

**ABSTRACT AND REFERENCES**  
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**ANALYSIS OF FREE OSCILLATIONS OF ROUND THIN PLATES OF VARIABLE THICKNESS WITH A POINT SUPPORT (p. 6–12)**

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This paper reports the derived general analytical solution to the IV-order differential equation with variable coefficients for the problem on free axisymmetric oscillations of a circular plate of variable thickness. The plate thickness changes along the radius  $p$  in line with the parabolic law  $h=H_0(1-\mu p)^2$ . When building a solution, the synthesis of the factorization method and the symmetry method was used. The factorization method has enabled us to represent the solution to the original IV-order equation as the sum of the solutions to the two respectively constructed II-order equations. The method of symmetry has produced precise solutions to these two equations.

The problem on a point fixation of the plate has been considered as a boundary case of the problem on the rigid fixation of the inner contour of a circular plate whose  $p \rightarrow 0$ . To this end, the general solution has been transformed into the form that pre-meets the conditions on a rigid point support. The result of such a transformation is a simpler solution, with only two permanent integration variables instead of four. As a result, the frequency equation for a plate under any conditions on the outer contour is significantly simplified because it is derived from the second-order determinant. The frequency equation for a plate with a point support and with a free edge at  $\mu=1.39127$ , which corresponds to the ratio of the limit thicknesses equal to 10.8, yielded the first five eigenvalues  $\lambda_i$  ( $i=1\div 5$ ). The oscillation shapes have been constructed as a graphic illustration for  $\lambda_i$  ( $i=1\div 3$ ). The numerical values of amplitude ratios have been given, as well as coordinates (relative radii) of the oscillation antinodes and nodal circles for each of the five oscillation shapes ( $i=1\div 5$ ). The derived numerical values of the oscillation parameters could in practice be used to initially identify the type of an oscillatory system and its possible characteristics for the case when a plate is fixed inside the inner contour of the small diameter. The criterial ratio of the fastening contour diameter to the plate diameter could serve the same purpose. If this ratio is equal to or less than 0.2, then it is permissible to assume that it is a point-type fastening. In this case, it is possible to calculate the circular plate oscillations with its internal contour fixed using the algorithm set out for a plate with a point support.

**Keywords:** free oscillations, symmetry method, thin plate, point support, analytical solution.

**References**

1. Trapezon, K. O. (2013). Method of symmetries at the vibrations of circular plates of variable thickness. Electronics and Communications, 17 (6), 66–77. doi: <https://doi.org/10.20535/2312-1807.2012.17.6.11401>
2. Trapezon, K. O., Trapezon, A. G. (2014). To the decision of task about the vibrations of circular plate with a thickness decreasing from a center on a protuberant parabola. Electronics and Communications, 18 (6), 44–53. doi: <https://doi.org/10.20535/2312-1807.2013.18.6.142696>
3. Trapezon, K. O. (2014). The decision of task about the axisymmetric natural vibrations of circular plate with a thickness decreasing from a center on a concave parabola. Electronics and Communications, 19 (5), 98–106. doi: <https://doi.org/10.20535/2312-1807.2014.19.5.38881>
4. Bitseno, K. B., Grammel', R. (1952). Tehnicheskaya dinamika. Vol. II. Moscow: GITTL, 638.
5. Hronin, D. V. (1970). Teoriya i raschet kolebaniy v dvigatelyah letatel'nykh apparatov. Moscow: Mashinostroenie, 412.
6. Timoshenko, S. P. (1975). Staticheskie i dinamicheskie problemy teorii uprugosti. Kyiv: Naukova dumka, 563.
7. Korenev, B. G. (1971). Vvedenie v teoriyu besselevykh funktsiy. Moscow: Nauka, 288.
8. Hache, F., Elishakoff, I., Challamel, N. (2017). Free vibration analysis of plates taking into account rotary inertia and shear deformation via three alternative theories: a Lévy-type solution. Acta Mechanica, 228 (10), 3633–3655. doi: <https://doi.org/10.1007/s00707-017-1890-8>
9. Jafari, N., Azhari, M. (2017). Bending Analysis of Moderately Thick Arbitrarily Shaped Plates with Point Supports Using Simple Hp Cloud Method. Iranian Journal of Science and Technology, Transactions of Civil Engineering, 41 (4), 361–371. doi: <https://doi.org/10.1007/s40996-017-0079-7>
10. Kiani, Y. (2016). Free vibration of carbon nanotube reinforced composite plate on point Supports using Lagrangian multipliers. Meccanica, 52 (6), 1353–1367. doi: <https://doi.org/10.1007/s11012-016-0466-3>
11. Li, R., Wang, B., Lv, Y., Zhang, Q., Wang, H., Jin, F. et. al. (2016). New analytic solutions for static problems of rectangular thin plates point-supported at three corners. Meccanica, 52 (7), 1593–1600. doi: <https://doi.org/10.1007/s11012-016-0500-5>
12. Li, R., Wang, B., Li, G. (2015). Analytic solutions for the free vibration of rectangular thin plates with two adjacent corners point-supported. Archive of Applied Mechanics, 85 (12), 1815–1824. doi: <https://doi.org/10.1007/s00419-015-1020-9>
13. Zhang, J., Ullah, S., Zhong, Y. (2019). Accurate free vibration solutions of orthotropic rectangular thin plates by straightforward finite integral transform method. Archive of Applied Mechanics, 90 (2), 353–368. doi: <https://doi.org/10.1007/s00419-019-01613-1>
14. Merneedi, A., RaoNalluri, M., Rao, V. V. S. (2017). Free vibration analysis of a thin rectangular plate with multiple circular and rectangular cut-outs. Journal of Mechanical Science and Technology, 31 (11), 5185–5202. doi: <https://doi.org/10.1007/s12206-017-1012-5>
15. Rahbar-Ranji, A., Shahbaztabar, A. (2016). Free Vibration Analysis of Moderately Thick Rectangular Plates on Pasternak Foundation with Point Supports and Elastically Restrained Edges by Using the Rayleigh–Ritz Method. Journal of Failure Analy-

- sis and Prevention, 16 (6), 1006–1023. doi: <https://doi.org/10.1007/s11668-016-0190-2>
16. Trapezon, K. A. (2015). Variant of method of symmetries in a task about the vibrations of circular plate with a decreasing thickness by law of concave parabola. Electronics and Communications, 20 (2), 90–99. doi: <https://doi.org/10.20535/2312-1807.2015.20.2.47781>
  17. Abramowitz, M., Stegun, I. (Eds.) (1972). Handbook of mathematical functions with formulas, graphs, and mathematical tables. Applied mathematics series - 55. Washington, D.C.
  18. Southwell, R. V. (1922). On the free transverse vibrations of a uniform circular disc clamped at its centre; and on the effects of rotation. Proceedings of the Royal Society of London. Series A, Containing Papers of a Mathematical and Physical Character, 101 (709), 133–153. doi: <https://doi.org/10.1098/rspa.1922.0032>

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## THE HOMOGENIZATION OF MULTI-MODULAR COMPOSITES AT THEIR LONGITUDINAL DEFORMATION (p. 13–19)

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A model has been proposed of the homogenization of a transversally-isotropic composite material, whose mechanical characteristics differ at the longitudinal stretching and compression. On its basis, the longitudinal elasticity module of the first kind has been derived, as well as a Poisson coefficient for a multimodular composite. These indicators are necessary to design structural elements made from composites. The object of the study is a unidirectional fibrous composite consisting of the isotropic elastic matrix and fibers. To determine the effective elastic constants, an approach has been suggested, which is based on the use of conditions for the alignment of the displacements of points in the homogenized composite, matrix, and fiber.

First, the displacements and stresses are determined for the matrix and fiber points at their joint axisymmetric stretching. An equation from the multimodular theory of elasticity was preliminarily obtained for solving this problem. Similar components of the stressed-strained state are determined at the same deformation of the cylindrical cell made from a homogeneous transversally-isotropic composite. The conditions for the displacement alignment, derived in solving the specified problems, are the equality of axial displacements in an arbitrary cross-section of the composite by a plane, parallel to the isotropy plane, and the radial displacements at the surface of the composite cell. The result of applying these conditions is the derived formulae for effective constants – the longitudinal module of elasticity of the first kind and a Poisson coefficient, which express these indicators through the mechanical characteristics of the matrix

and fiber, as well as the proportion of fibers in the composite cell volume. Similar formulae have been obtained for the longitudinal compression.

The derived effective elastic characteristics of a transversally-isotropic composite could be used when calculating the stressed-strained state of the structural elements made from it. In this case, one takes into consideration differences in the values of stresses and deformations under axial stretching and compression.

**Keywords:** homogenization, multimodular transversally-isotropic composite, stresses, displacement, deformation, effective constant.

