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New hyperbolic statistics for the equilibrium distribution function of interacting electrons

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New statistics of a low-parameter distribution of the sech (ε, μ) type are presented, which reproduce the results of plasma simulation by the method of dynamics of many particles (DMP) with high accuracy. The distribution is based on a conceptual model of a two-component plasma — virtual quasiparticles of negative energy (exciton phase $\varepsilon<0$); the scattering region of positive energy (gas phase $\varepsilon>0$). Optimization and elementary estimates of the applicability of the sech (ε, μ) distribution statistics were made after the results of DMP experiments. The sech (ε,μ) distribution reduces the number of parameters of the three-piece DMP distribution from 4 energy diffusion coefficients (D_1, D_2, D_3, D_4) to two — the chemical potential μ and the asymmetry coefficient α. The functional relationship D_1, D_2, D_3, D_4 with the chemical potential of the system μ in the sech (ε, μ) distribution is introduced in a similar way to the Einstein relation between mobility and energy diffusion constants. The functional variety of the differential equation belongs to the family of elliptic functions. It is much wider than the hyperbolic solution given, which has significant physical application for complex values of the energy ε. The proposed simplified scheme grounded in the physical interpretation of negative energies can be written for the electrometric electrons of the atmosphere, which previously presented significant methodological difficulties. The chemical potentials of the fluid (metastable states) and gas phases are presented as functions of the plasma imperfection parameter. The problem is posed as an application to the problem of electrometric electrons in the atmosphere. The proposed distribution is not represented in mathematical statistics and statistical physics; it is new and extremely relevant.

Key words: cold plasma, electron distribution function, elliptical functions, atmospheric electrons.

Excess electrons of metastable atoms and molecules lie in the vicinity of the ionization threshold with a width of ~kT, capture the region of the discrete and continuous spectrum, and have increased polarizability (~n^6) and coherence [Anderson, 2015], where n is the main quantum number. The single-particle distribution of such electrons in contact with the thermostat does not coincide with the single-particle Boltzmann energy distribution ε. The transition to an equilibrium distribution for a quasi-classical cold plasma was performed in the research on DMP simulation [Maiorov et al., 1991, 1992], where a fundamental result was obtained — the existence of a significant number of single-particle total electron energies. The numerical nonparametric DMP distribution has the classical form of mathematical statistics functions with a Gaussian vertex and separate asymmetric asymptotics in the region of negative states — the manifestation of quasi-stable structures (electrons, Boron atoms, classical negative ions). A characteristic factor for such a plasma was the presence of large fluctuations in the negative regions of the supra-threshold ionization energy. In this context, there was a problem of interpretation of the classical thermodynamic temperature and the residual number of parameters of the analytical distribution, which the authors solved [Maiorov et al., 1995] on the basis of the Focker-Planck process by obtaining a new three-piece distribution function with four energy diffusion parameters D_1, D_2, D_3, D_4, and the plasma imperfection parameter δ.

In engineering and physical terms, such a distribution accurately describes the equilibrium state of the plasma as a function of the parameters (D_1, D_2, D_3, D_4, δ). However, its classical application for the analysis of statistical plasma physics is complicated by the complexity of parametrization and the physical meaning of the process.

In this report, a small-parametric distribution of the sech (ε, μ) type is proposed, which allows reproducing three-piece distributions from the parameters (D_1, D_2, D_3, D_4, δ) [Maiorov et al., 1992] using only two parameters: μ — the chemical potential of the plasma and the asymmetry parameter α, in the vicinity of equilibrium.

The physical meaning of the new distribution corresponds to a two-component model of real plasma — virtual quasiparticles of negative energy, the scattering region of positive energy. Such a mechanism corresponds to the structure of the phase space of a multiparticle system adopted in ergodic theory — ergodic divergence of trajectories in the region of positive energies and homoclinic destruction of separatrices in the region of quasi-bound states. A characteristic feature of the sech(ε, μ) distribution is quasi-Fermi (the presence of unity in the denominator), which is due to the classical displacement of neutral particles from a dense plasma volume — the Van der Waals effect.

