

ABSTRACT AND REFERENCES

MATHEMATICS AND CYBERNETICS – APPLIED ASPECTS

DOI: 10.15587/1729-4061.2018.140717

CONSTRUCTION OF SPECTRAL DECOMPOSITION FOR NON-SELF-ADJOINT FRIEDRICHS MODEL OPERATOR (p. 6–18)**Evhen Cheremnikh**

National University «Lviv Polytechnic», Lviv, Ukraine

ORCID: <https://orcid.org/0000-0002-9034-8590>**Halyna Ivasyk**

National University «Lviv Polytechnic», Lviv, Ukraine

ORCID: <https://orcid.org/0000-0003-0587-0630>**Vladislav Alieksiev**

National University «Lviv Polytechnic», Lviv, Ukraine

ORCID: <https://orcid.org/0000-0003-0712-0120>**Mariia Kuchma**

National University «Lviv Polytechnic», Lviv, Ukraine

ORCID: <https://orcid.org/0000-0002-5563-3847>**Oksana Brodyak**

National University «Lviv Polytechnic», Lviv, Ukraine

ORCID: <http://orcid.org/0000-0002-9886-3589>

The spectral decomposition for the non-self-adjoint Friedrichs model is given and a generalization of the well-known Weyl function in non-self-adjoint cases is given. It is found that for the non-self-adjoint Friedrichs model, an arbitrary space element can be presented as a linear combination of the operator's eigenelements corresponding to the points of the spectrum. The spectral decomposition, that is, the representation of an arbitrary space element through its own functions, is constructed, which indicates the completeness of its eigenfunctions. This is done taking into account the spectral features (i.e., eigenvalues on a continuous spectrum) of the non-self-adjoint operator of the Friedrichs model. This model serves as an important tool for finding the solution of ordinary differential equations after the application of the corresponding Fourier transform.

A general method for constructing the spectral decomposition (that is, not bound only to the Friedrichs model) is proposed, which is based on the concept of so-called branching of the resolvent and which can be used for arbitrary non-self-adjoint operators, as well as for self-directed operators.

It is proved that under the conditions of the existence of the maximal operator, the resolvent allows separation of the branch. Sufficient conditions for the existence of the Weyl function $m(\zeta)$ for the operator of the non-self-adjoint Friedrichs model are given and formulas for its calculation through the resolvent are obtained.

It is shown that the Weyl function $m(\zeta)$ for the self-directed operator coincides with the classical Weyl function in the case of the Sturm-Liouville operator on the semiaxis. Two examples are given in which we find the generalized Weyl function $m(\zeta)$ for the non-self-adjoint Friedrichs model.

Keywords: Sturm-Liouville operator, Friedrichs model, Fourier transform, Weyl function, continuous spectrum, branching of the resolvent, maximal operator.

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DOI: 10.15587/1729-4061.2018.140634

DEVELOPMENT OF THE ANALYTICAL METHOD OF THE GENERAL MATHIEU EQUATION SOLUTION (p. 19–26)**Yurii Krutii**

Odessa State Academy of Civil Engineering and Architecture, Odessa, Ukraine

ORCID: <http://orcid.org/0000-0001-7105-3087>**Alexander Vasiliev**

Odessa I. I. Mechnikov National University, Odessa, Ukraine

ORCID: <http://orcid.org/0000-0002-3826-4883>

An analytical method for solving the general Mathieu differential equation in the initial form is proposed. The method is based on the corresponding exact solution, which is found for arbitrary

numerical parameters of the initial equation a and q . In turn, the exact solution is expressed through the fundamental functions, which are represented by series in powers of the parameters a and q with variable coefficients.

Along with Mathieu equation, the system of equivalent differential equations is also considered. It is shown that the Wronskian matrix, which is formed of the fundamental functions of the equation, is the transition matrix of the system. Thus, it is proved that the fundamental functions of the Mathieu equation satisfy the given conditions at the zero point.

In order to solve the problem of numerical realization of the exact formulas found, the fundamental functions are represented by power series. To calculate the coefficients of the power series, the corresponding recurrent relations are derived.

As a result of the research, finite analytical formulas for calculating the characteristic exponent ν , the determination of which is the central part of any problem, the mathematical model of which is the Mathieu equation are obtained. In fact, a direct analytical dependence of ν on the initial parameters of the equation a , q is established. This is especially important, since the parameter ν plays the role of an indicator of such properties of solutions of the Mathieu equation as boundedness and periodicity.

