ABSTRACT AND REFERENCES

MATHEMATICS AND CYBERNETICS – APPLIED ASPECTS

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DEVELOPMENT OF A MATHEMATICAL MODEL FOR PREDICTING POSTOPERATIVE PAIN AMONG PATIENTS WITH LIMB INJURIES (p. 4-9)

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A mathematical model is devised to predict the probability of development of postoperative pain among patients of young age, operated on in a planned manner for the limb injuries. As the model predictors we selected: the level of pain before operation, determined by the visual analog scale, result of evaluation of cognitive abilities by the Montreal scale and level of the mean blood pressure. The application of the developed model makes it possible to improve quality of providing the patients with anesthesiological assistance. The results obtained might be used in the development of information decision support system for a physician-anaesthesiologist for the objectification and automation of the process for determining the probability of development of postoperative pain syndrome. The introduction of such a system into clinical practice will make it possible to reduce the load on the medical staff and decrease the amount of anaesthetising preparations for patients, whose value of the level of pain before operation, determined by the visual analog scale after the operation, does not exceed 3 points, as well as to conduct more adequate analgesia among patients with a higher value of this indicator.

Keywords: postoperative pain, limb injury, patients of young age, anesthesia, prediction, logistic regression.

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CLUSTERING METHOD BASED ON FUZZY BINARY RELATION (p. 10-16)

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Heuristic methods of fuzzy clustering hold a special place in data mining. They are important in preliminary data analysis when the number of clusters, their structure and mutual arrangement are unknown.

The object clustering methods, based on fuzzy binary relations are generalized by developing clear and fuzzy single-level clustering methods, clear and fuzzy sequential multi-level clustering methods. Possible examples of fuzzy binary relations, which characterize similarity of objects by length, angle and distance of their vector features are presented. For this purpose, the Harrington type desirability function and scale, enabling effective analysis of clustering results are suggested.

Based on the proposed methods, the software systems that were effectively used for solving applied clustering problems are developed. Also, the study illustrated the clear single-level clustering method on a specific example.

It is shown that application of the apparatus of fuzzy binary relations in clustering provides an additional opportunity to study the dynamics of the number of clusters, their structure and determine the degree of similarity of objects in a cluster. The results can be used for preliminary data analysis and for holding the object clustering procedure.

Keywords: cluster analysis, automatic object classification, data mining, fuzzy clustering.

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DEVELOPMENT OF A VERIFICATION METHOD OF ESTIMATED INDICATORS FOR THEIR USE AS AN OPTIMIZATION CRITERION (p. 17-23)

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The optimization criterion is that reference point of the controlled systems, which provides the maximum coherence of the operational process results with the purpose of its owner. Difficulties of solving the tasks, connected with the choice of an adequate optimization criterion are caused by the fact that a large number of indicators intended for use as an optimization criterion have been developed and continue to be developed now.

The verification process of the indicator, which can potentially be used as an optimization criterion, is rather difficult. This difficulty is caused by the fact that the identification process is based on an exclusion method. Those indicators that have shown contradictory results on the classes of operations models identified by the rating efficiency should be excluded from the "contenders".

During the researches, the hypothesis on the possibility of creating the reference models, allowing the development of direct verification methods wasn't confirmed. However, the comparative research of cybernetic models of operations with different duration for consistency has allowed setting a restriction on the formation rules of models of this class and defining the rules of their equivalent converting.

Besides, during the researches, the method of creating a class of simple operation models with the predetermined identification, concerning their rating efficiency, has been developed. This means that the rating creation of these models doesn't require determination of local efficiency criteria.

Creation of a new class of reference models expands the possibilities of the estimated indicator verification method. At the same time, the positive effect of expansion of the opportunities of the verification method is shown not so much in an increase in the probability of excluding the inadequate estimated indicator, but in the introduction of a restriction on the rules of creating the classes of reference operation models.

Introduction of a restriction prevents the possibility of excluding the adequate estimated indicator from consideration.