Optimization and elementary estimates of the applicability of the sech (ε, μ) distribution statistics were performed using DMP experiments [Maiorov et al., 1992]. The numerical correspondence in accuracy between the sech(ε, μ) and DMP distributions is given in the Appendix.

DMP distribution of classical Coulomb plasma. This section shows the distribution function f_{DMP} (see expression (1)) from research [Maiorov et al., 1992], according to which the distribution function is optimized f_{ch} (see expression (2) below).

We quote according to the research [Maiorov et al., 1992]: «the study of classical Coulomb plasma by methods of dynamics of many particles (DMP) allowed us to obtain the electron distribution function over the total energy in a piecewise analytical form consisting of three asymptotics

$$f(y)=\frac{2A}{T_e\sqrt{\pi}}\begin{cases}\sqrt{y}\exp(-y)\\ D_3\exp\left(D_1y+D_2y^2/2\right),\\ D_4\exp\left(\beta y/\sqrt[3]{\delta}\right)\end{cases}\quad \begin{cases}y>\alpha\delta^{1/3}\\ |y|\le\alpha\delta^{1/3}\\ y<-\alpha\delta^{1/3}\end{cases},\qquad (1)$$

where A is the normalization constant, α and β are the adjustable constants, δ is the plasma imperfection parameter, and $y=\varepsilon/T_e$ is the dimensionless energy. The distribution describes the statistics of strongly interacting particles. It solves the problem of crosslinking the output of two-particle states from the Rydberg series ε<0 to the scattering region ε>0, using four energy diffusion coefficients $D_1(\delta_e)$, $D_2(\delta_e)$, $D_3(\delta_e)$, $D_4(\delta_e)$ depending on the plasma imperfection parameter δ:

$$D_1=\left[-1+1/\left(2\alpha\delta^{1/3}\right)+\beta/\delta^{1/3}\right]/2,$$

$$D_2=\left[-1+1/\left(2\alpha\delta^{1/3}\right)-\beta/\delta^{1/3}\right]/\left(2\alpha\delta^{1/3}\right),$$

$$D_3=\alpha^{1/2}\delta^{1/6}\exp\left[-\alpha\delta^{1/3}\left(1+D_1+D_2\alpha\delta^{1/3}/2\right)\right],$$

$$D_4=\alpha^{1/2}\delta^{1/6}\exp\left[\alpha\beta-\alpha\delta^{1/3}\left(1+2D_1\right)\right].$$

Note for Geophysics. The considered type of cold plasma is close in parameters to the electrometric cold plasma of dense layers of the atmosphere, clouds, and auroras, where the concentration of charged particles is of the order $N\sim10^2\div10^4$. It is obvious that such distributions can hardly be obtained by solving kinetic equations since relaxation is nonlinear in nature with delayed recombination, as noted in research [Tkachev, Yakovlenko, 1997]. It is delayed recombination that provides the electrical activity of the atmosphere.

Problem statement. Let us set the fraction of particles with positive and negative energies in the DMP simulation by p_+ and p_-. According to [Maiorov et al., 1995], the total number of particles in the simulation (512 cm^{-3}) will correspond to $p_+ + p_- = 1$. Then, for particles with negative energies $p_- = 200/512 = 0.4$, positive $p_+ = 0.6$.

Knowing these parameters, we can obtain a definite integral for the value p_+, ($y=x-\mu$) for p:

$$p_+ = 1 \Big/ \pi \int_0^{\infty} \frac{1}{\cosh(x-\mu)} dx = 1 \Big/ \pi \int_{-\mu}^{\infty} \frac{1}{\cosh(y)} dy .$$

Performing computations of the integral, we obtain the equation for the chemical potential μ providing p:

$$\mu \to \ln\left(\mathrm{Tan}\left(\frac{p\pi}{2}\right)\right).$$

If we substitute the parameters from [Maiorov et al., 1995], we get meaningful values of the chemical potential of the system μ: at $p=0.6$, $\mu=0.32$; at $p=0.84$, $\mu=1.36$.