## References

1. Klasztorny, M., Konderla, P., Piekarski, R. (2009). An exact stiffness theory for unidirectional xFRP composites. Mechanics of Composite Materials, 45 (1), 77–104. doi: <https://doi.org/10.1007/s11029-009-9064-y>
2. Cruz-González, O. L., Rodríguez-Ramos, R., Otero, J. A., Bravo-Castillero, J., Guinovart-Díaz, R., Martínez-Rosado, R. et. al. (2018). Viscoelastic effective properties for composites with rectangular cross-section fibers using the asymptotic homogenization method. Generalized Models and Non-Classical Approaches in Complex Materials 1, 203–222. doi: [https://doi.org/10.1007/978-3-319-72440-9\\_10](https://doi.org/10.1007/978-3-319-72440-9_10)
3. Daridon, L., Licht, C., Orankitjaroen, S., Pagano, S. (2016). Periodic homogenization for Kelvin-Voigt viscoelastic media with a Kelvin-Voigt viscoelastic interphase. European Journal of Mechanics - A/Solids, 58, 163–171. doi: <https://doi.org/10.1016/j.euromechsol.2015.12.007>
4. Pathan, M. V., Tagarielli, V. L., Patsias, S. (2017). Numerical predictions of the anisotropic viscoelastic response of uni-directional fibre composites. Composites Part A: Applied Science and Manufacturing, 93, 18–32. doi: <https://doi.org/10.1016/j.compositesa.2016.10.029>
5. Grebenyuk, S., Klymenko, M., Titova, O., Boguslavskaya, A. (2015). Effective longitudinal elastic modulus of the composite with viscoelastic matrix and transtropic fiber. 20th International Scientific Conference Mechanika. Kaunas: Kaunas University of Technology, 96–100.
6. Wang, Y., Huang, Z. (2017). A Review of Analytical Micromechanics Models on Composite Elastoplastic Behaviour. Procedia Engineering, 173, 1283–1290. doi: <https://doi.org/10.1016/j.proeng.2016.12.159>
7. Zhang, L., Yu, W. (2015). Variational asymptotic homogenization of elastoplastic composites. Composite Structures, 133, 947–958. doi: <https://doi.org/10.1016/j.compstruct.2015.07.117>
8. Ambartsumyan, S. A. (1982). Raznomodul'naya teoriya uprugosti. Moscow: Nauka, 320.
9. TSvelodu, I. Yu. (2008). O raznomodul'noy teorii uprugosti. Prikladnaya matematika i teoreticheskaya fizika, 49 (1), 157–164.
10. Yershova, A. U., Martirosov, M. I. (2015). Experimental study heterogeneous polymer composites with finely divided filler. Stroitel'naya mehanika inzhenernyh konstruktsiy i sooruzheniy, 5, 68–72.
11. Bessonov, D. E., Zezin, Yu. P., Lomakin, E. V. (2009). Multi-modulus Behavior of the Grained Composites on the Base of Unsaturated Polyethers. Izvestiya Saratovskogo universiteta. Novaya seriya. Seriya Matematika. Mehanika. Informatika, 9 (4), 9–13.
12. Pakhomov, B. M. (2017). Alternative model of isotropic material with different modulus. Herald of the Bauman Moscow State Technical University. Series Mechanical Engineering, 6, 35–45. doi: <https://doi.org/10.18698/0236-3941-2017-6-35-48>

13. Nassef, A. S. E., Dahim, M. A. (2016). New Bi-modular Material Approach to Buckling Problem of Reinforced Concrete Columns. *Mechanical Engineering Research*, 6 (1), 19–28. doi: <https://doi.org/10.5539/mer.v6n1p19>
14. Grebenyuk, S., Klymenko, M., Smoliankova, T., Koval, R. (2019). Effective Characteristics of the Multi-Modular Composites under Transverse Stretching. *Materials Science Forum*, 968, 511–518. doi: <https://doi.org/10.4028/www.scientific.net/msf.968.511>

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## DETERMINING THE FEATURES OF LOADING THE IMPROVED BEARING STRUCTURE OF A PLATFORM WAGON FOR THE TRANSPORTATION OF MILITARY EQUIPMENT (p. 20–26)

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The paper reports the improvement in the bearing structure of a platform wagon that transports military equipment and is involved in artillery fire. A special feature of the platform wagon is the presence of rotary sectors made from a composite material with viscous or elastic-viscous links, which makes it possible to absorb the kinetic energy that is transmitted to the frame when firing from the wagon, as well as enables the loading/unloading of military equipment from its side.

We have investigated the dynamic loading of the bearing structure of a platform wagon for military equipment transportation and combat operations. A mathematical model has been constructed, which takes into consideration the movement of the bearing structure of a platform wagon when firing from it. It has been considered that the platform wagon is loaded with two anti-aircraft guns. The mathematical model was solved in the Mathcad programming environment.

The study was conducted in a flat coordinate system. We have determined the accelerations that act on the bearing structure of a platform wagon. The maximum acceleration rate, in this case, is about  $3.6 \text{ m/s}^2$  at bouncing oscillations and  $4.0 \text{ m/s}^2$  at galloping oscillations. In other words, considering the proposed technical solutions, the dynamic loading of the bearing structure of a platform wagon in the vertical plane at firing decreases by almost 30 %. The magnitude of the acceleration is almost independent of the firing angle in this case.

The derived acceleration values have been taken into consideration in determining the strength indicators for the bearing structure of a platform wagon. Calculation was carried out by the method of finite elements in the CosmosWorks programming environment. The maximum equivalent stresses in the bearing structure of a platform wagon amounted to about 285 MPa; they

are concentrated in the region where the bearing structure rests on the trolley. Consequently, the durability of the bearing structure of a platform wagon is ensured.

Modal analysis of the bearing structure of a platform wagon has been conducted. The values of the natural oscillation frequencies are within allowable limits.

Our research would contribute to designing innovative structures for platform wagons.

**Keywords:** platform wagon, bearing structure, dynamic loading, modal analysis, combined transportation.

## References

1. Reidemeister, O. H., Kalashnyk, V. O., Shykunov, O. A. (2016). Modernization as a way to improve the use of universal cars. *Science and Transport Progress. Bulletin of Dnipropetrovsk National University of Railway Transport*, 2 (62), 148–156. doi: <https://doi.org/10.15802/stp2016/67334>
2. Kel'rich, M., Fedosov-Nikonov, D. (2016). The strength research of the long-wheelbase flatcar construction. *Visnyk Skhidnoukrainskoho Natsionalnoho universytetu imeni Volodymyra Dalia*, 1 (225), 90–94.
3. Donchenko, A. V., Fedosov-Nikonov, D. V. (2016). Method of the calculation-experimental research of the long-wheel-base platform design. *Zbirnyk naukovykh prats Derzhavnoho ekonomiko-tehnolohichnoho universytetu transportu. Seriya: Transportni sistemy i tekhnolohiyi*, 28, 53–60.
4. Šťastniak, P., Kurčík, P., Pavlík, A. (2018). Design of a new railway wagon for intermodal transport with the adaptable loading platform. *MATEC Web of Conferences*, 235, 00030. doi: <https://doi.org/10.1051/matecconf/20182350030>
5. Wiesław, K., Tadeusz, N., Michał, S. (2016). Innovative Project of Prototype Railway Wagon and Intermodal Transport System. *Transportation Research Procedia*, 14, 615–624. doi: <https://doi.org/10.1016/j.trpro.2016.05.307>
6. Ermolenko, I. Yu., Zheleznyak, V. N. (2016). Study on dynamics of rolling stock using an experimental laboratory car when driving on difficult sections of the road. *Sovremennye tehnologii. Sistemnyi analiz. Modelirovanie*, 4 (52), 199–203.
7. Divya Priya, G., Swarnakumari, A. (2014). Modeling and analysis of twenty tonnes heavy duty trolley. *International Journal of Innovative Technology and Research*, 2 (6), 1568–1580.
8. Chepurnoy, A. D., Litvinenko, A. V., Sheychenko, R. I., Grabrov, R. V., Chuban', M. A. (2015). Hodovye prochnostnye i dinamicheskie ispytaniya vagona-platformy. *Visnyk Natsionalnoho tekhnichnoho universytetu "KhPI". Seriya: Mashynoznavstvo ta SAPR*, 31 (1140), 111–128.
9. Domin, Yu. V., Cherniak, H. Yu. (2003). *Osnovy dynamiky vagonov*. Kyiv: KUETT, 269.
10. Fomin, O., Lovska, A., Masliyev, V., Tsymbaliuk, A., Burlutsky, O. (2019). Determining strength indicators for the bearing structure of a covered wagon's body made from round pipes when transported by a railroad ferry. *Eastern-European Journal of Enterprise Technologies*, 1 (7 (97)), 33–40. doi: <https://doi.org/10.15587/1729-4061.2019.154282>
11. Fomin, O. (2015). Improvement of upper bundling of side wall of gondola cars of 12-9745 model. *Metallurgical and Mining Industry*, 1, 45–48.
12. Lovskaya, A., Ryibin, A. (2016). The study of dynamic load on a wagon–platform at a shunting collision. *Eastern-European Journal of Enterprise Technologies*, 3 (7 (81)), 4–8. doi: <https://doi.org/10.15587/1729-4061.2016.72054>
13. Fomin, O. V., Burlutsky, O. V., Fomina, Yu. V. (2015). Development and application of cataloging in structural design of freight car building. *Metallurgical and Mining Industry*, 2, 250–256.

14. Kučera, P., Píštěk, V. (2017). Testing of the mechatronic robotic system of the differential lock control on a truck. International Journal of Advanced Robotic Systems, 14 (5), 172988141773689. doi: <https://doi.org/10.1177/1729881417736897>
15. Alyamovskiy, A. A. (2007). SolidWorks/COSMOSWorks 2006–2007. Inzhenernyi analiz metodom konechnykh elementov. Moscow, 784.
16. Alyamovskiy, A. A. (2010). COSMOSWorks. Osnovy rascheta konstruktsiy na prochnost' v srede SolidWorks. Moscow, 785.
17. Lovska, A. (2018). Simulation of Loads on the Carrying Structure of an Articulated Flat Car in Combined Transportation. International Journal of Engineering & Technology, 7 (4.3), 140–146. doi: <https://doi.org/10.14419/ijet.v7i4.3.19724>
18. Fomin, O., Gerlici, J., Lovska, A., Kravchenko, K., Prokopenko, P., Fomina, A., Hauser, V. (2019). Durability Determination of the Bearing Structure of an Open Freight Wagon Body Made of Round Pipes during its Transportation on the Railway Ferry. Communications-Scientific Letters of the University of Zilina, 21 (1), 28–34.
19. DSTU 7598:2014. Vahony vantazhni. Zahalni vymohy do rozrakhunkiv ta proektuvannia novykh i modernizovanykh vahoniv kol'yi 1520 mm (nesamokhidnykh) (2015). Kyiv, 162.
20. GOST 33211-2014. Vagony gruzovye. Trebovaniya k prochnosti i dinamicheskim kachestvam (2016). Moscow, 54.
21. BS EN 12663-2. Railway applications. Structural requirements of railway vehicle bodies. Freight wagons (2010). British Standards Document. doi: <https://doi.org/10.3403/30152552u>

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**CONSTRUCTION OF AN ALGORITHM FOR THE SELECTION OF RIGID STOPS IN STEEL-CONCRETE BEAMS UNDER THE ACTION OF A DISTRIBUTED LOAD (p. 27–35)**

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An algorithm has been developed to select rigid stops in steel-concrete beams under the action of distributed load. Concrete is connected rigidly to a steel sheet in order to perform the joint operation of the concrete and steel sheet. Such a connection in the beam is provided by rigid stops that prevent shifting efforts in the concrete and steel contact area. The efforts are determined through the turning angles between the two adjacent sections of the beam. A graph-analytical method for determining movements is used to determine the turning angles. In determining the deformations of a steel-concrete beam, the calculation is based on the reduced rigidities of cross-sections.

