

## ABSTRACT AND REFERENCES

## APPLIED MECHANICS

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## ANALYTICAL ESTIMATION OF INERTIAL PROPERTIES OF THE CURVED ROTATING SECTION IN A DRILL STRING (p. 6-14)

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We have proposed approaches to analytical assessment of the moment of inertia related to the curved rotary sections in a drill string. Research into rotation processes of the curvilinear sections in a drill string is associated at present with a particular difficulty, which arose because of the lack of precise expressions to evaluate the moments of inertia for a curved pipe based on the parameters of its deformation. Solution to such problems is important for the analysis of dynamic resistance of drill strings at rotor and rotor-turbine drilling techniques, when studying the stressed-strained state of its elements, refining the energy costs for rotation process of the curved sections in a well, as well as for analysis of critical rotation speeds. We have investigated the moment of inertia for a bent section in the rotating drill string using the models with concentrated and distributed masses. Based on the results, we have established the exact and asymptotic analytical dependences in order to determine inertial characteristics of the curvilinear sections in a drill string, as well as provided recommendations regarding the application of these dependences.

A current trend in the development and modernization of drilling equipment is the use of drill pipes made of unconventional materials. Given the scientific and practical interest in the application of these materials, we calculated the moments of inertia for the curved sections of drill strings, which can be equipped with steel, aluminum, titanium, or glass-plastic drill pipes. Analytical estimation of the moment of inertia of curved sections refers to a different scale of the deformed state of a drill string. The formula for the moment of inertia, established for simple models, holds in cases when the curved section of a drill string executes large displacements. For the case of small displacements, it is necessary to apply the analytical result derived when using a model with distributed parameters. The established patterns are essential for analyzing the dynamics of a drill string in deep conditionally vertical, inclined-directional, or horizontal wells with a complex mining-geological profile.

**Keywords:** drill string, drill pipe, curved rod, concentrated mass, distributed mass, moment of inertia.

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**THE STUDY OF MULTICOMPONENT LOADING EFFECT ON THINWALLED STRUCTURES WITH BOLTED CONNECTIONS (p. 15-25)**

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Features of influence of various factors on the stress-strain state of composite thin-walled structures with bolted connection of separate elements were studied on an example of the test problem. As an example of such structures, a metallic granary (a silo) consisting of panels connected with bolts was taken. The test structure contained two lapping narrow flat strips. A bolt is inserted in bolt holes bored in these strips and pre-tightened. Friction and slipping of the strips and the bolt, contact between the side surface of the bolt and the holes as well as mutual influence of bending and stretching were taken into consideration. Thus, the model has taken into consideration geometric, physical and structural nonlinearities. The system was subjected to a transverse load applied to one side of the strip. Staged loading of the systems was modeled. It was established that under load, the studied system acquires a deflection which unevenly increases with the load increase. This is determined by the fact that it is affected by both elastic deformation of the strips and mutual slip in the connection zone. When the gap between the bolt and the holes in the panels finally vanishes, mainly elastic deformation of the system takes place. Residual deflection was established in the system after the first unloading. It was also established that longitudinal forces act in the system. They can be much larger than transverse forces from the load. The system featured strong mutual influence

of bending and stretching of the strip. As a result of the studies, factors determining stress-strain state of the studied system were determined: geometric nonlinearity, contact interaction, friction and slip, connection between deflection and stretching. Thus, the design model for such thin-walled structures will be inadequate without all these factors, the results of calculations with its application will have significant errors and recommendations will be unreliable. The conducted studies have made it possible to develop more adequate models for analysis of reaction of composite thin-walled structures to the effect of loading.