To describe the equilibrium state of a plasma with a different number of particles for negative $\varepsilon<0$ and positive $\varepsilon>0$ energies, we write down the shape of the cosh(y–μ) distribution by entering a dimensionless parameter of fluctuations $y=\varepsilon/\varepsilon_0$:

$$f_{ch} = \frac{1}{\pi} \frac{1}{\cosh(y-\mu)} .$$

The function f_{ch} has a more acute maximum than the DMP distribution f_{DMP} (see expression (1)), which has a flat appearance due to an equilibrium state $\varepsilon=0$, to which a system of Coulomb particles with probability 1 tends. Optimization of distribution parameters f_{ch} can occur both by varying the asymmetry coefficient and by chemical potential. The results of such variations are shown in Fig. 1.

Let us write down f_{ch} the distribution for a system with effective three energy levels $\varepsilon=0$ and $\pm\varepsilon$:

$$f_{ch} = \frac{1}{1+\cosh(y-\mu)} .$$

Then, similarly to the distribution with a single ground state, we obtain the equation for the chemical potential μ:

$$\mu \to -ln\left(-\frac{-1+p}{p}\right).$$

Substituting the parameters p, we obtain acceptable estimates of the chemical potential μ: $p=0.6$, $\mu=0.41$; $p=0.84$, $\mu=1.65$.

It can be seen that by introducing different chemical potentials for the fluid and gas phases (the vicinity of the ionization threshold), it is possible to obtain a reasonable correspondence with the experimental distributions [Maiorov et al., 1992] by introducing only two parameters, the chemical potential μ and the asymmetry coefficient of the asymptotics α. Thus, we will write down the general form of statistics in the following form:

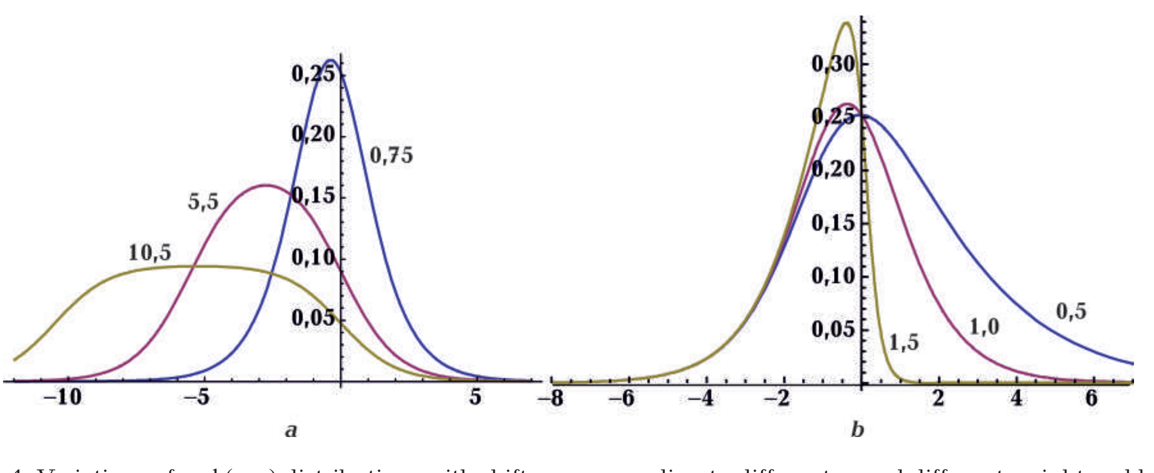

Fig. 1. Variations of cosh(y–μ) distributions with shifts corresponding to different μ, and different α right and left asymptotics f_{ch}: *a* — α=1, μ=0.75; μ=5.5; μ=10.5; *b* — μ=0.75, α=0.5; α=1; α=1.5.

$$f_{ch} = \frac{1}{S}\frac{1}{1+e^{-a(x-\mu_1)}+e^{(x-\mu_2)}}. \qquad (2)$$

Here α considers the different nature of ergodization for ε<0, actually, the asymptotics of the wave function of a quasi-stationary state (a materially dependent constant).

