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Gravity data reduction, Bouguer anomaly, and gravity disturbance

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Each point on the earth has a gravity and gravity potential value. Surfaces formed by connecting points with equal gravity potential values are called equipotential surfaces or level surfaces. Determination of gravity field of the earth and the geoid which is one of the earth's equipotential surfaces is very important for physical geodesy. Gravity values measured on the physical earth are not directly included in studies; firstly, they must be converted into gravity anomalies. For this, in this study precise leveling, gravity and GPS measurements were made in the field. Heights (H) with precise leveling measurements, gravity values (g) with gravity measurements and geographical latitudes (φ) with GPS measurements were recorded. Then, gravity reductions (free-air, Bouguer) were calculated at the points. The actual gravity g measured on Earth is not immediately directly comparable to the normal gravity of the ellipsoid surface. Gravity values must be reduced to the geoid. Since there are masses outside the geoid, reduction methods differ according to the way these topographic masses are handled. Bouguer gravity anomalies and gravity disturbances are derived. The gravity anomaly (Δg) is defined as the scalar difference between the Earth's gravity on the geoid and normal gravity on the surface of the reference ellipsoid. Gravity disturbance (δg) is defined as the difference between the actual gravity magnitude measured on Earth and its equivalent normal gravity in the normal gravity field for the same point. The changes and magnitudes of the calculated quantities are compared. Changes such as the observed gravity and height data, observed gravity changes versus calculated normal gravity changes, normal gravity on ellipsoid versus geographic latitude, observed gravity changes versus latitude changes, Bouguer gravity anomaly and gravity disturbance versus latitude and elevation, free-air reduction and Bouguer gravity reduction versus latitude and elevation have been investigated.

Key words: real gravity, normal gravity, ellipsoid, geoid, free-air gravity reduction, Bouguer gravity reduction, Bouguer gravity anomaly, gravity disturbance.

Introduction. Earth's gravity field is affected by various sources of tidal, subsurface density variation and topographic relief, and rotation of the Earth. Some sources will be removed using practical Earth's model from the measured gravity and retain only the subsurface effect for geophysical applications. The normal gravity potential is derived as the potential of a reference ellipsoid plus the rotational potential of the ellipsoid. The gravitational field of a reference ellipsoid of rotation that closely approximates the real Earth is called the normal gravity field. The anomalous field is the discrepancy between the real and the normal fields.

The intensity of gravity, or gravity g, is the magnitude of the gravity vector. The force per unit mass, or acceleration, is a unit of gravity. The value of the normal gravity, γ_P, is the magnitude of the normal gravity vector. It is the magnitude of the gradient of the normal potential (contains centrifugal potential). It

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can be calculated at a given point on the ellipsoid (λ, φ, h=0) and Earth's physical surface (λ, φ, h) [Alemu, 2021].

Topography plays an important role in solving many geodetic and geophysical problems. In the evaluation of a topographical effect, a planar model, a spherical model or an even more sophisticated model can be used. In most applications, the planar model is considered appropriate; recall the evaluation of gravity reductions of the free-air, Poincaré-Prey or Bouguer kind [Vaníček et al., 2001].

The magnitude $g = \|g\|$ of the gravity acceleration vector g along the plumb line is observed on (land,marine), above (air- and space borne), or below (sea bottom, borehole, mines) the Earth's surface. The gravimetric information is required for numerous applications and studies not only in the Earth sciences but also in metrology and planetary and space sciences. The gravity (also the gravity anomaly) values are commonly expressed in the CGS (centimeter-gram-second) acceleration unit of mGal (1 mGal=10 $\mu m/s^2$) [Oja et al., 2019].

Gravity anomaly, in geodesy, is the difference between the geoidal gravity and the normal gravity on the mathematical model ellipsoid. In the theory of *modern* physical geodesy, the normal gravity upon the reference ellipsoid was introduced as the reference gravity [Heiskanen, Moritz, 1967].