The purpose of this study is to optimize the structure of a steel-concrete beam by selecting the rational number and arrangement of rigid stops. This optimization would allow a more rational utilization of the structure's material – concrete and steel. That would reduce the cost of operations and the quantity of materials required in the production, installation, and operation of the considered structures.

An earlier proposed algorithm for the selection of rigid stops in steel-concrete beams under the action of a concentrated force has been expanded for the case of an evenly distributed load. When selecting the number of rigid stops, it is assumed that the magnitude of the distributed load acting on a beam, the mechanical characteristics of materials (steel and concrete), as well as the span of the beam and the size of its cross-section, are known. In contrast to the beams with a concentrated force in the middle, where the forces abide by a linear law, in the beams with an evenly distributed load the efforts in a steel strip change in line with a square parabola. Therefore, while the same step has been obtained for stops, it is not possible to achieve a situation at which efforts in all stops have the same value.

**Keywords:** steel-concrete beam, rigid stop, stop step, effort in a stop, reduced rigidity, graphic-analytical method.

**References**

1. Xing, Y., Han, Q., Xu, J., Guo, Q., Wang, Y. (2016). Experimental and numerical study on static behavior of elastic concrete-steel composite beams. Journal of Constructional Steel Research, 123, 79–92. doi: <https://doi.org/10.1016/j.jcsr.2016.04.023>
2. Patil, S. P., Sangle, K. K. (2016). Tests of steel fibre reinforced concrete beams under predominant torsion. Journal of Building Engineering, 6, 157–162. doi: <https://doi.org/10.1016/j.jobe.2016.02.004>
3. Vandolovskyi, S. S., Kostyuk, T. O., Rachkovskyi, O. V., Plakhotnikova, I. A. (2018). Technology of creation of steelfibrobeton with high strength to stretchings. Scientific Works of Kharkiv National Air Force University, 2 (56), 126–131. doi: <https://doi.org/10.30748/zhups.2018.56.18>
4. DBN V.2.6-160:2010. Stalelizobetonni konstruktsiyi (2011). Kyiv: Minrekhionbud Ukrainy, 93.
5. TKP EN 1994-1-1-2009 (02250). Eurocode 4: Design of composite steel and concrete structures Part 1-1. General rules and rules for buildings (2010). Minsk: Minstroyarhitektury, 95.
6. DSTU B V.2.6-216:2016. Rozrakhunok i konstruiuvannia ziednuvalnykh elementiv stale zalizobetonnykh konstruktsiy

- (2016). Kyiv: Ministerstvo rehionalnoho rozvytku, budivnytstva ta zhytlovo-komunalnogo hospodarstva Ukrayiny, 40.
7. Hsiao, P.-C., Lehman, D. E., Roeder, C. W. (2012). Improved analytical model for special concentrically braced frames. *Journal of Constructional Steel Research*, 73, 80–94. doi: <https://doi.org/10.1016/j.jcsr.2012.01.010>
  8. Mahmoud, A. M. (2016). Finite element modeling of steel concrete beam considering double composite action. *Ain Shams Engineering Journal*, 7 (1), 73–88. doi: <https://doi.org/10.1016/j.asej.2015.03.012>
  9. Luan, N. K., Bakhshi, H., Ronagh, H. R., Barkhordari, M. A., Amiri, G. G. (2011). Analytical solutions for the in-plane behavior of composite steel/concrete beams with partial shear interaction. *Asian Journal of Civil Engineering*, 12 (6), 751–771.
  10. Medvedev, V. N., Semeniuk, S. D. (2016). Durability and deformability of braced bending elements with external sheet reinforcement. *Magazine of Civil Engineering*, 3 (63), 3–15. doi: <https://doi.org/10.5862/mce.63.1>
  11. Zamaliev, F. S. (2018). Numerical and full-scale experiments of prestressed hybrid reinforced concrete-steel beams. *Vestnik MG-SU*, 13 (3 (114)), 309–321. doi: <https://doi.org/10.22227/1997-0935.2018.3.309-321>
  12. Rakhmonov, A. D., Solov'ov, N. P., Pozdeev, V. M. (2014). Computer modeling for investigating the stress-strain state of beams with hybrid reinforcement. *Vestnik MGSU*, 1, 187–195. doi: <https://doi.org/10.22227/1997-0935.2014.1.187-195>
  13. Utkin, V. A. (2010). The regulation of the neutral axis position when designing sections of steel reinforced concrete span structures. *Vestnik Sibirskoy gosudarstvennoy avtomobil'nodorozhnoy akademii*, 4 (18), 55–60.
  14. Storozhenko, L. I., Lapenko, O. I., Horb, O. H. (2010). Konstruktsiyi zalizobetonnykh perekryttiv po profilnomu nastylu iz zabezpecheniem sumisnoi roboty betonu i stali za dopomohoю skleuvannia. *Visnyk Natsionalnogo universytetu "Lvivska politehnika"*, 662, 360–365.
  15. Torkatyuk, V. I., Zolotova, N. M., Mel'man, V. A. (2003). Ispol'zovanie akrilovykh kleev dlya soedineniya betonnykh i zhelezobetonnykh konstruktsiy. *Municipal economy of cities*, 51, 61–68.
  16. Storozhenko, L. I., Krupchenko, O. A. (2010). Stalezalizobetonni balky iz zalizobetonnym verkhnim poiasom. *Visnyk Natsionalnogo universytetu "Lvivska politehnika"*, 662, 354–360.
  17. Bobalo, T. V., Blikharskyi, Z. Ya., Ilnytskyi, B. M., Kramarchuk, A. P. (2011). Osoblyvosti roboty stalebetonnykh balok armovanykh sterzhnevoi vysokomitsnoiu armaturoi riznykh klasiv. *Visnyk Natsionalnogo universytetu "Lvivska politehnika"*, 697, 35–48.
  18. Vahnenko, P. F., Hilobok, V. G., Andreyko, N. T., Yarovoy, M. L. (1987). *Raschet i konstruirovaniye chastej zhilyh i obshchestvennyh zdaniy*. Kyiv: Budlvel'nik, 424.
  19. Ying, H., Huawei, P., Xueyou, Q., Jun, P., Xiancun, L., Qiyun, P., Bao, L. (2017). Performance of Reinforced Concrete Beams Retrofitted by a Direct-Shear Anchorage Retrofitting System. *Procedia Engineering*, 210, 132–140. doi: <https://doi.org/10.1016/j.proeng.2017.11.058>
  20. John, A. T., Nwankwo, E., Orumu, S. T., Osuji, S. O. (2019). Structural Performance of Externally Strengthened Rectangular Reinforced Concrete Beams by Glued Steel Plate. *European Journal of Engineering Research and Science*, 4 (9), 101–106. doi: <https://doi.org/10.24018/ejers.2019.4.9.1480>
  21. Shkromada, O., Paliy, A., Nechyporenko, O., Naumenko, O., Nechyporenko, V., Burlaka, O. et. al. (2019). Improvement of functional performance of concrete in livestock buildings through the use of complex admixtures. *Eastern-European Journal of Enterprise Technologies*, 5 (6 (101)), 14–23. doi: <https://doi.org/10.15587/1729-4061.2019.179177>
  22. Petrov, A., Pavliuchenkov, M., Nanka, A., Paliy, A. (2019). Construction of an algorithm for the selection of rigid stops in steel concrete beams. *Eastern-European Journal of Enterprise Technologies*, 1 (7 (97)), 41–49. doi: <https://doi.org/10.15587/1729-4061.2019.155469>
  23. Petrov, A. N., Kobzeva, E. N., Krasyuk, A. G. (2015). Vybor optimal'nyh po stoimosti parametrov stalebetonnyh balok. Materialy III mizhnarodnoi naukovo-praktychnoi konferentsiyi. Kharkiv-Krasnyi Lyman, 330–336.
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- DOI: 10.15587/1729-4061.2020.204102**
- EXPERIMENTAL AND THEORETICAL BUBBLE GROWTH COMPARISON AT THE INITIAL STAGES OF HORIZONTAL INJECTION (p. 36–44)**
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- Two-phase liquid-gas injection constitutes an important industrial process that is used in most separators. At the early step of injection, a cylindrical bubble is formed. As time elapses, the bubble shape becomes more complex and very difficult to analyze. In this study, a simple analytical model is developed to explain bubble shape changes. The analytical model was developed based on water flow inertia that continually pushes the bubble while the drag force resists it so that the frontal area of the bubble increases. The bubble size and frontal area were estimated using the assumption of the equilibrium between inertia force and drag force neglecting viscous force. From the estimation, the role of the vortex ring from the difference between theoretical and experimental results can be identified. The analytical model was verified through experimental data collected on the shape deformation induced by bubble motion at the beginning of injection. The experimental data used as verification were measured from the bubble nose image with ten times repetition having the uncertainty of ±6 %. The experimental method is conducted by injecting a bubble along the horizontal direction into a water pool. The inertial force of the water flow in front of the bubble nose generates the bubble. The bubble suddenly changes its shape, moves in the form of a bubble jet, and undergoes gradual shape changes. The frontal area of the bubble increases and reaches a maximum at the terminal velocity point. The bubble shape deformation is affected by the inertial force of the water flow that pushes the bubble forward. Accordingly, the bubble changes its shape from cylindrical to spherical, and then to an ellipsoidal disk. When the bubble attains terminal velocity, the inertial force becomes equal to the drag force. The edge of the ellipsoidal disk bubble exhibits increased surface tension. The difference between experimental data and the analytical model

is due to the complex fluid and dynamic flow surrounding the bubble. The mathematical framework proposed in this work is envisaged to be an important tool for the prediction of the bubble frontal area.

