

**ABSTRACT AND REFERENCES**  
**APPLIED MECHANICS**

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**STABILITY OF STRUCTURAL ELEMENTS OF SPECIAL LIFTING MECHANISMS IN THE FORM OF CIRCULAR ARCHES (p. 4–10)**

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The system of differential equations of stability of circular arches with symmetric sections and the sixth-order resolving ordinary differential equation are derived. It is noted that these equations have variable coefficients and their analytical solution under existing external loads leads to serious mathematical difficulties. The problem of finding exact solutions can be substantially simplified if we use the numerical-analytical version of the boundary element method (BEM). Here it is necessary to have a solution of the resolving equation of the problem, but with constant coefficients. This problem is much simpler than the initial one and can be realized according to the known procedure for constructing the fundamental functions of an ordinary differential equation. In this regard, the constants for integrating the general solutions of the differential equation are determined for the two most common cases and rationing of the fundamental functions in the matrix resolving form is performed. Recommendations are given on the solution of various boundary-value problems of stability of the simple bending of arch elements of special lifting mechanisms using them.

**Keywords:** stability problems, system of differential equations with variable coefficients, fundamental functions, BEM.

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**THE INFLUENCE OF THE BLADE FEATHER CONSTRUCTIONAL INHOMOGENEITY ON THE TURBINE COOLING BLADES STRESS-STRAIN STATE (p. 11–17)**

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The problem of gas turbine engines inhomogeneous rotor blades stress-strain state has been studied. For this purpose, the new, more correct mathematical model, based on the special three-dimensional curvilinear finite elements has been used. Such elements have three modifications, applied for the blade feather and its transition zones correct modeling. The complex influence of vibration and heat loads on the blade feather has also been taken into consideration.

The values of maximum dynamic stresses and their localization zones have also been found. The main concentrators of stresses are located in the transfer zones between the cooling channels and blade feather surfaces. The blade output edge is another zone of maximum dynamic stresses localization.

It has also been found that the influence of the geometric parameters of cooling channels in the blade feather cavity on the value of maximum dynamic stresses is sharper than the influence of cooling holes on the blade output edge.

By comparing the obtained calculated results with the experimental data, we can state the high adequacy and reliability of the developed mathematical model. All calculations and experimental procedures were held by equal boundary conditions. The results of

the research can be used as a base for further studying of the whole rotor stress-strain state and processes of blades fatigue destruction.

**Keywords:** turbine engine blades, geometric parameters, three-dimensional finite elements, dynamic stresses.

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## INVESTIGATION OF THE WORK OF THE ROAD CONSTRUCTION AT THE SITES BY PIPE DRENES FROM MATERIALS OF DIFFERENT ORIGIN (p. 18–26)

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The paper reports research into operation of road structures with tubular drains made from materials with different physical-mechanical properties, which makes it possible to identify basic factors that affect operational conditions under the influence of own mass and the rated load from rolling stock in accordance with the building norms of Ukraine. Non-standard road structures were simulated in the SCAD environment. The results of numerical simulation allowed us to derive diagrams of normal stresses and deformations of structural layers in road surfacing, as well as in the body of tubular drains. The calculations were performed both for the standard and the actual compaction of material used as a trench backfill, for a PVC pipe and a concrete pipe.

Consideration of tubular openings in solid layered road structures made it possible to estimate the actual stressed-deformed state at the sections of roads that require control over a water-heat mode. The proposed method of study enables the selection of individual design-structural parameters for drainages of shallow laying for general-purpose public roads of different technical categories as opposed to the standard approaches implied by the building regulations of Ukraine.

**Keywords:** road structure, drainage structure of shallow laying, tubular drain, polyvinylchloride pipe, concrete pipe.

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## NUMERICAL SIMULATION OF TWO-DIMENSIONAL PROBLEMS OF CREEP CRACK GROWTH WITH MATERIAL DAMAGE CONSIDERATION (p. 27–33)

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Approach for numerical simulation of the process of the creep crack growth taking into account the hidden material damage is proposed. The approach is based on the application of finite element creep modeling, accompanied by damage. For calculations, the FEM Creep software package is used. Using the proposed algorithm for rebuilding the grid with the removal of the destroyed elements, the current picture of deformation and fracture is analyzed. This takes into account the growing level of damage during the crack motion in each element. Numerical fracture simulation data are used to determine the constants in the differential creep fracture propagation equation. As an example, the creep fracture of planar specimens with sharp notches in their plane is considered. The material of the specimens is a high-temperature nickel-based alloy EI 867 at a temperature of 950 °C. Calculations are carried out for different values of the load. For different times, finite element grids with remote elements are shown. Graphs of the dependence of crack length on time are built. Comparison of numerical and calculated data obtained with the motion equation of a crack shows their acceptable coincidence. The possibility of using the proposed approach for obtaining constants in the equation of crack motion as an alternative to the existing experimental one is discussed.

**Keywords:** creep, damage, creep crack growth, finite element calculation model.

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## DESIGN OF THE LABORATORY BENCH FOR A HYDROVOLUMETRIC-MECHANICAL TRANSMISSION OF THE TRACKED TRACTOR (p. 34–43)

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Double-flow hydrovolumetric mechanical transmissions is an advanced technical solution that aims to increase productivity, improve efficiency and convenience of control over wheeled and tracked tractors. Their arrangement renders considerable potential for their modernization and makes it possible to introduce recuperation systems that would enhance their performance efficiency coefficient to the level of mechanical transmissions. Innovativeness of these transmissions and the lack of a sufficient number of prototypes largely hinder their implementation in production.