The proposed analytical method is a real alternative to the application of approximate methods in solving any problems that are reduced to the Mathieu equation. The presence of finite analytic formulas will allow avoiding the procedure of finding the solutions of the equation in the future. Instead, to solve the problem in each specific case, it is enough to implement the obtained analytical formulas numerically.

Keywords: general Mathieu equation, analytical method, fundamental functions, characteristic exponent

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DOI: 10.15587/1729-4061.2018.140686

THE RESEARCH OF POSSIBILITIES FOR FAST CALCULATION OF MEDIAN CONSENSUS RANKINGS (p. 27–35)

Viktor Boltenkov

Institute of Computer Systems, Odessa National Polytechnic University, Odessa, Ukraine
ORCID: <http://orcid.org/0000-0003-3366-974X>

Varvara Kuvaieva

Institute of Computer Systems, Odessa National Polytechnic University, Odessa, Ukraine
ORCID: <http://orcid.org/0000-0002-9350-1108>

Oleg Galchonkov

Institute of Computer Systems, Odessa National Polytechnic University, Odessa, Ukraine
ORCID: <http://orcid.org/0000-0001-5468-7299>

Alesya Ishchenko

Institute of Computer Systems, Odessa National Polytechnic University, Odessa, Ukraine
ORCID: <http://orcid.org/0000-0002-7882-4718>

We investigated the possibility of fast computation of collective expert estimates of the median type. Despite the widespread use of the Kemeny-Snell and Cook-Seiford medians for calculating the collective expert estimates, the possibilities to reduce the time required to calculate median consensus ranking through the application of the assignment problem and known algorithms for solving it, have been insufficiently investigated. In contrast to the most known methods, the method proposed in this paper is not approximated and retains the original median axiomatics of Kemeny. We investigated a possibility for calculating the Kemeny-Snell and Cook-Seiford medians using the assignment problem applying methods of computer experiment. We estimated the time required to calculate median rankings by four different algorithms for solving the assignment problem. It was established that the proposed method, at a moderate number of alternatives ($n < 50$), computes median rankings over the time close to a real time mode. It is also shown that, in contrast to other methods, the calculation of median rankings using the assignment problem does not depend on coherence degree of individual expert rankings. The results obtained are useful for the practical application of the examined procedure in network expert systems. In such systems, the computation time of a consensus ranking should be close to real time. In addition, in the network expertise systems, due to the random gathering of a team of experts, there is a possibility of the low level of coherence in individual rankings. For the examined procedure, that does not affect the duration of computation. This

allows us to recommend the developed computational procedure for a fast search for median consensus rankings by Kemeny-Snell and Cook-Seiford for practical application in the systems of network collective expertise.

Keywords: collective expert estimation, median consensus rankings, assignment problem.

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DOI: 10.15587/1729-4061.2018.140649

**EXAMINING THE KALMAN FILTER
IN THE FIELD OF NOISE AND
INTERFERENCE WITH THE NON-GAUSSIAN
DISTRIBUTION (p. 36–42)**

Olga Oliynyk

Ukrainian State University
of Chemical Technology, Dnipro, Ukraine
ORCID: <http://orcid.org/0000-0003-2666-3825>

Yuri Taranenko

Ukrainian State University
of Chemical Technology, Dnipro, Ukraine
ORCID: <http://orcid.org/0000-0003-4072-011X>

Dmitriy Losikhin

Ukrainian State University
of Chemical Technology, Dnipro, Ukraine
ORCID: <http://orcid.org/0000-0002-6325-7263>

Alexander Shvachka

Ukrainian State University
of Chemical Technology, Dnipro, Ukraine
ORCID: <http://orcid.org/0000-0003-1076-6950>

We have developed a sequential recursive Kalman Filter algorithm to filter data in the field of the non-Gaussian noise distribution to be used in measurement instruments. A special feature of the constructed Kalman Filter algorithm to filter data with the non-Gaussian noises is the absence of a need to determine a priori the statistical characteristics of noise.

The applicability of the developed Kalman filtering procedure was tested by processing different distribution laws: the Cauchy, Pareto noises, normal and logistic distributions. The effectiveness of the devised filtering procedure is confirmed by applying the filter when processing experimental data with different laws of noise distribution. We have conducted approbation of the developed procedure for the Kalman filtering based on data obtained experimentally, with respect to the superposition of noise distribution laws. The a priori

estimate for a filtering error when the number of iterations exceeds 30 tends to zero.