The classical Bouguer anomaly has been defined upon the geoid by the difference between the observed gravity reduced to the geoid and the reference gravity upon the geoid. The reference gravity has been equated to the standard gravity. No distinction was made between the reference and standard gravity [Nozaki, 2006].

In geodetic applications, Bouguer gravity data is primarily used for gravimetric geoid modeling. In contrast, its use in geophysical applications is often related to the modeling and interpreting of inner structures (e.g., sediment basin basements) and processes (e.g., flexural displacement of the lithosphere due to loading). Bouguer gravity data is obtained from observed (free-air) gravity data by applying a topographic gravity correction. Different methods have been developed and applied to compute this correction. When adopting the most basic approximation of the actual topography by an infinite Bouguer plate of a constant thickness and density, the application of the incomplete planar topographic correction to the free-air gravity data yields the simple planar Bouguer gravity data [Tenzer et al., 2019].

Mean free-air gravity anomalies are often needed in geodesy for gravity field modelling. Two possible ways of compiling the mean free-air gravity anomalies are discussed. One way is via simple Bouguer gravity anomalies; the other, more time-consuming, is via refined Bouguer gravity anomalies [Janák,Vaníček, 2005].

Methodology. ***Normal gravity (Theoretic gravity).*** Normal gravity is also known as the latitude correction. Its correction varies as a function of latitude caused by the centrifugal acceleration [Bramanto et al., 2021].

The normal (ellipsoidal) gravity on the ellipsoid can be calculated by Somigliana formula as below:

$$\gamma = \gamma_e \frac{1 + k \sin^2 \varphi}{\sqrt{1 - e^2 \sin^2 \varphi}}, \qquad (1)$$

where γ_e=9.780 325 3359 ms^{-2} is the gravity at the equator, k=0.001 931 852 652 is the normal gravity constant and e^2=0.006 694 379 990 14is the first eccentricity squared (where a and b are the semi-major and semi-minor axes of the WGS84 reference ellipsoid) [Kılıçoğlu et al., 2010] (Table 1).

The normal gravity outside the ellipsoid can be computed with (2) equation as below:

$$\gamma(H) = \gamma\left[1 - \frac{2}{a}\left(1 + f + m' - 2f\sin^2\varphi\right)H + \frac{3}{a^2}H^2\right], \qquad (2)$$

where

f=$(a-b)/a$, m'=$(\omega^2 a^2 b)/GM$=0.00335281066478 (for WGS84) is the ratio between the gravitational and centrifugal forces at the equator and H is the normal orthometric height in meters [Heiskanen, Moritz, 1967].

Table 1. Comparison of GRS80 and WGS84 parameters [Kılıçoğlu et al., 2010]

Parameter	GRS80	WGS84
a	6 378 137 m	6 378 137 m
GM	3 986 005×10^8 m^3 s^{-2}	3 986 004.418×$10^8$$m^3$ s^{-2}
$J2$	108 263×10^{-8}	108 187.4×10^{-8}
ω	7 292 115×10^{-11}rad s^{-1}	7 292 115×10-11 rad s^{-1}
b	6 356 752.3141 m	6 356 752.3142 m
E	521 854.0097 m	521 854.0084 m
c	6 399 593.6259 m	6 399 593.6258 m
e^2	0.006 694 380 022 90	0.006 694 379 990 14
e'^2	0.006 739 496 775 48	0.006 739 496 742 28
f	0.003 352 810 681 18	0.003 352 810 664 78
f^1	298.257 222 101	298.257 223 563
$R1$	6 371 008.7714 m	6 371 008.7714 m
$R2$	6 371 007.1810 m	6 371 007.1809 m
$R3$	6 371 000.7900 m	6 371 000.7900 m
$U0$	6 263 6860.850 m^2s^{-1}	6 263 6851.7146 m^2s^{-1}
γe	9.780 326 7715 ms^{-2}	9.780 325 3359 ms^{-2}
γp	9.832 186 3685 ms^{-2}	9.832 184 9378 ms^{-2}
k	0.001 931 851 353	0.001 931 852 652