**Keywords:** thin-walled structure, bolted connection, stress-strain state, metallic granary, geometric nonlinearity.

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### MODELING THE ELASTIC IMPACT OF A BODY WITH A SPECIAL POINT AT ITS SURFACE (p. 25-32)

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We have considered the elastic straight impact along a flat border of the stationary half-space of the body bounded in a zone of contact interaction by the surface of rotation, whose order is smaller

than two. The feature of the problem is that for the selected case an infinite curvature of the boundary surface at a point of initial contact, from which the process of dynamic compression of bodies in time starts. In addition to basic assumptions from the quasi-static theory of elastic impact between solid bodies, we have used a known solution to the static axisymmetric contact problem from the theory of elasticity. The process of an impact at a small initial velocity is divided into two stages: the dynamic compression and the dynamic decompression. For each of them, we have built an analytic solution to the nonlinear differential equation of relative convergence of the centers of bodies' masses in time. A solution to the non-linear problem with initial conditions for the differential equation of second order at the first stage was expressed through the Ateb-sinus, and at the second stage – through the Ateb-cosine. To simplify calculations, we have compiled separate tables for the specified special functions, as well as proposed their compact approximations using basic functions. It was established that an error of analytical approximations of both special functions is less than one percent. We have also derived closed expressions for computing the maximum values: compression of a body, impact strength, radius of the circular contact area, and pressure, which is limited in the center of this area. We have considered a numerical example related to the impact of a rigid elastic body against a rubber half-space. Problems of this type arise when modeling the dynamic action of pieces of a solid mineral on rubber, when they fall on the rolls of a vibratory classifier lined with rubber. Based on the results from comparing the calculated parameters of an impact, we have received good agreement between numerical results, obtained from the constructed analytical solutions, and the integration of a nonlinear equation at a computer. This confirms the reliability of the built analytical solutions to the problem on impact, which provide for the convolution of a brief process over time.

**Keywords:** elastic impact, special point at the contact surface, periodic Ateb-functions.

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**DETERMINING STRENGTH INDICATORS FOR THE BEARING STRUCTURE OF A COVERED WAGON'S BODY MADE FROM ROUND PIPES WHEN TRANSPORTED BY A RAILROAD FERRY (p. 33-40)**

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Improving the efficiency of transportation process through international transport corridors promotes the development of interoperable systems. Successful functioning of the interoperability of transportation is possible at reliable and well-coordinated work of individual components. In this regard, it is necessary to introduce into service a new generation of rolling stock with improved techno-economic and performance indicators. We have designed a supporting structure of the covered wagon, whose special feature is that the elements of the body are made of round tubes; in order to ensure the reliability of its fastening to the deck of a rail ferry, the nodes for fastening chain couplers are arranged at the pivot beams. To refine the determination of indicators for the strength of the body of a covered wagon, we have investigated its dynamic loading under the most unfavorable estimation scheme – angular displacements of the railroad ferry relative to its longitudinal axis (equivalent to lateral pitching oscillations in the dynamics of railroad cars). We have determined the maximum magnitude of accelerations using mathematical modeling of a railroad ferry oscillations with wagons placed on its decks, applying a second-order Lagrange method. Solving differential equations of a railroad ferry oscillations, with railroad cars on it, employed the Runge-Kutta method in the programming environment MathCad. When determining the total magnitude of acceleration acting on the body of a covered wagon when transported by a railroad ferry, we also accounted for the horizontal component of a free fall acceleration, predetermined by the tilt angle (heeling) of the railroad ferry. The resulting value for acceleration as a component of dynamic loading was taken into account while studying the strength of a load-bearing bodywork of the covered wagon. The calculation employed a finite element method in the programming environment CosmosWorks. To this end, we developed a model of strength of a load-bearing bodywork of the covered wagon made from round tubes when transported by a railroad ferry. It has been established that the maximum equivalent stresses do not exceed those permissible for the grade of steel used for metallic structures of the body and are about 280 MPa. We have determined a design service life of the node for fastening chain screeds at the body of a covered wagon when transported by a railroad ferry. Results of this research could be applied when designing railroad cars of the new generation with improved techno-economic and performance indicators.

**Keywords:** covered wagon, bearing structure, dynamic loading, railroad and water transport, railroad-ferry transportation.