Keywords: local efficiency criterion, estimated indicators verification method, optimization criterion.

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OPTIMIZATION OF KNOWLEDGE BASES ON THE BASIS OF FUZZY RELATIONS BY THE CRITERIA "ACCURACY – COMPLEXITY" (p. 24-31)

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The method of optimization of fuzzy classification knowledge bases by the criteria "inference accuracy – complexity" was proposed. A relational fuzzy model, which corresponds to the fuzzy classification knowledge base, was developed. The matrix of fuzzy relations in the form of one-dimensional projection "input terms – output classes" is a simplified representation of the system of classification rules. A problem on the optimization of a knowledge base is reduced to the problem on the min-max clustering and comes down to selecting such partition matrices "inputs – output" that provide for the required or extreme levels of inference accuracy and the number of rules.

In the relational models, a question about optimal choice of the number of output terms remains open. A selection of output classes, input terms and rules is reduced to the problem on discrete optimization of the algorithm reliability indicators, in order to solve which, we employed the gradient method. The number and location of hyperboxes are determined by the relations matrix, and the sizes of hyperboxes are defined as a result

of tuning of the triangular membership functions. A selection of the number of input and output terms in the partition matrices may be performed both under the offline mode and by adaptive adding/removing of terms.

Known methods of the min-max clustering apply heuristic procedures for the selection of the number of rules (classes). The proposed method generates variants of fuzzy knowledge bases in accordance with the formalized procedures of reliability analysis and synthesis of algorithmic processes. This resolves a general problem on the methods of min-max clustering related to the minimization of the number of input terms without losing inference accuracy.

A transition to the relational fuzzy model provides simplification of the process of the knowledge bases tuning both for the assigned and unknown output classes.

Keywords: optimization of fuzzy knowledge bases, min-max clustering, fuzzy relational models.

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FINDING THE PROBABILITY DISTRIBUTION OF STATES IN THE FUZZY MARKOV SYSTEMS (p. 32-38)

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A problem on finding the stationary distributions of probabilities of states for the Markov systems under conditions of uncertainty is solved. It is assumed that parameters of the analyzed Markov and semi-Markov systems (matrix of transition intensities, analytical description of distribution functions of the durations of being in states of the system before exiting, as well as a matrix of transition probabilities) are not clearly assigned. In order to describe the fuzziness, we employ the Gaussian membership functions, as well as functions of the (L–R) type. The appropriate procedure of systems analysis is based on the developed technology for solving the systems of linear algebraic equations with fuzzy coefficients. In the problem on analysis of a semi-Markov system, the estimation of components of the stationary distribution of probabilities of states of the system is obtained by the minimization of a complex criterion. The criterion considers the measure of deviation of the desired distribution from the modal one, as well as the level of compactness of membership functions of the fuzzy result of solution. In this case, we apply the rule introduced for the calculation of expected value of fuzzy numbers. The criterion proposed is modified through the introduction of weight coefficients, which consider possible differences in the levels of requirements to different components of the criterion.

Keywords: Markov and semi-Markov systems, complex criterion, deviation of solution from the modal one, compactness measure of solution.

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RECURRENT TRANSFORMATION OF THE DYNAMICS MODEL FOR AUTONOMOUS UNDERWATER VEHICLE IN THE INERTIAL COORDINATE SYSTEM (p. 39-47)

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We considered the motion equation for an autonomous underwater vehicle (AUV) with a manipulator onboard in the inertial coordinate system (ICS). A nonlinear system of differential equations takes into account in the form of attached mass coefficients the impact of infiltration effects and dissymmetry of the outer shell of the body. The work of manipulator and fixing elements is accounted for by additional forces and moments that occur as a consequence of their relative motion. The expressions of forces and moments are presented for the given kinematic schemes. A recurrent approximation method is applied, as result of which we transformed a solution for the system of nonlinear differential equations to the recurrent sequence of analytical expressions. The constructed sequence describes the dynamics of AUV with regard to the angular position and kinematic parameters without simplifications in the kinematic matrix. The algorithm that was synthesized based on this model is presented, which provides for presenting, in the form of recurrent sequence of actions and calculations, the expressions for analytical representation of solution for the direct problem on the AUV dynamics. Based on the analytical approximations of the model, we present expressions for an error that occurs as a consequence of angular deviations and simplifications in the kinematic matrices.