Gravity Reduction. The reduction of surface gravity data to on the geoid or another equipotential surface removes the gravitational effects of topography and distance from the geocentric. The reduction itself involves the application of a series of corrections. These corrections account for the vertical gradient of gravity near the Earth's surface, the gravimetric attraction of the topography, and the centrifugal acceleration and oblate ellipticity of the figure of the Earth [Featherstone, Dentith, 1997].

Bouguer gravity reduction. The Bouguer gravity reduction aims to completely remove the topographic masses outside the geoid. In this method, the surroundings of the Earth point P is considered flat and horizontal, and the masses between the geoid and the earth are considered to have a constant densityρ. In this case, the gravitational effect of the masses outside the geoid can be calculated as the gravitational effect of a cylinder of infinite radius with thickness. This cylinder is called the Bouguer plate. Thus, the gravitational effect of the Bouguer plate can be calculated as follows:

$$A_B=2\pi k\rho h, \tag{3}$$

Where ρ=2.67 g/cm^3 with standard density, h is height in meters, and k is the Newton's gravitational constant. So, the gravitational effect of the Bouguer can be written as follows:

$$A_B=0.1119h \text{ mGal}. \tag{4}$$

Removing the plate means removing this gravitational effect from the measured gravity.

*Free-air reduction***.** The Earth point P, measured by removing the topographic masses, remains in the air (space). This point needs to be lowered to the geoid (P_0). This is called free-air reduction. Free-air reduction is as follows:

$$F=-\frac{\partial g}{\partial h}h\,. \tag{5}$$

Normal gravity gradient can be written as follows:

$$F\approx-\frac{\partial\gamma}{\partial h}h\approx+0.3086h \;\text{ mGal}. \tag{6}$$

This combined operation of removing topographic masses and applying free air reduction is called the completed Bouguer reduction. The Bouguer gravity (g_B) at P_0 on the geoid is expressed as:

$$g_B = g - A_B + F = g + 0.1967h, \qquad (7)$$

g_B is on the surface of the geoid.

The gravity anomaly and gravity disturbance. In geodesy, the gravity anomaly (Δg) is the scalar difference between the Earth's gravity on the geoid and the normal gravity on the surface of the reference ellipsoid at the observation latitude.

A simple Bouguer anomaly is given by:

$$\Delta g_B = g_B - \gamma. \qquad (8)$$

Gravity disturbance is given by [Heiskanen, Moritz, 1967]:

$$\delta g = g - \gamma(H). \qquad (9)$$

Study area. This application was made in the form of a route. The route extends from Çamoba Street above Trabzon Atatürk Mansion to the coastline of the Ayasofya neighbourhood. The study area is relatively mountainous and rugged, with a route length of 4.78 km and a maximum height difference of around 385 m between points. The measurement route is shown in Fig. 1.

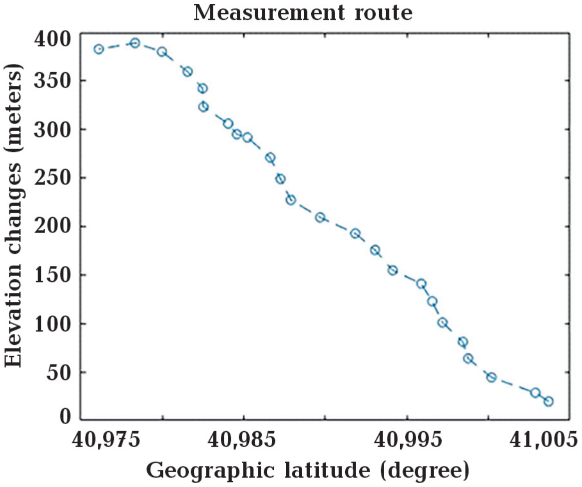

Fig. 1. Between Çamoba and Ayasofya Measurement Route.