The devised filtering procedure employing the Kalman filter could be used when performing the metrological certification of measuring instruments under industrial conditions. Under such circumstances, measuring information could become noisy due to various noises, including those that are not governed by the Gaussian distribution law. The filter could be used when processing data from control systems over state parameters, implemented on the principle of a magnitude threshold control.

The applied aspect of the scientific result obtained implies the possibility of extending the scope of application of the classic Kalman filter in measurement instruments. This is a prerequisite for the development of a generic filtering algorithm using the Kalman filter.

Keywords: Kalman filter, recursive algorithm, Python, non-Gaussian noise, distribution law.

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DOI: 10.15587/1729-4061.2018.140641

TRANSFORMATION OF OPERATIONS WITH FUZZY SETS FOR SOLVING THE PROBLEMS ON OPTIMAL MOTION OF CREWLESS UNMANNED VEHICLES (p. 43–50)

Alexander Trunov

Petro Mohyla Black Sea National University, Mykolaiv, Ukraine

ORCID: <http://orcid.org/0000-0002-8524-7840>

Solution of an optimization problem of the minimum time of motion of the crewless and unmanned aerial vehicle (CUV) was stated and analyzed. A connection was established between projections of the velocity vector as a condition for solving the problem of minimum travel time. It was proposed to construct an algorithm of correcting parameters of the optimal path. It was shown that if a scale of the value « c » is introduced along the path between two transverse derivatives of the velocity vector module in two orthogonal directions and ensure action of forces which will connect the second derivatives of coordinates in these directions with the same scale of « c », then such path will minimize the total travel time. Separation of motions forms the possibility of control based on video images at a condition of satisfying restrictions of the magnitude of the « c » scale and imposes restrictions on the work of propellers. It was established that calibration of motions makes it possible to determine the « c » constant.

Control actions were formed: forces and moments for a hydrodynamic model of the CUV. It was proposed to represent control actions through the number of revolutions of the propeller axle. The control action through a membership function and maximum and minimum ratings of propeller axle rotation speeds was presented.

New qualitative concepts were introduced. They are specified by membership functions: speed of rotation of the propeller axle to the values realizable by the motor, $\mu_r(n_s/n_{max})$; propeller thrust which will provide accelerated motion of the CUV in accordance with rating, $\mu_{sx}(x^*,t)$; lifting force which provides its excess, $\mu_{sy}(x^*)$; speed of rotation of the propeller axle which provides mechanical power at an economical consumption of electric power.

The process of choosing speed of rotation of the propeller axle during spatial motion of the CUV has been simulated taking into account influence of such qualitative factors. Simplification of the process of choosing relative speeds of the propeller axes during the period of CUV control has been demonstrated. Based on numerical examples, independence and stability of the calculated value of the function of the intersection belonging and the selected relative propeller axle rotation speeds from the choice of the angles of orientation of the propeller axle were shown.

Keywords: optimal path, division of motions, control actions, fuzzy sets, transformation of operations.

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DOI: 10.15587/1729-4061.2018.140722

DEVELOPMENT OF THE MATHEMATICAL MODEL AND THE METHOD TO SOLVE A PROBLEM ON THE OPTIMIZATION OF PACKING THE ELLIPSOIDS INTO A CONVEX CONTAINER (p. 51–58)

Olha Kh lud

A. N. Podgorny Institute for Mechanical Engineering Problems of the National Academy of Sciences of Ukraine, Kharkiv, Ukraine
ORCID: <http://orcid.org/0000-0003-1838-4121>

Olexander Pankratov

A. N. Podgorny Institute for Mechanical Engineering Problems of the National Academy of Sciences of Ukraine, Kharkiv, Ukraine
ORCID: <http://orcid.org/0000-0002-2958-8923>

Tetyana Romanova

A. N. Podgorny Institute for Mechanical Engineering Problems of the National Academy of Sciences of Ukraine, Kharkiv, Ukraine
ORCID: <http://orcid.org/0000-0002-8618-4917>

This paper addresses the problem on the optimal packing of the predefined set of ellipsoids into a convex container of minimum volume. The ellipsoids are assigned by the dimensions of semi-axes and arrangement parameters in the local coordinate system and may permit continuous rotation and translation. The container could be a cuboid (rectangular parallelepiped), a cylinder, a sphere, an ellipsoid, or a convex polyhedron. To analytically describe the non-overlapping relations between ellipsoids, we use the quasi-phi-functions. To model the inclusion relations, we apply the quasi-phi-functions or phi-functions depending on the shape of a container. By employing the appropriate modeling tools, we construct a mathematical model in the form of a non-linear programming task.