**Keywords:** injection, deformation, bubble shape, frontal area.

## References

1. Chu, P., Waters, K. E., Finch, J. A. (2016). Break-up in formation of small bubbles: Break-up in a confined volume. *Colloids and Surfaces A: Physicochemical and Engineering Aspects*, 503, 88–93. doi: <https://doi.org/10.1016/j.colsurfa.2016.05.037>
2. Movafaghian, S., Jaua-Marturet, J. A., Mohan, R. S., Shoham, O., Kouba, G. E. (2000). The effects of geometry, fluid properties and pressure on the hydrodynamics of gas–liquid cylindrical cyclone separators. *International Journal of Multiphase Flow*, 26 (6), 999–1018. doi: [https://doi.org/10.1016/s0301-9322\(99\)00076-2](https://doi.org/10.1016/s0301-9322(99)00076-2)
3. Rosa, E. S., França, F. A., Ribeiro, G. S. (2001). The cyclone gas–liquid separator: operation and mechanistic modeling. *Journal of Petroleum Science and Engineering*, 32 (2-4), 87–101. doi: [https://doi.org/10.1016/s0920-4105\(01\)00152-8](https://doi.org/10.1016/s0920-4105(01)00152-8)
4. Bozzano, G., Dente, M. (2001). Shape and terminal velocity of single bubble motion: a novel approach. *Computers & Chemical Engineering*, 25 (4-6), 571–576. doi: [https://doi.org/10.1016/s0098-1354\(01\)00636-6](https://doi.org/10.1016/s0098-1354(01)00636-6)
5. Emami, A., Briens, C. (2008). Study of downward gas jets into a liquid. *AICHE Journal*, 54 (9), 2269–2280. doi: <https://doi.org/10.1002/aic.11524>
6. Tomiyama, A., Celata, G. P., Hosokawa, S., Yoshida, S. (2002). Terminal velocity of single bubbles in surface tension force dominant regime. *International Journal of Multiphase Flow*, 28 (9), 1497–1519. doi: [https://doi.org/10.1016/s0301-9322\(02\)00032-0](https://doi.org/10.1016/s0301-9322(02)00032-0)
7. Bari, S. D., Robinson, A. J. (2013). Experimental study of gas injected bubble growth from submerged orifices. *Experimental Thermal and Fluid Science*, 44, 124–137. doi: <https://doi.org/10.1016/j.expthermflusci.2012.06.005>
8. Rassame, S., Hibiki, T., Ishii, M. (2016). Void penetration length from air injection through a downward large diameter submerged pipe in water pool. *Annals of Nuclear Energy*, 94, 832–840. doi: <https://doi.org/10.1016/j.anucene.2016.04.046>
9. Bai, H., Thomas, B. G. (2001). Bubble formation during horizontal gas injection into downward-flowing liquid. *Metallurgical and Materials Transactions B*, 32 (6), 1143–1159. doi: <https://doi.org/10.1007/s11663-001-0102-y>
10. Mandal, A. (2010). Characterization of gas-liquid parameters in a down-flow jet loop bubble column. *Brazilian Journal of Chemical Engineering*, 27 (2), 253–264. doi: <https://doi.org/10.1590/s0104-66322010000200004>
11. Liu, Z., Reitz, R. D. (1997). An analysis of the distortion and breakup mechanisms of high speed liquid drops. *International Journal of Multiphase Flow*, 23 (4), 631–650. doi: [https://doi.org/10.1016/s0301-9322\(96\)00086-9](https://doi.org/10.1016/s0301-9322(96)00086-9)
12. Liu, L., Yan, H., Zhao, G. (2015). Experimental studies on the shape and motion of air bubbles in viscous liquids. *Experimental Thermal and Fluid Science*, 62, 109–121. doi: <https://doi.org/10.1016/j.expthermflusci.2014.11.018>
13. Hinze, J. O. (1955). Fundamentals of the hydrodynamic mechanism of splitting in dispersion processes. *AICHE Journal*, 1 (3), 289–295. doi: <https://doi.org/10.1002/aic.690010303>
14. Han, L., Luo, H., Liu, Y. (2011). A theoretical model for droplet breakup in turbulent dispersions. *Chemical Engineering Science*, 66 (4), 766–776. doi: <https://doi.org/10.1016/j.ces.2010.11.041>
15. Cihonski, A. J., Finn, J. R., Apte, S. V. (2013). Volume displacement effects during bubble entrainment in a travelling vortex ring. *Journal of Fluid Mechanics*, 721, 225–267. doi: <https://doi.org/10.1017/jfm.2013.32>
16. Gao, L., Yu, S. C. M. (2010). A model for the pinch-off process of the leading vortex ring in a starting jet. *Journal of Fluid Mechanics*, 656, 205–222. doi: <https://doi.org/10.1017/s0022112010001138>
17. Jiang, X. F., Zhu, C., Li, H. Z. (2017). Bubble pinch-off in Newtonian and non-Newtonian fluids. *Chemical Engineering Science*, 170, 98–104. doi: <https://doi.org/10.1016/j.ces.2016.12.057>
18. Tomiyama, A., Kataoka, I., Zun, I., Sakaguchi, T. (1998). Drag Coefficients of Single Bubbles under Normal and Micro Gravity Conditions. *JSME International Journal Series B*, 41 (2), 472–479. doi: <https://doi.org/10.1299/jsmeb.41.472>
19. Vincent, F., Le Goff, A., Lagubeau, G., Quéré, D. (2007). Bouncing Bubbles. *The Journal of Adhesion*, 83 (10), 897–906. doi: <https://doi.org/10.1080/00218460701699765>
20. Walter, J. F., Blanch, H. W. (1986). Bubble break-up in gas–liquid bioreactors: Break-up in turbulent flows. *The Chemical Engineering Journal*, 32 (1), B7–B17. doi: [https://doi.org/10.1016/0300-9467\(86\)85011-0](https://doi.org/10.1016/0300-9467(86)85011-0)
21. Moore, D. W. (1965). The velocity of rise of distorted gas bubbles in a liquid of small viscosity. *Journal of Fluid Mechanics*, 23 (4), 749–766. doi: <https://doi.org/10.1017/s0022112065001660>
22. Aoyama, S., Hayashi, K., Hosokawa, S., Tomiyama, A. (2016). Shapes of ellipsoidal bubbles in infinite stagnant liquids. *International Journal of Multiphase Flow*, 79, 23–30. doi: <https://doi.org/10.1016/j.ijmultiphaseflow.2015.10.003>
23. Hreiz, R., Lainé, R., Wu, J., Lemaitre, C., Gentric, C., Fünfschilling, D. (2014). On the effect of the nozzle design on the performances of gas–liquid cylindrical cyclone separators. *International Journal of Multiphase Flow*, 58, 15–26. doi: <https://doi.org/10.1016/j.ijmultiphaseflow.2013.08.006>
24. Tomita, Y., Robinson, P. B., Tong, R. P., Blake, J. R. (2002). Growth and collapse of cavitation bubbles near a curved rigid boundary. *Journal of Fluid Mechanics*, 466, 259–283. doi: <https://doi.org/10.1017/s0022112002001209>
25. Fei, Y., Pang, M. (2019). A treatment for contaminated interfaces and its application to study the hydrodynamics of a spherical bubble contaminated by surfactants. *Chemical Engineering Science*, 200, 87–102. doi: <https://doi.org/10.1016/j.ces.2019.01.052>
26. Chen, Y., Groll, M. (2006). Dynamics and shape of bubbles on heating surfaces: A simulation study. *International Journal of Heat and Mass Transfer*, 49 (5-6), 1115–1128. doi: <https://doi.org/10.1016/j.ijheatmasstransfer.2005.07.053>
27. Gharib, M., Rambod, E., Shariff, K. (1998). A universal time scale for vortex ring formation. *Journal of Fluid Mechanics*, 360, 121–140. doi: <https://doi.org/10.1017/s0022112097008410>
28. Canedo, E. L., Favelukis, M., Tadmor, Z., Talmon, Y. (1993). An experimental study of bubble deformation in viscous liquids in simple shear flow. *AICHE Journal*, 39 (4), 553–559. doi: <https://doi.org/10.1002/aic.690390403>
29. Uchiyama, T., Sasaki, S. (2014). Experimental Investigation of the Interaction between Rising Bubbles and Swirling Water Flow. *International Journal of Chemical Engineering*, 2014, 1–10. doi: <https://doi.org/10.1155/2014/358241>
30. Yuan, D., Xiao, Z., Chen, D., Zhong, Y., Yan, X., Xu, J., Huang, Y. (2016). Numerical Investigation on Bubble Growth and Sliding Process of Subcooled Flow Boiling in Narrow Rectangular Channel. *Science and Technology of Nuclear Installations*, 2016, 1–12. doi: <https://doi.org/10.1155/2016/7253907>