Land surveys. The measurements were collected over 2004—2005 years. Precise leveling and gravity measurements were made along Çamoba and Ayasofya Measurement Route. The latitude was determined by a hand GPS. Leveling measurements along the routes were made according to the round-trip measurement plan. It is envisaged that the round-trip difference in dimensions should be smaller than $6\sqrt{K}$ mm (leveling passage length in K, km). Leveling measurements were made with the DL-101C Electronic Digital Level. In the measurements, attention was paid to ensure that the invar rods were on metal shoes and that the distance between the instrument and the invar rods was equal and did not exceed approximately 30 m.

Master-type Worden Gravimeter was used for gravity measurements. The measurements taken with this are presented in quadrant scales. To convert them in milligals, the instrument constant is used.

Latitude values were measured using SPORTRAK Map (MAGELLAN) handheld GPS.

Results and discussion. Normal gravity on the ellipsoid was calculated by equation (1), outside the ellipsoid — by equation (2). Free-Air gravity reduction was calculated by equation (6) and the Bouguer gravity reduction, by equation (4). Bouguer gravity anomaly was calculated by equation (8), gravity disturbance by equation (9). These results are shown in Table 2.

Some basic statistical information regarding these calculations is shown in Table 3.

From the measured gravity and height data, the variation of gravity with height is plotted in Fig. 2.

The variation between gravity measured in the field and the normal gravity calculated at the same points (outside ellipsoid) is plotted in Fig. 3.

The variation of the normal gravity calculated on the ellipsoid with respect to the latitude is shown in Fig. 4.

The variation of the real gravity measured on the land with respect to the latitude is shown in Fig. 5.

The variations of the Bouguer gravity anomaly and gravity disturbance with respect to the latitude and elevation are shown in Fig. 6.

The variations of Free-air gravity reduction and Bouguer gravity reduction with respect to latitude and elevation are shown in Fig. 7.

Table 2. Observed and calculated gravity data, Bouguer gravity anomalies and gravity disturbances