For symmetric α and α=1, the normalization in (2) is given as $N=1/S$:

$$S \cong \frac{\pi}{a\sqrt{-1+4\mathrm{e}^{\alpha(\mu_1-\mu_2)}}},$$

where S is the distribution area. The normalization calculation using the contour integral (phase shift method) is given in the Appendix.

Let us note that with the chemical potentials and asymmetries estimated above, the normalization varies within 0.8—1.5, which corresponds in order of magnitude to the complex normalization of the DMP distribution [Maiorov et al., 1992]. Within the accuracy of the experimental spread, the distribution (1) has already been normalized, which facilitates the optimization procedure for obtaining parameters during reparametrization (2) by the DMP distribution (1).

Numerical reparametrization of the DMP distribution. The problem of reparametrization of distribution (2)

$$f_{ch} = \frac{1}{S}\frac{1}{e^{-a_1(x-\mu_1)}+e^{a_2(x-\mu_2)}}$$

by piecewise analytical distribution (1) is solved. The initial conditions and plasma parameters (electron concentration — N_e; temperature — T_e; imperfection parameter — δ) from the research [Maiorov et al., 1992] for a series of experiments (26, 30, 40, 43) were

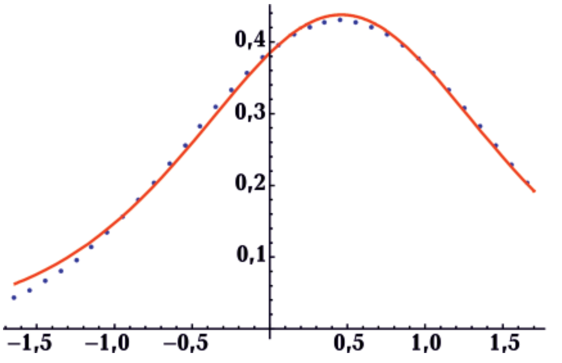

Fig. 2. Graphical representation of the machine optimization of the distribution f_{ch} (1) (black line), according to the DMP distribution (2) (dots).

selected as optimization. The optimization results are summarized in Table. The graphical match is shown in Fig. 2. Optimization was performed by an interactive machine method with a variation of the main parameters and normalization, which in order of magnitude, always lies within unity.

As shown in Fig. 2, the correspondence between distributions (1) and (2) matches the experimental accuracy. Boltzmann asymptotics merge graphically.

Functional reparametrization in the vicinity of equilibrium. Einstein-type relations. Using the asymptotics α=β/γ and the expansion of the function in a series in the vicinity of the maximum points, we obtain functional dependencies between the distribution parameters f_{ch} — (2) and f_{DMP} — (1). The maximum points for (1) and (2) will be

$$\max\left(f_{\mathrm{DMP}}\right) = -\frac{D_1}{D_2}, \quad \max\left(f_{ch}\right) = \frac{\mu_1+\mu_2}{2}.$$

Then the values of the function at the maximum points will be written as:

Reparametrization f_{ch} (x, α, μ_1, μ_2) for different coefficients of asymmetry of the right and left parts of the distribution (α_1, α_2, μ_1, μ_2) and for the symmetric case with different shifts in energy (α, μ_1, μ_2)

N	N_e, cm^{-3*}	T_e, eV*	δ*	α_1	α_2	μ_1	μ_2	α	μ_1	μ_2
26	10^{14}	0.018	0.102	0.15	0.49	0.97	1.01	0.024	0.20	0.79
40	10^{14}	0.02	0.074	0.02	0.04	0.07	0.60	0.031	0.258	0.820
30	10^{15}	0.038	0.108	0.06	0.05	0.07	0.66	0.057	0.165	0.757
43	10^{17}	0.20	0.074	0.361	0.263	0.13	0.67	0.323	0.234	0.780