**DOI: 10.15587/1729-4061.2020.203131****A ROBOMECH CLASS PARALLEL MANIPULATOR WITH THREE DEGREES OF FREEDOM (p. 44–56)****Zh. Baigunchekov**Al-Farabi Kazakh National University, Almaty,  
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This paper presents the methods of structural-parametric synthesis and kinematic analysis of a parallel manipulator with three degrees of freedom working in a cylindrical coordinate system. This parallel manipulator belongs to a RoboMech class because it works under the set laws of motions of the end-effector and actuators, which simplifies the control system and improves its dynamics. Parallel manipulators of a RoboMech class work with certain structural schemes and geometrical parameters of their links. The considered parallel manipulator is formed by connecting the output point to a base using one passive and two active closing kinematic chains (CKC). Passive CKC have zero degree of freedom and it does not impose a geometrical constraint on the movement of the output point, so the geometrical parameters of the links of the passive CKC are freely varied. Active CKCs have active kinematic pairs and they impose geometrical constraints on the movement of the output point. The geometrical parameters of the links of the active CKCs are determined on the basis of the approximation problems of the Chebyshev and least-square approximations. For this, the equations of geometrical constraints are derived in the forms of functions of weighted differences, which are presented in the forms of generalized (Chebyshev) polynomials. This leads to linear iterative problems.

The direct and inverse problems of the kinematics of the investigated parallel manipulator are solved. In the direct kinematics problem, the coordinates of the output point are determined by the given position of the input links. In the inverse kinematics problem, the positions of the input links are determined by the coordinates of the output point. The direct and inverse problems of the kinematics of the investigated parallel manipulator are reduced to solving problems on the positions of Sylvester dyads. Numerical results of structural-parametric synthesis and kinematic analysis of the considered parallel manipulator are presented. The numerical results of the kinematic analysis show that the maximum deviation of the movement of the output point from the orthogonal trajectories is 1.65 %.

**Keywords:** parallel manipulator, RoboMech, cylindrical coordinate systems, Chebyshev and least-square approximations.

**References**

- Baigunchekov, Z., Kalimoldayev, M., Ibrayev, S., Izmambetov, M., Baigunchekov, T., Naurushev, B., Aisa, N. (2016). Parallel Manipulator of a Class RoboMech. Mechanism and Machine Science, 547–557. doi: [https://doi.org/10.1007/978-981-10-2875-5\\_45](https://doi.org/10.1007/978-981-10-2875-5_45)
- Baigunchekov, Z., Ibrayev, S., Izmambetov, M., Baigunchekov, T., Naurushev, B., Mustafa, A. (2019). Synthesis of Cartesian Manipulator of a Class RoboMech. Mechanisms and Machine Science, 69–76. doi: [https://doi.org/10.1007/978-3-030-00365-4\\_9](https://doi.org/10.1007/978-3-030-00365-4_9)
- Baigunchekov, Z., Izmambetov, M., Zhumasheva, Z., Baigunchekov, T., Mustafa, A. (2019). Parallel manipulator of a class RoboMech for generation of horizontal trajectories family. Mechanisms and Machine Science, 1395–1402. doi: [https://doi.org/10.1007/978-3-030-20131-9\\_137](https://doi.org/10.1007/978-3-030-20131-9_137)
- Assur, L. V. (1913). Investigation of Plane Hinged Mechanisms with Lower Pairs from the Point of View of their Structure and Classification (in Russian): Part I. Bull. Petrograd Polytech. Inst., 20, 309–386.
- Assur, L. V. (1914). Investigation of Plane Hinged Mechanisms with Lower Pairs from the Point of View of their Structure and Classification (in Russian): Part II. Bull. Petrograd Polytech. Inst., 21, 187–283.
- Yang, T.-L., Sun, D.-J. (2012). A General Degree of Freedom Formula for Parallel Mechanisms and Multiloop Spatial Mechanisms. Journal of Mechanisms and Robotics, 4 (1). doi: <https://doi.org/10.1115/1.4005526>
- Kutzbach, K. (1933). Einzelfragen aus dem Gebiet der Maschinenteile. Zeitschrift der Verein Deutscher Ingenieur, 77, 1168–1169.
- Meng, X., Gao, F., Wu, S., Ge, Q. J. (2014). Type synthesis of parallel robotic mechanisms: Framework and brief review. Mechanism and Machine Theory, 78, 177–186. doi: <https://doi.org/10.1016/j.mechmachtheory.2014.03.008>
- Burmeister, L. (1988). Lehrbuch der Kinematik. Leipzig.
- Schoenflies, A. (1886). Geometric der Bewegung in Synthetischer Darstellung. Leipzig.
- Bottema, O., Roth, B. (1979). Theoretical Kinematics. North-Holland Publishing Company, 558.
- Chebyshev, P. L. (1879). Sur Les Parallélogrammes Composés de Trois Éléments Quelconques. Mémoires de l'Académie des Sciences de Saint-Pétersbourg, 36, Suppl. 3.
- Levitskii, N. I. (1950). Design of Plane Mechanisms with Lower Pairs. Moscow-Leningrad, 182.
- Sarkisyan, Y. L., Gupta, K. C., Roth, B. (1973). Kinematic Geometry Associated With the Least-Square Approximation of a Given Motion. Journal of Engineering for Industry, 95 (2), 503–510. doi: <https://doi.org/10.1115/1.3438183>
- Sarkissyan, Y. L., Gupta, K. C., Roth, B. (1973). Spatial Least-Square Approximation of a Motion. IFFToM Int. Symposium on Linkages and Computer Design Methods. Vol. B. Bucharest, 512–521.
- Sarkisyan, Y. L., Gupta, K. C., Roth, B. (1979). Chebychev Approximations of Finite Point Sets with Application to Planar Kinematic Synthesis. Journal of Mechanical Design, 101 (1), 32–40. doi: <https://doi.org/10.1115/1.3454021>
- Sarkisyan, Y. L., Gupta, K. C., Roth, B. (1979). Chebychev Approximations of Spatial Point Sets Using Spheres and Planes. Journal of Mechanical Design, 101 (3), 499–503. doi: <https://doi.org/10.1115/1.3454084>
- McCarthy, J. M., Bodduluri, R. M. (2000). Avoiding singular configurations in finite position synthesis of spherical 4R linkages. Mechanism and Machine Theory, 35 (3), 451–462. doi: [https://doi.org/10.1016/s0094-114x\(99\)00005-1](https://doi.org/10.1016/s0094-114x(99)00005-1)

19. Plecnik, M. M., Michael McCarthy, J. (2015). Computational Design of Stephenson II Six-Bar Function Generators for 11 Accuracy Points. *Journal of Mechanisms and Robotics*, 8 (1). doi: <https://doi.org/10.1115/1.4031124>
20. Plecnik, M. M., McCarthy, J. M. (2016). Design of Stephenson linkages that guide a point along a specified trajectory. *Mechanism and Machine Theory*, 96, 38–51. doi: <https://doi.org/10.1016/j.mechmachtheory.2015.08.015>
21. Plecnik, M. M., McCarthy, J. M. (2016). Kinematic synthesis of Stephenson III six-bar function generators. *Mechanism and Machine Theory*, 97, 112–126. doi: <https://doi.org/10.1016/j.mechmachtheory.2015.10.004>

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**DESIGNING THE STRUCTURES OF DISCRETE SOLID-ALLOY ELEMENTS FOR BROACHING THE HOLES OF SIGNIFICANT DIAMETER BASED ON THE ASSESSMENT OF THEIR STRENGTH (p. 57–65)**

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This paper addresses the issues related to designing and estimating the strength of solid-alloy elements in the deforming broaches of significant diameter (exceeding 150 mm) for the developed process of discrete broaching. The tool limit condition was assessed based on two strength criteria: the specific potential energy of shape change and the maximum tangent stresses. Numerical modeling using the finite element method has made it possible to derive the distribution of equivalent stresses in the tool elements and the contact stresses at the surface of the contact between a solid-alloy insert and the body, which enabled the analysis of tool strength under loading. The simulation was performed under a single normal load, which ensured the versatility of the calculation for any contact pressure values. We have derived for-

mulae to calculate the acceptable contact pressure depending on unit load. The effect of the insert protrusion height over the body on the strength of tool elements has been established. We have derived engineering dependences that determine the required magnitude of insert protrusion over the body depending on the ultimate load. An example of calculating the strength of a prefabricated deforming element in the machining of a sleeve made from gray modified cast iron of hardness HB230 has been considered. Our calculations have shown that the deforming element designed for the new technological process corresponds to the conditions of strength, provided the ratio  $h_1/h=0.15$  is maintained (where  $h_1$  is the insert height above the body,  $h$  is the insert height). The results obtained could be used in engineering calculations when designing the prefabricated tool for discrete deformation, as well as to assess the strength of prefabricated tools, such as cutters, core drills, reamers, when refining external loads.

