

**DOI: 10.15587/1729-4061.2020.203590**  
**PERFORMING ARITHMETIC OPERATIONS OVER**  
**THE ( $L$ - $R$ )-TYPE FUZZY NUMBERS (p. 6–11)**

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The issue of constructing a system of rules to perform binary operations over fuzzy numbers has been formulated and considered. The set problem has been solved regarding the ( $L$ - $R$ )-type fuzzy numbers with a compact carrier. Such a problem statement is predetermined by the simplicity of the analytical notation of these numbers, thereby making it possible to unambiguously set a fuzzy number by a set of values of its parameters. This makes it possible, as regards the ( $L$ - $R$ )-type numbers, to reduce the desired execution rules for fuzzy numbers to the rules for simple arithmetic operations over their parameters. It has been established that many cited works provide ratios that describe the rules for performing operations over the ( $L$ - $R$ )-type fuzzy numbers that contain errors. In addition, there is no justification for these rules in all cases.

In order to build a correct system of fuzzy arithmetic rules, a set of metarules has been proposed, which determine the principles of construction and the structure of rules for operation execution. Using this set of metarules has enabled the development and description of the system of rules for performing basic arithmetic operations (addition, subtraction, multiplication, division). In this case, different rules are given for the multiplication and division rules, depending on the position of the number carriers involved in the operation, relative to zero. The proposed rule system makes it possible to correctly solve many practical problems whose raw data are not clearly defined. This system of rules for fuzzy numbers with a compact carrier has been expanded to the case involving a non-finite carrier. The relevant approach has been implemented by a two-step procedure. The advantages and drawbacks of this approach have been identified.

**Keywords:** ( $L$ - $R$ )-type fuzzy numbers, compact carrier, rules for performing arithmetic operations.

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**DOI: 10.15587/1729-4061.2020.203865**  
**DEVELOPMENT OF A DUMMY GUIDED**  
**FORMULATION AND EXACT SOLUTION METHOD**  
**FOR TSP (p. 12–19)**

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A traveling salesman problem (TSP) is a problem whereby the salesman starts from an origin node and returns to it in such a way that every node in the network of nodes is visited once and that the total distance travelled is minimized. An efficient algorithm for the

TSP is believed not to exist. The TSP is classified as NP-hard and coming up with an efficient solution for it will imply  $NP=P$ . The paper presents a dummy guided formulation for the traveling salesman problem. To do this, all sub-tours in a traveling salesman problem (TSP) network are eliminated using the minimum number of constraints possible. Since a minimum of three nodes are required to form a sub-tour, the TSP network is partitioned by means of vertical and horizontal lines in such a way that there are no more than three nodes between either the vertical lines or horizontal lines. In this paper, a set of all nodes between any pair of vertical lines or horizontal lines is called a block. Dummy nodes are used to connect one block to the next one. The reconstructed TSP is then used to formulate the TSP as an integer linear programming problem (ILP). With branching related algorithms, there is no guarantee that numbers of sub-problems will not explode to unmanageable levels. Heuristics or approximating algorithms that are sometimes used to make quick decisions for practical TSP models have serious economic challenges. The difference between the exact solution and the approximated one in terms of money is very big for practical problems such as delivering household letters using a single vehicle in Beijing, Tokyo, Washington, etc. The TSP model has many industrial applications such as drilling of printed circuit boards (PCBs), overhauling of gas turbine engines, X-Ray crystallography, computer wiring, order-picking problem in warehouses, vehicle routing, mask plotting in PCB production, etc.

**Keywords:** traveling salesman problem, sub-tour, block, integer linear program, dummy.

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DOI: 10.15587/1729-4061.2020.205107

#### CONSTRUCTING A METHOD FOR SOLVING THE RICCATI EQUATIONS TO DESCRIBE OBJECTS PARAMETERS IN AN ANALYTICAL FORM (p. 20–26)

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This paper reports the established feature of non-linear differential equations as those that most adequately describe the properties of objects. Possible methods of their linearization have been analyzed. The issues related to solving the original equations in a linearized form have been defined. The Riccati equation has been given as an example.

For a special type Riccati equation, a method to solve it has been constructed, whereby the results are represented in an analytical form. It is based on the use of linearization and a special method of nondimensionalization.

A special feature of the constructed method is determined by its application not to the original equation but to its discrete analog. The result of solving it is an analytical expression based on elementary functions. It is derived from using the existing analytical solution (supporting, basic) to one of the equations of the examined type. All the original equations of the examined type have the same type of solution. This also applies to equations that had no previous analytical solution.

A formalized procedure for implementing the devised method has been developed. It makes it possible to link the analytical type of solution to the examined equation and known analytical solution to the basic one. The link is possible due to the equality of discrete analogs of the considered and basic equations. The equality of discrete analogs is provided by using a special nondimensionalization method.

The applicability of the method and the adequacy of the results obtained have been shown by comparing them with existing analytical solutions to two special type Riccati equations. In one case, the solution has movable special points. In the second case, a known solution has an asymptote but, at the positive values of the argument, has no special points.

The possibility of using the constructed method to solve the general Riccati equation has been indicated.

**Keywords:** Riccati equation, special points, linearization, nondimensionalization, analytical solution, elementary functions.