* — plasma parameters are taken as an example from [Maiorov et al., 1992, table 1, p. 21].

$$f_{\mathrm{DMP}}=\frac{1}{A}D_3\exp\left(-\frac{D_1^2}{2D_2}\right),$$

$$f_{ch}=\frac{1}{S}\frac{1}{2\exp\left(\frac{\alpha\mu_1}{2}-\frac{\alpha\mu_2}{2}\right)},$$

where A, S are normalizations; D_1, D_2, D_3 are energy diffusion coefficients; μ_1, μ_2 are chemical potentials; α is the asymmetry coefficient.

Given the values of the function at the points of maxima, two functional equations can be obtained to determine the parameters μ_1 and μ_2:

$$\ln(2D_3)-\frac{D_1^2}{2D_2}=-\frac{\beta/\gamma(-\mu_1+\mu_2)}{2},$$

$$-\frac{D_1}{D_2}=\frac{\mu_1+\mu_2}{2}.$$

Considering the normalization of order 1, the system is solved analytically and has roots μ_1 and μ_2:

$$\mu_1=-\frac{2D_1\beta+D_1^2\gamma-2D_2\gamma\ln(2D_3)}{2D_2\beta},$$

$$\mu_2=-\frac{2D_1\beta-D_1^2\gamma+2D_2\gamma\ln(2D_3)}{2D_2\beta}. \qquad (3)$$

Substituting the plasma parameters from experiment 43 (see Table), we determine the energy diffusion coefficients and the chemical potential of the system: D_1=0.598; D_2=–1.198; D_3=0.369; D_4=0.429; β=0.4; γ=0.333; δ=0.074; μ_1=0.371; μ_2=0.627.

As can be seen from the estimates of the chemical potential by expression (3), they are close to the optimization ones but do not coincide (see Table). Reduction to equilibrium parameters is stable within the limits of the DMP experiment.

Conclusions. The report shows that the kinetic distribution is *significantly simplified* in the vicinity of equilibrium — the number of energy diffusion parameters is reduced, similar to Einstein's relation between mobility and diffusion $D/\mu=kT$. This suggests that in the vicinity of equilibrium, the statistical process has a simplified Fokker-Planck nature. The type f_{ch} distribution identically satisfies a nonlinear cubic equation of the second order for the distribution function, similar to nonlinear noise generators. A similar cubic equation for the distribution function is found in research on Rydberg plasma [Morrison et al., 2012; Aghigh et al., 2020] and quantum optics [Tkachev, Yakovlenko, 1993, 2001]. The distributions (2) given in the research have a computational advantage — analytics for application in statistical physics.

It should be particularly noted that the functional variety of the differential equation belongs to the family of elliptic functions and is much wider than the hyperbolic solution given, which has significant physical application for complex values of the energy ε. Moments are considered parametric differentiation of a given distribution (Appendix).

The proposed simplified scheme grounded on the physical interpretation of negative energies can be written for the electrometric electrons of the atmosphere, which presented significant methodological difficulties previously. The authors suggest that the complexity of parametrization for the case of equilibrium of nonperfect plasma can hardly be obtained by solving a kinetic problem without partial simulation, and the presented equilibrium distribution is a good candidate for simplifying the results of research (S.A. Maiorov, A.N. Tkachev, S.I. Yakovlenko from period 1992—2008) on the equilibrium properties of a nonperfect plasma. The chemical potentials of the fluid (metastable states) and the gas phase are presented as functions of the plasma imperfection parameter. The problem is posed as an application to the problem of electrometric electrons in the atmosphere.