The dynamics of AUV was modeled and quantitative values of error were obtained as a function of operating and kinematic parameters of AUV in ICS. The derived models of dynamics and expressions for errors are important for decision support systems because they allow the representation of information about the motion of AUV and manipulator aboard in a uniform ICS. The possibilities obtained have eliminated obstacles to comprehensive modeling of technological AUV and creation of ACS. Using the results of modeling the impact of manipulator's work aboard AUV, we established factors that influence the magnitude of error when calculating by the simplified kinematic matrices.

Keywords: autonomous underwater vehicle, inertial coordinate system, recurrent sequence, kinematic matrix.

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DEVELOPMENT OF PROJECTION TECHNIQUE FOR DETERMINING THE NON-CHAOTIC OSCILLATION TRAJECTORIES IN THE CONSERVATIVE PENDULUM SYSTEMS (p. 48-57)

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We developed a technique to determine the non-chaotic oscillations of loads in the conservative pendulum systems by using the graphic technology of projection focusing. In this case, phase trajectories of the differential equations of oscillation are considered as projections of integral curves from the phase space onto the phase plane. The effect exerted by the value of one of the system's parameters on the image of phase trajectories was examined (at stable values of other parameters). By using projection focusing, the element that defines the critical value of variable parameter is selected among a family of phase trajectories, which, in a combination with other parameters, allows us to describe a non-chaotic trajectory in the load motion.

The need for such studies is predetermined by the absence, in practice, of an engineering method for computing the non-chaotic trajectory of the load motion for a certain pendulum system.

We proposed the notion of a focus-line of the parametric family of curves and the technique of projection focusing, which is based on it. We constructed integral curves in the phase space based on the numerical solution of second order Lagrange differential equations. A procedure is presented to determine the critical value of pendulum oscillation parameter by using the graphic notion of projection focusing of phase trajectories in the

solutions of second order Lagrange differential equations. The examples are presented of determining the parameters of certain pendulums, which would provide for the non-chaotic trajectory of the load oscillations.

The developed computerized projection technique for the simulation of oscillations in the pendulum mechanical systems makes it possible to choose the required values of parameters and initial conditions for initiating the oscillations, which provide for the non-chaotic technological character of oscillation trajectory of their elements, which is important for the practical implementation in the designs of pendulum systems.

Keywords: pendulum systems, conservative systems, integral curves, phase trajectories, projection focusing.

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CONSTRUCTION OF A NUMERICAL METHOD FOR FINDING THE ZEROS OF BOTH SMOOTH AND NONSMOOTH FUNCTIONS (p. 58-64)

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Here we report building a numerical method for finding the zeros of a function of one real variable using the apparatus of nonclassical Newton's minorants and diagrams of functions', given in the tabular form. The examples of the search for zeros of functions are given.

A problem on finding the roots of equations belongs to important problems of applied mathematics. Classical methods of finding the zeroes of functions require first to isolate the roots and then to find them. In order to find a separate root with a given accuracy, it is necessary to choose one of the points in the vicinity that contains the root as the initial approximation and to employ an appropriate iterative process.

The numerical method constructed does not require additional information about the location of roots and has many advantages over other methods for finding the zeros of functions, in particular: simplicity and visual representation of the method. Because of this, it can gain a widespread application in many areas, such as physics, mechanics, and natural sciences. By using the method built, it is possible to find the roots in a linear time, which is rather fast. The practical value of a numerical method is largely determined by the speed of obtaining the solution.

Keywords: minorant of a function, zero of a function, Chebyshev polynomial, Newton's diagram, smooth and nonsmooth function.

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