**Keywords:** deforming broaching, stressed state, solid alloy, discrete deforming element, element strength.

**References**

1. Smerichevskyi, S., Kryvoviaziuk, I., Raicheva, L. et al. (2017). Research on the development of the machine-building industry of Ukraine: state and prospects. Riga, 200. Available at: <https://philpapers.org/archive/SAR-41.pdf>
2. Nemyrovskyi, Y., Posvyatenko, E., Sardak, S. (2019). Technical-Economic Aspects of the Use of Technological Process of Deforming Broaching. *Advances in Design, Simulation and Manufacturing II*, 238–247. doi: [https://doi.org/10.1007/978-3-030-22365-6\\_24](https://doi.org/10.1007/978-3-030-22365-6_24)
3. Hosseini, A., Kishawy, H. A., Moetakef-Imani, B. (2016). Effects of Broaching Operations on the Integrity of Machined Surface. *Procedia CIRP*, 45, 163–166. doi: <https://doi.org/10.1016/j.procir.2016.02.352>
4. Sheikin, S. E., Pashchenko, E. A., Rostotsky, I. Yu., Gavrilova, V. S., Protishin, V. T. (2014). Process lubricant for deforming drawing of pieces made of titanium. *Metallurgy and Mining*, 4, 38–43.
5. Shepelenko, I. V., Warouma, A., Sherkin, V. V. (2016). Restoration of bronze bushes by the method of surface plastic deformation. *International Journal of Engineering & Technology*, 5 (1), 29. doi: <https://doi.org/10.14419/ijet.v5i1.5651>
6. Grushko, A. V. (2014). Determination of work hardening during deforming broaching of thick-walled pipes. *Scientific Herald of the Donbass State Engineering Academy*, 2 (14E), 18–26.
7. Zanger, F., Boev, N., Schulze, V. (2014). Surface Quality after Broaching with Variable Cutting Thickness. *Procedia CIRP*, 13, 114–119. doi: <https://doi.org/10.1016/j.procir.2014.04.020>
8. Studenets, S. F., Yeromin, P. M., Chernyavsky, O. V. (2015). Influence of deformation conditions during the machining of hard-alloy combined broaches on the structure and hardening of the surface layer of cast irons. *Superhard materials*, 4, 91–99.
9. Posviatenko, E. K., Nemyrovskyi, Ya. B., Cherniavskyi, O. V., Yeromin, P. M. (2017). *Mekhanika kombinovanoho protiahuvannia hrafitovimisnykh chavuniv*. Kropyvnytskyi, 286.
10. Rozenberh, A. M., Rozenberh, O. A. (1990). *Mekhanyka plastycheskoho deformyrovaniya v protsessakh rezanyia y deformyruishchego protiahuvannya*. Kyiv: Naukova dumka, 320.
11. Rozenberg, A. M., Rozenberg, O. A., Posvyatenko, E. K. et. al. (1978). *Raschet i proektirovanie tverdosplavnih deformiruyushchikh protyazhek i protessha protyagivaniya*. Kyiv: Naukova dumka, 256.
12. Nemirovskiy, Ya. B., Derevets, L. I., Polotnyak, S. B. (2004). *Vliyanie geometrii deformiruyushchego elementa na ego prochnost'*. *Sverhtverdyie materialy*, 2, 60–65.

13. Balaganskaya, E. A., Golodenko, B. A., Nemirovskiy, Ya. B., Tsehanov, Yu. A. (2001). Matematicheskoe modelirovanie protsessa deformiruyushchego protyagivaniya. Voronezh, 194.
14. Rozenberg, O. A., Nemirovskiy, Ya. B., Sheykin, S. E., Vlas'yuk, Z. G. (1987). Primenenie iznosostoykih pokrytiy na rabochih elementah deformiruyushchih protyazhek. Sverhtverdye materialy, 1, 36–41.
15. Protosenya, A., Karasev, M., Ochkurov, V. (2017). Introduction of the method of finite-discrete elements into the Abaqus/Explicit software complex for modeling deformation and fracture of rocks. Eastern-European Journal of Enterprise Technologies, 6 (7 (90)), 11–18. doi: <https://doi.org/10.15587/1729-4061.2017.116692>
16. Polotnyak, S. B. (2008). Metodika chisel'nogo modeliuvannya protsesiv martensitnih fazovih peretvoreni' u malih obsyagah materialiv pri deformuvannii na almaznih kovadlah. Sverhtverdye materialy, 2, 13–28.
17. Lavrikov, S. A., Rukin, A. Yu. (1994). Konechnoelementnoe modelirovanie staticheskikh i dinamicheskikh lineynyh i nelineynyh termomechanicheskikh protsessov v trehmernyh konstruktsiyah proizvol'noy formy na personal'nyh EVM FEM\_TOOLS. Katalog. Programmye produkty Ukrayiny. TEHNO, 54.
18. Loshak, M. G. (1984). Prochnost' i dolgovechnost' tverdyh splavov. Kyiv: Naukova dumka, 328.

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## THE REFINED STRENGTH CALCULATION AND OPTIMIZATION OF THE INNER GEOMETRY OF CYLINDRICAL BEARING UNITS (p. 66–78)

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Closed bearing units for railway rolling stock shall operate over 800,000 km or during 8 years of operating life (and, in the near future, 1 million km and 10 years) without any maintenance. In order to achieve such high operational indicators, it is necessary, already at the design stage of closed bearing units, to ensure almost absence of wear during the entire specified operating life.

This paper reports the results of the optimal design of the elements in the internal geometry of closed bearings based on refined mathematical models using an example of the cylindrical axlebox bearing unit "DUPLEX" for 1520 gauge rolling stock. The chosen principal mathematical model was a geometrically nonlinear contact problem from the theory of elasticity, which was solved using a finite element method.

An original non-linear finite-element model of the multi-contact problem has been developed, taking into consideration the contact deformations "rail-wheel", the deformation of a wheel-set axis, the deformation of the axlebox and bearing rings during contact with all rollers. The model makes it possible to clarify the distribution of loads in the circumferential direction and, accordingly, the maximum load on a roller. The same model could be used, among other things, to analyze the wear of a wheel flange and the effect of gap difference on the bearing wear.

A mathematical model and an objective function have been constructed to optimize the profile of a roller ("crown", the generatrix of the lateral surface of rotation) considering the ac-

cumulation of damage as a result of the "irregular" loading the roller surface points due to contacts with both the outer and inner rings.

The shapes of the roller's face and the ring's operating flange have been optimized that are in contact in the axial direction, which has helped establish that the "anthropologically shaped" convex face of the roller and the concave flange of the ring are optimal. To simplify the structure technologically, a variant with a conical surface of the flange with the optimal "camber" value has been accepted instead of the concave flange of the ring.

**Keywords:** closed bearing unit, crown, roller, multi-contact problem, finite element method, mathematical model.

## References

1. Harris, T. A., Kotzalas, M. N. (2006). Essential Concepts of Bearing Technology. CRC Press, 392. doi: <https://doi.org/10.1201/9781420006599>
2. Brändlein, J., Eschmann, P., Hasbargen, L., Weigahd, K. (1999). Ball and Roller Bearings: Theory, Design and Application. Wiley, 642.
3. Harnoy, A. (2002). Bearing Design in Machinery. Engineering Tribology and Lubrication. CRC Press, 664. doi: <https://doi.org/10.1201/9780203909072>
4. Mason, M. A., Cartin, C. P., Shahidi, P., Fett, M. W., Wilson, B. M. (2014). Hertzian Contact Stress Modeling in Railway Bearings for Assorted Load Conditions and Geometries. 2014 Joint Rail Conference. doi: <https://doi.org/10.1115/jrc2014-3846>
5. Yang, K., Zhang, G., Wang, Y. W., Cai, S. (2019). Finite element analysis on contact stress of high-speed railway bearings. IOP Conference Series: Materials Science and Engineering, 504, 012073. doi: <https://doi.org/10.1088/1757-899x/504/1/012073>
6. Gopalakrishnan, T., Murugesan, R. (2015). Contact analysis of roller bearing using finite element method. Vels Journal of Mechanical Engineering, 2 (2), 30–33. Available at: [https://www.researchgate.net/publication/305768098\\_CONTACT\\_ANALYSIS\\_OF\\_ROLLER\\_BEARING\\_USINGFINITE\\_ELEMENT\\_METHOD](https://www.researchgate.net/publication/305768098_CONTACT_ANALYSIS_OF_ROLLER_BEARING_USINGFINITE_ELEMENT_METHOD)
7. Pandiyarajan, R., Starvin, M. S., Ganesh, K. C. (2012). Contact Stress Distribution of Large Diameter Ball Bearing Using Hertzian Elliptical Contact Theory. Procedia Engineering, 38, 264–269. Available at: <https://cyberleninka.org/article/n/531103>
8. Shah, D. B., Patel, K. M., Trivedi, R. D. (2016). Analyzing Hertzian contact stress developed in a double row spherical roller bearing and its effect on fatigue life. Industrial Lubrication and Tribology, 68 (3), 361–368. doi: <https://doi.org/10.1108/ilt-06-2015-0082>
9. Nabhan, A., Ghazaly, N. (2015). Contact Stress Distribution of Deep Groove Ball Bearing Using ABAQUS. Journal of the Egyptian Society of Tribology, 12 (1), 49–61. Available at: [https://www.academia.edu/25070023/Contact\\_Stress\\_Distribution\\_of\\_Deep\\_Groove\\_Ball\\_Bearing\\_Using\\_ABAQUS](https://www.academia.edu/25070023/Contact_Stress_Distribution_of_Deep_Groove_Ball_Bearing_Using_ABAQUS)
10. Puşcaşu, A. M., Lupescu, O., Bădănaç, A. (2017). Analysis of cylindrical roller bearings design in order to optimize the classical process using FEM. MATEC Web of Conferences, 112, 06017. doi: <https://doi.org/10.1051/matecconf/201711206017>
11. Li, S., Motooka, M. (2017). A finite element method used for contact analysis of rolling bearings. The 8th International Conference on Computational Methods (ICCM2017). Available at: <https://www.sci-en-tech.com/ICCM2017/PDFs/2199-9211-1-PB.pdf>
12. Demirhan, N., Kanber, B. (2008). Stress and Displacement Distributions on Cylindrical Roller Bearing Rings Using FEM#. Mechanics Based Design of Structures and Machines, 36 (1), 86–102. doi: <https://doi.org/10.1080/15397730701842537>