APPENDIX

The right-hand side of distribution (2) has a materially dependent exponential decline connected with the asymptotics of an atom or molecule immersed in a medium. The absence of f_{ch} 1 in the denominator in the distribution makes it possible to perform analytical taking of integrals by the phase shift method. The main integral is taken along the real axis, the second integral is shifted by the width of the distribution. This will correspond to the composite state of ionization of a quantum system with a thermostat. For supercooled plasma, it is necessary to introduce the fullness of the ground state, which is close to 1. In other words, even in the absence of Fermi degeneracy ($\varepsilon_f \sim 10^{-3} \div 10^{-5}$), this value will be much less than kT. Proximity to the ionization threshold leads to a quasi-Fermi distribution with 1 in the denominator. The normalization of distribution (2) is obtained by contour integration with a phase shift of the function of a complex variable; the moments are considered parametric differentiation of this distribution.

The integral of distribution (2) is estimated by a series of geometric progression or reparametrization of the analytical expression. Below are 1—3 points and the general form of the record:

$$I_1 = \pi \frac{\left(b^2/a^2\right)^{-\frac{\alpha}{1+\alpha}}}{a^2(1+\alpha)\sin\left(\pi\frac{\alpha}{1+\alpha}\right)}, \quad (4)$$

$$I_2 = \pi \frac{\left(b^2/a^2\right)^{-\frac{2\alpha}{1+\alpha}}(1-\alpha)}{a^4(1+\alpha)^2\sin\left(\pi\frac{2\alpha}{1+\alpha}\right)}, \quad (5)$$

$$I_3 = \pi \frac{\left(b^2/a^2\right)^{-\frac{3\alpha}{1+\alpha}}\left(2-5\alpha+2\alpha^2\right)}{2a^6(1+\alpha)^3\sin\left(\pi\frac{3\alpha}{1+\alpha}\right)}, \quad (6)$$

$$I_n = \pi \frac{\left(b^2/a^2\right)^{-\frac{n\alpha}{1+\alpha}}\left(\frac{\alpha-n}{\alpha+1}\right)_n}{2a^{n+1}\sin\left(\pi\frac{n\alpha}{1+\alpha}\right)}, \quad (7)$$

where $F=\left(\frac{\alpha-n}{\alpha+1}\right)_n$ is the Pochhammer function.

Let us consider the accuracy of the calculation results for expressions (4)—(6) with the results of numerical integration of expression (2), with symmetric α, when there is no 1 in the denominator

$$f_s = \frac{1}{a^2e^{ax}+b^2e^{-x}}. \quad (8)$$

We will consider I_1, I_2, and I_3 — calculation by analytical expressions (4)—(6), and I_{2N}, I_{3N} is a calculation of numerical integration. Let us substitute the parameters (α=1.5, a=1, b=2) into expressions (4)—(8) and get estimates:

$$I_1 = \pi \frac{\left(b^2/a^2\right)^{-\frac{a}{1+a}}}{a^2(1+a)\sin\left(\frac{a}{1+a}\pi\right)} = 0.575132.$$

Numerical integration:

$$\int_{x_{\min}}^{x_{\max}} \frac{1}{a^2e^{\alpha x}+b^2e^{-x}}dx,$$

the area between $x_{\min}$ and 0

I_{1-}=0.234361297341549,

the area between 0 and $x_{\max}$

I_{1+}=0.3407707179217075.

I_{1N}=0.234361297341 + 0.3407707179217= = 0.5751320152.

We have for a square:

$$I_2 = \pi \frac{\left(b^2/a^2\right)^{-\frac{2\alpha}{1+\alpha}}(1-\alpha)}{a^4(1+\alpha)^2\sin\left(\pi\frac{2\alpha}{1+\alpha}\right)} = 0.08101196.$$

Numerical integration:

the area between $x_{\min}$ and 0 I_{2-}=0.02564997810,

the area between 0 and $x_{\max}$ I_{2+}=0.05536199032.

I_{2N}=0.02564997810+0.05536199032 = =0.08101196.