Number	Latitude, degree	Height differences, m	Observed gravity change, mGal	Normal gravity on the ellipsoid, Gal	Normal gravity outside the ellipsoid, Gal	Normal gravity change outside the ellipsoid, mGal	Free-Air Reduction, mGal	Bouguer Reduction, mGal	Bouguer gravity anomalies, mGal	Gravity disturbance, mGal
1	41.00369	19.56684	–2.39100	978.84706	978.84430	–6.05335	8.80397	3.19237	1435.98144	1439.15713
2	41.00288	28.56673	–4.42300	978.84703	978.83824	–8.91368	11.58133	4.19945	1435.80854	1439.98604
3	41.00019	44.35777	–6.50600	978.84694	978.83538	–13.82453	16.45445	5.96647	1436.87882	1442.81404
4	40.99875	63.87434	–10.30300	978.84690	978.83047	–19.84617	22.47726	8.15037	1436.93097	1445.03855
5	40.99843	81.04502	–13.89400	978.84689	978.82445	–25.17597	27.77614	10.07177	1436.75815	1446.77693
6	40.99718	100.99988	–17.42600	978.84684	978.81912	–31.34279	33.93421	12.30472	1437.17250	1449.41233
7	40.99655	122.81546	–21.64300	978.84682	978.81295	–38.08401	40.66649	14.74589	1437.26851	1451.93643
8	40.99588	141.03063	–25.82600	978.84680	978.80621	–43.75165	46.28769	16.78416	1436.72599	1453.42122
9	40.99412	154.79384	–27.18800	978.84674	978.80055	–48.02605	50.53502	18.32427	1438.10575	1456.33277
10	40.99304	175.81391	–31.65400	978.84671	978.79627	–54.53984	57.02181	20.67641	1437.81445	1458.38086
11	40.99183	192.82171	–33.73700	978.84667	978.78976	–59.84877	62.27042	22.57959	1439.14772	1461.60694
12	40.98967	209.42819	–36.54000	978.84660	978.78445	–65.02223	67.39518	24.43785	1439.67022	1463.97755
13	40.98787	227.37643	–39.23100	978.84654	978.77928	–70.57068	72.93401	26.44626	1440.52987	1466.83458
14	40.98724	249.08706	–42.75400	978.84652	978.77373	–77.27783	79.63391	28.87568	1441.29816	1470.01889
15	40.98661	271.13713	–45.51400	978.84650	978.76702	–84.11456	86.43856	31.34308	1442.92117	1474.09563
16	40.98521	291.82234	–49.38800	978.84645	978.76018	–90.50650	92.82201	33.65776	1443.13705	1476.61329
17	40.98456	295.11020	–51.68600	978.84643	978.75379	–91.53664	93.83665	34.02567	1441.50420	1475.34630
18	40.98403	306.13198	–52.62800	978.84641	978.75276	–94.98074	97.23797	35.25900	1442.77899	1477.84754
19	40.98251	323.38147	–56.05700	978.84636	978.74932	–100.29454	102.56116	37.18922	1442.74486	1479.73279
20	40.98246	342.45219	–56.64800	978.84636	978.74400	–106.19785	108.44639	39.32324	1445.93476	1485.04467
21	40.98154	359.68200	–62.31400	978.84633	978.73810	–111.55574	113.76351	41.25125	1443.70983	1484.73687
22	40.97997	380.33004	–64.91100	978.84628	978.73274	–117.96864	120.13549	43.56177	1445.22820	1488.55263
23	40.97832	389.27493	–67.72300	978.84623	978.72633	–120.79640	122.89589	44.56270	1444.24928	1488.56897
24	40.97609	382.91790	–66.72000	978.84615	978.72350	–118.89865	120.93410	43.85135	1444.06165	1487.67407

Table 3. Basic statistical information

Parameter	Min	Max	Mean
Height Difference, m	19.56684	389.27493	214.742416
Observed gravity change, mGal	–67.72300	–2.391	–36.962708
Calculated normal gravity change outside the ellipsoid, mGal	–120.7964	–6.05335	–66.63033
Free-Air Reduction, mGal	8.80397	122.89589	69.0351512
Bouguer Reduction, mGal	3.19237	44.562701	25.0325127
Bouguer gravity anomalies, mGal	1435.98144	1445.93476	1440.26505
Gravity disturbance, mGal	1439.15713	1488.569	1465.16279

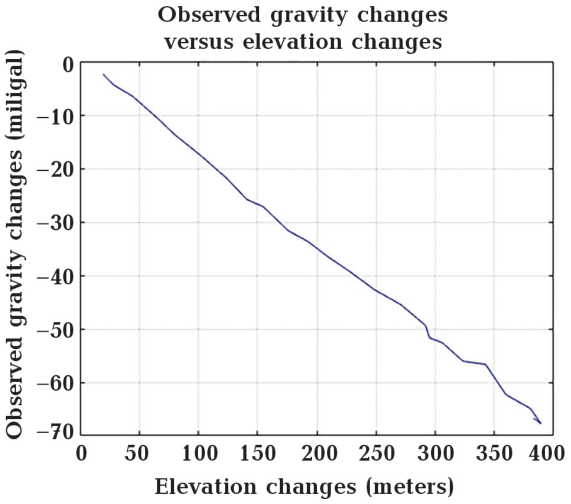

Fig. 2. Observed gravity changes versus elevation changes.