13. Hao, X., Gu, X., Zhou, X., Liao, X., Han, Q. (2018). Distribution characteristics of stress and displacement of rings of cylindrical roller bearing. Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science, 233 (12), 4348–4358. doi: <https://doi.org/10.1177/0954406218820551>
14. Lin, F., Zhao, Y. X. (2008). Finite Element Analysis on the Fatigue Stresses of a Railway Vehicle Roller Bearing. Advanced Materials Research, 44-46, 935–941. doi: <https://doi.org/10.4028/www.scientific.net/amr.44-46.935>
15. Chen, G., Wang, H. (2016). Contact stress and radial stiffness of a cylindrical roller bearing with corrected roller generator. Transactions of the Canadian Society for Mechanical Engineering, 40 (5), 725–738. doi: <https://doi.org/10.1139/tcsme-2016-0059>
16. Tong, V.-C., Hong, S.-W. (2017). Modeling and analysis of double-row cylindrical roller bearings. Journal of Mechanical Science and Technology, 31 (7), 3379–3388. doi: <https://doi.org/10.1007/s12206-017-0627-x>
17. Simson, E. A., Anatskiy, Yu. P., Ovcharenko, V. V., Trohman, M. V., Zenkevich, Yu. A. (2009). Optimizatsiya obrazuyushchey poverhnosti rolika podshipnika kacheniya. Vestnik Nats. tehn. un-ta "KhPI", 30, 8–11. Available at: [http://library.kpi.kharkov.ua/files/Vestniki/2009\\_30.pdf](http://library.kpi.kharkov.ua/files/Vestniki/2009_30.pdf)
18. Simson, E. A., Anatskiy, Yu. P., Ovcharenko, V. V., Trohman, M. V., Zenkevich, Yu. A. (2009). Optimizatsiya bortov kolets i tortsevoy poverhnosti rolika podshipnika kacheniya. Vestnik Nats. tehn. un-ta "KhPI", 42, 8–11. Available at: [http://library.kpi.kharkov.ua/files/Vestniki/2009\\_42.pdf](http://library.kpi.kharkov.ua/files/Vestniki/2009_42.pdf)
19. Wang, Z. W., Meng, L. Q., Hao, W. S., Zhang, E. (2010). Finite Element Method Analysis and Optimal Design of Roller Convexity of Tapered Roller Bearing. Advanced Materials Research, 139-141, 1079–1083. doi: <https://doi.org/10.4028/www.scientific.net/amr.139-141.1079>
20. Girshfeld, A. M., Anatskiy, Yu. P., Simson, E. A., Ovcharenko, V. V. (2009). Pat. No. 44588. Method for designing optimal geometric parameters of contacting end surfaces of roller bearing. No. u200903804; declared: 17.04.2009; published: 12.10.2009, Bul. No. 19.
21. Ovcharenko, V. V., Simson, E. A., Girshfeld, A. M., Anatskiy, Yu. P. (2009). Pat. No. 44969. Method for designing optimal geometric parameters of generatrix of working surface of roller of roller bearing. No. u200903761; declared: 17.04.2009; published: 26.10.2009, Bul. No. 20.
22. Girshfeld, A. M., Rukavishnikov, V. F., Semykin, S. I., Shcherbinina, A. V. (2009). Pat. No. 2425767 RF. Buksovi podshipnikoviy uzel. declared: 14.12.2009; published: 10.08.2011.
23. Girshfeld, A. M., Simson, E. A., Ovcharenko, V. V. (2010). Pat. No. 98790 RF. Rolikoviy podshipnik.

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## STUDYING THE STEADY-STATE VIBRATIONS OF A TWOMASS VIBRATORY MACHINE EXCITED BY A PASSIVE AUTO-BALANCER (p. 79–86)

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Analytical-numerical methods have been applied to investigate the steady-state vibrations of a two-mass vibratory machine with rectilinear translational motion of platforms and a vibration exciter in the form of a ball, a roller, or a pendulum auto-balancer.

A procedure for studying the modes of load jamming has been devised for the systems similar to the one under consideration. The procedure is based on the idea of parametric solution to the problem of finding the frequencies of load jamming and a bifurcation theory of motion.

It has been established that a two-mass vibratory machine has two resonance frequencies of rotor rotation and two corresponding shapes of platform oscillations. The use of the procedure has shown that for the case of small resistance forces, a vibratory machine:

- has five possible modes of load jamming, with the first shape of resonance vibrations of platforms being excited under modes 1 and 2, the second shape – 3 and 4, and, under the mode 5, the frequency of load jamming is close to the frequency of rotor rotation;

- demonstrates stable jamming modes under the odd (1, 3, 5) load jamming modes;

- shows that the jamming modes 1 and 2 are suitable to excite the resonance oscillations of platforms and for industrial application;

- exhibits that increasing the rotor speed monotonously increases the amplitudes of platform oscillations corresponding to a certain jamming mode;

- proves that the amplitude of resonance platform oscillations can be controlled by changing the rotor rotation velocity.

The viscous resistance forces acting on a first platform reduce (up to the complete elimination) the first range of rotor speeds, at which the first resonance shape of platform oscillations is excited.

The internal forces of viscous resistance, acting between the platforms, reduce (up to the complete elimination) the second range of rotor speeds, at which the second shape of resonance platform oscillations is excited.

The viscous resistance forces acting on the loads at motion relative to an auto-balancer reduce both ranges.

**Keywords:** inertial vibration exciter, two-frequency vibrations, resonance vibratory machine, auto-balancer, two-mass vibratory machine, Sommerfeld effect.

## References

1. Gorlach, E. A., Stepanova, N. Yu. (2016). Ispol'zovanie netraditsionnogo rastitel'nogo syr'ya v proizvodstve varenih kolbas. Izvestiya Sankt-Peterburgskogo gosudarstvennogo agrarnogo universiteta, 36 (77) – 37 (78). Available at: <http://ir.nmu.org.ua/handle/123456789/3086>
2. Gursky, V., Kuzio, I., Korendiy, V. (2018). Optimal Synthesis and Implementation of Resonant Vibratory Systems. Universal Journal of Mechanical Engineering, 6 (2), 38–46. doi: <https://doi.org/10.13189/ujme.2018.060202>
3. Gursky, V. M., Kuzio, I. V., Lanets, O. S., Kisała, P., Tolegenova, A., Syzdykpayeva, A. (2019). Implementation of dual-frequency resonant vibratory machines with pulsed electromagnetic drive. Przeglad Elektrotechniczny, 4, 43–48. doi: <https://doi.org/10.15199/48.2019.04.08>
4. Fedorenko, I. Y., Gnezdilov, A. A. (2016). The dynamic properties of a two-mass vibration technological machine. Vestnik Altayskogo gosudarstvennogo agrarnogo universiteta, 3 (137), 179–183.
5. Lanets, O. S., Hurskyi, V. M., Lanets, O. V., Shpak, Ya. V. (2014). Obgruntuvannia konstruktsiyi ta modeliuvannia roboty rezonansnoho dvomasovoho vibrostola z inertsiynym pryvodom. Visnyk Natsionalnoho universytetu "Lvivska politekhnika", 788, 28–36. Available at: <http://ena.lp.edu.ua:8080/bitstream/ntb/24646/1/6-28-36.pdf>

6. Makarenkov, O. Y. (2013). The asymptotic stability of the oscillations of a two-mass resonance sifter. *Journal of Applied Mathematics and Mechanics*, 77 (3), 287–295. doi: <https://doi.org/10.1016/j.jappmathmech.2013.09.004>
7. Antipov, V. I., Palashova, I. V. (2010). Dynamics of a two-mass parametrically excited vibration machine. *Journal of Machinery Manufacture and Reliability*, 39 (3), 238–243. doi: <https://doi.org/10.3103/s1052618810030052>
8. Zhao, J., Liu, L., Song, M., Zhang, X. (2015). Influencing Factors of Anti-Resonant Inertial Resonant Machine Vibration Isolation System. 2015 3rd International Conference on Computer and Computing Science (COMCOMS). doi: <https://doi.org/10.1109/comcoms.2015.22>
9. Xiaohao, L., Tao, S. (2016). Dynamic performance analysis of nonlinear anti-resonance vibrating machine with the fluctuation of material mass. *Journal of Vibroengineering*, 18 (2), 978–988. Available at: <https://www.jvejournals.com/article/16559/pdf>
10. Filimonikhin, G., Yatsun, V. (2015). Method of excitation of dual frequency vibrations by passive autobalancers. *Eastern-European Journal of Enterprise Technologies*, 4 (7 (76)), 9–14. doi: <https://doi.org/10.15587/1729-4061.2015.47116>
11. Sommerfeld, A. (1904). Beitrage zum dinamischen Ausbay der Festigkeitslehre. *Zeitschrift des Vereins Deutsher Jngenieure*, 48 (18), 631–636
12. Lu, C.-J., Tien, M.-H. (2012). Pure-rotary periodic motions of a planar two-ball auto-balancer system. *Mechanical Systems and Signal Processing*, 32, 251–268. doi: <https://doi.org/10.1016/j.ymssp.2012.06.001>
13. Artyunin, A. I., Eliseyev, S. V. (2013). Effect of “Crawling” and Peculiarities of Motion of a Rotor with Pendular Self-Balancers. *Applied Mechanics and Materials*, 373–375, 38–42. doi: <https://doi.org/10.4028/www.scientific.net/amm.373-375.38>
14. Yaroshevich, N. P., Silivoniuk, A. V. (2013). About some features of run-updynamic vibration machines with self-synchronizing inertion vibroexciters. *Naukovi visnyk Natsionalnoho hirnychoho universytetu*, 4, 70–75. Available at: [http://nbuv.gov.ua/UJRN/Nvngu\\_2013\\_4\\_14](http://nbuv.gov.ua/UJRN/Nvngu_2013_4_14)
15. Kuzo, I. V., Lanets, O. V., Gursky, V. M. (2013). Synthesis of low-frequency resonance vibratory machines with an aeroinertia drive. *Naukovi visnyk Natsionalnoho hirnychoho universytetu*, 2, 60–67. Available at: [http://nbuv.gov.ua/UJRN/Nvngu\\_2013\\_2\\_11](http://nbuv.gov.ua/UJRN/Nvngu_2013_2_11)
16. Artyunin, A. I., Barsukov, S. V., Sumenkov, O. Y. (2019). Peculiarities of Motion of Pendulum on Mechanical System Engine Rotating Shaft. Proceedings of the 5th International Conference on Industrial Engineering (ICIE 2019), 649–657. doi: [https://doi.org/10.1007/978-3-030-22041-9\\_70](https://doi.org/10.1007/978-3-030-22041-9_70)
17. Yatsun, V., Filimonikhin, G., Dumenko, K., Nevdakha, A. (2017). Equations of motion of vibration machines with a translational motion of platforms and a vibration exciter in the form of a passive auto-balancer. *Eastern-European Journal of Enterprise Technologies*, 5 (1 (89)), 19–25. doi: <https://doi.org/10.15587/1729-4061.2017.111216>
18. Yatsun, V., Filimonikhin, G., Dumenko, K., Nevdakha, A. (2018). Search for the dualfrequency motion modes of a dualmass vibratory machine with a vibration exciter in the form of passive autobalancer. *Eastern-European Journal of Enterprise Technologies*, 1 (7 (91)), 47–54. doi: <https://doi.org/10.15587/1729-4061.2018.121737>
19. Yatsun, V., Filimonikhin, G., Podoprygora, N., Pirogov, V. (2019). Studying the excitation of resonance oscillations in a rotor on isotropic supports by a pendulum, a ball, a roller. *Eastern-European Journal of Enterprise Technologies*, 6 (7 (102)), 32–43. doi: <https://doi.org/10.15587/1729-4061.2019.182995>
20. Filimonikhin, G., Yatsun, V., Filimonikhina, I. (2020). Investigation of oscillations of platform on isotropic supports excited by a pendulum. E3S Web of Conferences, 168, 00025. doi: <https://doi.org/10.1051/e3sconf/202016800025>