We have for a cube:

$$I_3 = \pi \frac{\left(b^2/a^2\right)^{-\frac{3\alpha}{1+\alpha}}(2-5\alpha+2\alpha^2)}{2a^6(1+\alpha)^3\sin\left(\pi\frac{3\alpha}{1+\alpha}\right)} = 0.0141050029.$$

Numerical integration:

the area between $x_{\min}$ and 0 I_{3-}=0.0036319352,

the area between 0 and $x_{\max}$ I_{3+}=0.0104730676.

As we can see, the accuracy of matching the results is of an order of magnitude 10^{-10}. Let us estimate numerically the integral of expression (8) when there is 1 in the denominator:

$$\int_{x_{\min}}^{x_{\max}} \frac{1}{a^2\left(1+e^{\alpha x}+b^2e^{-x}\right)}dx. \quad (9)$$

Integral (9) in the form of a geometric progression up to the third order gives the following approximation: 0.575132015–0.081011968+0.014105002=0.508225049.

Comparing it with the exact value (I_1–I_2+I_3=0.506002817) we obtain a discrepancy of geometric progression up to the third order equal to: 0.506002817–0.5082250497= =–0.00222223269.

The calculation of the symmetric integral gives a good approximation for the area with a functional simplification of the integration result (2), namely: $\frac{\pi}{\sqrt{-1+4a^2b^2\alpha}}$.

Substituting the values (a=1, b=2, α=1.5) we have a numerical result of 0.5. Then the discrepancy of the symmetric approximation will be: 0.5060028–0.5407704=–0.034767.

In the area of a particular asymmetry coefficient α, a good approximation for accurate normalization can be obtained if it is renormalized. In our case (a=1, b=2, α=1.59606), an increase of 0.1 gives the following result:

$$\frac{\pi}{\sqrt{-1+4a^2b^2\alpha}} = 0.5069723 .$$

The symmetric approximation makes sense to simplify the analysis of calculating averages within the framework of statistical mechanics. In our case, the coefficients a and b are obviously related to the parameters of the model: $a = \exp^{-\frac{\alpha\mu_2}{2}}$, $b = \exp^{\frac{\alpha\mu_1}{2}}$.

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Нові гіперболічні статистики для рівноважної функції розподілу взаємодіючих електронів

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Подано нові статистики малопараметричного розподілу типу sech (ε,μ), які з високою точністю відтворюють результати моделювання плазми методом динаміки багатьох частинок (ДБЧ). В основу нового розподілу покладено концептуальну модель двохкомпонентної плазми — віртуальні квазічастинки негативної енергії (екситона фаза $\varepsilon<0$); область розсіювання позитивної енергії (газова фаза $\varepsilon>0$). Оптимізація та елементарні оцінки застосування статистики sech (ε,μ) розподілу проведені за результатами експериментів ДБЧ. Функція розподілу sech (ε,μ) скорочує кількість параметрів трикускового ДБЧ-розподілу з чотирьох коефіцієнтів енергетичної дифузії (D_1, D_2, D_3, D_4) до двох — хімічний потенціал μ і коефіцієнт асиметрії α. Функціональний зв'язок D_1, D_2, D_3, D_4 з хімічним потенціалом системи μ у розподілі sech (ε, μ) вводиться аналогічно співвідношенню Ейнштейна між рухливістю та константами енергетичної дифузії. Функціональне різноманіття диференціального рівняння належить до сімейства еліптичних функцій та значно ширше наведеного гіперболічного розв'язання, що має суттєво фізичний додаток для комплексних значень енергії ε. Запропонована спрощена схема на основі фізичної інтерпретації негативних енергій може бути записана для електрометричних електронів атмосфери, що раніше становило значні методичні складності. Хімічні потенціали флюїдної (метастабільні стани) та газової фаз представлені як функції параметра неідеальності плазми. Завдання поставлено як додаток до проблеми електрометричних електронів атмосфери. Запропонований розподіл, не представлений у математичній статистиці та статистичній фізики, є новим і вкрай актуальним.

Ключові слова: холодна плазма, функція розподілу електронів, еліптичні функції, електрони атмосфери.