A large number of possible HT circuits necessitates the creation of an original bench-prototype for each of them. In order to solve this

task, we proposed in this paper a kinematic circuit of the universal testing laboratory bench for HT of the tracked tractor. Its design makes it possible to study HT circuits of the type with a differential «at the output», the type with a differential «at the input» for both wheeled and tracked vehicles, and to simulate the process of work of a hydrovolumetric turning mechanism. Introduction of electric generators to the design makes it possible to estimate in practice the effectiveness of recuperation of kinetic energy at braking, as well as parasite power that circulates in the closed circuit of HT during acceleration and braking.

We give quasi-static characteristics of basic HT circuits that are simulated at the laboratory bench. The results were obtained based on an improved mathematical model that makes it possible to determine the volumetric, mechanical, and full performance efficiency coefficient of separate hydraulic machines in the direct and reverse flows of power and special working regions of hydraulic gear. For the transmissions with a differential «at the output», maximal performance efficiency coefficient reaches 83 %, indicating the proper selection of gear ratios and standard size of hydraulic machines.

The results obtained are of interest for industrial and design organizations, specializing in the development of transmissions for transportation vehicles. Employing the developed bench would significantly reduce the time between designing and industrial implementation. There is a possibility to confirm experimentally the efficiency and high technical-economic indicators of the proposed transmission. Such a bench will make it possible for scientific institutions to discover new phenomena and processes in HT, systematize the influence of gear ratios of reducers, planetary mechanisms, and their number, on the working processes in HT.

**Keywords:** tractor, transmission, laboratory bench, planetary mechanism, circulation of power, hydrovolumetric turning mechanism.

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## GEOMETRICAL MODELING OF THE SHAPE OF A MULTILINK ROD STRUCTURE IN WEIGHTLESSNESS UNDER THE INFLUENCE OF PULSES ON THE END POINTS OF ITS LINKS (p. 44–58)

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We have examined a geometrical model of the new technique for unfolding a multilink rod structure under conditions of weightlessness. Displacement of elements of the links occurs due to the action of pulses from pyrotechnic jet engines to the end points of links in a structure. A description of the dynamics of the obtained inertial unfolding of a rod structure is performed using the Lagrange equation of second kind, built using the kinetic energy of an oscillatory system only.

The relevance of the chosen subject is indicated by the need to choose and explore a possible engine of the process of unfolding a rod structure of the pendulum type. It is proposed to use pulse pyrotechnic jet engines installed at the end points of links in a rod structure. They are lighter and cheaper as compared, for example, with electric motors or spring devices. This is economically feasible when the process of unfolding a structure in orbit is scheduled to run only once.

We have analyzed manifestations of possible errors in the magnitudes of pulses on the geometrical shape of the arrangement of links in a rod structure, acquired as a result of its unfolding. It is shown at the graphical level that the error may vary within one percent of the estimated value of the magnitude of a pulse. To determine the moment of fixing the elements of a multilink structure in the preset unfolded state, it is proposed to use a «stop-code». It is a series of numbers, which, by using functions of the generalized coordinates of the Lagrange equation of second kind, define the current values of angles between the elements of a rod structure.

Results are intended for geometrical modeling of the unfolding of large-size structures under conditions of weightlessness, for example, power frames for solar mirrors, or cosmic antennae, as well as other large-scale orbital facilities.

**Keywords:** rod structure, process of unfolding in space, multilink rod structure, Lagrange equation of second kind.

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**ON STABILITY OF THE DUAL-FREQUENCY  
MOTION MODES OF A SINGLE-MASS VIBRATORY  
MACHINE WITH A VIBRATION EXCITER IN THE  
FORM OF A PASSIVE AUTO-BALANCER (p. 59–67)**

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By employing computational experiments, we investigated stability of the dual-frequency modes of motion of a single-mass vibratory machine with translational rectilinear motion of the platform and a vibration exciter in the form of a passive auto-balancer.

For the vibratory machines that are actually applied, the forces of external and internal resistance are small, with the mass of loads much less than the mass of the platform. Under these conditions, there are three characteristic rotor speeds. In this case, at the rotor speeds:

- lower than the first characteristic speed, there is only one possible frequency at which loads get stuck; it is a pre-resonance frequency;

- positioned between the first and second characteristic speeds, there are three possible frequencies at which loads get stuck, among which only one is a pre-resonant frequency;

- positioned between the second and third characteristic speeds, there are three possible frequencies at which loads get stuck; all of them are the over-resonant frequencies;

- exceeding the third characteristic speed, there is only one possible frequency at which loads get stuck; it is the over-resonant frequency and it is close to the rotor speed.

Under a stable dual-frequency motion mode, the loads: create the greatest imbalance; rotate synchronously as a whole, at a pre-resonant frequency. The auto-balancer excites almost perfect dual-frequency vibrations. Deviations of the precise solution (derived by integration) from the approximated solution (established previously using the method of the small parameter) are equivalent to the ratio of the mass of loads to the mass of the entire machine. That is why, for actual machines, deviations do not exceed 2 %.

There is the critical speed above which a dual-frequency motion mode loses stability. This speed is less than the second characteristic speed and greatly depends on all dimensionless parameters of the system.

At a decrease in the ratio of the mass of balls to the mass of the entire system, critical speed tends to the second characteristic speed. However, this characteristic speed cannot be used for the approximate computation of critical speed due to an error, rapidly increasing at an increase in the ratio of the mass of balls to the mass of the system. Based on the results of a computational experiment, we have derived a function of dimensionless parameters, which makes it possible to approximately calculate the critical speed.

**Keywords:** inertial vibration exciter, dual-frequency vibrations, auto-balancer, single-mass vibratory machine, Sommerfeld effect, motion stability.

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