The solution strategy is devised based on the method of a multistart. We propose a fast algorithm for generating the starting points from the region of feasible solutions, as well as the specialized optimization procedure that reduces the problem of large dimensionality $O(n^2)$ with a large number of nonlinear inequalities to a sequence of sub-tasks in nonlinear programming with a smaller dimensionality $O(n)$ with fewer non-linear inequalities.

The optimization procedure makes it possible to significantly reduce (by 10 % to 90 %, depending on the dimensionality of a problem) computing resources, such as time and memory. Depending on the shape of a container, constraints for the orientation of ellipsoids (continuous turns, fixed orientation) and features in metric characteristics of ellipsoids, the result of solving the problem is the derived locally optimal or good feasible solutions. In the work we report numerical experiments for different containers (including a cylinder, a cuboid, a sphere, an ellipsoid).

Keywords: optimal packing, ellipsoids, convex container, method of phi-functions, modeling the arrangement relations, nonlinear optimization.

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DOI: 10.15587/1729-4061.2018.140636

DEVELOPMENT AND INVESTIGATION OF METHODS OF GRAPHIC-FUNCTIONAL MODELING OF DISTRIBUTED SYSTEMS (p. 59–69)

Anatolii Boinik

Ukrainian State University of Railway Transport, Kharkiv, Ukraine
ORCID: <http://orcid.org/0000-0001-7773-9055>

Oleksii Progonnij

Ukrainian State University of Railway Transport, Kharkiv, Ukraine
ORCID: <http://orcid.org/0000-0002-4777-0729>

Oleksandr Kameniev

Ukrainian State University of Railway Transport, Kharkiv, Ukraine
ORCID: <http://orcid.org/0000-0001-5372-5628>

Anton Lapko

Ukrainian State University of Railway Transport, Kharkiv, Ukraine
ORCID: <http://orcid.org/0000-0003-2881-1238>

Viktor Kustov

Scientific and Production Enterprise «SATEP», Ltd,
Kharkiv, Ukraine
ORCID: <http://orcid.org/0000-0002-9773-5470>

Dmytro Kuzmenko

Scientific and Production Association «Railwayautomatic», Ltd,
Kharkiv, Ukraine
ORCID: <http://orcid.org/0000-0002-6978-8954>

Olena Shcheblykina

Scientific and Production Enterprise «SATEP», Ltd,
Kharkiv, Ukraine
ORCID: <http://orcid.org/0000-0002-8304-2921>

During the research, the method of geometric modeling of distributed systems and related technological objects was developed. The method is based on the use of functional graphs. In the context of the research, the main differences between such graphs are: modeling of technological objects of distributed systems only by vertices without the use of edges to reproduce the mentioned objects; using the edges exclusively for the reproduction of the connections between the objects. Weighing of the vertices of the specified graphs is performed using the assigned functions or functionals with a complete absence of weights of the edges.

In contrast to the closest analogues, the basis of the analytical interpretation of the formed graphic models in the proposed method is formed not by the incidence matrices, but by the parametric-topological adjacency matrices. In such conditions, the principles of assigning weight coefficients of graph elements change significantly: instead of the positional distribution of the weight sets elements between the matrix cells, the assignment of the specified elements as function arguments as a part of functional vertices is used. In the mentioned approach, a diagonal way of prescribing functions or functionals of vertices in the adjacency matrix is used. The assignment of the connections between the graph elements in the analytical interpretation is performed according to the introduced positional principle with the use of positive or negative logic. With this approach, the possibility of analytical formation of multiple connections between the vertices with an arbitrary number and direction, which was not used in the matrix of adjacency previously, is reached. Moreover, assigning functional dependencies to the graph elements allows the reproduction of not only static, but also the dynamic characteristics of the modeled objects in the geometric model.

The practical value of the proposed method is increasing the universality and simplifying the automated configuration of software management systems. Achieving this result is possible by reducing the amount of input data and the possibility of introducing additional functions of control objects without the modification of the output code. In addition, the improvement of the formalized drawing up of technical tasks in the development of technical documentation and hardware distribution systems is provided. Furthermore, the integration of the method into existing CAE and CAD systems is possible, which provides opportunities for building up and creating fundamentally new similar systems.

Further development of the proposed method consists in solving the problems related to the optimization of the distribution of arguments of vertex functions by cells of parametric-topological matrices.

Keywords: graphic model, functional graph, parametric-topological matrix, weight parameters, distributed system.

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