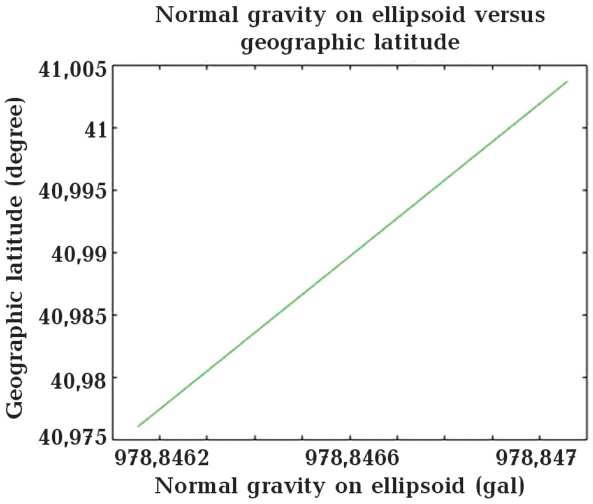

Fig. 4. Normal gravity on ellipsoid versus the geographic latitude.

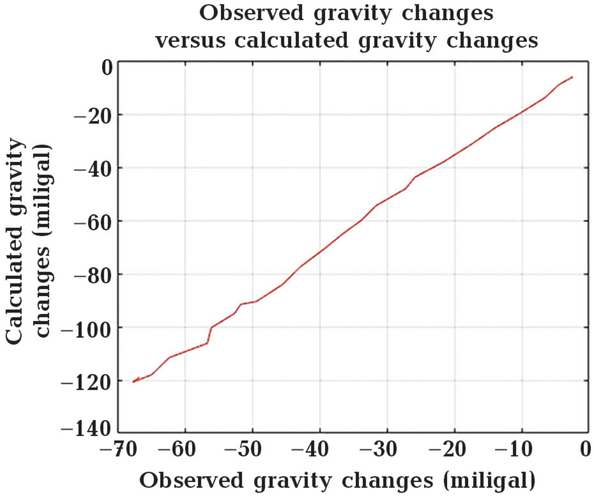

Fig. 3. Observed gravity changes versus calculated normal gravity changes.

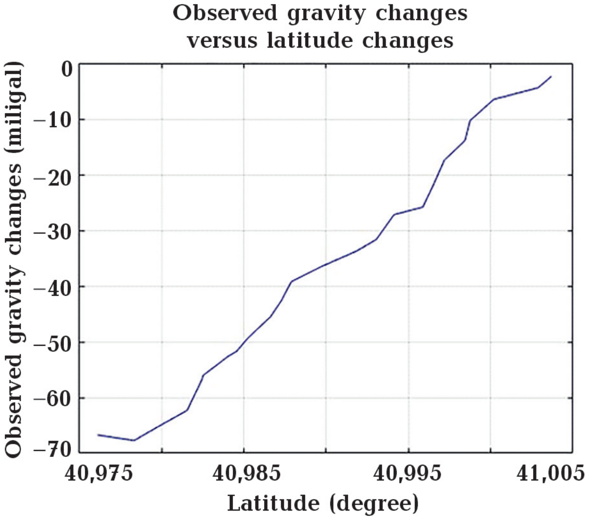

Fig. 5. Observed gravity changes versus latitude changes.

Conclusions. The heights in the study area vary from about 19 m to 389 m. The latitude spans 41.0037 degrees to 40.9761 degrees North.

- From Table 2 and Fig. 2, it is seen that the gravity difference measured in the field is inversely proportional to the height, and the real gravity difference decreases as the

