Ukraine
ORCID: http://orcid.org/0000-0001-5954-0712

We have constructed a stochastic model of a water supply network with leaks, which, compared to the previously proposed models (excluding leaks), more adequately describes the processes of transportation and water distribution in water supply systems. Mathematical modeling of water supply networks is associated with difficulties related to the huge dimensionality of actual water supply networks, limited information resources and operational data, which does not make it possible to assess parameters of the technological equipment and structure of a water supply network adequately enough. Therefore, an equivalent scheme is built for an actual water supply network based on its dictation points, which is then used for subsequent calculations. The task on building a scheme for an equivalent water supply network consists of three problems: identification of the structure, parameters, and state of a water supply network. The proposed method for leaks detection is based on the comparison of change in the magnitude of head at pumping stations and at the dictation points of a water supply network. Based on the stochastic model of a water supply network with leaks, we have constructed a method for calculating the magnitude of leaks, which implies the following: by knowing the head of water at the nodes of an equivalent water supply network and the approximate diameters of leaks at nodes, new values of heads at nodes of the equivalent water supply network are calculated. Then we again compute the magnitude of a leak by knowing the new head at the node and the diameter of the leaks. Upon completion of several such iterations, a conclusion is drawn on that starting at a certain step the magnitude of leaks and the heads at the nodes of the equivalent water supply network stop changing. By knowing the magnitude of leaks and head at each node within the equivalent water supply network, we determine the actual diameter of fistulas at each node. The proposed method for calculating the magnitude of leaks does not require financial costs or the use of additional equipment; it could be used by water utilities to detect and calculate the magnitude of leaks.

Keywords: water supply network, leaks, equivalent water supply network, stochastic model, pumping station.

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DEVELOPMENT OF THE CORRELATION METHOD FOR OPERATIVE DETECTION OF RECURRENT STATES (p. 39-46)

Boris Pospelov
Scientific-Methodical Center of Educational Institutions in the Sphere of Civil Defence, Kharkiv, Ukraine
ORCID: http://orcid.org/0000-0002-0957-3839

Vladimir Andronov
National University of Civil Defence of Ukraine, Kharkiv, Ukraine
ORCID: http://orcid.org/0000-0001-7486-482X

Evgeniy Rybka
National University of Civil Defence of Ukraine, Kharkiv, Ukraine
ORCID: http://orcid.org/0000-0002-5396-5151

Olekcii Krainiukov
V. N. Karazin Kharkiv National University, Kharkiv, Ukraine
ORCID: http://orcid.org/0000-0002-5264-3118

Kostiantyn Karpets
V. N. Karazin Kharkiv National University, Kharkiv, Ukraine
ORCID: http://orcid.org/0000-0001-6388-7647

Oleksandr Pirohov
National University of Civil Defence of Ukraine, Kharkiv, Ukraine
ORCID: http://orcid.org/0000-0002-0958-0801

Iryna Semenyshyna
State Agrarian and Engineering University in Podilya, Kamianets-Podilskyi, Ukraine
ORCID: http://orcid.org/0000-0001-9300-8914

Ruslan Kapitan
Cherkasy State Technological University, Cherkasy, Ukraine
ORCID: http://orcid.org/0000-0003-1039-6988

Alona Promska
National University of Civil Defence of Ukraine, Kharkiv, Ukraine
ORCID: http://orcid.org/0000-0003-0425-8917

Oleksii Horbov
National Technical University "Kharkiv Polytechnic Institute", Kharkiv, Ukraine
ORCID: http://orcid.org/0000-0002-8326-9413

The correlation method for operative detection of recurrent states in complex dynamical systems at irregular measurements was proposed. The concepts of correlation for the case of the vectors of states of the trajectory of dynamics of complex systems and estimates of vectors correlation for a fixed length fragment moving along the trajectory were generalized. The space with scalar product of states vectors is used to implement the method. Estimation of the magnitudes of correlations of state vectors makes it possible to interpret them as corresponding levels of energy interaction of states vectors and to detect degree of their recurrence. In this case, calculation of the magnitudes of correlation are carried out only based on the known measurements of the state vector and does not require determining the threshold and the method of distance calculation, traditionally used in the methods of recurrent plots. The efficiency of the proposed method was tested on a specific example of experimental data of the actual dynamics of the vector of states of pollution of the urban atmosphere. The following gas pollutants were considered as components of the vector of state: formaldehyde, ammonia and carbon dioxide. The obtained results in general indicate the efficiency of the proposed method. It was established experimentally that the correlation method in case of irregular measurements of atmospheric contaminations ensures the authenticity of detection of recurrent states, corresponding to maximum correlation of states. In this case, the correlation assessment should be conducted for a movable fragment of a trajectory of the states vector. The length of the fragment should not be more than 10 responses.

Keywords: correlation of states, energy interaction, fragment of trajectory of states, recurrent states, complex dynamical systems, gas atmospheric pollution.