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**CONSTRUCTION OF AN ALGORITHM FOR THE SELECTION OF RIGID STOPS IN STEEL CONCRETE BEAMS (p. 41-49)****Anatoliy Petrov**

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Calculation of steel-concrete beams is performed with a rigid connection between concrete and a steel strip. This is possible if one installs hard stops that prevent the displacement of the strip with respect to concrete. The force acting on the stop, the number of hard stops and their pitch, are determined through the rotation angles between two adjacent stops. To determine the efforts that act on hard stops, as well as a step, one must first find a rotation angle between two cross-sections within the beams. The rotation angles of cross-sections are derived using a graph-analytical method. Calculation for the deformations of reinforced-concrete and steel-concrete beams is performed based on the reduced rigidities of cross-sections.

When one chooses a step for hard stops and their number, it is necessary to strive for the optimization of a structure of steel-concrete beams. Optimization implies that the maximum stresses in a steel strip are equal to its limiting value while the effort acting in stops, and the step of stops, are the same. In order for the efforts in each stop to be the same, one must fabricate a zero section less than the others.

In the course of our study we have developed an algorithm for selecting the number, a step of hard stops, and the efforts in them. The choice is based on the assigned characteristics of materials used, the acting external load, the length of a beam, known size of the cross-section of concrete and a steel strip. In this case, efforts in all stops are identical, the step of stops is constant except for the zero section, maximum effort in the steel strip, occurring in the middle of the span, does not exceed the boundary value obtained in the calculation. The reported algorithm makes it possible to calculate hard stops at the assigned value for efforts that act on them under existing load.

**Keywords:** steel-concrete beam, hard stop, step between stops, effort in a stop, steel sheet, reduced rigidity, graph-analytical method.

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**STUDYING THE RAILROAD TRACK GEOMETRY DETERIORATION AS A RESULT OF AN UNEVEN SUBSIDENCE OF THE BALLAST LAYER (p. 50-59)****Olga Nabochenko**

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A method for calculating impairment of the track geometry under influence of dynamic loads in the course of passing the track unevenness by the rolling stock was developed. The method takes into consideration interrelated short-term processes of dynamic interaction and long-term processes of subsidence of the ballast layer in a mutual influence on each other. Mathematical model of dynamic interaction of the track in the form of a planar three-layer continual beam system with a two-mass discrete system corresponding to the rolling stock is the basis of the first part of the method. This model makes it possible to simulate dynamic loads from individual sleepers to the ballast when the rolling stock passes geometric unevennesses and the track elasticity unevennesses.

The second part of the method is based on the phenomenological mathematical model of accumulation of residual deformations formed using the results of laboratory studies of subsidence of individual sleepers in the ballast layer. Peculiarity of this model consists in taking into consideration not only uniform accumulation of residual subsidence from the passed tonnage but also presence of a plastic component of subsidence which depends on the maximum stresses in the history of ballast loading by each sleeper.

A new theoretical mechanism of development of the track unevenness was proposed. It takes into consideration not only residual subsidences of the ballast layer but also appearance of gaps under sleepers resulting in a local change of the track elasticity. This mechanism enables taking into consideration the ambiguous influence of subsidences with occurrence of gaps under the sleepers. Subsidence causes an increase in dynamic loads on the track and the ballast layer on the one hand and onset of the gap causes a decrease in the track rigidity and corresponding reduction of dynamic loads on the other hand.

Practical application of the developed method was demonstrated on an example of quantitative estimation of long-term uneven subsidences of the ballast layer when changing the sleeper diagram.