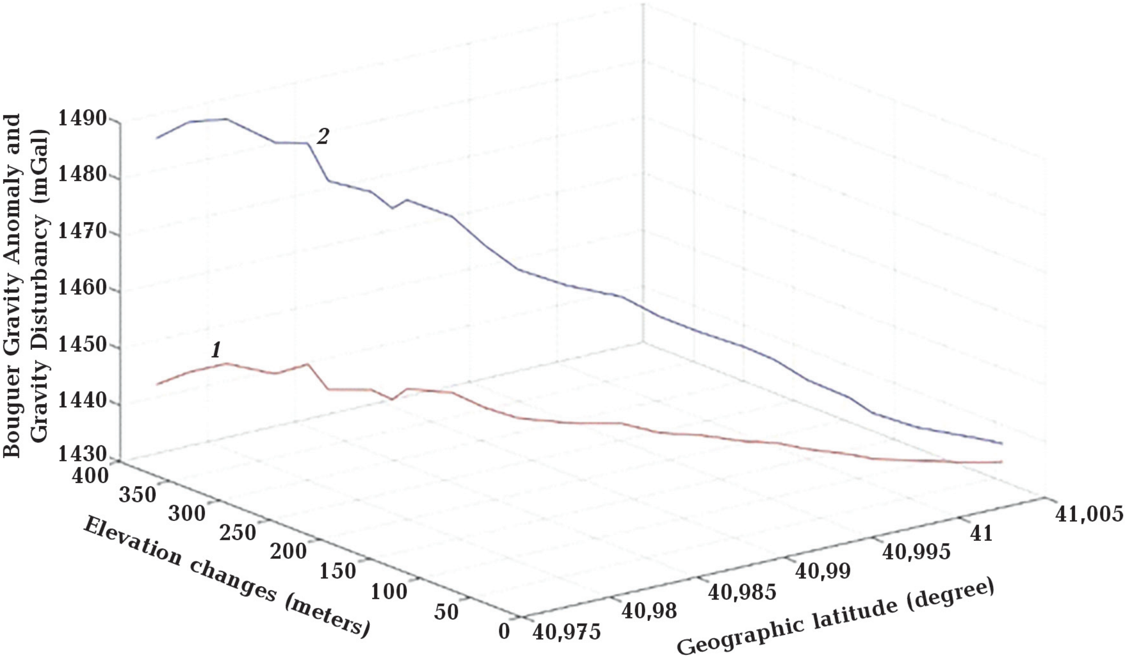

Fig. 6. Bouguer gravity anomaly (*1*) and gravity disturbance (*2*) versus latitude and elevation.