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**DOI: 10.15587/1729-4061.2020.205048**  
**DEVELOPMENT OF ITERATIVE ALGORITHMS**  
**FOR SOLVING THE INVERSE PROBLEM USING**  
**INVERSE CALCULATIONS (p. 27–34)**

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Iterative algorithms for solving the inverse problem, presented as a quadratic programming problem, developed by modifying algorithms based on the inverse calculation mechanism are proposed. Iterative algorithms consist in a sequential change of the argument values using iterative formulas until the function reaches the value that most corresponds to the constraint. Two solutions are considered: by determining the shortest distance to the line of the given level determined by the constraint, and by moving along the gradient. This approach was also adapted to solve more general nonlinear programming optimization problems. The solution of four problems is considered: formation of production output and storage costs, op-

timization of the securities portfolio and storage costs for the given volume of purchases. It is shown that the solutions obtained using iterative algorithms are consistent with the result of using classical methods (Lagrange multiplier, penalty), standard function of the MathCad package. In this case, the greatest degree of compliance was obtained using the method based on constructing the level line; the method based on moving along the gradient is more universal.

The advantage of the algorithms is a simpler computer implementation of iterative formulas, the ability to get a solution in less time than known methods (for example, the penalty method, which requires multiple optimizations of a modified function with a change in the penalty parameter). The algorithms can also be used to solve other nonlinear programming problems of the presented kind.

The paper can be useful for specialists when solving problems in the field of economics, as well as developing decision support systems.

**Keywords:** inverse calculations, function optimization, nonlinear programming, gradient method, inverse problem.

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**DOI: 10.15587/1729-4061.2020.205843**  
**ROBUST ESTIMATION OF THE AREA OF ADEQUACY OF FORECASTING ONE-PARAMETER MODEL OF EXPONENTIAL SMOOTHING (p. 35–42)**

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The problem of parameter synthesis of a forecasting one-parameter model of exponential smoothing for predictive estimation of indicators of the organizational and technical system is considered.

To select intervals of a given quality in the range of admissible values of the internal parameter, the criterion of absolute error of multiple forecasts is selected. It allowed the formation of an analytical retrospective model with «soft» constraints. As a result, a method of robust estimation of the adequacy area of the forecasting one-parameter exponential smoothing model is developed, which allows one to analytically evaluate the limits of the adequacy area of the forecasting model depending on the requirements for its retrospective accuracy. The proposed method allows the user to specify a set of permissible retrospective errors depending on the requirements of forecasting specifications. The proposed method can be used for parameter adjustment of one-parameter forecasting models and serves as a decision support tool in the forecasting process. The simulation results are interval estimates, which are preferable to point ones in the process of parameter synthesis. Unlike search methods, the analytical form of retrospective dependencies allows you to obtain a solution with high accuracy and, if necessary, provides the analyst with the opportunity for graphical analysis of the adequacy area of the model. The example shows the fragment of estimating the dynamics of the time series in a retrospective analysis with a depth of three values and specified limit relative errors of 1–4 %. Under such conditions, the area for a reasonable selection of the adjustment parameter is determined by the combined intervals of a width of about 20 % of the initial range of acceptable values.

**Keywords:** exponential smoothing, inverse verification, forecasting model adequacy, robust interval estimation.

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**DOI: 10.15587/1729-4061.2020.206308**  
**THE ALGORITHM FOR MINIMIZING BOOLEAN FUNCTIONS USING A METHOD OF THE OPTIMAL COMBINATION OF THE SEQUENCE OF FIGURATIVE TRANSFORMATIONS (p. 43–60)**

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The reported study has established the possibility of improving the productivity of an algorithm for the minimization of Boolean functions using a method of the optimal combination of the sequence of logical operations applying different techniques for gluing the variables – simple gluing and super-gluing.

The correspondence of intervals  $I(\alpha, \beta)$  in the Boolean space  $B^n$  has been established, given by a pair of Boolean vectors  $\alpha$  and  $\beta$ , such that  $\alpha \leq \beta$ , with a complete combinatorial system with the repeated  $2-(n, b)$ -designs. The internal components of the interval  $I(\alpha, \beta)$  correspond to the complete  $2-(n, b)$ -design system while external ones are determined by calculating the number of zeros or unities in the columns of the truth table of the assigned logical function. This makes it possible to use the theory of  $I(\alpha, \beta)$  intervals in the mathematical apparatus of  $2-(n, b)$ -design combinatorial systems to minimize Boolean functions by the method of equivalent figurative transformations, in particular, to perform automated search for the  $2-(n, b)$ -design systems in the structure of a truth table.

Experimental study has confirmed that the combinatorial  $2-(n, b)$ -design system and the consistent alternation of logical operations of super-gluing the variables (if such an operation is possible) and the simple gluing of variables in the first truth table improves the efficiency of the process and the reliability of results from minimizing the Boolean functions. This simplifies algorithmizing the search for the  $2-(n, b)$ -design system in the structure of a truth table of the assigned logical function, which would provide for the tool to further automate the search for the  $2-(n, b)$ -design system. In comparison with analogs, it enables increasing the productivity of the Boolean function minimization process by 100–200 % by using an optimum

alternation of operations of super gluing and simple gluing of variables by the method of equivalent figurative transformations.

There are reasons to argue about the possibility to improve the productivity of the Boolean function minimization process by the optimal combination of the sequences of the logical operations of super-gluing the variables and the simple gluing of variables by the method of equivalent figurative transformations.

**Keywords:** Boolean function minimization, optimal combination of the sequence of figurative transformations, Mahoney map.

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