**DOI: 10.15587/1729-4061.2020.202483****SUBSTANTIATION OF THE STABILITY OF HAULAGE DRIFTS WITH PROTECTIVE STRUCTURES OF DIFFERENT RIGIDITY (p. 87–96)****Igor Iordanov**LLC “MC ELTEKO”, Kostiantynivka, Ukraine  
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The purpose of the current research is to substantiate the conditions for the stability of haulage drifts when developing steep coal seams.

The process of modeling the stability of haulage drifts has established that the stressed-strained state of side rocks in a coal-producing array that includes workings depends on the physical and mechanical properties of the roof and soil of the worked coal seam, the rigidity of protective structures, and the length of the roof section supported by a protective structure. Increasing the length of the roof section supported by a protective structure, at the minimal rigidity of pliable supports, increases the zone of smooth bending of the side rocks over a haulage drift and decreases the level of their stressed-strained state.

It has been proven that when maintaining mining workings in deep mines, a reduction in the stressed-strained state of the side rocks, when applying the filling of the worked space, occurs as a result of the sealing of the filling massif on which the roof rocks are based when the values of the compaction factor of the source material accept maximum values equal to  $k_{comp}=1.5–1.53$ . When using artificial pliable protective structures, erected above a drift, a change in the stressed-strained state occurs as a result of compression of the supports, when the movement of rocks of the roof and soil is limited and the area of contact between side rocks and the erected protective structures increases.

When choosing a protection technique for haulage drifts, it is necessary to take into consideration the parameters of the protective structures, because the impact of the size of the same supports, at the same rigidity, on the distribution of stresses in a coal-rock massif is diverse.

In order to ensure the operational condition of the district preparatory workings in the development of steep coal seams,

it is advisable to use pliable protective structures located above a haulage drift, which limit the movement of side rocks in the worked space.

**Keywords:** mining pressure, breakage face, side rock collapse, the filling of worked space, pliable supports.

## References

1. Zinchenko, Yu. I., Sudin, M. S., Zinchenko, A. Yu. (2011). Perspektiva razvitiya shaht TSentral'nogo rayona Donbassa. *Ugol' Ukrayny*, 12, 35–38.
2. Liashok, Y., Iordanov, I., Chepiga, D., Podkopaiev, S. (2018). Experimental studies of the seam openings competence in different methods of protection under pitch and steep coal seams development. *Mining of Mineral Deposits*, 12 (4), 9–19. doi: <https://doi.org/10.15407/mining12.04.009>
3. Sotskov, V., Gusev, O. (2014). Features of using numerical experiment to analyze the stability of development workings. *Progressive Technologies of Coal, Coalbed Methane, and Ores Mining*, 401–404. doi: <https://doi.org/10.1201/b17547-68>
4. Dzyuba, S. V., Shmelev, N. A., Koval', N. V. (2012). Analysis of technology of underground development of mineral deposits for mining work in difficult geological conditions. *Geotekhnicheskaya mehanika*, 101, 284–291.
5. Zhukov, V. E. (2001). Ob odnoy strategicheskoy oshibke v razreshenii problemy razrabotki krutyh plastov. *Ugol' Ukrayny*, 7, 6–10.
6. Krupnik, L. A., Shaposhnik, Y. N., Shaposhnik, S. N., Tursunbaeva, A. K. (2013). Backfilling technology in Kazakhstan mines. *Journal of Mining Science*, 49 (1), 82–89. doi: <https://doi.org/10.1134/s1062739149010103>
7. Blyuss, B., Semenenko, Eu., Nykyforova, N. (2008). The calculation procedure of hydrotransport parameters of bulk solids using hydrodynamically active additives solutions. Papers presented at the 14th International Conference on Transport and Sedimentation of Solid Particles. Saint Petersburg, 41–48.
8. Podkopaiev, S., Gogo, V., Yefremov, I., Kipko, O., Iordanov, I., Simonova, Y. (2019). Phenomena of stability of the coal seam roof with a yielding support. *Mining of Mineral Deposits*, 13 (4), 28–41. doi: <https://doi.org/10.33271/mining13.04.028>
9. Shpakov, V. P.; Fedyukin, D. L. (Ed.) (1986). *Klassifikatsiya pnevmaticheskikh konstruktsiy. Primenenie RTI v narodnom hozyaystve*. Moscow: Himiya, 240.
10. Henderson, Dzh., Nashif, A., Dzhounis, D. (1988). *Dempfirovanie kolebaniy*. Moscow: Mir, 448.
11. Haimova-Mal'kova, R. I. (1970). Metodika issledovaniy napryazheniy polyarizatsionno-opticheskim metodom. Moscow: Nauka, 116.
12. Glushihin, F. P., Kuznetsov, G. N., Shklyarskiy, M. F. et. al. (1991). *Modelirovanie v geomehanike*. Moscow: Nedra, 240.
13. Basov, V. V., Rib, S. V. (2016). Selection of equivalent material for physical modeling of geomechanical processes in the vicinity of the mine workings of coal mines. *Bulletin of the Siberian State Industrial University*, 4, 32–35.
14. Shashenko, A. N., Pustovoytenko, V. P., Sdvizhkhova, E. A. (2016). *Geomehanika*. Kyiv: Noviy druk, 528.
15. Borshch-Komponiets, V. I. (2013). *Prakticheskaya mehanika gornyh porod*. Moscow: Gornaya kniga, 322.
16. Hrebonkin, S. S., Havrysh, M. M. (Eds.) (2004). *Mekhanika hirskykh porid. Vol. 1*. Donetsk: DonNTU, 169.
17. Baklashov, I. V. (1988). *Deformirovanie i razrushenie porodnyh massivov*. Moscow: Nedra, 271.
18. Kleppner, D., Kolenkow, R. J. (2012). *An Introduction to Mechanics*. Cambridge University Press. doi: <https://doi.org/10.1017/cbo9780511794780>
19. Aliev, T. I. (2009). *Osnovy modelirovaniya diskretnyh sistem*. Sankt-Peterburg, 363.
20. Iordanov, I. V., Simonova, Yu. I., Polozhy, A. V., Podkopayev, Ye. S., Skyrda, A. Ye., Kayun, A. P. (2020). A comprehensive study of the stability of lateral rocks with a supple support. *World Science*, 1 (1 (53)), 4–17. doi: [https://doi.org/10.31435/rsglobal\\_ws/31012020/6889](https://doi.org/10.31435/rsglobal_ws/31012020/6889)
21. Obiralov, A. I., Limonov, A. N., Gavrilova, L. A. (2006). *FotoGRAMMETRIYA i distantsionnoe zondirovanie*. Moscow: Koloss, 335.
22. TSigler, F. (2002). *Mehanika tverdyh tel i zhidkostey*. Izhevsk, 912.
23. Akimov, V. A. et. al. (2010). *Teoreticheskaya mehanika. Dinamika. Praktikum. Chast' 2. Dinamika material'noy sistemy. Analiticheskaya mehanika*. Minsk: Novoe znanie; Moscow: TSUPL, 863.
24. Strelkov, S. P. (2005). *Vvedenie v teoriyu kolebaniy*. Sankt-Peterburg: Lan', 440.
25. Gogo, V., Kipko, A., Vlasenko, N., Simonova, Y., Polozhy, A. (2019). Features of the stressed-deformed state of the side breeds in the assessment of the operational state of mining operations. *Journal of Donetsk Mining Institute*, 1, 53–64. doi: <https://doi.org/10.31474/1999-981x-2019-1-53-64>