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OPTIMIZING THE STRATEGY OF ACTIVITIES USING NUMERICAL METHODS FOR DETERMINING EQUILIBRIUM (p. 47-56)

Iryna Sievidova
Kharkiv Petro Vasylenko National Technical University of Agriculture, Kharkiv, Ukraine
ORCID: http://orcid.org/0000-0003-3703-4610

Tamila Oliynyk
Kharkiv National Agrarian University named after V. V. Dokuchaiev, p/o "Dokuchaevske-2", Kharkiv dist., Kharkiv reg., Ukraine
ORCID: http://orcid.org/0000-0002-3312-3133

Oleksandra Mandych
Kharkiv Petro Vasylenko National Technical University of Agriculture, Kharkiv, Ukraine
ORCID: http://orcid.org/0000-0002-4375-2208

Tetyana Kvyatko
Kharkiv Petro Vasylenko National Technical University of Agriculture, Kharkiv, Ukraine
ORCID: http://orcid.org/0000-0001-8963-3696

Iryna Romaniuk
Kharkiv Petro Vasylenko National Technical University of Agriculture, Kharkiv, Ukraine
ORCID: http://orcid.org/0000-0002-9257-2043

Larisa Leshchenko
Kharkiv National Agrarian University named after V. V. Dokuchaiev, p/o "Dokuchaevske-2", Kharkiv dist., Kharkiv reg., Ukraine
ORCID: http://orcid.org/0000-0002-6395-4827

Serhiy Vynohradenko
Kharkiv National Agrarian University named after V. V. Dokuchaiev, p/o "Dokuchaevske-2", Kharkiv dist., Kharkiv reg., Ukraine
ORCID: http://orcid.org/0000-0002-8520-6504

Serhii Plyhun
Kharkiv National Agrarian University named after V. V. Dokuchaiev, p/o "Dokuchaevske-2", Kharkiv dist., Kharkiv reg., Ukraine
ORCID: http://orcid.org/0000-0001-5251-0553

The paper considers issues on the theoretical substantiation of options for choosing an optimal strategy to integrate an agricultural enterprise into the wholesale market by using methodological tools of the non-cooperative game theory. We have proposed modeling the behavior of an agrarian enterprise in the market by achieving a Nash equilibrium under various scenarios of competitors' activities and volumes of information on market conditions.

The methodology has been substantiated to apply the iterative algorithms to calculate equilibria in a general class of non-quadratic convex polyhedra in order to form the methodologies and construct algorithms for a behavior of agricultural enterprises in market activity. It was determined that decision-making occurs in parallel to the real conditions of activity of an agricultural enterprise in the wholesale market. The comprehensive application of numerical methods based on solving the optimization problems provides a smooth approach to the Nash equilibrium. A game can have multiple isolated Nash equilibria if players have non-quadratic payment functions when solving such problems. Based on the above, the results were determined of local convergence, since global results have strong constraints in non-quadratic problems. However, there is a connection with semi-global practical asymptotic stability if players have quadratic payoff functions. It has been shown that there is a shift in the convergence in proportion to the amplitudes of disturbance signals and the third derivative of payoff functions for non-quadratic payoff functions. This shift in the convergence corresponds to the shift in a numerical example.

It has been determined that the learning strategy developed in accordance with the main provisions of the theory of games remains attractive if one has partial information on the state of the market. Application of the indicated action strategy provides a company with a possibility to improve its initial position by measuring its own payoff values only and not using estimates of potentially uncertain parameters. It has been proposed to use applied tools from the game theory to determine an optimal action strategy for an agricultural enterprise for its integration into the wholesale market of vegetable products.

Keywords: game theory, action strategy, agricultural enterprise, wholesale market, Nash equilibrium.

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FORMING A METHODOLOGY OF BASIC MATRICES IN THE STUDY OF POORLY CONDITIONED LINEAR SYSTEMS (p. 57-67)

Volodymyr Kudin
Taras Shevchenko National University of Kyiv, Kyiv, Ukraine
ORCID: http://orcid.org/0000-0002-5665-0868

Viacheslav Onotskyi
Taras Shevchenko National University of Kyiv, Kyiv, Ukraine
ORCID: http://orcid.org/0000-0002-1920-0905

Andriy Onyshchenko
Taras Shevchenko National University of Kyiv, Kyiv, Ukraine
ORCID: http://orcid.org/0000-0002-3194-1116

Yurii Stupak
National Metallurgical Academy of Ukraine, Dnipro, Ukraine
ORCID: http://orcid.org/0000-0002-7199-057X

Algorithm of the basic matrix method for analysis of properties of the system of linear arithmetic equation (SLAE) in various changes introduced in the model, in particular, when

including-excluding a group of rows and columns (based on "framing") without re-solving the problem from beginning has been improved. Conditions of compatibility (incompatibility) of restrictions were established and vectors of the fundamental solution system in a case of compatibility were established. Influence of accuracy of representing the model elements (mantis length, order value, thresholds of machine zero and overflow) and variants of computation organization on solution properties was studied. Specifically, effect of magnitude and completeness of rank was studied on an example of a SLAE with a poorly conditioned constraint matrix. A program was developed for implementation of conducting calculations using the basic matrix methods (BMM) and Gauss method, that is, long arithmetic was used for models with rational elements. Algorithms and computer-aided implementation of Gaussian methods and artificial basic matrices (as a variant of the basic matrix method) in MATLAB and Visual C++ environments with the use of the technology of exact calculation of the method elements, first of all, for poorly conditioned systems with different dimensions were proposed.

Using as an example Hilbert matrices, which are characterized as "inconvenient" matrices, an experiment was conducted to analyze properties of a linear system at different dimensions, accuracy of the input data and computation scenarios. Formats ("exact" and "inexact") of representation of model elements (mantis length, order value, thresholds of machine zero and overflow) as well as variants of organization of basic computation operations during calculation and their influence on solution properties have been developed. In particular, influence of rank magnitude and completeness was traced on an example of an SLAE with a poorly conditioned constraint matrix.

Keywords: basic matrix method, rectangular constraint matrix, poorly conditioned SLAE.

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