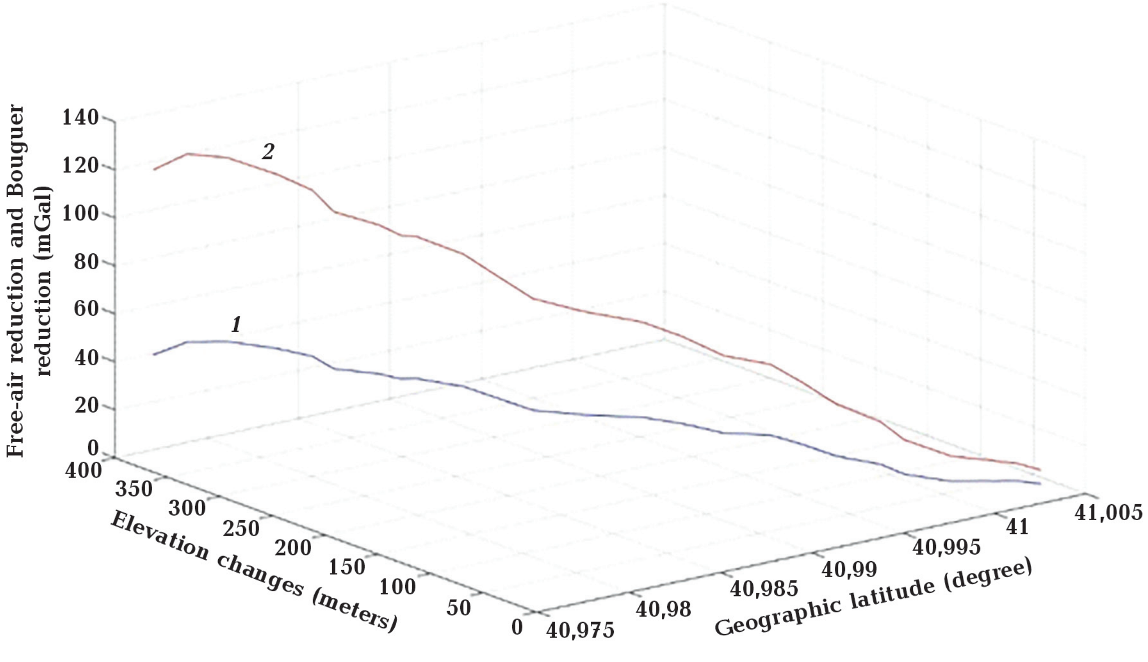

Fig. 7. Free-air reduction (*1*) and Bouguer gravity reduction (*2*) versus latitude and elevation.

height increases. This difference is approximately 65 mGal at 370 m height.

- From Table 2 and Fig. 3, it is seen that as the gravity difference measured in the field decreases, so does the normal gravity difference.
- As the height increases, the normal gravity calculated on the ellipsoid and outside the ellipsoid decreases (Table 2).
- As the latitude increases, the normal gravity calculated on the ellipsoid and outside the ellipsoid increases. At the same time, the real gravity is increasing. It can be seen from Fig. 4 and Fig. 5.

- From Table 2, Fig. 6, Fig. 7, it is seen that as the height increases and latitude decreases, both free-air and Bouguer gravity reduction increase. At the same time, Bouguer gravity anomaly and gravity disturbance increase, too.

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Обробка гравітаційних даних, аномалія Буге та гравітаційне збурення

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Кожна точка на Землі має гравітаційне значення. Поверхні, утворені сполученнями точок з однаковими значеннями гравітаційного потенціалу, називаються еквіпотенціальними або поверхнями рівня. Визначення гравітаційного поля Землі та геоїда, що є однією з еквіпотенціальних поверхонь Землі, має велике значення для фізичної геодезії. Значення сили тяжіння, виміряні на фізичній Землі, не включені безпосередньо в це дослідження. По-перше, їх необхідно перерахувати на гравітаційні аномалії. Для цього в польових умовах виконано точні вимірювання рівня, гравітації та GPS. Були записані висоти (*H*) з точними вимірюваннями рівня, значення сили тяжіння (*g*) з вимірюваннями сили тяжіння та географічні широти (φ) з вимірюваннями GPS. Після того в точках розраховували зниження сили тяжіння (Free-air, Буге). Фактична сила цього *g*, виміряна на Землі, не може бути безпосередньо співставлена з нормальною гравітацією поверхні еліпсоїда. Необхідно звести *g* до геоїда. Оскільки за межами геоїда існують маси, методи приведення різняться залежно від того, було оброблено ці топографічні маси. Виведено гравітаційні аномалії Буге та гравітаційні збурення. Аномалія сили тяжіння (Δg) визначається як скалярна різниця між силою тяжіння Землі на геоїді та нормальної сили тяжіння на поверхні опорного еліпсоїда. Збурення гравітації (δg) визначається як різниця між фактичною величиною гравітації, виміряною на Землі, та її еквівалентною нормальною гравітацією в нормальному гравітаційному полі для тієї ж точки. Порівнюються зміни та значення розрахованих величин, а саме: дані гравітації та висоти, зміни сили тяжіння порівняно з розрахованими нормальними змінами сили тяжіння, нормальна сила тяжіння на еліпсоїді залежно від географічної широти, спостережені зміни сили тяжіння залежно від змін широти, гравітаційна аномалія Буге та гравітаційне збурення залежно від широти й висоти, зменшення вільного повітря. Також було досліджено зменшення сили тяжіння Буге залежно від широти й висоти.

Ключові слова: реальна гравітація, нормальна гравітація, еліпсоїд, геоїд, зниження сили тяжіння у вільному повітрі, зменшення сили тяжіння Буге, гравітаційна аномалія Буге, збурення гравітації.