**Keywords:** railway track, ballast layer, railway rolling stock, geometric unevenness of the track.

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**DEVELOPMENT OF A METHOD FOR COMPUTER**  
**SIMULATION OF A SWINGING SPRING LOAD**  
**MOVEMENT PATH (p. 60-73)**

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Studies of geometric modeling of non-chaotic periodic paths of movement of loads attached to a variety of mathematical pendulums were continued. Pendulum oscillations in a vertical plane of a suspended weightless spring which maintains straightness of its axis were considered. In literature, this type of pendulum is called a swinging spring. The sought path of the load of the swinging spring was modeled with the help of a computer using values of the load weight, stiffness of the spring and its length without load. In addition, initial values of oscillation of the swinging spring were used: initial angle of deviation of the spring axis from the vertical, initial rate of change of this angle as well as initial parameter of the spring elongation and initial rate of elongation change. Calculations were performed using Lagrange equation of the second kind. Variants of finding conditionally periodic paths of movement of a point load attached to a swinging spring with a movable fixing point were considered.

Relevance of the topic was determined by necessity of study and improvement of new technological schemes of mechanical devices which include springs, in particular, the study of conditions of detuning from chaotic oscillations of the elements of mechanical structures

and determination of rational values of parameters to ensure periodic paths of their oscillation.

A method for finding values of a set of parameters for providing a nonchaotic periodic path of a point load attached to a swinging spring was presented. The idea of this method was explained by the example of finding a periodic path of the second load of the double pendulum.

Variants of calculations for obtaining periodic paths of load movement for the following set parameters were given:

- length of the spring without load and its stiffness at an unknown value of the load weight;
- length of the spring without load and the value of the load weight at unknown spring stiffness;
- value of the load weight and stiffness of the spring at an unknown length of the spring without load.

As an example, determination of the values of a set of parameters to provide a non-chaotic, conditionally periodic path of movement of a point load attached to a swinging spring with a movable attachment point was considered.

Phase paths of functions of generalized coordinates (values of angles of deflection of the swinging spring axis from the vertical and extension of the spring) were constructed with the help of which it is possible to estimate ranges of these values and rates of their variation.

The results can be used as a paradigm for studying nonlinear coupled systems as well as in calculating variants of mechanical devices where springs affect oscillation of their elements when it is necessary to detune from chaotic movements of loads in the technologies using mechanical devices and provide periodic paths of their movement.

**Keywords:** pendulum oscillations, periodic paths of movement, swinging spring, Lagrange equation of the second kind.

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**ANALYTICAL METHOD TO STUDY A MATHEMATICAL MODEL OF WAVE PROCESSES UNDER TWOPOINT TIME CONDITIONS (p. 74-83)**

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Research and analysis of dynamic processes in oscillatory systems are closely connected to the establishment of exact or approximate analytical solutions to the problems of mathematical physics, which model such systems. The mathematical models of wave propagation in oscillatory systems under certain initial conditions at a fixed time are well known in the literature. However, wave processes in lengthy structures subject to an external force only and at the assigned states of the process at two points in time have been insufficiently studied. Such processes are modeled by a two-point time problem for the inhomogeneous wave equation in an unbounded domain  $t > 0, x \in \mathbb{R}^s$ . The model takes into consideration the assignment of a linear combination with unknown amplitude of oscillations and the rate of its change at two points in time. A two-point problem, generally speaking, is the ill-posed boundary value problem, since the respective homogeneous problem has non-trivial solutions. A class of quasi-polynomials has been established as the class of the existence of a single solution to the problem. This class does not contain the non-trivial elements from the problem's kernel, which ensures the uniqueness of solution to the problem. We have proposed a precise method to build the solution in the specified class. The essence of the method is that

the problem's solution is represented as the action of a differential expression, whose symbol is the right-hand side of the equation, on some function of parameters. The function is constructed in a special way using the equation and two-point conditions, and has special features associated with zeroes of the denominator – the characteristic determinant of the problem.

The method is illustrated by the description of oscillatory processes within an infinite string and a membrane.

The main practical application of the constructed method is the possibility to adequately mathematically model the oscillatory systems, which takes into consideration a possibility to control the system's parameters. Such a control over parameters makes it possible to perform optimal synthesis and design of parameters for the relevant technical systems in order to analyze and account for special features in the dynamic modes of oscillations.

**Keywords:** oscillatory systems, mathematical models of wave processes, differential-symbol method, two-point problem, wave equation.

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